Predicate Logic

Why is logic important in AI?

Predicate Logic

- Propositional logic lacks expressivity.
- Propositional logic is included in predicate logic.
- Predicate logic allows us to express sophisticated properties, about many objects (even infinitely many) in one go.
- **Predicate Logic** is also called *first order Logic*, **first order predicate calculus**, and *Predicate Calculus*.
- Predicate calculus formulas are built on predicates, variables, terms, functions and connectives. They are evaluated to true or false.
  - A **predicate** "is a quantified proposition with variables".
  - Quantifiers are $\forall$ (for all) and $\exists$ (exists).
  - T and F are predicates.
  - Variables are x, y, z etc.
  - Functions are of the form $f(x1, x2, .., xn)$. f is of arity n

- Constants are functions of arity 0.
- Terms are either constants, variables or function expressions.
- Connectives are $\wedge$, $\vee$, $\neg$, $\rightarrow$, and $\iff$. They are used to create predicate formulas.

Examples

- π and a are constants.
- The expressions below are predicate formulas.
- p (refer to propositional logic)
- $p(x)$
- $\geq (x, y)$ (for $x \geq y$)
- $= (x, y)$ (for $x = y$)
- $= (f(x), z)$ (for $f(x) = z$)
- $parentof(x, y)$ is the same as $\forall x, \forall y, parentof(x, y)$
- $fatherof(x, y)$
- $speaks(x, y)$
- $prime(n)$
- $\forall x(speaks(x, Japanese))$
- $\exists x(speaks(x, Japanese))$

- $\forall x \exists y (speaks(x, y))$

- There exist a unique person who cannot read
  $\exists ! x (cannotread(x)$

- $father(x, y) \wedge man(x)$

- $p(x, y) \rightarrow (\exists z) p(x, z) \wedge p(z, y)$

- There are infinitely many prime numbers.
  $\forall q \exists p \forall x, y, (p \geq q \wedge (x, y \geq 1 \rightarrow xy \neq p))$

- Fermat's Last Theorem
  $\forall a, b, c, n((a, b, c \geq 0 \wedge n \geq 2) \rightarrow a^n + b^n \neq c^n)$

$\boxed{\text{Aristotle Syllogism}}$

Problem

All men are mortal.

Socrates is a man.

Therefore, Socrates is mortal.

$$\downarrow$$

Model the problem

Hypotheses:

$$man(x) \rightarrow mortal(x)$$

$$man(socrates)$$

Goal to prove:

$$mortal(socrates)$$

$$\downarrow$$

Proof

Deduction / Theorem Proving

Semantics

- One of the important tasks in predicate logic is to provide meaning to the formulas.

- A predicate formula is **satisfiable** if for some particular assignment of values to its variables (**interpretation**) the predicate is true. A **domain** needs to be considered for the values.

  The semantics of a predicate logic formula α is given in terms of all possible interpretations, including domains.

- A predicate formula is **valid** if for all assignments of values to its variables the predicate is true.

- Examples:

  – $p(x,y) \rightarrow (\exists z)p(x,z) \wedge p(z,y)$
    The formula is satisfiable using 1 and 2, but not 3.

    * Interpretation 1: We interpret p as $<$ and the domain as the real numbers. We can pick $Z = (X+Y)/2$

    * Interpretation 2: We interpret p as true for the pairs aa, ab, ba, bc, cb,and c on the domain $\{a, b, c\}$.

- Interpretation 3: We interpret p as $<$ and the domain as the natural numbers. If $x = 1$ and $y = 2$, the formula is not true.

– $p(x) \vee \neg p(x)$ is valid

Modus Ponens with Variables

$$\frac{\begin{array}{l} man(X) \rightarrow mortal(X) \\ man(socrates) \end{array}}{\therefore mortal(socrates)}$$

- We need to unify $man(X)$ and $man(socrates)$.
- We obtain a substitution $sigma = \{x \mapsto socrates\}$.
- We apply the substitution on $mortal(X)$, i.e. $\sigma(mortal(X))$ which is $mortal(socrates)$.

Resolution

Simplified version

- Formulas are in Conjunctive Normal Form (CNF)
- The resolution inference rule in propositional logic is the following:

  From $p_1 \vee p_2$ *and* $\neg p_1 \vee p_3$
  derive $p_2 \vee p_3$.

$$\begin{array}{l} p_1 \vee p_2 \\ \neg p_1 \vee p_3 \\ \hline \therefore p_2 \vee p_3 \end{array}$$

- In the case with variables, we need to unify predicates and terms.

$\boxed{\textbf{Resolution}}$

Example with variables

$$\begin{array}{l} p(john, jim) \\ \neg p(x,y) \vee \neg p(y,z) \vee g(x,z) \\ \hline \therefore \neg p(bob, z) \vee g(john, z) \end{array}$$

Results

- There is a distinction between what is provable and what is true by proof systems.

  Sound / correct: If something is provable, then it is true. (required)

  Complete: If something is true, then it is provable. (optional)

- Predicate logic has a complex axiomatisation.

- Predicate logic is complete (considering resolution or natural deduction.

- **Goedel's incompleteness theorem** states that elementary number theory (i.e., arithmetic for the nonnegative integers) contains true expressions that cannot be proved.

- Turing's theorem describes a formal model of a computer called a "Turing machine" and says that there are problems that cannot be solved by a computer. Predicate logic is undecidable. Some domain-specific problems can be solved.

PROLOG

Paradigm

- Declarative programming paradigm
  - The programmer declares the goals of the computation rather than the detailed algorithm by which these goals can be achieved.
- Logic programming is based on:
  - unification (Robinson, 1965) and
  - resolution (Robinson, 1965)
- Two important features of logic programming are:
  - non-determinism and
  - backtracking
- Popular in artificial intelligence
- Applications:
  - Natural language processing
  - Theorem proving
  - Databases
  - Expert systems
- PROLOG is a logic programming language (Colmerauer, 1972)

Normal Forms

- *Normal forms* are equivalent formulas of a certain syntactic form. We consider **Conjunctive** and **disjunctive forms**.

- They permit us to answer certain questions more easily.

- A propositional formula is said to be in **conjunctive normal form** (CNF) if
  1. it contains only the logical connectives $\neg$, $\wedge$ and $\vee$,
  2. no logical connective occurs inside of a negation.
  3. no conjunction occurs inside of a disjunction.

- $(\neg p \vee q) \wedge (\neg p \vee \neg r \vee q)$ is a conjunctive normal form

- We speak of a **disjunctive normal form** (DNF) if the last condition is replaced by the condition that *no disjunction occur inside any conjunction*.

- $(\neg p \wedge q) \vee (p \wedge r)$ is a disjunctive normal form

- Any formula can be transformed to a CNF (or DNF).

- Exercise: Transform $\neg((p \vee q) \iff (p \rightarrow (q \wedge True)))$ into a CNF.

Clauses

- A **literal** is either a predicate or the negation of a predicate.

- Disjunctions of literals, $L_1 \vee \cdots \vee L_n$, are also called **clauses**.

- If a clause contains *at most* one positive literal, then it is called a **Horn clause**.

  – For example, $\neg p \vee \neg q$ and $\neg p \vee \neg q \vee r$ are Horn clauses, but $p \vee q$ is not a Horn clause.

- Horn clauses can be interpreted as program rules and used for computation, as it is done in **logic programming**.

Logic Program

- A **Horn clause** $\neg p_1 \vee \cdots \vee \neg p_n \vee q$ is logically equivalent to the implication $(p_1 \wedge \cdots \wedge p_n) \rightarrow q$.

- If the implication is known to be true, and one wishes to prove q, then it sufficient to show that $p_1, \ldots, p_n$ are all true; an observation that provides the logical basis for logic programming.

- A **logic program** is a set of Horn clauses, each containing exactly one positive literal (and zero or more negative literals). Such Horn clauses are usually written as backward implications

  $$q \leftarrow p_1, \ldots, p_n$$

  and called **program rules**. More specifically, q is called the **head** of the rule, and the sequence $p_1, \ldots, p_n$ the **body** of the rule.

- Each rule must have a head, but the body may be empty and in that case the rule is called a **fact**. For instance $q \leftarrow$ is a fact.

- A logic program is composed of rules and facts.

Notations

- A Horn clause is a rule and it is written as:

  $$q \leftarrow p_1, \ldots, p_n$$

  It means the same as:

  $$\neg p_1 \vee \cdots \vee \neg p_n \vee q$$

- If $n = 0$, the clause is a fact and is written: $q \leftarrow$.
  $q \leftarrow$ is the same as q.

- $\leftarrow p$ is the negation of the goal (the query) and it is the same as $\neg p$.

Logic program

Propositional case

$$\begin{array}{l} e \leftarrow \\ f \leftarrow \\ b \leftarrow \\ c \leftarrow a, b \\ a \leftarrow e, f \end{array}$$

- is a propositional logic program of 5 rules. The first 3 rules have an empty body and represent **facts**.

- In addition to the program rules one needs to specify a **goal** (or a list of goals) that we want to prove.

  Example: If we want to prove c, the goal is c.

- A computation with a logic program represents an attempt to derive the goal from the program rules (in an indirect way by deriving a **contradiction** in the form of the "empty clause " (represented by $\square$) from the **negation** of the goal).

- The logical inference rule underlying such computations is called **resolution**.

Logic program

With variables

p(edward7, george5) $\leftarrow$
p(victoria, edward7)$\leftarrow$
p(alexandra, george5)$\leftarrow$
p(george6, elizabeth2)$\leftarrow$
p(george5, george6)$\leftarrow$
$g(X, Y) \leftarrow p(X, Z), p(Z, Y)$

- is a logic program of 6 rules. The first 5 rules have an empty body and represent facts (about the British royal family).

- The last rule defines the *grandparent relation* in terms of the *parent relation*: a person X is a grandparent of Y if there is a third person Z, such that X is the parent of Z, and Z the parent of Y.

- Informally, the rule $g(X, Y) \leftarrow p(X, Z), p(Z, Y)$ may be thought of as a schema representing all clauses obtained by substituting specific values for the variables, e.g.,

  $g(victoria, george5) \leftarrow p(victoria, edward7), p(edward7, george5)$

  X = victoria, Z = edward7, Y = george5

- In addition to the program rules one needs to specify a **goal** (or a list of goals) that we want to prove.

  Example: If we want to prove that the grandfather of George V is Victoria then the goal is g(victoria, george5).

- A computation with a logic program represents an attempt to derive the goal from the program rules (in an indirect way by deriving a **contradiction** in the form of the "empty clause" ($\square$) from the **negation** of the goal).

- The logical inference rule underlying such computations is called **resolution**.

$\boxed{\text{Unification}}$

- **Unification** is a pattern-matching process that determines what particular instantiation can be made to variables to make two predicates equal. This instantiation is called a **substitution**.

- Examples:
  - How to make $brotherof(john, X)$ and $brotherof(Y, bill)$ equal?
    With the substitution: $X \mapsto bill$, $Y \mapsto john$
  - How to make b and b equal?
    With the substitution: id (identity)

Unification algorithm

Delete		$P \wedge s =^? s$	
	$\mapsto$	P	
Decompose		$P \wedge f(s_1, \ldots, s_n) =^? f(t_1, \ldots, t_n)$	
	$\mapsto$	$P \wedge s_1 =^? t_1 \wedge \ldots \wedge s_n =^? t_n$	
Conflict		$P \wedge f(s_1, \ldots, s_n) =^? g(t_1, \ldots, t_p)$	
	$\mapsto$	$\mathbf{F}$	if $f \neq g$
Coalesce		$P \wedge x =^? y$	
	$\mapsto$	$\{x \mapsto y\}P \wedge x =^? y$	if $x, y \in \mathcal{V}ar(P)$ *and* $x \neq y$
Check*		$P \wedge x_1 =^? s_1[x_2] \wedge \ldots$ $\ldots \wedge x_n =^? s_n[x_1]$	
	$\mapsto$	$\mathbf{F}$	if $s_i \notin \mathcal{X}$ *for some* $i \in [1..n]$
Merge		$P \wedge x =^? s \wedge x =^? t$	
	$\mapsto$	$P \wedge x =^? s \wedge s =^? t$	if $0 < \lvert s \rvert \leq \lvert t \rvert$
Check		$P \wedge x =^? s$	
	$\mapsto$	$\mathbf{F}$	if $x \in \mathcal{V}ar(s)$ *and* $s \notin \mathcal{X}$
Eliminate		$P \wedge x =^? s$	
	$\mapsto$	$\{x \mapsto s\}P \wedge x =^? s$	if $x \notin \mathcal{V}ar(s), s \notin \mathcal{X}, x \in \mathcal{V}ar(P)$

SyntacticUnification: Rules for syntactic unification

$\boxed{\textbf{Resolution}}$

Propositional case

- The propositional version of resolution for Horn clauses is:

  From $\leftarrow p_1, \ldots, p_n$ *and* $p_1 \leftarrow q_1, \ldots, q_k$
  derive $\leftarrow q_1, \ldots, q_k, p_2, \ldots, p_n$.

$$\frac{\begin{array}{l}\leftarrow p_1, \ldots, p_n \\ p_1 \leftarrow q_1, \ldots, q_k\end{array}}{\therefore \leftarrow q_1, \ldots, q_k, p_2, \ldots, p_n}$$

  – What is the rule if $n = 1$ and $k = 1$? It's the Modus Ponens.

$$\frac{\begin{array}{l}\leftarrow p_1 \\ p_1 \leftarrow q_1\end{array}}{\therefore \leftarrow q_1}$$

  – What is the rule if $n = 1$ and $k = 0$?

$$\frac{\begin{array}{l}\leftarrow p_1 \\ p_1 \leftarrow\end{array}}{\therefore \Box}$$

- **Example:** Assume we want to prove c.
  - The negation of the goal c is written as a negative clause
$$\leftarrow c.$$
  - We have also seen that c is the head of a rule $(c \leftarrow a, b)$.
  - This indicates that the given goal may be *reduced* to subgoals (by the resolution rule)
$$\leftarrow a, b.$$
  - We have also seen that a is the head of a rule $(a \leftarrow e, f)$.
  - This indicates that the given goal may be *reduced* to subgoals (by the resolution rule)
$$\leftarrow e, f, b.$$
    where a is replaced by e, f.

- The three subgoals are present as facts and hence can be deleted, which results in the empty clause ($\Box$).

- We conclude that the original goal logically follows from the program clauses.

- But much of the power of logic programming derives from the fact that resolution can be generalized to effectively handle clauses with variables.

Resolution

With variables

- Assume we want to prove that Victoria is the grandmother of George.

- The negation of the above goal is written as a negative clause

$$\leftarrow g(victoria, george5).$$

- We have also seen that suitable values may be substituted for the variables in the last program rule, so that the head is g(victoria, george5) (X=victoria and Y = george5).

- This indicates that the given goal may be *reduced* to subgoals (backward reasoning)

$$\leftarrow p(victoria, edward7), p(edward7, george5).$$

- Both subgoals are present as facts and hence can be deleted, which results in the empty clause ($\square$).

- We conclude that the original goal logically follows from the program clauses.

- Goals with variables are also possible.

  Example: If one specifies the goal

  $$\leftarrow g(victoria, X)$$

  the result of the computation will be a list of all grandchildren of Victoria. A discussion of these aspects of logical programming is beyond the scope of this course.

PROLOG

- SWI-prolog
  - Download Prolog here: https://www.swi-prolog.org
  - or use Prolog online here: https://swish.swi-prolog.org/example/examples.swinb

- Prolog files have $.pl$ as extensions. Let's takes a file $likes.pl$ as an example.

- To run PROLOG type: *swipl*, then:

- To load the *likes.pl* file, type: *[likes].* or *consult(likes).*.

- You can also use *swipl likes.pl* to run likes directly.

- Exemple: Let's consider the *likes.pl* file. There are 3 facts.

  ```
  likes(john,mary).
  likes(mary,sue).
  likes(mary,tom).
  ```

  You can now play with Prolog and makes queries: Who are the people that Mary likes?

  ```
  likes(mary,X).
  ```

X is a variable and must be written using a capital letter. Constants are written in lower cases.

To have all the solutions to the $likes(mary, X)$ goal, type n (for next) after each solution.

- In Prolog:
  - A variable begins with a capital letter.
  - A constant is written in lower cases.
  - Underscore characters are considered as variables.
  - All facts, rules and queries end with a period.
  - Closed world assumption: if we cannot prove something, it is false.
  - Prolog may return all possible answers (ways) to prove the goal.

Prolog language

- Prolog reads the facts and rules in the order they are defined.
- Each clause is looked at from left to right.
- Numbers: 3, 2.5
- Strings: "" (e.g., "$Hello$")
- Assignment: is (e.g., X is 4+5.)
- Predefined functions: $-$, $+$, $*$, $/$, ^ , mod, abs, min, max, sign, random, sqrt, sin, cos, tan, log, exp (e.g., X is sin(pi/2).)
- Comparisons: $=:=$, $\backslash==$, $=\backslash=$, $>$, $<$, $>=$, $=<$
- Checking the types: var, nonvar, integer, float, number, atom, string (e.g., number(5))

Examples of programs

- Explicit definition 1:

```
f(x) = if x=0 then 1 else 5

PROLOG:
f(0,1).
f(X,5) :- X>0.
```

- Explicit definition 2:

```
g(x) = 2*x

PROLOG:
g(X,Y) :- Y is 2*X.
```

- Example:

  ```
  PROLOG:
  speaks(allen,russian).
  speaks(bob,english).
  speaks(mary,russian).
  speaks(mary,english).
  talkswith(Person1,Person2):-speaks(Person1,L),
  speaks(Person2,L), Person1 \= Person2.
  ```

  How to know who talks with who?

- Recursive definition 1:

  ```
  fact(n) = if n=0 then 1 else n*fact(n-1)

  PROLOG:
  factorial(0,1).
  factorial(N,Result) :- N>0, M is N-1,
  factorial(M,SubResult), Result is N*SubResult.
  ```

- Recursive definition 2:

  ```
  fib(n) = if n=0 then 1 else if n=1 then 1
  else fib(n-1)+fib(n-2)

  PROLOG:
  fib(0,1).
  fib(1,1).
  fib(N,R) :- N>1, N1 is N-1, N2 is N-2, fib(N1,R1),
  fib(N2,R2), R is R1+R2.
  ```

Tracing in PROLOG

- To trace a particular predicate p use:

  `trace(p/2). or trace, p/2`

- Example:

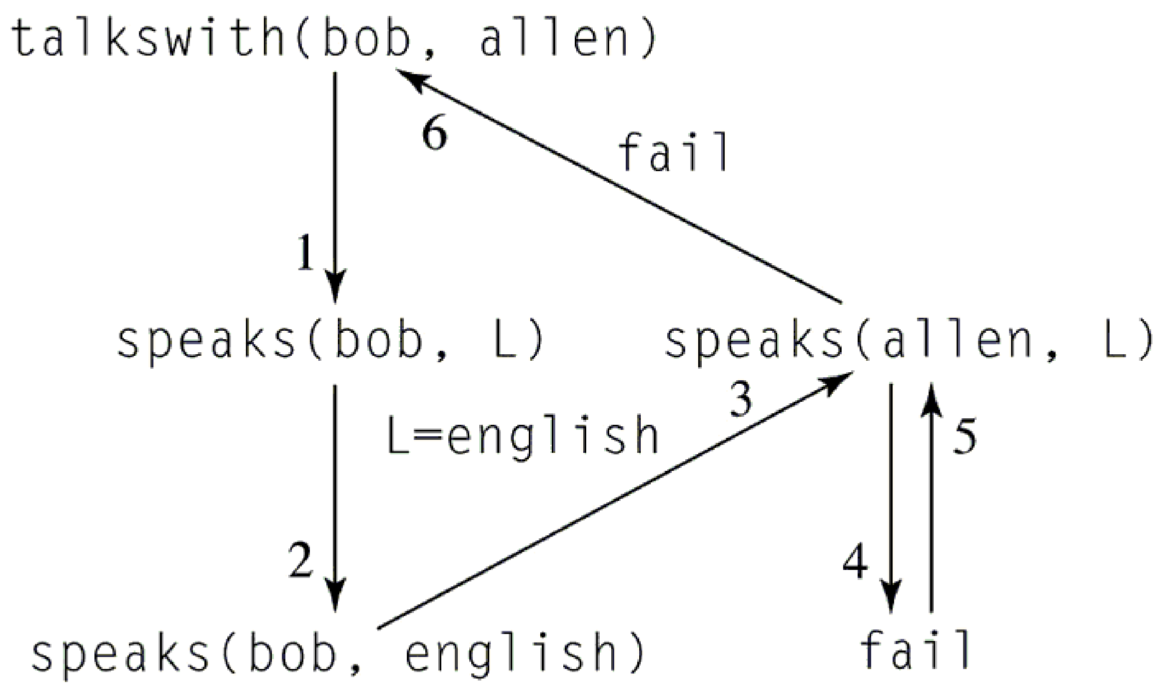
  `trace(factorial/2).`

```
?- factorial(4, X).                  N  M   P     Result
Call:  (  7) factorial(4, _G173)  4  3  _G173  4*P
Call:  (  8) factorial(3, _L131)  3  2  _L131  3*P
Call:  (  9) factorial(2, _L144)  2  1  _L144  2*P
Call:  ( 10) factorial(1, _L157)  1  0  _L157  1*P
Call:  ( 11) factorial(0, _L170)  0     _L170
Exit:  ( 11) factorial(0, 1)                   1
Exit:  ( 10) factorial(1, 1)                   1*1 = 1
Exit:  (  9) factorial(2, 2)                   2*1 = 2
Exit:  (  8) factorial(3, 6)                   3*2 = 6
Exit:  (  7) factorial(4, 24)                  4*6 = 24
```

Unification, Evaluation, Backtracking

Goal without variables

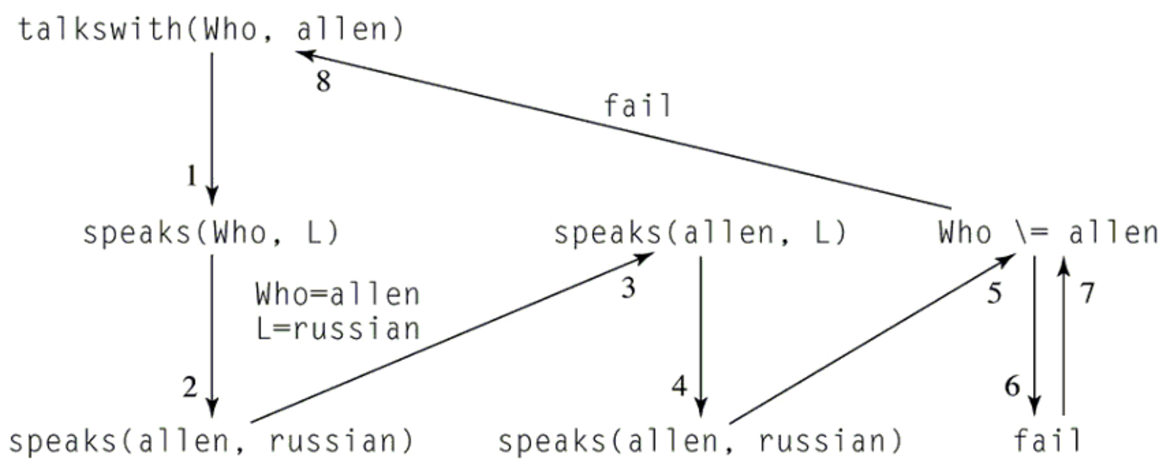
`talkswith(bob,allen).`



Unification, Evaluation, Backtracking

Goal with variables

`talkswith(Who,allen).`



Lists in PROLOG

- The basic data structure in PROLOG is the *list*.
  - [] is the empty list
  - $[X, Y]$ is a list with 2 elements
  - $[\_, \_, Y]$ is a list with 3 elements
  - $[X|Y]$ denotes a list with head X and tail Y.
- Some built-in functions on lists:
  - $append(?List1, ?List2, ?List3)$
  - $length(?List1, ?Int)$
  - $reverse(+List1, -List2)$
  - $member(?Elem, ?List)$
  - $sort(+List, -Sorted)$ (to sort a list – it removes the duplicates)
- + arguments are seen as input arguments, - arguments as output arguments, ? arguments as both input and output arguments.
- Definition of functions on lists:
  - member:

```
member(X,[X|_]).
member(X,[_|Y]) :- member(X,Y).
```

– append:

```
append([],X,X).
append1([H|T],Y,[H|Z]) :- append1(T,Y,Z).
```

```
append([english, russian], [spanish], L).
   |
   | H = english, T = [russian], Y = [spanish], L = [english | Z]
 1 v
append([russian], [spanish], [Z]).
   |
   | H = russian, T = [], Y = [spanish], [Z] = [russian | Z']
 2 v
append([], [spanish], [Z']).
   |
   | X = [spanish], Z' = spanish
 3 v
append([], [spanish], [spanish]).
```

Cut

- The **cut** permits us to force the evaluation of a series of subgoals on the right-hand side of a rule not to be retried if the right-hand side succeeds once.

- You can thing about the cut as a *conditional statement*.

- The cut is implemented by !.

- Example 1:

```
f(x) = if x=0 then 1 else 5

PROLOG:
f(0,1).
f(X,5) :- X>0.
is the same as:
f(0,1) :- !.
f(X,5) :-.
```

- Example 2: Bubble Sort

```
bsort(L,S) :- append(U,[A,B|V],L), B<A, !,
append(U,[B,A|V],M), bsort(M,S).
bsort(L,L).
```

```
?- bsort([5,2,3,1], Ans).
Call:  (  7) bsort([5, 2, 3, 1], _G221)
Call:  (  8) bsort([2, 5, 3, 1], _G221)
Call:  (  9) bsort([2, 3, 5, 1], _G221)
Call:  ( 10) bsort([2, 3, 1, 5], _G221)
Call:  ( 11) bsort([2, 1, 3, 5], _G221)
Call:  ( 12) bsort([1, 2, 3, 5], _G221)
Redo:  ( 12) bsort([1, 2, 3, 5], _G221)
Exit:  ( 12) bsort([1, 2, 3, 5], [1, 2, 3, 5])
Exit:  ( 11) bsort([2, 1, 3, 5], [1, 2, 3, 5])
Exit:  ( 10) bsort([2, 3, 1, 5], [1, 2, 3, 5])
Exit:  (  9) bsort([2, 3, 5, 1], [1, 2, 3, 5])
Exit:  (  8) bsort([2, 5, 3, 1], [1, 2, 3, 5])
Exit:  (  7) bsort([5, 2, 3, 1], [1, 2, 3, 5])

Ans = [1, 2, 3, 5] ;

No
```

Natural language processing

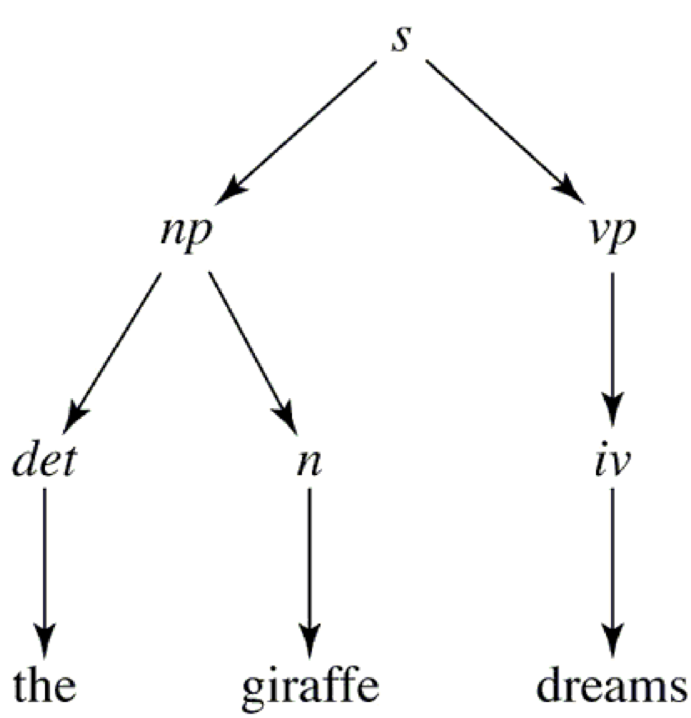
Prolog for NLP

- PROLOG is very useful to analyze and generate sentences according to a syntax.
- It permits to define vocabulary and the corresponding grammatical rules of the language.
- It relies on using lists (*difference lists*) to parse sentences.
- It has limitations for expressivity.

From BNF to NLP

- Write a Prolog program that is effectively a BNF grammar, which, when executed, will parse sentences in natural language.
- Consider the following BNF grammar below. We did not use | to be closed to the translation of the BNF rules to Prolog.
- This language generates 78 sentences.

$s \rightarrow np\ vp$
$np \rightarrow det\ n$
$vp \rightarrow tv\ np$
$\rightarrow iv$
$det \rightarrow$ the
$\rightarrow$ a
$\rightarrow$ an
$n \rightarrow$ giraffe
$\rightarrow$ apple
$iv \rightarrow$ dreams
$tv \rightarrow$ eats
$\rightarrow$ dreams



Representation in Prolog 1

- We represent a sentence using a list.
- We can write Prolog rules that partition a sentence into its grammatical categories using the structure defined by the BNF grammar.
- For example:

  ```
  s -> np vp
  ```

  is represented by:

  ```
  s(X,Y) :- np(X,U),vp(U,Y).
  ```

  X is the sentence being parsed and Y represents the resulting tail of the list that will remain to be parsed if this rule succeeds (to be applied).

- For example:

  ```
  det -> the | a | an
  ```

  is represented by:

  ```
  det([the | Y], Y).
  det([a | Y], Y).
  det([an | Y], Y).
  ```

- Assume we want to parse "the giraffe dreams". We write the query:

  `s([the,giraffe,dreams],X).`

```
?- s([the, giraffe, dreams],[]).
Call:  (  7) s([the, giraffe dreams], []) ?
Call:  (  8) np([the, giraffe, dreams], _L131) ?
Call:  (  9) det([the, giraffe, dreams], _L143) ?
Exit:  (  9) det([the, giraffe, dreams], [giraffe, dreams]) ?
Call:  (  9) n([giraffe, dreams], _L131) ?
Exit:  (  9) n([giraffe, dreams], [dreams]) ?
Exit:  (  8) np([the, giraffe, dreams], [dreams]) ?
Call:  (  8) vp([dreams], []) ?
Call:  (  9) iv([dreams], []) ?
Exit:  (  9) iv([dreams], []) ?
Exit:  (  8) vp([dreams], []) ?
Exit:  (  7) s([the, giraffe, dreams], []) ?

Yes
```

  X is [].

- Assume we want to parse "the giraffe sleeps".We write the query:

  `s([the,giraffe,sleeps],X).`

  The result is "false".

- Assume we want all the sentences parsed by the grammar.

  `s(Sentence,[]).`

- Complete program:

```
s(X, Y) :- np(X, U), vp(U, Y).
np(X, Y) :- det(X, U), n(U, Y).
vp(X, Y) :- tv(X, U), np(U, Y).
vp(X, Y) :- iv(X, Y).
det([the | Y], Y).
det([a | Y], Y).
det([an | Y], Y).
n([giraffe | Y], Y).
n([apple | Y], Y).
iv([dreams | Y], Y).
tv([eats | Y], Y).
tv([dreams | Y], Y).
```

Representation in Prolog 2

- We use a notation called **Definite Clause Grammar** (DCG).
- This notation is close from the notation of context-free grammars rules.
- We use the operator $--\>$ instead of $: -$.
- We remove the variables from the rules.
- But the meaning and the arity of the predicates do not change.
- For example:

  ```
  s(X, Y) :- np(X, U), vp(U, Y).
  ```

  becomes:

  ```
  s --> np,vp.
  ```

- DCG representation of the previous BNF grammar:

```
s --> np,vp.
np --> det,n.
vp --> tv,np.
vp --> iv.
det --> [the].
det --> [a].
det --> [an].
n --> [giraffe].
n --> [apple].
iv --> [dreams].
tv --> [eats].
tv --> [dreams].
```

- Queries are the same as previously:

```
s([the, giraffe, dreams], []).
s([the, giraffe, sleeps], []).
s(X, []).
```

Representation in Prolog 3

- If we modify slightly each rule, we can add the capability to generate a **parse tree** directly from the grammar.

  We use the notation below to represent the parse tree (recursive definition).

- For example, the parse tree of "giraffe" can be represented by:

  `n(giraffe)`

- For example, the parse tree of "the giraffe" can be represented by:

  `np(det(the),n(giraffe))`

- For example, the parse tree of "the giraffe dreams" can be represented by:

  `s(np(det(the),n(giraffe)),vp(iv(dreams)))`

- To generate the parse tree, we add a parameter with the appropriate syntax to store the tree and the intermediate values that are derived.

  The tree is the first parameter.

- For example:

  ```
  s --> np,vp.
  ```

  becomes:

  ```
  s(s(NP,VP)) --> np(NP),vp(VP).
  ```

- Complete program:

  ```
  s(s(NP,VP)) --> np(NP),vp(VP).
  np(np(DET,N)) --> det(DET),n(N).
  vp(vp(tv(TV),np(NP))) --> tv(TV),np(NP).
  vp(vp(VP)) --> iv(VP).
  det(det(the)) --> [the].
  det(det(a)) --> [a].
  det(det(a)) --> [an].
  n(n(giraffe)) --> [giraffe].
  n(n(apple)) --> [apple].
  iv(iv(dreams)) --> [dreams].
  tv(tv(eats)) --> [eats].
  tv(tv(dreams)) --> [dreams].
  ```

  Here are some possible queries:

  ```
  s(Tree,[the,giraffe,dreams],X).
  s(Tree,Sentence,[]).
  ```

Small French Example

- Vocabulary

  Each word is represented by a list.

```
det([le|X],X).
det([la|X],X).
n([souris|X],X).
n([chat|X],X).
v([mange|X],X).
v([trottine|X],X).
```

- Syntax

  A phrase is a noun (*sn*) followed by a verb (*sv*). It is represented as a list.

  A noun (sn) is a determinant (*det*) followed by a noun (*n*).

  A verb can be intransitive (only a verb (*v*)) or transitive (a verb (*v*) followed by a noun).

```
p(X,Y) :- sn(X,U), sv(U,Y).
sn(X,Y) :- det(X,U), n(U,Y).
sv(X,Y) :- v(X,Y).
sv(X,Y) :- v(X,U), sn(U,Y).
```

- Query

```
p([le, chat, trottine],[]).
p(X,[]).
```

- What is the associated BNF?
- How many sentences are recognized by this language?
- Write the program using the DCG notation.
- Write the program using the DCG notation to generate the parse tree.

- There are limitations in the grammar:
  - Agreement between the determinant and the noun.
    le souris
  - Agreement between the subject and the object.
    la souris trottine le chat
  - Subjects and objects are the same.
    la souris trottine la souris
  - W need to introduce semantics constraints. So define predicates to represent them.