$x(D-\sqrt{5}) = 0$, $D = 0, D = \sqrt{5}$

Differential Equation - I

Lect # 1

Types of variables:-

There are two types of variable,

i) Dependent variable

ii) Independent variable.

Differential equation:-

An equation involving derivative of dependent variable w.r.t. one or more independent variable is called defferential equation

e.g

$$\frac{dy}{dx} + y\sin x = 0$$

Where y is a dependent variable and x is independent variable.

Types of differential equation:-

i). ordinary diff eq:-

The differential equation involving derivative of dependent variable w.r.t single independent variable is called ordinary differential equation for e.g

$$\frac{dy}{dx} + y\sin x = 0$$

Scanned with CamScanner

ii) Partial differential equation:-

The equation involving derivative of dependent variable w.r.t more than one independent variable is called Paritial differential equation e.g

$$\frac{\partial y}{\partial x} + \frac{\partial y}{\partial z} = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

order of diff-eq :-

The order of a diff-eq is the order of the heighest derivative term in that equation e.g

$$\frac{d^2y}{dx^2} + 2x\left(\frac{dy}{dx}\right)^3 = 0$$

above eq has order '2'.

Degree of diff-eq

The degree of a diff-eq is degree of highest order derivative term e.g

$$\frac{d^2y}{dx^2} + 2x\left(\frac{dy}{dx}\right)^2 = 0$$

The above equation has degree 1

Ordinary linear diff - eq.

An ordinary eq of the form

$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots \frac{d^ny}{dx^n}\right)$ is said to be linear if F is a linear function of $\left(x, \frac{dy}{dx}, \dots\right)$

Condition for linear diff - eq :-

i) A diff - eq is said to be linear if the degree of dependent variable or degree of all of it's derivation is one otherwise it's non-linear

$y^2 \frac{dy}{dx} + 5x = 0$ → [Non-Linear as y was degree 2]

$x^2 \frac{dy}{dx^2} + 5x = 0$ linear.

ii) There should not be product of dependent variable with it's own derivative or with itself.

$y^2 \frac{dy}{dx} + 5x = 0$ Non - Linear

$y \frac{d^2y}{dx^2} + \sin x = 0$ Non - Linear

$x \frac{dy}{dx} + 5x = 0$ Linear.

iii) There should not any trignometric function of dependent variable (y)

$$\frac{d^2y}{dx^2} + \cot y = \text{Non-Linear}$$

$$\frac{d^2y}{dx^2} + \tan x = 0 \quad \text{Linear.}$$

Scanned with CamScanner

Equation of Ist order and Ist degree

Separable equation

Question 1

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

S.V

$$y\,dy = \frac{x^2}{1+x^3}\,dx$$

$$\int y\,dy = \int \frac{x^2}{1+x^3}\,dx.$$

$$\frac{y^2}{2} = \frac{1}{3}\int \frac{3x^2}{1+x^3}\,dx.$$

$$\frac{y^2}{2} = \frac{1}{3}\ln(1+x^3) + c.$$

Question 2:

$$\frac{dy}{dx} + y^2 \sin x = 0$$

Sol:-

$$\frac{dy}{dx} = -y^2 \sin x.$$

S.V

$$\frac{dy}{y^2} = -\sin x\,dx.$$

$$\int y^{-2}\,dy = -\int \sin x\,dx.$$

$$-\frac{1}{y} = \cos x + c.$$

Scanned with CamScanner

Question[3]:- $\frac{dy}{dx} = 1+x+y^2+xy^2$

$$\frac{dy}{dx} = (1+x) + y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

S.V

$$\frac{dy}{1+y^2} = (1+x)\,dx$$

$$\int \frac{dy}{1+y^2} = \int (1+x)\,dx$$

$$\tan^{-1} y = x + \frac{x^2}{2} + c$$

Question[4]:- $x\sin y\,dx + (x^2+1)\cos y\,dy = 0$

Sol:- $\div$ by $\sin y\,(x^2+1)$

$$\frac{x\cancel{\sin y}}{\cancel{\sin y}(x^2+1)}\,dx + \frac{\cancel{(x^2+1)}}{\sin y\cancel{(x^2+1)}}\cos y\,dy = 0$$

$$\frac{x}{x^2+1}\,dx + \frac{\cos y}{\sin y}\,dy = 0$$

$$\frac{1}{2}\int \frac{2x}{x^2+1}\,dx + \int \frac{\cos y}{\sin y}\,dy = 0$$

$$\frac{1}{2}\ln(x^2+1) + \ln \sin y = c$$

Sol:- $\cancel{x \sin y\, dx + (x^2+1)\cos y\, dy = 0}$

$\cancel{\div \text{ by } y}$

Question 5:- $(xy+2x+y+2)dx + (x^2+2x)dy = 0$

Sol:- $(xy+2x+y+2)dx + (x^2+2x)dy = 0$

$$[x(y+2)+1(y+2)]dx + (x^2+2x)dy = 0$$

$$(x+1)(y+2)\,dx + (x^2+2x)\,dy = 0$$

$$(x^2+2x)dy = -(x+1)(y+2)dx$$

S.V

$$\frac{dy}{y+2} = -\left(\frac{x+1}{x^2+2x}\right)dx.$$

$$\int \frac{1}{y+2}\,dy = -\frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx.$$

$$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x)+C$$

Question 6:-

$$\frac{dy}{dx} = 2x^2+y-x^2y+xy-2x-2.$$

$$= 2x^2-2x-2-x^2y+xy+y$$

$$= 2(x^2-x-1)-y(x^2-x-1)$$

$$= (2-y)(x^2-x-1)$$

S.V

Scanned with CamScanner

$$\frac{dy}{2-y} = (x^2 - x - 1)\,dx.$$

$$-\int \frac{-dy}{2-y} = \int (x^2 - x - 1)\,dx.$$

$$-\ln(2-y) = \frac{x^3}{3} - \frac{x^2}{2} - x + c.$$

Question 7:- $\operatorname{cosec} y\,dx + \sec x\,dy = 0$

Sol:- $\frac{1}{\sin y}\,dx + \frac{1}{\cos x}\,dy = 0$

$$\frac{1}{\sin y}\,dx = -\frac{1}{\cos x}\,dy$$

$$\cos x\,dx = -\sin y\,dy$$

$$\int \cos x\,dx = -\int \sin y\,dy$$

$$\sin x = -(-\cos y) + c$$

$$\sin x = \cos y + c.$$

Question 8:-

$$y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$$

Sol:-

$$y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$$

$\div$ by xy

$$\frac{\sqrt{1+x^2}}{x}\,dx + \frac{\sqrt{1+y^2}}{y}\,dy = 0$$

$$\int \frac{\sqrt{1+x^2}}{x}\,dx = $$

Let $\sqrt{1+x^2} = t$

$1+x^2 = t^2 \Rightarrow x^2 = t^2-1$

$\cancel{2}x\,dx = \cancel{2}t\,dt \Rightarrow dx = \frac{t}{x}\,dt$

$$\int \frac{\sqrt{1+x^2}}{x}\,dx = \int \frac{t}{x} \cdot \frac{t}{x}\,dt$$

$$= \int \frac{t^2}{x^2}\,dt$$

$$= \int \frac{t^2}{t^2-1}\,dt$$

$$= \int \left(1 + \frac{1}{t^2-1}\right) dt$$

$$= \int dt + \int \frac{1}{t^2-1}\,dt$$

$$= t + \frac{1}{2} \ln \frac{t-1}{t+1}$$

$$= \sqrt{1+x^2} + \frac{1}{2} \ln \frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}$$

$$= \sqrt{1+x^2} + \frac{1}{2} \ln \frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1} \times \frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}-1}$$

$$= \sqrt{1+x^2} + \frac{1}{2} \ln \frac{\left(\sqrt{1+x^2}-1\right)^2}{\cancel{1}+x^2-\cancel{1}}$$

$$= \sqrt{1+x^2} + \frac{1}{2}\ln\frac{\left(\sqrt{1+x^2}-1\right)^2}{x^2}$$

$$= \sqrt{1+x^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{|x|}\right)^2$$

Similarly

$$\int\frac{\sqrt{1+y^2}}{y}\,dy = \sqrt{1+y^2} + \frac{1}{2}\ln\frac{\sqrt{1+y^2}-1}{|y|}$$

G. Sol is

$$\sqrt{1+x^2} + \sqrt{1+y^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{|x|}\right)^2 + \frac{1}{2}\ln\frac{\sqrt{1+y^2}-1}{|y|}$$

Question 9.

$$\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\frac{dy}{\sqrt{1-y^2}} = -\frac{dx}{\sqrt{1-x^2}}$$

$$\int\frac{dy}{\sqrt{1-y^2}} = -\int\frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1}y = -\sin^{-1}x + c$$

$$c = \sin^{-1}x + \sin^{-1}y$$

Question 10: $(e^x+1)\,y\,dy = (y+1)e^x\,dx.$

Sol

$$\frac{y\,dy}{y+1} = \frac{e^x}{e^x+1}\,dx.$$

$$\int\left(1-\frac{1}{y+1}\right)dy = \int\frac{e^x}{e^x+1}\,dx$$

$$y - \ln(y+1) = \ln(e^x+1) + \ln c$$

$$y = \ln(y+1) + \ln(e^x+1) + \ln c$$

$$e^y = (y+1)(e^x+1)\,c.$$

$$ce^y = (y+1)(e^x+1)$$

Question 11: $\dfrac{dy}{dx} = \dfrac{y^3+2y}{x^2+3x}.$

Sol:

$$\frac{dy}{y^3+2y} = \frac{dx}{x^2+3x}.$$

$$\int\frac{dy}{y(y^2+2)} = \int\frac{dx}{x(x+3)}$$

Let

$$\frac{1}{y(y^2+2)} = \frac{A}{y} + \frac{By+C}{y^2+2}.$$

$$1 = A(y^2+2) + (By+C)y$$

Put $y=0$

$1 = 2A \Rightarrow A = 1/2$

$1 = A(y^2+2) + B(y^2) + Cy$.

comparing the co-eff of y^2

$0 = A + B \Rightarrow 0 = \frac{1}{2} + B$

$B = -1/2$

Comparing the co-eff of y

$0 = C$

$$\int \frac{1}{y(y^2+2)} dy = \int \frac{1}{2} \frac{dy}{y} - \frac{1}{2} \int \frac{y}{y^2+2} dy$$

$$= \frac{1}{2} \ln y - \frac{1}{2 \cdot 2} \int \frac{2y}{y^2+2} dy$$

$$= \frac{1}{2} \ln y - \frac{1}{4} \ln(y^2+2)$$

$$\int \frac{dx}{x(x+3)} = \int$$

Let $\frac{1}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3}$.

$1 = A(x+3) + Bx$

put $x = 0, -3$

$1 = 3A \Rightarrow A = 1/3$

$1 = -3B \Rightarrow B = -1/3$

$$\int \frac{1}{x(x+3)} dx = \frac{1}{3} \int \frac{dx}{x} - \frac{1}{3} \int \frac{dx}{x+3}$$

$$= \frac{1}{3} \ln x - \frac{1}{3} \ln(x+3)$$