3600
2500
1100

Ex 10.1

Diff eq of Higher order.

Question:-

$(9D^2 - 12D + 4)y = 0$

Sol:-

The characteristic eq is.

$9D^2 - 12D + 4 = 0$

$9D^2 - 6D - 6D + 4 = 0$

$3D(3D-2) - 2(3D-2) = 0$

$(3D-2)(3D-2) = 0$

$3D - 2 = 0 \Rightarrow 3D = 2 \Rightarrow D = 2/3, 2/3.$

The G.S is

$y = (c_1 + c_2 x)e^{2/3 x}$

Question.

$(75D^2 + 50D + 12)y = 0$

Sol:- The C.E is.

$75D^2 + 50D + 12 = 0$

$a = 75, \quad b = 50, \quad c = 12.$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$D = \frac{-50 \pm \sqrt{2500 - 3600}}{2(75)}$$

$$= \frac{-50 \pm \sqrt{1100}}{150}$$

$$= \frac{-50 \pm \sqrt{100 \times 11}}{150}$$

$$= \frac{-50 \pm 10\sqrt{11}}{150}$$

$$= \frac{10(-5 \pm \sqrt{11})}{15(10)} \Rightarrow \frac{-5 \pm \sqrt{11}}{15}$$

G.S is.

$$y = e^{x/3}\left(c_1 \cos\frac{\sqrt{11}}{15}x + c_2 \sin\frac{\sqrt{11}}{15}x\right)$$

Question..

$$(D^3 - 4D^2 + D + 6)y = 0$$

Sol:- The C.E is

$$D^3 - 4D^2 + D + 6 = 0$$

	1	-4	1	6
-1		-1	5	-6
	1	-5	6	0

$$D^2 - 5D + 6 = 0$$

$$D^2 - 3D - 2D + 6 = 0$$

$$D(D-3) - 2(D-3) = 0$$

$(D-2)(D-3)=0$

$D=2,3.$

$D=-1,2,3.$

G.S

$$y = c_1e^{-x} + c_2e^{2x} + c_3e^{3x}.$$

Question:

$$(D^3+D^2+D+1)y=0$$

Sol:-

$$D^3+D^2+D+1=0$$

	1	1	1	1
-1		-1	0	-1
	1	0	1	0

$D=-1$

$D^2+1=0 \Rightarrow D^2=-1$

$D=\pm i$

$D=-1, \pm i$

$$y = c_1e^{-x} + c_2\sin x + c_3\sin x.$$

$D^4+4=0$

	1	0	0	4
2		2	4	
	1	2	4	-2

	1	0	0	4
-2		-2	+4	-8
	1	-2	4	-4

Question. $(D^3-6D^2+12D-8)y=0$

Sol:-

The C.E is.

$D^3-6D^2+12D-8=0$

	1	-6	12	-8
2		2	-8	8
	1	-4	4	0

$D^2-4D+4=0$

$D^2-2D-2D+4=0$

$D(D-2)-2(D-2)=0$

$(D-2)(D-2)=0$

$D=2,2,$

$D=2,2,2.$

C.S

$$y=(c_1+c_2x+c_3x^2)e^{2x}.$$

	1	0	0	4
-1		-1	1	-1
	1	-1	1	4

$D^4+4=0$

$D^4=-4$

$D^4=4i^2$

$D^2=\pm 2i$

$D=2i$ ~~$D=-2i$~~

$D=\sqrt{2i}$

$D^4+4=0$

$D^4=4$

$D^2=\pm 2$

$D=2$ | $D^2=2i$

$D=\sqrt{2i}$ | $D=\pm\sqrt{2i}$

Question. $(D^3-6D^2+3D+10)y=0$

Sol.- $D^3-6D^2+3D+10=0$

	1	-6	3	10
-1		-1	+7	-10
	1	-7	10	0

$D^2 - 7D + 10 = 0$

$D^2 - 5D - 2D + 10 = 0$

$D(D-5) - 2(D-5) = 0$

$(D-2)(D-5) = 0$

$D = 2, 5$

$D = -1, 2, 5$

G.S

$y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{5x}$.

Question:- $(D^3 - 27)y = 0$

Sol:- $D^3 - 27 = 0$

$(D)^3 - (3)^3 = 0$

$(D-3)(D^2 + 3D + 9) = 0$

$D - 3 = 0 \Rightarrow D = 3$.

$D^2 + 3D + 9 = 0$

$a = 1, \quad b = 3, \quad c = 9$.

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{9 - 36}}{2}$$

$$= \frac{-3 \pm \sqrt{-27}}{2}$$

$\sqrt{9x-3} =$

$= \frac{-3 \pm 3\sqrt{3}i}{2}$

$= \frac{+3(-1 \pm \sqrt{3}i)}{2} \Rightarrow \frac{-3 \pm 3\sqrt{3}i}{2}$

$y = c_1 e^{3x} + e^{-3x/2}\left(c_2 \cos\left(\frac{3\sqrt{3}}{2}\right)x + c_3 \sin\left(\frac{3\sqrt{3}}{2}\right)x\right)$

Question:

$(4D^4 - 4D^3 - 3D^2 + 4D - 1)y = 0$

Sol:

The C.E is.

$4D^4 - 4D^3 - 3D^2 + 4D - 1 = 0$

	4	-4	-3	4	-1
-1		-4	8	-5	1
	4	-8	5	-1	0
1		4	-4	1	
	4	-4	1	0	

$4D^2 - 4D + 1 = 0$

$4D^2 - 2D - 2D + 1 = 0$

$2D(2D-1) - 1(2D-1) = 0$

$(2D-1)(2D-1) = 0$

$D = 1/2, 1/2.$

$D = 1, -1, 1/2, 1/2.$

G.S is

$$y = c_1 e^{x} + c_2 e^{-x} + (c_3 + c_4 x) e^{x/2}.$$

Question:-

$$(D^4 + 2D^3 - 2D^2 - 6D + 5)y = 0.$$

Sol:-

The C.E is.

$$D^4 + 2D^3 - 2D^2 - 6D + 5 = 0$$

	1	2	-2	-6	5
1		1	3	1	-5
1	1	3	1	-5	0
		1	4	5	
	1	4	5	0	

$$D^2 + 4D + 5 = 0$$

$a = 1, b = 4, c = 5$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$D = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$$

$$D = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$D = \frac{-4 \pm \sqrt{-4}}{2}$$

$$D = \frac{-4 \pm 2i}{2}$$

$$D = -2 \pm i$$

$$D = 1, 1, -2 \pm i$$

G.S is

$$y = (C_1 + C_2 x)e^{x} + (C_3 \cos x + C_4 \sin x)e^{-2x}$$

Question:-

$$(D^4 - 5D^3 + 6D^2 + 4D - 8)y = 0$$

Sol:-

The C.E is.

$$D^4 - 5D^3 + 6D^2 + 4D - 8 = 0$$

	1	-5	6	4	-8
-1		-1	6	-12	8
	1	-6	12	-8	0
2		2	-8	8	
	1	-4	4	0	

$$D^2 - 4D + 4 = 0$$

$D^2 - 2D - 2D + 4 = 0$

$D(D-2) - 2(D-2) = 0$

$(D-2)(D-2) = 0$

$D = 2, 2$

$D = -1, 2, 2, 2$.

G.S is.

$y = c_1 e^{-x} + (c_2 + c_3 x + c_4 x^2) e^{2x}$.

Question.. $(D^4 - 4D^3 - 7D^2 + 22D + 24) y = 0$

Sol. The C.E is.

$D^4 - 4D^3 - 7D^2 + 22D + 24 = 0$

	1	-4	-7	22	24.
-1		-1	5	2	-24
	1	-5	-2	24	0
-2		-2	14	-24	
	1	-7	12	0	

$D^2 - 7D + 12 = 0$

$D^2 - 4D - 3D + 12 = 0$

$D(D-4) - 3(D-4) = 0$

$(D-3)(D-4) = 0$

$D = 3, 4$.

$D = -1, -2, 3, 4.$

G.S is

$$y = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{3x} + c_4 e^{4x}.$$

Question :- $(D^4 - D^3 - 3D^2 + D + 2)y = 0$

$D = 1, 2, -1, -1$

Question

$$(16D^6 + 8D^4 + D^2)y = 0.$$

$D = 0, 0, \pm\frac{1}{2}i, \pm\frac{1}{2}i$

G.S $= (c_1 + c_2 x) + (c_3 + c_4 x)\cos x/2 + (c_5 + c_6 x)$ $\sin x/2$

Question

$$(D^4 + 6D^3 + 15D^2 + 20D + 12)y = 0$$

$D = -2, -2, -1 \pm \sqrt{2}i$

G.S $y = (c_1 + c_2 x)e^{-2x} + (c_3 \cos\sqrt{2}x + c_4 \sin\sqrt{2}x)e^{-x}$
