THE BERNOULLI EQUATION

(9.21) Definition. An equation of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

is called the Bernoulli differential equation. This equation is linear if n = 0 or linear equation. Dividing by y^n , (1) become not zero or 1, then (1) is reducible to a linear equation. Dividing by y_1^n , (1) becomes

$$y^{-n}\frac{dy}{dx}+P(x)y^{1-n} = Q(x)$$

In (2), put $v = y^{1-n}$ then it reduces to the reduces to the reduces to the reduces of the reduces to the r

$$\frac{dv}{dx} + (1-n)P(x)v = (1-n)Q(x)$$

which is linear in v 398 gaigles)

Note. Consider the equation was as we not so the grant grant

$$f'(y)\frac{dy}{dx} + P(x)f(y) = Q(x) \theta \text{ not } \theta \text{ see } \tau + \frac{\tau b}{\theta + b} \theta \text{ see}.$$

Letting v = f(y), this equation becomes

$$\frac{dv}{dx} + P(x)v = Q(x)^2 - [0 \cos x] \frac{h}{\theta h}$$

which is linear in v.

Example 26. Solve:
$$\frac{dy}{dx} + \frac{xy}{1-x^2} = xy^{\frac{1}{2}}$$
.

Solution. Dividing by $y^{\frac{1}{2}}$, (1) becomes

$$y^{-\frac{1}{2}}\frac{dy}{dx} + \frac{x}{1-x^2}y^{\frac{1}{2}} = x.$$

Put

$$y^{\frac{1}{2}} = v$$

or

$$\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = \frac{dv}{dx}.$$

Then (2) reduces to

$$\frac{dv}{dx} + \frac{x}{2(1-x^2)}v = \frac{x}{2}$$

integrating, we obtain

his is linear in
$$\nu$$
.
I.F. = $\exp\left[\int \frac{x}{2(1-x^2)} dx\right] = \exp\left[\frac{-1}{4} \ln(1-x^2)\right] = (1-x^2)^{-\frac{1}{4}}$.

(Itiplying (3) by
$$(1 - x^2)^{-\frac{1}{4}}$$
, we get

I.F. =
$$\exp\left[\int \frac{1}{2(1-x^2)^{4x}}\right]$$
 supplying (3) by $(1-x^2)^{-\frac{1}{4}}$, we get

(1-x²) $-\frac{1}{4}\frac{dv}{dx} + \frac{x}{2(1-x^2)^{5/4}}v = \frac{x}{2(1-x^2)^{1/4}}$

$$\frac{(1-x^2)^{-\frac{1}{4}}\frac{dv}{dx} + \frac{1}{2(1-x^2)^{5/4}} \cdot 2(1-x^3)^{1/4}}{\frac{d}{dx}\left[(1-x^2)^{-\frac{1}{4}}v\right]} = \frac{-1}{4}\left[-2x(1-x^2)^{-\frac{1}{4}}\right]$$

Integrating, we have

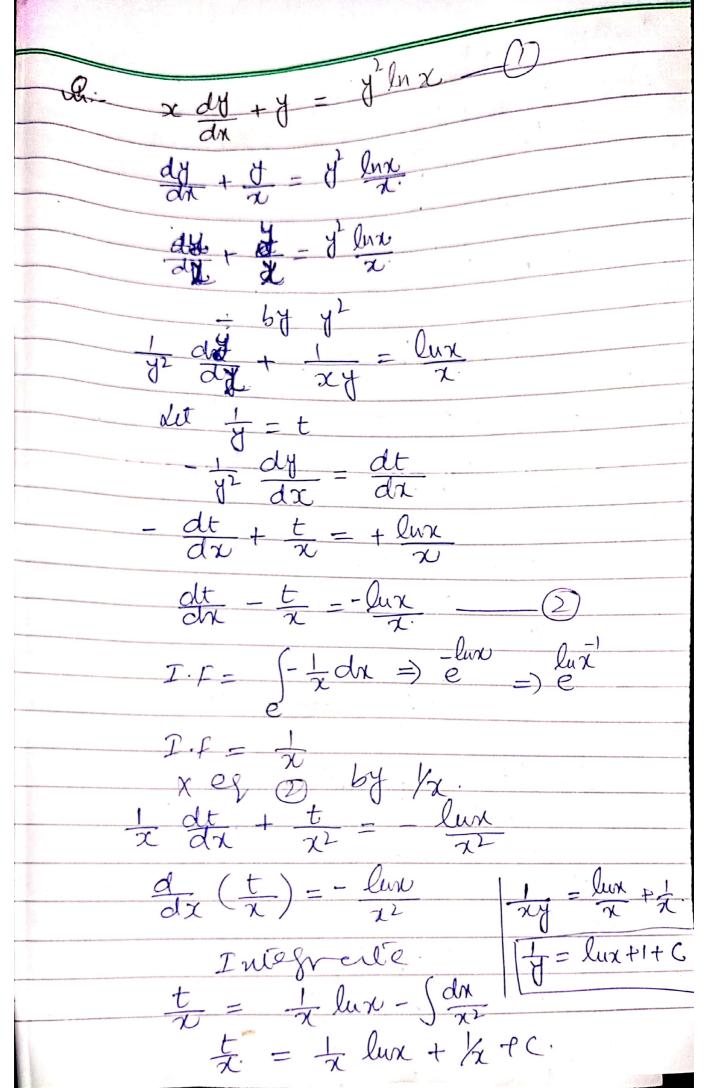
rating, we have
$$v(1-x^2)^{-\frac{1}{4}} = \frac{-1}{4} \frac{(1-x^2)^{3/4}}{3/4} + c$$

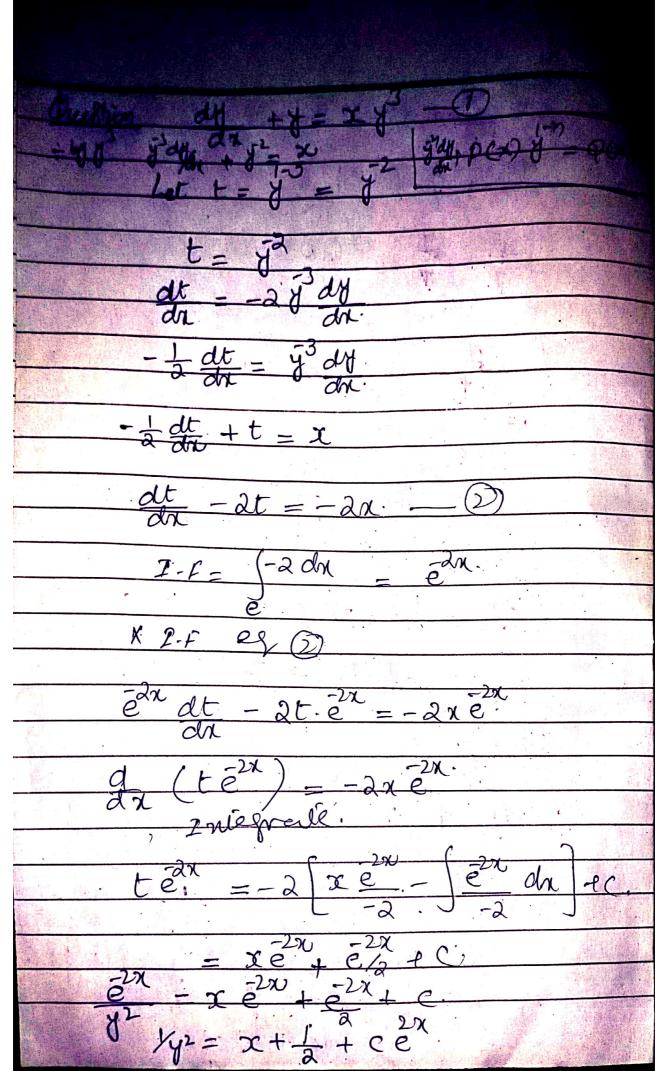
$$v = c(1-x)^{-\frac{1}{4}} \frac{1-x^2}{3}$$

$$\frac{1}{v^2} = c(1-x^2)^{\frac{1}{4}} = \frac{1-x^2}{3}$$

equired solution of (1).

CISE 9.6





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Question dy +y = xy3 -
$$\sqrt{3}$$
 $\frac{1}{3}$
 $\frac{1}{3}$

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