(Formulas)

... Derivatives ...

(i) $\frac{d}{dx}(\sin x) = \cos x$

(ii) $\frac{d}{dx}(\cos x) = -\sin x$

(iii) $\frac{d}{dx}(\tan x) = \sec^2 x$

(iv) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(v) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(vi) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x.\cot x$

$y = \sin 2x$
Diff with.r.t x
$y = \cos 2x . 2$

$y = 2\cos 2x$

(7) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

(8) $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

(9) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

(10) $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

(11) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

(12) $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$

(13) $\ln(e) = 1$

(14) $\ln(1) = 0$

(15) $\ln(0) = 1$

(16) $\int dx$, $\int 2dy$
$= x + c$ Ans $= 2y + c$
Constant

(Intigration)

(17) $\int \sin x dx = -\cos x + c$

(18) $\int \cos x dx = \sin x + c$

(19) $\int \sec^2 x dx = \tan x + c$

(20) $\int \sec x \tan x dx = \sec x + c$

(21) $\int \operatorname{cosec}^2 x dx = -\cot x + c$

(22) $\int \operatorname{cosec} x.\cot x dx = -\operatorname{cosec} x + c$

(23) $y = (x+1)^x$ variable, Power is variable ہے

$\ln y = \ln (x+1)^x$ Then

$\ln y = x \ln (x+1)$

(24) $\int \frac{1}{t^2-1}\,dt = \frac{1}{2} \ln \frac{t-1}{t+1}$ → formula

(25) $y = \ln(x+1)$ | ~~tan~~ $\sec^2 x = 1 + \tan^2 x$

$e^y = x+1$ | $\operatorname{cosec}^2 x = 1 + \cot^2 x$

(26) جب بھی integration میں $\sin^2 x$, $\cos^2 x$ یا $\tan^2 x$ آئے تو فورا ان کی
جگہ یہ فارمولا لکھیں گے

$\sin^2 x = \frac{1-\cos 2x}{2}$, $\cos^2 x = \frac{1+\cos 2x}{2}$, $\tan^2 x = \sec^2 x - 1$

(27) $\sin 2x = 2 \sin x . \cos x$

(28)

Intigration by Parts.

Intigration of the Product of two function.

Formula

⇓

1st function × Integral of 2nd function $-\int$ (derivative of 1st function)(Integral of

(i) '1' is always taken as 2nd function.

(ii) e^x, e^{2x}, e^{3x} etc are always taken as 2nd function.

(iii) $\sec^2 x$, $\operatorname{cosec}^2 x$, $\sec x \tan x$, $\operatorname{cosec} x \cot x$ are taken as 2nd function

(iv) Term of 'ln' is always taken as 1st function.

(v) Inverse trignometric function ($\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ etc) are always taken as 1st function.

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(vi) If one function is x, x^2, x^3, x^4, etc and other function is trigonometric function Then trignometric function is taken as 2nd function.

(29) $= e^{\ln x}$

$= x$

(30) **The following differential formulas are useful in the calculation of Certain exact equations.**

(i) $d\left(\frac{y}{x}\right) = \frac{x\,dy - y\,dx}{x^2}$ (ii) $d\left(\frac{x}{y}\right) = \frac{y\,dx - x\,dy}{y^2}$

(iii) $d(xy) = x\,dy + y\,dx$

(iv) $d(x^2+y^2) = 2x\,dx + 2y\,dy$

$= 2(x\,dx + y\,dy)$ or

(v) $d\left(\ln \frac{x}{y}\right) = \frac{1}{\frac{x}{y}} \cdot \frac{y\,dx - x\,dy}{y^2}$

$d\left(\ln \frac{x}{y}\right) = \frac{y\,dx - x\,dy}{xy}$

Note

$\arctan x = \tan^{-1} x$

2nd function) dx

(vi) $d\left(\arctan \frac{x}{y}\right) = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{y\,dx - x\,dy}{y^2}$

$= \frac{1}{\frac{y^2+x^2}{y^2}} \cdot \frac{y\,dx - x\,dy}{y^2}$

$= \frac{\cancel{y^2}}{y^2+x^2} \cdot \frac{y\,dx - x\,dy}{\cancel{y^2}}$

$d\left(\arctan \frac{x}{y}\right) = \frac{y\,dx - x\,dy}{y^2+x^2}$

Formulas.

(i) $\sin\alpha - \sin\beta = 2 . \sin\dfrac{\alpha-\beta}{2} . \cos\dfrac{\alpha+\beta}{2}$

(ii) $\cos\alpha - \cos\beta = -2 . \sin\dfrac{\alpha+\beta}{2} . \sin\dfrac{\alpha+\beta}{2}$

"Hyperbolic Function"

called

(i) $\sinh x = \dfrac{e^{x} - e^{-x}}{2}$, $\operatorname{cosech} x = \dfrac{2}{e^{x} - e^{-x}}$

(ii) $\cosh x = \dfrac{e^{x} + e^{-x}}{2}$, $\operatorname{sech} x = \dfrac{2}{e^{x} + e^{-x}}$

(iii) $\tanh x = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$, $\coth x = \dfrac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$

(iv) $\sinh 2x = \dfrac{e^{2x} - e^{-2x}}{2}$

(v) $\cosh 2x = \dfrac{e^{2x} + e^{-2x}}{2}$

(vi) $\tanh 2x = \dfrac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$

Note:

$$\cosh^2 x - \sinh^2 x = 1$$

Note

i) change in x is denoted by δx.

ii) " " y " " " δy.

iii) " " Z " " " δZ.

iv) " " t " " " δt.

v) " " u " " " δu.

Note

i) $\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx}$

ii) $\lim_{\delta y \to 0} \frac{\delta x}{\delta y} = \frac{dx}{dy}$

iii) $\lim_{\delta t \to 0} \frac{\delta x}{\delta t} = \frac{dx}{dt}$

iv) $\lim_{\delta x \to 0} \frac{\delta \theta}{\delta x} = \frac{d\theta}{dx}$

Note

1) Derivative of y w.r.t $x = \frac{dy}{dx}$

2) " " x " $t = \frac{dx}{dt}$

3) " " x " $\theta = \frac{dx}{d\theta}$

4) " " x " $u = \frac{dx}{du}$

5) " " x " $x = \frac{dx}{dx} = 1$

6) " " y " $y = \frac{dy}{dy} = 1$

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