

❖ Derivatives ❖ (Chapter no. 2) ①

Derivative of a function:- Let $f(x)$ be a real valued function then derivative of $f(x)$ denoted by $f'(x)$ & is defined as

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Some standard formulae of differentiation:- Set ①

$$① \frac{d}{dx}(x^n) = nx^{n-1}$$

$$② \frac{d}{dx}(c) = 0 \quad \text{where } c \text{ is any const.}$$

$$③ \frac{d}{dx}(x) = 1$$

$$④ \frac{d}{dx}(a^{bx}) = a^{bx} \cdot b \cdot \ln a$$

$$⑤ \frac{d}{dx}(e^{ax}) = a e^{ax}$$

$$⑥ \frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$⑦ \frac{d}{dx}(ax+b)^n = n(ax+b)^{n-1} \cdot a$$

$$⑧ \frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$⑨ \frac{d}{dx}(u \cdot v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

$$⑩ \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

$$⑪ \frac{d}{dx}(cu) = c \cdot \frac{du}{dx}$$



Set (2)

(2)

$$① \quad \frac{d}{dx}(\sin x) = \cos x$$

$$② \quad \frac{d}{dx}(\cos x) = -\sin x$$

$$③ \quad \frac{d}{dx}(\tan x) = \sec^2 x$$

$$④ \quad \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$⑤ \quad \frac{d}{dx}(\sec x) = \sec x \cdot \tan x$$

$$⑥ \quad \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$$

Set (3)

$$① \quad \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$② \quad \frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$③ \quad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$④ \quad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$⑤ \quad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$⑥ \quad \frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

Set (4)

$$① \quad \frac{d}{dx}(\sinh x) = \cosh x$$

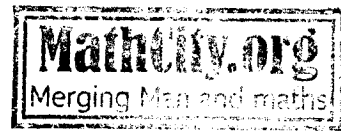
$$② \quad \frac{d}{dx}(\cosh x) = \sinh x$$

$$③ \quad \frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$④ \quad \frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x$$

$$⑤ \quad \frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \cdot \tanh x$$

$$⑥ \quad \frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \cdot \coth x$$



Set ⑤

③

$$① \quad \frac{d}{dx} (\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$$

$$② \quad \frac{d}{dx} (\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$$

$$③ \quad \frac{d}{dx} (\tanh^{-1} x) = \frac{1}{1-x^2} \quad |x| < 1$$

$$④ \quad \frac{d}{dx} (\coth^{-1} x) = \frac{1}{1-x^2} \quad |x| > 1$$

$$⑤ \quad \frac{d}{dx} (\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}$$

$$⑥ \quad \frac{d}{dx} (\operatorname{cosech}^{-1} x) = \frac{-1}{x\sqrt{1+x^2}}$$

④

EXERCISE 2.2

Differentiate w.r.t. x , (Problems 1 - 14)

1. $\sqrt{a^2 + x^2}$

Sol. Let $y = \sqrt{a^2 + x^2}$

or $y = (a^2 + x^2)^{1/2}$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{1}{2}(a^2 + x^2)^{\frac{1}{2}-1} \cdot \frac{d}{dx}(a^2 + x^2)$$

⑤

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2} (a^2 + x^2)^{-1/2} \cdot 2x \\ &= \frac{x}{(a^2 + x^2)^{1/2}} \\ &= \frac{x}{\sqrt{a^2 + x^2}}\end{aligned}$$

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2. $\sqrt[3]{x^3 + x + 1}$

Sol. Let $y = (x^3 + x + 1)^{1/3}$

$$\begin{aligned}\text{Diff. w.r.t. } x \quad \frac{dy}{dx} &= \frac{1}{3} (x^3 + x + 1)^{1/3 - 1} \cdot \frac{d}{dx} (x^3 + x + 1) \\ &= \frac{1}{3} (x^3 + x + 1)^{-2/3} \cdot (3x^2 + 1) \\ &= \frac{1}{3} \cdot \frac{1}{(x^3 + x + 1)^{2/3}} \cdot (3x^2 + 1) \\ &= \frac{3x^2 + 1}{3(x^3 + x + 1)^{2/3}}\end{aligned}$$

3. $\frac{\sqrt{a+x} - \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}$

Sol. Let $y = \frac{\sqrt{a+x} - \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}$

$$\begin{aligned}\text{Multiplying numerator \& denominator by } \sqrt{a+x} - \sqrt{a-x} \\ &= \frac{\sqrt{a+x} - \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}} \times \frac{\sqrt{a+x} - \sqrt{a-x}}{\sqrt{a+x} - \sqrt{a-x}} \\ &= \frac{(\sqrt{a+x} - \sqrt{a-x})^2}{(\sqrt{a+x})^2 - (\sqrt{a-x})^2} = \frac{(a+x) + (a-x) - 2\sqrt{a+x}\sqrt{a-x}}{(a+x) - (a-x)} \\ &= \frac{2a - 2\sqrt{(a+x)(a-x)}}{a+x - a+x} = \frac{2a - 2\sqrt{a^2 - x^2}}{2x}\end{aligned}$$

$$y = \frac{a - \sqrt{a^2 - x^2}}{x}$$

$$\frac{dy}{dx} = \frac{x \left[0 - \frac{1}{2} (a^2 - x^2)^{-1/2} (-2x) \right] - [a - \sqrt{a^2 - x^2}] \cdot 1}{x^2}$$

$$= \frac{x \cdot \frac{x}{\sqrt{a^2 - x^2}} - a + \sqrt{a^2 - x^2}}{x^2}$$

$$= \frac{\frac{x^2}{\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2} - a}{x^2} = \frac{\frac{x^2 + a^2 - x^2}{\sqrt{a^2 - x^2}} - a}{x^2}$$

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$$4. \quad y = \frac{\sqrt{\sin x}}{\sin \sqrt{x}} = \frac{\frac{a^2}{\sqrt{a^2-x^2}} - a}{x^2} = \frac{a^2 - a\sqrt{a^2-x^2}}{x^2 \sqrt{a^2-x^2}} = \frac{a(a - \sqrt{a^2-x^2})}{x^2 \sqrt{a^2-x^2}}$$

Sol. Let $y = \frac{\sqrt{\sin x}}{\sin \sqrt{x}}$

Diff. w.r.t. x

Sol.

$$\frac{dy}{dx} = \frac{\sin \sqrt{x} \left\{ \frac{1}{2\sqrt{\sin x}} \cdot \cos x \right\} - \sqrt{\sin x} \left\{ \cos \sqrt{x} \cdot \frac{1}{2\sqrt{x}} \right\}}{(\sin \sqrt{x})^2}$$

$$= \frac{\sin \sqrt{x} \cdot \frac{\cos x}{2\sqrt{\sin x}} - \frac{\sqrt{\sin x} \cdot \cos \sqrt{x}}{2\sqrt{x}}}{(\sin \sqrt{x})^2}$$

$$= \frac{\frac{\sqrt{x} \sin \sqrt{x} \cos x - \sin x \cos \sqrt{x}}{2\sqrt{x} \sqrt{\sin x}}}{(\sin \sqrt{x})^2}$$

$$= \frac{\sqrt{x} \sin \sqrt{x} \cos x - \sin x \cos \sqrt{x}}{2\sqrt{x} \sqrt{\sin x} \sin^2 \sqrt{x}}$$

$$5. \quad \sqrt{\log_{10}(x^2+1)}$$

Sol. Let $y = \sqrt{\log_{10}(x^2+1)}$

$$\text{or } y = \sqrt{\frac{\log_e(x^2+1)}{\log_{10} e}} = \sqrt{\frac{\ln(x^2+1)}{\ln 10}} = \frac{\sqrt{\ln(x^2+1)}}{\sqrt{\ln 10}}$$

$$\text{or } y = \frac{1}{\sqrt{\ln 10}} \cdot \sqrt{\ln(x^2+1)}$$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{1}{\sqrt{\ln 10}} \cdot \frac{1}{2\sqrt{\ln(x^2+1)}} \cdot \frac{1}{x^2+1} \cdot 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{\sqrt{\ln 10} \cdot \sqrt{\ln(x^2+1)} \cdot (x^2+1)} \quad \text{Ans.}$$

$$6. \quad y = \tan(\sin x)$$

Diff. w.r.t. x

Sol. $\frac{dy}{dx} = \sec^2(\sin x) \cdot \frac{d}{dx}(\sin x)$

$$= \sec^2(\sin x) \cdot \cos x$$

$$= \cos x \cdot \sec^2(\sin x)$$

Note

$$\frac{\log_a x}{\log_a y} = \log_y x$$

(7)

7. $y = \arctan \left(\frac{x \sin a}{1 - x \cos a} \right)$
 Diff. w.r.t. x

$$\begin{aligned} \text{Sol. } \frac{dy}{dx} &= \frac{1}{1 + \left(\frac{x \sin a}{1 - x \cos a} \right)^2} \cdot \frac{d}{dx} \left(\frac{x \sin a}{1 - x \cos a} \right) \\ &= \frac{1}{1 + \frac{(x \sin a)^2}{(1 - x \cos a)^2}} \cdot \frac{(1 - x \cos a) \sin a - x \sin a (-\cos a)}{(1 - x \cos a)^2} \\ &= \frac{1}{\frac{(1 - x \cos a)^2 + (x \sin a)^2}{(1 - x \cos a)^2}} \cdot \frac{\sin a - x \sin a \cos a + x \sin a \cos a}{(1 - x \cos a)^2} \\ &= \frac{\sin a}{1 - 2x \cos a + x^2 \cos^2 a + x^2 \sin^2 a} = \frac{\sin a}{1 - 2x \cos a + x^2 (\cos^2 a + \sin^2 a)} \\ &= \frac{\sin a}{1 - 2x \cos a + x^2} \end{aligned}$$

8. $y = \ln \frac{x^2 + x + 1}{x^2 - x + 1}$

Sol. $y = \ln (x^2 + x + 1) - \ln (x^2 - x + 1)$
 Diff. w.r.t. x

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{(x^2 + x + 1)} \frac{d}{dx} (x^2 + x + 1) - \frac{1}{(x^2 - x + 1)} \frac{d}{dx} (x^2 - x + 1) \\ &= \frac{2x + 1}{x^2 + x + 1} - \frac{2x - 1}{x^2 - x + 1} \\ &= \frac{(2x + 1)(x^2 - x + 1) - (2x - 1)(x^2 + x + 1)}{(x^2 + x + 1)(x^2 - x + 1)} \\ &= \frac{(2x^3 - 2x^2 + 2x + x^2 - x + 1) - (2x^3 + 2x^2 + 2x - x^2 - x - 1)}{(x^2 + x + 1)(x^2 - x + 1)} \\ &= \frac{2x^3 - x^2 + x + 1 - 2x^3 - x^2 - x + 1}{(x^2 + x + 1)(x^2 - x + 1)} = \frac{-2x^2 + 2}{(x^2 + x + 1)(x^2 - x + 1)} \\ &= \frac{2(1 - x^2)}{(x^2 + x + 1)(x^2 - x + 1)} \end{aligned}$$

9. $y = x^{x^2}$

Sol. Let $y = x^{x^2}$

taking log on both sides

$$\ln y = \ln x^{x^2}$$

$$\ln y = x^2 \ln x$$

Diff. w.r.t. x

$$\frac{1}{y} \cdot \frac{dy}{dx} = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$$

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$$= x + 2x \ln x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = x(1 + 2 \ln x)$$

$$\Rightarrow \frac{dy}{dx} = y \cdot x(2 \ln x + 1)$$

$$= x^{x^2} \cdot x(2 \ln x + 1)$$

$$= x^{x^2+1} \cdot (2 \ln x + 1)$$

10. $\ln(x^2 + x)$

Sol. Let $y = \ln(x^2 + x)$

Differentiating w.r.t x , we have

$$\frac{dy}{dx} = \frac{1}{(x^2 + x)} \cdot \frac{d}{dx}(x^2 + x)$$

$$= \frac{1}{x^2 + x} \cdot (2x + 1)$$

$$= \frac{2x + 1}{x(x + 1)}$$

11. $y = (\arcsin x)^{x^{1/2}}$

Sol. Taking logarithm of both sides

$$\ln y = \ln(\sin^{-1} x)^{x^{1/2}}$$

$$\ln y = x^{1/2} \cdot \ln(\sin^{-1} x)$$

Diff. w.r.t. x

$$\frac{1}{y} \frac{dy}{dx} = x^{1/2} \cdot \frac{1}{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}} + \ln \sin^{-1} x \cdot \frac{d}{dx}(x^{1/2}) \quad \text{--- (1)}$$

Now, let $u = x^{1/2}$

$$\ln u = \ln x^{1/2}$$

$$\text{or } \ln u = \frac{1}{2} \ln x$$

Diff. w.r.t. x

$$\frac{1}{u} \frac{du}{dx} = \frac{1}{x} \cdot \frac{1}{2} + \ln x \left(-\frac{1}{x^2} \right)$$

$$= \frac{1}{x^2} + \ln x \left(-\frac{1}{x^2} \right)$$



9

$$\begin{aligned}\frac{du}{dx} &= u \left(\frac{1}{x^2} - \frac{1}{x^2} \ln x \right) \\ &= x^{1/2} \cdot \frac{1}{x^2} (1 - \ln x)\end{aligned}$$

$$\frac{du}{dx} = x^{\frac{1}{2}-2} (1 - \ln x)$$

$$\text{or } \frac{d}{dx}(x^{1/2}) = x^{\frac{1}{2}-2} (1 - \ln x)$$

Putting in (1)

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= x^{\frac{1}{2}} \cdot \frac{1}{\sin^{-1} x \cdot \sqrt{1-x^2}} + \ln \sin^{-1} x \cdot x^{\frac{1}{2}-2} (1 - \ln x) \\ \Rightarrow \frac{dy}{dx} &= y \cdot \left[x^{\frac{1}{2}} \cdot \frac{1}{\sin^{-1} x \cdot \sqrt{1-x^2}} + x^{\frac{1}{2}-2} \cdot \ln \sin^{-1} x \cdot \ln(1-x) \right] \\ &= (\sin^{-1} x)^{\frac{1}{2}} \left[x^{\frac{1}{2}} \cdot \frac{1}{\sin^{-1} x \cdot \sqrt{1-x^2}} + x^{\frac{1}{2}-2} \cdot \ln \sin^{-1} x \cdot \ln(1-x) \right]\end{aligned}$$

$$12. \quad y = |x^2 - 9|$$

Sol. Given $y = |x^2 - 9|$

$$\begin{aligned}\text{Here } y &= x^2 - 9, \quad \text{if } |x| \geq 3 \\ &= -(x^2 - 9), \quad \text{if } |x| < 3\end{aligned}$$

Diff. w.r.t. x

$$\frac{dy}{dx} = \begin{cases} 2x & \text{if } |x| \geq 3 \\ -2x & \text{if } |x| < 3 \end{cases} \quad \text{--- Ans.}$$

Note Let $x \in \mathbb{R}$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$13. \quad y = \sqrt{x + \sqrt{x + \sqrt{x}}}$$

Sol. Let $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$ Diff. w.r.t. x

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \cdot \frac{d}{dx}(x + \sqrt{x + \sqrt{x}}) \\ &= \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \cdot \left(1 + \frac{1}{2\sqrt{x + \sqrt{x}}} \cdot \left(1 + \frac{1}{2\sqrt{x}} \right) \right) \\ &= \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \cdot \left(1 + \frac{1}{2\sqrt{x + \sqrt{x}}} \right) \cdot \left(1 + \frac{1}{2\sqrt{x}} \right) \quad \text{--- Ans.}\end{aligned}$$

$$14. y = (x + |x|)^{1/2}$$

$$\text{Sol. Here, } y = (x+x)^{1/2}, \text{ if } x \geq 0 \\ = (x-x)^{1/2} \text{ if } x < 0$$

$$\text{or } y = (2x)^{1/2} \quad \text{if } x \geq 0 \\ = 0 \quad \text{if } x < 0$$

Diff. w.r.t. x

$$15. \text{ Differentiate } \frac{dy}{dx} = \frac{1}{2}(2x)^{-1/2} \cdot 2 = \frac{1}{\sqrt{2x}} \quad \text{if } x > 0 \\ = 0 \quad \text{if } x < 0$$

$$\arctan \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \text{ w.r.t. } \arccos x^2$$

$$\text{Sol. Let } y = \tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right) \text{ \& } U = \cos^{-1} x^2 \text{ then } \frac{dy}{du} = ?$$

$$\text{as } U = \cos^{-1} x^2 \Rightarrow x^2 = \cos U \text{ Put in eq.}$$

$$y = \tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$$

$$= \tan^{-1} \left(\frac{\sqrt{1+\cos U} - \sqrt{1-\cos U}}{\sqrt{1+\cos U} + \sqrt{1-\cos U}} \right)$$

$$= \tan^{-1} \left(\frac{\sqrt{2 \cos^2 \frac{U}{2}} - \sqrt{2 \sin^2 \frac{U}{2}}}{\sqrt{2 \cos^2 \frac{U}{2}} + \sqrt{2 \sin^2 \frac{U}{2}}} \right)$$

$$= \tan^{-1} \left(\frac{\sqrt{2} \cos \frac{U}{2} - \sqrt{2} \sin \frac{U}{2}}{\sqrt{2} \cos \frac{U}{2} + \sqrt{2} \sin \frac{U}{2}} \right)$$

$$= \tan^{-1} \left(\frac{\cos \frac{U}{2} - \sin \frac{U}{2}}{\cos \frac{U}{2} + \sin \frac{U}{2}} \right)$$

Dividing numerator & denominator by $\cos \frac{U}{2}$

$$= \tan^{-1} \left(\frac{1 - \tan \frac{U}{2}}{1 + \tan \frac{U}{2}} \right)$$

$$y = \tan^{-1} \left(\frac{\tan \frac{\pi}{4} - \tan \frac{U}{2}}{1 + \tan \frac{\pi}{4} \cdot \tan \frac{U}{2}} \right) = \tan^{-1} \tan \left(\frac{\pi}{4} - \frac{U}{2} \right)$$

$$\text{So } y = \frac{\pi}{4} - \frac{U}{2}$$

Diff. w.r.t. U

$$\frac{dy}{du} = -\frac{1}{2} \text{ --- Ans.}$$

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16. $y = x^{\sin y}$

Sol. $y = x^{\sin y}$

Taking logarithm of both sides, we have

$$\ln y = \ln x^{\sin y}$$

$$\text{or } \ln y = \sin y \cdot \ln x$$

Diff. w.r.t. x

$$\frac{1}{y} \cdot \frac{dy}{dx} = \sin y \cdot \frac{1}{x} + \ln x \cdot \cos y \cdot \frac{dy}{dx}$$

Multiplying both sides by xy

$$x \frac{dy}{dx} = y \sin y + xy \ln x \cdot \cos y \cdot \frac{dy}{dx}$$

$$x \frac{dy}{dx} - xy \cos y \cdot \ln x \cdot \frac{dy}{dx} = y \sin y$$

$$\frac{dy}{dx} (x - xy \cos y \cdot \ln x) = y \sin y$$

$$17. \quad x^y = e^{y \ln x} \Rightarrow \frac{dy}{dx} = \frac{y \sin y}{x - xy \cos y \cdot \ln x}$$

Sol. Taking logarithm of both sides, we have

$$\ln x^y = \ln e^{y \ln x}$$

$$y \ln x = (y \ln x) \cdot \ln e$$

$$\text{or } y \ln x = x - y$$

$$y \ln x + y = x$$

$$y (\ln x + 1) = x$$

$$\Rightarrow y = \frac{x}{1 + \ln x}$$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{(1 + \ln x) \cdot 1 - x \cdot \frac{1}{x}}{(1 + \ln x)^2}$$

$$= \frac{1 + \ln x - 1}{(1 + \ln x)^2}$$

$$\frac{dy}{dx} = \frac{\ln x}{(1 + \ln x)^2}$$

18.

$$y^x + x^y = C$$

Sol. Given $y^x + x^y = C$

Sol. Let $u = y^x$ and $v = x^y$

Taking logarithm of both sides of the first equation

$$\ln u = \ln y^x$$

$$\ln u = x \ln y$$

Differentiating w.r.t. x , we have

$$\frac{1}{u} \frac{du}{dx} = x \cdot \frac{1}{y} \frac{dy}{dx} + \ln y$$

$$\frac{du}{dx} = u \left(\frac{x}{y} \frac{dy}{dx} + \ln y \right)$$

$$= y^x \left(\frac{x}{y} \frac{dy}{dx} + \ln y \right)$$

Now from $v = x^y$, taking logarithm, we get

$$\ln v = \ln x^y$$

$$\ln v = y \ln x$$

Differentiating w.r.t. x , we obtain

$$\frac{1}{v} \frac{dv}{dx} = \frac{y}{x} + \ln x \cdot \frac{dy}{dx}$$

$$\frac{dv}{dx} = v \left[\frac{y}{x} + \ln x \cdot \frac{dy}{dx} \right]$$

$$= x^y \left[\frac{y}{x} + \ln x \cdot \frac{dy}{dx} \right]$$

The given equation is

$$u + v = c$$

Differentiating w.r.t. x , we get

$$\frac{du}{dx} + \frac{dv}{dx} = 0$$

Putting the values of $\frac{du}{dx}$ and $\frac{dv}{dx}$ from (1) and (2) into (3), we have

$$y^x \left[\frac{x}{y} \frac{dy}{dx} + \ln y \right] + x^y \left[\frac{y}{x} + \ln x \cdot \frac{dy}{dx} \right] = 0$$

$$x y^{x-1} \frac{dy}{dx} + y^x \ln y + x^y \frac{y}{x} + x^y \ln x \cdot \frac{dy}{dx} = 0$$

or

$$(x y^{x-1} + x^y \ln x) \frac{dy}{dx} + \left[y^x \ln y + x^y \cdot \frac{y}{x} \right] = 0$$

$$(x y^{x-1} + x^y \ln x) \cdot \frac{dy}{dx} = - (y^x \ln y + y \cdot x^{y-1})$$

✓ **19.** or $\frac{dy}{dx} = -\frac{y^x \ln y + y x^{y-1}}{x y^{x-1} + y^x \ln x}$

$\frac{x+y}{x-y} = x^2 + y^2$

Sol. Differentiating w.r.t. x , we have

$$\frac{(x-y)(1+y') - (x+y)(1-y')}{(x-y)^2} = 2x + 2yy'$$

$$\text{or } \frac{(x-y) + (x-y)y' - (x+y) + y'(x+y)}{(x-y)^2} = 2x + 2yy'$$

$$\frac{(x-y-x-y) + y'(x-y+x+y)}{(x-y)^2} = 2(x+yy')(x-y)^2$$

$$\frac{-2y + y'(2x)}{(x-y)^2} = 2(x+yy')(x-y)^2$$

$$\frac{-y + xy'}{(x-y)^2} = (x+yy')(x-y)^2$$

$$\frac{xy' - y}{(x-y)^2} = x(x-y)^2 + yy'(x-y)^2$$

$$\frac{xy' - y}{(x-y)^2} = x(x-y)^2 + yy'(x-y)^2$$

$$\frac{dy}{dx} = \frac{y + x(x-y)^2}{x-y(x-y)^2}$$

20. $x + \arcsin y = xy$

Sol. $x + \sin^{-1} y = xy$

Differentiating (1) w.r.t. x , we have

$$1 + \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = x \frac{dy}{dx} + y \cdot 1$$

$$\frac{1}{\sqrt{1-y^2}} \cdot \frac{dy}{dx} - x \frac{dy}{dx} = y - 1$$

$$\text{or } \frac{dy}{dx} \left(\frac{1}{\sqrt{1-y^2}} - x \right) = y - 1$$

$$\text{or } \frac{dy}{dx} \left(\frac{1 - x\sqrt{1-y^2}}{\sqrt{1-y^2}} \right) = y - 1$$

Therefore,

$$\frac{dy}{dx} = \frac{(y-1)\sqrt{1-y^2}}{1-x\sqrt{1-y^2}}$$

In Problems 21-30, find $f'(x)$:

21. $f(x) = \ln(x + \sqrt{x^2 - 1})$

Sol. Here $f(x) = \ln(x + \sqrt{x^2 - 1})$

Diff. w.r.t. x

$$\begin{aligned} f'(x) &= \frac{1}{(x + \sqrt{x^2 - 1})} \cdot \frac{d}{dx} (x + \sqrt{x^2 - 1}) \\ &= \frac{1}{(x + \sqrt{x^2 - 1})} \cdot \left(1 + \frac{1}{2\sqrt{x^2 - 1}} \cdot 2x\right) \\ &= \frac{1}{(x + \sqrt{x^2 - 1})} \cdot \left(1 + \frac{x}{\sqrt{x^2 - 1}}\right) \\ &= \frac{1}{(x + \sqrt{x^2 - 1})} \cdot \left[\frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}}\right] \\ &= \frac{1}{\sqrt{x^2 - 1}} \end{aligned}$$

22. $f(x) = \ln \frac{e^x}{1 + e^x}$

Sol. $f(x) = \ln \frac{e^x}{1 + e^x}$

$$= \ln e^x - \ln(1 + e^x)$$

$$f(x) = x - \ln(1 + e^x)$$

Diff. w.r.t. x

$$f'(x) = 1 - \frac{1}{1 + e^x} \cdot e^x = 1 - \frac{e^x}{1 + e^x}$$

$$= \frac{1 + e^x - e^x}{1 + e^x} = \frac{1}{1 + e^x}$$

23. $f(x) = x^{\ln x}$

Sol. Taking \ln of both sides, we get

$$\ln(f(x)) = \ln x^{\ln x}$$

$$\text{or } \ln f(x) = \ln x \cdot \ln x$$

Diff. w.r.t. x

$$\frac{1}{f(x)} \cdot f'(x) = \ln x \cdot \frac{1}{x} + \ln x \cdot \frac{1}{x}$$

$$f'(x) = f(x) \left[\frac{\ln x}{x} + \frac{\ln x}{x} \right]$$

$$= \ln x \left[\frac{2 \ln x}{x} \right]$$

$$= x^{\ln x - 1} \cdot (2 \ln x)$$

$$24. f(x) = \ln \frac{1+\sqrt{x}}{1-\sqrt{x}}$$

$$\text{Sol. } f(x) = \ln(1+\sqrt{x}) - \ln(1-\sqrt{x})$$

Diff. w.r.t. x

$$\begin{aligned} f'(x) &= \frac{1}{1+\sqrt{x}} \cdot \left(0 + \frac{1}{2\sqrt{x}}\right) - \frac{1}{1-\sqrt{x}} \cdot \left(0 - \frac{1}{2\sqrt{x}}\right) \\ &= \frac{1}{2\sqrt{x}(1+\sqrt{x})} + \frac{1}{2\sqrt{x}(1-\sqrt{x})} \\ &= \frac{1}{2\sqrt{x}} \left[\frac{1}{1+\sqrt{x}} + \frac{1}{1-\sqrt{x}} \right] \\ &= \frac{1}{2\sqrt{x}} \left[\frac{1-\sqrt{x}+1+\sqrt{x}}{(1+\sqrt{x})(1-\sqrt{x})} \right] \\ &= \frac{1}{2\sqrt{x}} \cdot \frac{2}{(1-x)} \\ &= \frac{1}{\sqrt{x}(1-x)} \end{aligned}$$

$$25. f(x) = e^{ax} \cos(b \arctan x)$$

$$\text{Sol. } f(x) = e^{ax} \cos(b \tan^{-1} x)$$

Diff. w.r.t. x

$$\begin{aligned} f'(x) &= e^{ax} \cdot \sin(b \tan^{-1} x) \cdot b \cdot \frac{1}{1+x^2} + \cos(b \tan^{-1} x) \cdot e^{ax} \cdot a \\ &= e^{ax} \left[a \cos(b \tan^{-1} x) - \frac{b}{1+x^2} \sin(b \tan^{-1} x) \right] \\ &= e^{ax} \left[\frac{a(1+x^2) \cos(b \tan^{-1} x) - b \sin(b \tan^{-1} x)}{(1+x^2)} \right] \end{aligned}$$

$$26. f(x) = \frac{1}{\sqrt{b^2-a^2}} \ln \frac{\sqrt{b+a} + \sqrt{b-a} \tan\left(\frac{x}{2}\right)}{\sqrt{b+a} - \sqrt{b-a} \tan\left(\frac{x}{2}\right)}$$

Sol.

$$\begin{aligned} \text{Here } f(x) &= \frac{1}{\sqrt{b^2-a^2}} \left[\ln(\sqrt{b+a} + \sqrt{b-a} \tan \frac{x}{2}) - \ln(\sqrt{b+a} - \sqrt{b-a} \tan \frac{x}{2}) \right] \\ &\quad \text{Diff. w.r.t. x} \\ f'(x) &= \frac{1}{\sqrt{b^2-a^2}} \left[\frac{1}{\sqrt{b+a} + \sqrt{b-a} \tan \frac{x}{2}} \cdot \sqrt{b-a} \cdot \sec^2 \frac{x}{2} \cdot \frac{1}{2} - \frac{-\sqrt{b-a} \cdot \sec^2 \frac{x}{2} \cdot \frac{1}{2}}{\sqrt{b+a} - \sqrt{b-a} \tan \frac{x}{2}} \right] \end{aligned}$$

$$\begin{aligned}
 f'(x) &= \frac{1}{\sqrt{b^2 - a^2}} \left[\frac{\frac{1}{2} \sqrt{b-a} \cdot \sec^2 \frac{x}{2}}{\sqrt{b+a} + \sqrt{b-a} \tan \frac{x}{2}} + \frac{\frac{1}{2} \sqrt{b-a} \cdot \sec^2 \frac{x}{2}}{\sqrt{b+a} - \sqrt{b-a} \tan \frac{x}{2}} \right] \quad 16 \\
 &= \frac{\sqrt{b-a} \sec^2 \frac{x}{2}}{2 \sqrt{(b-a)(b+a)}} \left[\frac{1}{\sqrt{b+a} + \sqrt{b-a} \tan \frac{x}{2}} + \frac{1}{\sqrt{b+a} - \sqrt{b-a} \tan \frac{x}{2}} \right] \\
 &= \frac{\sqrt{b-a} \sec^2 \frac{x}{2}}{2 \sqrt{b-a} \sqrt{b+a}} \cdot \left[\frac{\sqrt{b+a} - \sqrt{b-a} \tan \frac{x}{2} + \sqrt{b+a} + \sqrt{b-a} \tan \frac{x}{2}}{(\sqrt{b+a} + \sqrt{b-a} \tan \frac{x}{2})(\sqrt{b+a} - \sqrt{b-a} \tan \frac{x}{2})} \right] \\
 &= \frac{\sec^2 \frac{x}{2}}{2 \sqrt{b-a}} \cdot \frac{2 \sqrt{b+a}}{(b+a) - (b-a) \cdot \frac{\sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2}}} \\
 &= \frac{\sec^2 \frac{x}{2}}{\frac{(b+a) \cos^2 \frac{x}{2} - (b-a) \sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2}}} = \frac{\sec^2 \frac{x}{2} \cdot \cos^2 \frac{x}{2}}{b \cos^2 \frac{x}{2} + a \cos^2 \frac{x}{2} - b \sin^2 \frac{x}{2} + a \sin^2 \frac{x}{2}} \\
 &= \frac{1}{a (\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}) + b (\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2})} \\
 &= \frac{1}{a + b \cos x}
 \end{aligned}$$

27. $f(x) = x a^x \sinh x$

Sol: $f(x) = x \cdot a^x \cdot \sinh x$
Diff. w.r.t. x

$$\begin{aligned}
 f'(x) &= x \cdot a^x \cdot \cosh x + x \sinh x \cdot a^x \cdot \ln a + 1 \cdot a^x \cdot \sinh x \\
 &= a^x [x \cosh x + x \sinh x \cdot \ln a + \sinh x]
 \end{aligned}$$

28. $f(x) = -\frac{\cos x}{2 \sin^2 x} + \frac{1}{2} \ln \tan \frac{x}{2}$

Sol: Here $f(x) = -\frac{1}{2} \cdot \frac{\cos x}{\sin^2 x} + \frac{1}{2} \ln \tan \frac{x}{2}$

$$\begin{aligned}
 \text{Diff. w.r.t. } x \\
 f'(x) &= -\frac{1}{2} \left[\frac{\sin^2 x \cdot (-\sin x) - \cos x \cdot 2 \sin x \cos x}{\sin^4 x} \right] + \frac{1}{2} \cdot \frac{1}{\tan \frac{x}{2}} \cdot \sec^2 \frac{x}{2} \cdot \frac{1}{2} \\
 &= -\frac{1}{2} \left[\frac{-\sin^3 x - 2 \sin x \cos^2 x}{\sin^4 x} \right] + \frac{1}{4} \cdot \frac{\sec^2 \frac{x}{2}}{\tan \frac{x}{2}} \\
 &= -\frac{1}{2} \left[\frac{-\sin^2 x - 2 \cos^2 x}{\sin^3 x} \right] + \frac{1}{4} \cdot \frac{\cos^2 x/2}{\frac{\sin x/2}{\cos x/2}} \\
 &= -\frac{1}{2} \left[\frac{-\sin^2 x - 2 \cos^2 x}{\sin^3 x} \right] + \frac{1}{4} \cdot \frac{\cos^2 x/2}{\sin x/2 \cos x/2}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{2} \left[\frac{-\sin^2 x - 2\cos^2 x}{\sin^3 x} \right] + \frac{1}{(4\sin \frac{x}{2} \cos \frac{x}{2})} \quad 17 \\
&= -\frac{1}{2} \left[\frac{-\sin^2 x - 2\cos^2 x}{\sin^3 x} \right] + \frac{1}{2(2\sin \frac{x}{2} \cos \frac{x}{2})} \\
&= \frac{\sin^2 x + 2\cos^2 x}{2\sin^3 x} + \frac{1}{2\sin x} \\
&= \frac{\sin^2 x + 2\cos^2 x + \sin^2 x}{2\sin^3 x} = \frac{2\sin^2 x + 2\cos^2 x}{2\sin^3 x} \\
&= \frac{2(\sin^2 x + \cos^2 x)}{2\sin^3 x} = \frac{1}{\sin^3 x} = \operatorname{cosec}^3 x
\end{aligned}$$

29. $f(x) = \operatorname{arcsec}(\csc x + \sqrt{x})$

Sol. Here $f(x) = \sec^{-1}(\operatorname{cosec} x + \sqrt{x})$

Diff. w.r.t. x

$$\begin{aligned}
f'(x) &= \frac{1}{(\operatorname{cosec} x + \sqrt{x}) \sqrt{(\operatorname{cosec} x + \sqrt{x})^2 - 1}} \cdot \frac{d}{dx} (\operatorname{cosec} x + \sqrt{x}) \\
&= \frac{1}{(\operatorname{cosec} x + \sqrt{x}) \sqrt{(\operatorname{cosec} x + \sqrt{x})^2 - 1}} \times \left(-\operatorname{cosec} x \cot x + \frac{1}{2\sqrt{x}} \right) \\
&= \frac{1}{(\operatorname{cosec} x + \sqrt{x}) \sqrt{\operatorname{cosec}^2 x + x + 2\sqrt{x} \operatorname{cosec} x - 1}} \cdot \left[\frac{-2\sqrt{x} \operatorname{cosec} x \cot x + 1}{2\sqrt{x}} \right] \\
&= \frac{1 - 2\sqrt{x} \operatorname{cosec} x \cot x}{2\sqrt{x} (\operatorname{cosec} x + \sqrt{x}) \sqrt{\operatorname{cosec}^2 x + x + 2\sqrt{x} \operatorname{cosec} x - 1}}
\end{aligned}$$

30. $f(x) = \left(1 + \frac{1}{x}\right)^{x^2}$

Sol. Taking \ln of both sides, we have

$$\ln f(x) = \ln \left(1 + \frac{1}{x}\right)^{x^2}$$

$$\ln f(x) = x^2 \ln \left(\frac{x+1}{x}\right)$$

Diff. w.r.t. x

$$\frac{1}{f(x)} \cdot f'(x) = x^2 \cdot \frac{1}{\frac{x+1}{x}} \cdot \left[\frac{x \cdot 1 - (x+1) \cdot 1}{x^2} \right] + \ln \left(\frac{x+1}{x}\right) \cdot 2x$$

$$\begin{aligned}
\frac{1}{f(x)} \cdot f'(x) &= \frac{x}{x+1} \left[\frac{x - x - 1}{x^2} \right] + 2x \ln \left(\frac{x+1}{x}\right) \\
&= \frac{-1}{x(x+1)} + 2x \ln \left(\frac{x+1}{x}\right)
\end{aligned}$$

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$$\begin{aligned}\Rightarrow f'(x) &= f(x) \left[2x \ln\left(\frac{x+1}{x}\right) - \frac{1}{x(x+1)} \right] \\ &= \left(1 + \frac{1}{x}\right) \left[2x \ln\left(\frac{x+1}{x}\right) - \frac{1}{x(x+1)} \right]\end{aligned}$$

Differentiate with respect to x each of the following
(Problems 31-42)

31. $\arctan\left(\frac{1+2x}{2-x}\right)$

Sol.

Let $y = \tan^{-1}\left(\frac{1+2x}{2-x}\right)$

Diff. w.r.t. x

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{1 + \left(\frac{1+2x}{2-x}\right)^2} \cdot \frac{d}{dx} \left(\frac{1+2x}{2-x} \right) \\ &= \frac{1}{1 + \frac{(1+2x)^2}{(2-x)^2}} \cdot \frac{(2-x) \cdot 2 - (1+2x) \cdot (-1)}{(2-x)^2} \\ &= \frac{1}{\frac{(2-x)^2 + (1+2x)^2}{(2-x)^2}} \cdot \frac{4-2x+1+2x}{(2-x)^2} \\ &= \frac{(2-x)^2}{(2-x)^2 + (1+2x)^2} \cdot \frac{5}{(2-x)^2} = \frac{5}{5+5x^2} = \frac{5}{5(1+x^2)} = \frac{1}{1+x^2}\end{aligned}$$

32. $\ln(\arcsin e^x) + yx^2 = 1$

Sol. Given $\ln(\sin^{-1} e^x) + yx^2 = 1$

Diff. w.r.t. x

$$\frac{1}{\sin^{-1}(e^x)} \cdot \frac{d}{dx} (\sin^{-1} e^x) + \frac{d}{dx} (yx^2) = 0$$

$$\frac{1}{\sin^{-1}(e^x)} \cdot \frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x + y \cdot 2x + x^2 \frac{dy}{dx} = 0$$

$$\frac{1}{\sin^{-1}(e^x)} \cdot \frac{e^x}{\sqrt{1-e^{2x}}} + 2xy + x^2 \frac{dy}{dx} = 0$$

$$x^2 \frac{dy}{dx} = -2xy - \frac{e^x}{\sin^{-1}(e^x) \sqrt{1-e^{2x}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x^2} \left[-2xy - \frac{e^x}{\sin^{-1}(e^x) \sqrt{1-e^{2x}}} \right] = -\frac{2y}{x} - \frac{e^x}{x^2 \sin^{-1}(e^x) \sqrt{1-e^{2x}}}$$

33. $y = (\arcsin x^2)^\pi$

Sol. Let $y = (\sin^{-1} x^2)^\pi$

Diff. w.r.t. x

$$\begin{aligned}\frac{dy}{dx} &= \pi (\sin^{-1} x^2)^{\pi-1} \cdot \frac{d}{dx} (\sin^{-1} x^2) \\ &= \pi (\sin^{-1} x^2)^{\pi-1} \cdot \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x \\ &= \pi (\sin^{-1} x^2)^{\pi-1} \cdot \frac{2x}{\sqrt{1-x^4}} \\ &= \frac{2x\pi (\sin^{-1} x^2)^{\pi-1}}{\sqrt{1-x^4}}\end{aligned}$$

34. $\arctan\left(\frac{y}{x}\right) + yx^2 = 1$

Sol. Given $\tan^{-1}(y/x) + yx^2 = 1$

Diff. w.r.t. x

$$\begin{aligned}\frac{1}{1+(y/x)^2} \cdot \left(\frac{x \frac{dy}{dx} - y \cdot 1}{x^2} \right) + y \cdot 2x + x^2 \frac{dy}{dx} &= 0 \\ \frac{1}{1+y^2/x^2} \cdot \left(\frac{x \frac{dy}{dx} - y}{x^2} \right) + 2xy + x^2 \frac{dy}{dx} &= 0 \\ \frac{1}{\frac{x^2+y^2}{x^2}} \cdot \left(\frac{x \frac{dy}{dx} - y}{x^2} \right) + 2xy + x^2 \frac{dy}{dx} &= 0 \\ \frac{x^2}{x^2+y^2} \cdot \frac{x \frac{dy}{dx} - y}{x^2} + 2xy + x^2 \frac{dy}{dx} &= 0 \\ \frac{x \frac{dy}{dx} - y}{x^2+y^2} + x^2 \frac{dy}{dx} &= -2xy\end{aligned}$$

35. $y = \frac{1 - \cosh x}{1 + \cosh x}$

Sol.

Given $y = \frac{1 - \cosh x}{1 + \cosh x}$

Diff. w.r.t. x

$$\begin{aligned}\frac{dy}{dx} &= \frac{(1 + \cosh x) \cdot \frac{d}{dx} (1 - \cosh x) - (1 - \cosh x) \cdot \frac{d}{dx} (1 + \cosh x)}{(1 + \cosh x)^2} \\ &= \frac{(1 + \cosh x)(-\sinh x) - (1 - \cosh x) \cdot \sinh x}{(1 + \cosh x)^2} \\ &= \frac{-\sinh x - \cosh x \cdot \sinh x - \sinh x + \cosh x \cdot \sinh x}{(1 + \cosh x)^2} = \frac{-2\sinh x}{(1 + \cosh x)^2}\end{aligned}$$

$$= \frac{-2 \cdot 2 \sinh^{1/2} \cosh^{1/2}}{(2 \cosh^{1/2})^2} = \frac{-4 \sinh^{1/2} \cosh^{1/2}}{4 \cosh^{1/2}} = -\frac{\sinh^{1/2}}{\cosh^{1/2}} = -\tanh^{1/2} \operatorname{sech}^{1/2} x$$

36. $y = \ln(\tanh 2x)$

Sol. $y = \ln(\tanh 2x)$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{1}{\tanh 2x} \cdot \frac{d}{dx}(\tanh 2x)$$

$$= \frac{1}{\tanh 2x} \cdot \operatorname{sech}^2 2x \cdot 2$$

$$= \coth 2x \cdot \operatorname{sech}^2 2x \cdot 2$$

$$= \frac{\cosh 2x}{\sinh 2x} \cdot \frac{1}{\cosh^2 2x} \cdot 2$$

$$= \frac{2}{\sinh 2x \cdot \cosh 2x} = \frac{4}{2 \sinh 2x \cdot \cosh 2x}$$

$$= \frac{4}{\sinh 4x} = 4 \operatorname{cosech} 4x$$

37. $\log_{10}\left(\frac{x+1}{x}\right)$

Sol. $y = \log_{10}\left(\frac{x+1}{x}\right)$

$$y = \frac{\log_e\left(\frac{x+1}{x}\right)}{\log_e 10} = \frac{\ln\left(\frac{x+1}{x}\right)}{\ln 10}$$

$$\frac{dy}{dx} = \frac{1}{\ln 10} \cdot \frac{1}{\frac{x+1}{x}} \cdot \left[\frac{x \cdot 1 - (x+1) \cdot 1}{x^2} \right]$$

$$= \frac{x}{(x+1) \cdot \ln 10} \cdot \left(\frac{x - x - 1}{x^2} \right) = \frac{-x}{(x+1) \cdot \ln 10 \cdot x^2}$$

$$= \frac{-1}{(x+1) \cdot \ln 10 \cdot x} \quad \text{Ans.}$$

38. $\arccos \sqrt{1-x^2}$

Sol. Let $y = \cos^{-1} \sqrt{1-x^2}$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(\sqrt{1-x^2})^2}} \cdot \frac{d}{dx}(\sqrt{1-x^2})$$

$$= \frac{-1}{\sqrt{1-(1-x^2)}} \cdot \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) = \frac{x}{\sqrt{1-x^2} \cdot \sqrt{1-x^2}}$$

$$= \frac{x}{\sqrt{x^2}} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{x}{|x|} \cdot \frac{1}{\sqrt{1-x^2}}$$

39. $\text{arc Sec}(\sinh x)$

Sol. Let $y = \text{Sec}^{-1}(\sinh x)$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{1}{\sinh x \sqrt{\sinh^2 x - 1}} \cdot \frac{d}{dx}(\sinh x)$$

$$= \frac{1}{\sinh x \sqrt{\sinh^2 x - 1}} \cdot \cosh x$$

$$= \frac{\cosh x}{\sqrt{\sinh^2 x - 1}}$$

40. $\text{arcsin}(\text{arccot} \ln x)$

Sol. Let $y = \text{Sin}^{-1}(\text{Cot}^{-1} \ln x)$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - (\text{Cot}^{-1} \ln x)^2}} \cdot \frac{d}{dx}(\text{Cot}^{-1} \ln x)$$

$$= \frac{1}{\sqrt{1 - (\text{Cot}^{-1} \ln x)^2}} \cdot \frac{-1}{1 + (\ln x)^2} \cdot \frac{1}{x}$$

$$= \frac{-1}{x(1 + \ln^2 x) \sqrt{1 - (\text{Cot}^{-1} \ln x)^2}}$$

41. $\text{Cosh}^{-1}(1+x^2)$

Sol. Let $y = \text{Cosh}^{-1}(1+x^2)$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{1}{\sqrt{(1+x^2)^2 - 1}} \cdot \frac{d}{dx}(1+x^2)$$

$$= \frac{1}{\sqrt{1+2x^2+x^4-1}} \cdot 2x = \frac{2x}{\sqrt{2x^2+x^4}} = \frac{2x}{|x| \sqrt{2+x^2}}$$

42. $\text{Sinh}^{-1}(\tanh x)$

Sol. Let $y = \text{Sinh}^{-1}(\tanh x)$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + \tanh^2 x}} \cdot \frac{d}{dx}(\tanh x) = \frac{\text{sech}^2 x}{\sqrt{1 + \tanh^2 x}} \quad \text{Ans.}$$

Differentiate (logarithmically) with respect to x
(Problems 43-47)

$$43. y = \sqrt[3]{\frac{x(x^2+1)}{(x-1)^2}}$$

Sol.

$$\text{Here } y = \left[\frac{x(x^2+1)}{(x-1)^2} \right]^{1/3}$$

taking \ln on both sides

$$\ln y = \ln \left[\frac{x(x^2+1)}{(x-1)^2} \right]^{1/3}$$

$$= \frac{1}{3} \ln \left[\frac{x(x^2+1)}{(x-1)^2} \right]$$

$$= \frac{1}{3} [\ln x + \ln(x^2+1) - \ln(x-1)^2]$$

$$\ln y = \frac{1}{3} [\ln x + \ln(x^2+1) - 2 \ln(x-1)]$$

Diff. w.r.t. x

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{3} \left[\frac{1}{x} + \frac{1}{x^2+1} \cdot 2x - 2 \cdot \frac{1}{x-1} \cdot 1 \right] = \frac{1}{3} \left[\frac{1}{x} + \frac{2x}{x^2+1} - \frac{2}{x-1} \right]$$

$$= \frac{1}{3} \left[\frac{(x^2+1)(x-1) + 2x \cdot x(x-1) - 2x(x^2+1)}{x(x^2+1)(x-1)} \right] = \frac{1}{3} \left[\frac{x^3 - x^2 + x - 1 + 2x^2 - 2x^3 - 2x^2 - 2x}{x(x^2+1)(x-1)} \right]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{3} \left[\frac{x^3 - 3x^2 - x - 1}{x(x^2+1)(x-1)} \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{3} \left[\frac{x^3 - 3x^2 - x - 1}{x(x^2+1)(x-1)} \right] = \frac{y^{1/3} \cdot (x^2+1)^{1/3} \cdot x^{3/3} \cdot (x^2+1)^{1/3} \cdot (x-1)^{1/3}}{3(x-1)^{2/3} \cdot x(x^2+1)(x-1)}$$

$$= \frac{x^3 - 3x^2 - x - 1}{3x^{1/3} \cdot (x-1)^{2/3} \cdot (x^2+1)^{2/3}} = \frac{x^3 - 3x^2 - x - 1}{3x^{1/3} \cdot (x-1)^{2/3} \cdot (x^2+1)^{2/3}}$$

$$44. y = \frac{\sqrt{x}(1-2x)^{2/3}}{(2-3x)^{3/4}(3-4x)^{4/3}}$$

Sol. Taking logarithm of both sides, we get

$$\ln y = \ln \left[\frac{\sqrt{x}(1-2x)^{2/3}}{(2-3x)^{3/4}(3-4x)^{4/3}} \right]$$

$$= \ln(\sqrt{x}(1-2x)^{2/3}) - \ln((2-3x)^{3/4}(3-4x)^{4/3})$$

$$= [\ln \sqrt{x} + \ln(1-2x)^{2/3}] - [\ln(2-3x)^{3/4} + \ln(3-4x)^{4/3}]$$

$$= \ln x^{1/2} + \ln(1-2x)^{2/3} - \ln(2-3x)^{3/4} - \ln(3-4x)^{4/3}$$

$$\ln y = \frac{1}{2} \ln x + \frac{2}{3} \ln(1-2x) - \frac{3}{4} \ln(2-3x) - \frac{4}{3} \ln(3-4x)$$

Diff. w.r.t. x

$$\begin{aligned}
 \frac{1}{y} \cdot \frac{dy}{dx} &= \frac{1}{2} \cdot \frac{1}{x} + \frac{2}{3} \cdot \frac{1}{1-2x} \cdot (-2) - \frac{3}{4} \cdot \frac{1}{2-3x} \cdot (-3) - \frac{4}{3} \cdot \frac{1}{3-4x} \cdot (-4) \\
 &= \frac{1}{2x} - \frac{4}{3(1-2x)} + \frac{9}{4(2-3x)} + \frac{16}{3(3-4x)} \\
 &= \frac{1}{2x} + \frac{9}{4(2-3x)} + \frac{-4}{3(1-2x)} + \frac{16}{3(3-4x)} \\
 &= \frac{2(2-3x) + 9x}{4x(2-3x)} + \frac{-4(3-4x) + 16(1-2x)}{3(1-2x)(3-4x)} \\
 &= \frac{4-6x+9x}{4x(2-3x)} + \frac{-12+16x+16-32x}{3(1-2x)(3-4x)} \\
 \frac{1}{y} \cdot \frac{dy}{dx} &= \frac{4+3x}{4x(2-3x)} + \frac{4-16x}{3(1-2x)(3-4x)} \\
 \frac{dy}{dx} &= y \left[\frac{4+3x}{4x(2-3x)} + \frac{4-16x}{3(1-2x)(3-4x)} \right] \\
 &= \frac{\sqrt{x}(1-2x)^{4/3}}{(2-3x)^{3/4}(3-4x)^{3/4}} \left[\frac{4+3x}{4x(2-3x)} + \frac{4(1-4x)}{3(1-2x)(3-4x)} \right] \text{ --- Ans.}
 \end{aligned}$$

$$45. y = (\tan x)^{\cot x} + (\cot x)^{\tan x} \quad \text{--- (1)}$$

Sol. Let $u = (\tan x)^{\cot x}$
 taking \ln on both sides
 $\ln u = \ln(\tan x)^{\cot x}$
 $\ln u = \cot x \ln(\tan x)$

Differentiating, w.r.t. x, we have

$$\frac{1}{u} \cdot \frac{du}{dx} = \cot x \cdot \frac{1}{\tan x} \cdot \sec^2 x + \ln(\tan x) \cdot -\operatorname{cosec}^2 x$$

$$\frac{du}{dx} = u \left[\cot x \cdot \cot x \cdot \sec^2 x - \ln(\tan x) \cdot \operatorname{cosec}^2 x \right]$$

$$= (\tan x)^{\cot x} \left[\cot^2 x \cdot \sec^2 x - \ln(\tan x) \cdot \operatorname{cosec}^2 x \right]$$

$$= (\tan x)^{\cot x} \left[\frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos^2 x} - \ln(\tan x) \cdot \operatorname{cosec}^2 x \right]$$

$$\frac{du}{dx} = (\tan x)^{\cot x} \left[\operatorname{cosec}^2 x - \ln(\tan x) \cdot \operatorname{cosec}^2 x \right]$$

$$dV = (\cot x)^{\tan x}$$

$$\ln v = \ln(\cot x)^{\tan x}$$

$$\ln v = \tan x \cdot \ln(\cot x)$$

Diff. w.r.t. x

$$\frac{1}{v} \cdot \frac{dv}{dx} = \tan x \cdot \frac{1}{\cot x} \cdot -\operatorname{cosec}^2 x + \ln(\cot x) \cdot \sec^2 x$$

$$\frac{dv}{dx} = v \left[-\tan x \cdot \tan x \cdot \operatorname{cosec}^2 x + \ln(\cot x) \cdot \sec^2 x \right]$$

$$= v \left[-\tan^2 x \cdot \operatorname{cosec}^2 x + \ln(\cot x) \cdot \sec^2 x \right]$$

$$= (\cot x)^{\tan x} \left[-\frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x} + \ln(\cot x) \cdot \sec^2 x \right]$$

$$\frac{dv}{dx} = (\cot x)^{\tan x} \left[-\sec^2 x + \ln(\cot x) \cdot \sec^2 x \right]$$

Now Eq. (1) is

$$y = U + V$$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{dU}{dx} + \frac{dV}{dx}$$

Putting values

$$\frac{dy}{dx} = (\tan x)^{\cot x} \left[\csc^2 x - \ln(\tan x) \cdot \csc^2 x \right] + (\cot x)^{\tan x} \left[-\sec^2 x + \ln(\cot x) \cdot \sec^2 x \right]$$

$$46. y = x^x \cdot e^x \sin x \cdot \ln x$$

Sol. Taking logarithm of both sides, we get

$$\ln y = \ln (x^x \cdot e^x \sin x \cdot \ln x)$$

$$= \ln x^x + \ln e^x + \ln \sin x + \ln (\ln x)$$

$$= x \ln x + x \ln e + \ln \sin x + \ln (\ln x)$$

$$\ln y = x \ln x + x + \ln \sin x + \ln (\ln x)$$

Diff. w.r.t. x

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x \cdot (1) + 1 + \frac{1}{\sin x} \cdot \cos x + \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = y \left[1 + \ln x + 1 + \cot x + \frac{1}{x \ln x} \right]$$

$$= (x^x \cdot e^x \sin x \cdot \ln x) \left[2 + \ln x + \cot x + \frac{1}{x \ln x} \right] \quad \text{Ans}$$

In Problems 48 - 60, find $\frac{dy}{dx}$:

$$47. y = \frac{(x+2)^2}{(x+1)(x^2+3)^3}$$

Sol. Taking ln of both sides, we have

$$\ln y = \ln \left[\frac{(x+2)^2}{(x+1)(x^2+3)^3} \right] = \ln(x+2)^2 - \ln(x+1) - \ln(x^2+3)^3$$

$$\ln y = 2 \ln(x+2) - \ln(x+1) - 3 \ln(x^2+3)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 2 \cdot \frac{1}{x+2} - \frac{1}{x+1} - 3 \cdot \frac{1}{x^2+3} \cdot 2x$$

$$\frac{dy}{dx} = y \left[\frac{2}{x+2} - \frac{1}{x+1} - \frac{6x}{x^2+3} \right] = y \left[\frac{2(x+1)(x^2+3) - (x+2)(x^2+3) - 6x(x+1)(x^2+3)}{(x+1)(x+2)(x^2+3)} \right]$$

$$= y \left[\frac{2x^3 + 6x + 2x^2 + 6 - x^3 - 3x - 2x^2 - 6x(x^2+3x+2)}{(x+1)(x+2)(x^2+3)} \right]$$

$$= y \left[\frac{x^3 + 3x - 6x^3 - 18x^2 - 12x}{(x+1)(x+2)(x^2+3)} \right] = \frac{(x+2)^2}{(x+1)(x^2+3)^3} \left[\frac{-5x^3 - 18x^2 - 9x}{(x+1)(x+2)(x^2+3)} \right]$$

$$48. \sqrt{x} + \sqrt{y} = \sqrt{a}$$

Sol. Differentiating both sides w.r.t. x, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$$

$$\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} \cdot \frac{dy}{dx} = 0$$

$$\frac{1}{\sqrt{y}} \cdot \frac{dy}{dx} = -\frac{1}{\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\text{or } \frac{dy}{dx} = -\sqrt{\frac{y}{x}}$$

$$49. x^3 + y^3 - 3axy = 0$$

Sol. Differentiating w.r.t. x , we get

$$3x^2 + 3y^2 \frac{dy}{dx} - 3a \left[x \frac{dy}{dx} + y \cdot 1 \right] = 0$$

$$\text{or } x^2 + y^2 \frac{dy}{dx} - a \left[x \frac{dy}{dx} + y \cdot 1 \right] = 0$$

$$x^2 + y^2 \frac{dy}{dx} - ax \frac{dy}{dx} - ay = 0$$

$$\frac{dy}{dx} (y^2 - ax) = ay - x^2$$

$$\text{or } \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$$

$$50. y - \cos(x+y) = 0$$

Sol. It can be rewritten as

$$y = \cos(x+y)$$

Differentiating both sides w.r.t. x , we get

$$\frac{dy}{dx} = -\sin(x+y) \left[1 + \frac{dy}{dx} \right]$$

$$\frac{dy}{dx} = -\sin(x+y) - \sin(x+y) \frac{dy}{dx}$$

$$\frac{dy}{dx} + \sin(x+y) \frac{dy}{dx} = -\sin(x+y)$$

$$\frac{dy}{dx} (1 + \sin(x+y)) = -\sin(x+y)$$

$$\text{or } \frac{dy}{dx} = -\frac{\sin(x+y)}{1 + \sin(x+y)}$$

$$51. \arctan(x+y) = \arcsin(e^y + x)$$

$$\text{Sol. } \tan^{-1}(x+y) = \sin^{-1}(e^y + x)$$

Diff. w.r.t. x

$$\frac{1}{1+(x+y)^2} (1 + \frac{dy}{dx}) = \frac{1}{\sqrt{1-(e^y+x)^2}} (e^y \frac{dy}{dx} + 1)$$

$$\frac{1}{1+(x+y)^2} + \frac{1}{1+(x+y)^2} \frac{dy}{dx} = \frac{1}{\sqrt{1-(e^y+x)^2}} e^y \frac{dy}{dx} + \frac{1}{\sqrt{1-(e^y+x)^2}}$$

$$\frac{1}{1+(x+y)^2} \frac{dy}{dx} - \frac{e^y}{\sqrt{1-(e^y+x)^2}} \frac{dy}{dx} = \frac{1}{\sqrt{1-(e^y+x)^2}} - \frac{1}{1+(x+y)^2}$$

$$\frac{dy}{dx} \left[\frac{1}{1+(x+y)^2} - \frac{e^y}{\sqrt{1-(e^y+x)^2}} \right] = \frac{1+(x+y)^2 - \sqrt{1-(e^y+x)^2}}{\sqrt{1-(e^y+x)^2} (1+(x+y)^2)}$$

$$\frac{dz}{dx} \left[\frac{\sqrt{1-(e^y+x)^2} - e^y [1+(x+y)^2]}{[1+(x+y)^2] \sqrt{1-(e^y+x)^2}} \right] = \frac{[1+(x+y)^2] - \sqrt{1-(e^y+x)^2}}{\sqrt{1-(e^y+x)^2} [1+(x+y)^2]} \quad 26$$

$$\frac{dz}{dx} \left[\sqrt{1-(e^y+x)^2} - e^y (1+(x+y)^2) \right] = 1+(x+y)^2 - \sqrt{1-(e^y+x)^2}$$

$$\text{or } \frac{dz}{dx} = \frac{1+(x+y)^2 - \sqrt{1-(e^y+x)^2}}{\sqrt{1-(e^y+x)^2} - e^y (1+(x+y)^2)}$$

52. $x = a(t - \sin t)$, $y = a(1 - \cos t)$

Sol. Differentiating w.r.t. t , we get

$$\frac{dx}{dt} = a(1 - \cos t)$$

$$\text{and } \frac{dy}{dt} = a(\sin t) = a \sin t$$

$$\text{Now } \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= \frac{a \sin t}{a(1 - \cos t)}$$

$$= \frac{2 \sin \frac{t}{2} \cos \frac{t}{2}}{2 \sin^2 \frac{t}{2}} = \frac{\cos \frac{t}{2}}{\sin \frac{t}{2}} = \cot \frac{t}{2}$$

53. $x = a \cos^3 t$, $y = b \sin^3 t$

Sol. $x = a \cos^3 t$ & $y = b \sin^3 t$

Diff. w.r.t. t

$$\begin{aligned} \frac{dx}{dt} &= a \cdot 3 \cos^2 t \cdot (-\sin t) \\ &= -3a \cos^2 t \cdot \sin t \end{aligned}$$

4 as $y = b \sin^3 t$

Diff. w.r.t. t

$$\begin{aligned} \frac{dy}{dt} &= b(3 \sin^2 t \cdot \cos t) \\ &= 3b \sin^2 t \cos t \end{aligned}$$

$$\text{Now } \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= \frac{3b \sin^2 t \cos t}{-3a \cos^2 t \sin t}$$

$$= -\frac{b}{a} \cdot \frac{\sin t}{\cos t} = -\frac{b}{a} \tan t$$

$$54. \quad x = \frac{3at}{1+t^2}, \quad y = \frac{3at^2}{1+t^2}$$

Sol. Diff. w.r.t. t

$$\begin{aligned} \frac{dx}{dt} &= 3a \left[\frac{(1+t^2) \cdot 1 - t \cdot (2t)}{(1+t^2)^2} \right] \\ &= 3a \left[\frac{1+t^2-2t^2}{(1+t^2)^2} \right] \end{aligned}$$

$$\frac{dx}{dt} = \frac{3a(1-t^2)}{(1+t^2)^2}$$

$$\begin{aligned} \text{Now } \frac{dy}{dt} &= 3a \left[\frac{(1+t^2) \cdot 2t - t^2(2t)}{(1+t^2)^2} \right] \\ &= 3a \left[\frac{2t+2t^3-2t^3}{(1+t^2)^2} \right] \\ &= \frac{3a(2t)}{(1+t^2)^2} \end{aligned}$$

$$\frac{dy}{dt} = \frac{6at}{(1+t^2)^2}$$

$$\begin{aligned} \text{Now } \frac{dy}{dx} &= \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{6at}{(1+t^2)^2} \cdot \frac{(1+t^2)^2}{3a(1-t^2)} \\ &= \frac{2t}{(1-t^2)} \quad \text{Ans.} \end{aligned}$$

$$55. \quad y = (\sec x^3 + \operatorname{arcsec} x)^2$$

$$\text{Sol. } y = (\sec x^3 + \sec^{-1} x)^2$$

$$\begin{aligned} \text{Diff. w.r.t. } x \\ \frac{dy}{dx} &= 2(\sec x^3 + \sec^{-1} x) \cdot \frac{d}{dx} (\sec x^3 + \sec^{-1} x) \\ &= 2(\sec x^3 + \sec^{-1} x) \cdot \left[\sec x^3 \cdot \tan x^3 \cdot 3x^2 + \frac{1}{x\sqrt{x^2-1}} \right] \\ &= 2(\sec x^3 + \sec^{-1} x) \left[3\sec x^3 \cdot \tan x^3 \cdot x^2 + \frac{1}{x\sqrt{x^2-1}} \right] \quad \text{Ans.} \end{aligned}$$

$$56. \quad y = \exp\left(\operatorname{arccsc}\left(\frac{1}{x}\right)\right)$$

$$\text{Sol. } y = e^{\operatorname{arccsc}(1/x)}$$

$$\begin{aligned} \text{Diff. w.r.t. } x \\ \frac{dy}{dx} &= e^{\operatorname{arccsc}(1/x)} \cdot \frac{-1}{\frac{1}{x}\sqrt{\left(\frac{1}{x}\right)^2-1}} \cdot \frac{-1}{x^2} \\ &= e^{\operatorname{arccsc}(1/x)} \cdot \frac{1}{x\sqrt{\frac{1}{x^2}-1}} = e^{\operatorname{arccsc}(1/x)} \cdot \frac{1}{x\sqrt{\frac{1-x^2}{x^2}}} \end{aligned}$$

$$\frac{dy}{dx} = \frac{\csc^{-1}(1/x)}{e} \cdot \frac{1}{x\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{\csc^{-1} \frac{1}{x}}{e} \cdot \frac{1}{\sqrt{1-x^2}} \quad \text{Ans.}$$

57. $y = \arcsin(\ln x) - \ln(\arctan x)$

Sol. $y = \sin^{-1}(\ln x) - \ln(\tan^{-1} x)$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sqrt{1-(\ln x)^2}} \cdot \frac{1}{x} - \frac{1}{\tan^{-1} x} \cdot \frac{1}{1+x^2} \\ &= \frac{1}{x\sqrt{1-(\ln x)^2}} - \frac{1}{(1+x^2)\tan^{-1} x} \end{aligned}$$

58. $y \arcsin x - x \arctan y = 1$

Sol. $y \sin^{-1} x - x \tan^{-1} y = 1$

$$y \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1} x \cdot \frac{dy}{dx} - x \cdot \frac{1}{1+y^2} \cdot \frac{dy}{dx} - \tan^{-1} y = 0$$

$$\frac{dy}{dx} \left(\sin^{-1} x - \frac{x}{1+y^2} \right) + \frac{y}{\sqrt{1-x^2}} - \tan^{-1} y = 0$$

$$\frac{dy}{dx} \cdot \left(\frac{(1+y^2)\sin^{-1} x - x}{(1+y^2)} \right) = -\frac{y}{\sqrt{1-x^2}} + \tan^{-1} y$$

$$\frac{dy}{dx} \cdot \left(\frac{(1+y^2)\sin^{-1} x - x}{1+y^2} \right) = \frac{-y + \sqrt{1-x^2} \tan^{-1} y}{1+y^2}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1+y^2}{(1+y^2)\sin^{-1} x - x} \cdot \frac{-y + \sqrt{1-x^2} \tan^{-1} y}{1+y^2} \\ &= \frac{(1+y^2)(\sqrt{1-x^2} \tan^{-1} y - y)}{\sqrt{1-x^2} [(1+y^2)\sin^{-1} x - x]} \quad \text{Ans.} \end{aligned}$$

59. $\arcsin(\ln xy) = x + y^2$

Sol. $\sin^{-1}(\ln xy) = x + y^2$

$$\frac{1}{\sqrt{1-(\ln xy)^2}} \cdot \frac{d}{dx}(\ln xy) = 1 + 2y \cdot \frac{dy}{dx}$$

$$\frac{1}{\sqrt{1-(\ln xy)^2}} \cdot \frac{1}{xy} \cdot \left[x \frac{dy}{dx} + y \cdot 1 \right] = 1 + 2y \cdot \frac{dy}{dx}$$

$$x \frac{dy}{dx} + y = xy \sqrt{1-(\ln xy)^2} \left[1 + 2y \cdot \frac{dy}{dx} \right]$$

$$x \frac{dy}{dx} + y = xy \sqrt{1-(\ln xy)^2} + 2xy^2 \sqrt{1-(\ln xy)^2} \cdot \frac{dy}{dx}$$

$$x \frac{dy}{dx} - 2xy^2 \sqrt{1-(\ln xy)^2} \cdot \frac{dy}{dx} = xy \sqrt{1-(\ln xy)^2} - y$$

$$x \frac{dy}{dx} (1 - 2y^2 \sqrt{1 - (\ln xy)^2}) = y (x \sqrt{1 - (\ln xy)^2} - 1) \quad (29)$$

$$\frac{dy}{dx} = \frac{y (x \sqrt{1 - (\ln xy)^2} - 1)}{x (1 - 2y^2 \sqrt{1 - (\ln xy)^2})} = \frac{y (x \sqrt{1 - \ln^2 xy} - 1)}{x (1 - 2y^2 \sqrt{1 - \ln^2 xy})}$$

$$60. \operatorname{arcsec}(x^2 + y) - e^x = \frac{1}{x + y}$$

$$\text{Sol. } \sec^{-1}(x^2 + y) - e^x = \frac{1}{x + y}$$

Diff. w.r.t. x

$$\frac{1}{(x^2 + y) \sqrt{(x^2 + y)^2 - 1}} \cdot \frac{d}{dx}(x^2 + y) - e^x = \frac{d}{dx}(x + y)^{-1}$$

$$\frac{1}{(x^2 + y) \sqrt{(x^2 + y)^2 - 1}} \cdot (2x + \frac{dy}{dx}) - e^x = -(x + y)^{-2} \cdot (1 + \frac{dy}{dx})$$

$$\frac{2x + \frac{dy}{dx}}{(x^2 + y) \sqrt{(x^2 + y)^2 - 1}} - e^x = \frac{-1 - \frac{dy}{dx}}{(x + y)^2}$$

$$\frac{2x + \frac{dy}{dx} - e^x (x^2 + y) \sqrt{(x^2 + y)^2 - 1}}{(x^2 + y) \sqrt{(x^2 + y)^2 - 1}} = \frac{-1 - \frac{dy}{dx}}{(x + y)^2}$$

$$(x + y)^2 (2x + \frac{dy}{dx}) - (x + y)^2 \cdot e^x (x^2 + y) \sqrt{(x^2 + y)^2 - 1} = (-1 - \frac{dy}{dx}) (x^2 + y) \sqrt{(x^2 + y)^2 - 1}$$

$$2x(x + y)^2 + (x + y)^2 \cdot \frac{dy}{dx} - (x + y)^2 \cdot e^x (x^2 + y) \sqrt{(x^2 + y)^2 - 1} = -(x^2 + y) \sqrt{(x^2 + y)^2 - 1} - \frac{dy}{dx} (x^2 + y) \sqrt{(x^2 + y)^2 - 1}$$

$$(x + y)^2 \cdot \frac{dy}{dx} + \frac{dy}{dx} (x^2 + y) \sqrt{(x^2 + y)^2 - 1} = (x + y)^2 \cdot e^x (x^2 + y) \sqrt{(x^2 + y)^2 - 1} - (x^2 + y) \sqrt{(x^2 + y)^2 - 1} - 2x(x + y)^2$$

$$\frac{dy}{dx} ((x + y)^2 + (x^2 + y) \sqrt{(x^2 + y)^2 - 1}) = e^x (x + y)^2 (x^2 + y) \sqrt{(x^2 + y)^2 - 1} - (x^2 + y) \sqrt{(x^2 + y)^2 - 1} - 2x(x + y)^2$$

$$\frac{dy}{dx} = \frac{e^x (x + y)^2 (x^2 + y) \sqrt{(x^2 + y)^2 - 1} - (x^2 + y) \sqrt{(x^2 + y)^2 - 1} - 2x(x + y)^2}{(x + y)^2 + (x^2 + y) \sqrt{(x^2 + y)^2 - 1}} \quad \text{Ans}$$