Question:

$$(D^2+3D-4)y = 15e^x$$

Sol:

The C.F is

$$D^2+3D-4=0$$

$$D^2+4D-D-4=0$$

$$D(D+4)-1(D+4)=0$$

$$(D-1)(D+4)=0$$

$$D=1, -4$$

$$y_c = c_1 e^x + c_2 e^{-4x}$$

$$y_p = \frac{15e^x}{D^2+3D-4}$$

$$= \frac{15e^x}{(D+4)(D-1)}$$

$$= \frac{{}^{3}\cancel{15}e^x}{\cancel{5}(D-1)}$$

$$= \frac{3e^x}{D-1}$$

$$y_p = 3xe^x$$

G.S is

$$y = y_c + y_p$$

$$= c_1 e^x + c_2 e^{-4x} + 3xe^x$$

Question: $(D^2 - 3D + 2)y = e^x + e^{2x}$.

Sol: The C.E is.

$D^2 - 3D + 2 = 0$

$D^2 - 2D - D + 2 = 0$

$D(D-2) - 1(D-2) = 0$

$(D-1)(D-2) = 0$

$D = 1, 2$.

$y_c = c_1 e^x + c_2 e^{2x}$.

$$y_p = \frac{e^x + e^{2x}}{(D-1)(D-2)}$$

$$= \frac{e^x}{(D-1)(D-2)} + \frac{e^{2x}}{(D-1)(D-2)}$$

$$= \frac{e^x}{(D-1)(1-2)} + \frac{e^{2x}}{(2-1)(D-2)}$$

$$= \frac{-e^x}{D-1} + \frac{e^{2x}}{D-2}$$

$$= -xe^x + xe^{2x}$$

G.S is

$Y = y_c + y_p$

$= c_1 e^x + c_2 e^{2x} - xe^x + xe^{2x}$.

0315-0711681

Question.. $(D^2-2D-3)y = 2e^x - 10\sin x$.

Sol.. The C.E is.

$$D^2-2D-3=0$$

$$D^2-3D+D-3=0$$

$$D(D-3)+1(D-3)=0$$

$$(D+1)(D-3)=0$$

$$D=-1,\ 3.$$

$$y_c = c_1 e^{-x} + c_2 e^{3x}.$$

$$y_p = \frac{2e^x - 10\sin x}{(D-3)(D+1)}.$$

$$= \frac{2e^x}{(D-3)(D+1)} - \frac{10\sin x}{(D-3)(D+1)}.$$

$$= \frac{2e^x}{(1-3)(1+1)} - \frac{10\sin x}{D^2-2D-3}.$$

$$= \frac{\cancel{2}e^x}{(-2)(\cancel{2})} - \frac{10\sin x}{-1-2D-3}.$$

$$= -\frac{e^x}{2} - \frac{10\sin x}{-4-2D}.$$

$$= -\frac{e^x}{2} + \frac{5\sin x}{2+D}$$

$$= -\frac{e^x}{2} + \frac{5(D-2)}{(D+2)(D-2)}\sin x.$$

$$= -\frac{e^x}{2} + \frac{5(D-2)}{D^2-4}\sin x.$$

$$= -\frac{e^x}{2} + \frac{5(D-2)}{-1-4} \sin x.$$

$$= -\frac{e^x}{2} - (D-2)\sin x.$$

$$= -\frac{e^x}{2} - \cos x + 2\sin x.$$

G.S is.

$$y = y_c + y_p.$$

$$= c_1 e^{-x} + c_2 e^{3x} - \frac{e^x}{2} - \cos x + 2\sin x.$$

Question. $(D^4 - 2D^3 + D)y = x^4 + 3x + 1$

Sol..

the C.F is

$$D^4 - 2D^3 + D = 0$$

$$D(D^3 - 2D^2 + 1) = 0$$

$$D = 0$$

1	1	-2	0	1
		1	-1	-1
	1	-1	-1	0

$D = 1$

$$D^2 - D - 1 = 0$$

$$a = 1,\ b = -1,\ c = -1$$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{1 \pm \sqrt{1+4}}{2}$$

$4x^3+3$
$12x^2+0$

$$D = \frac{1 \pm \sqrt{5}}{2}$$

$$D = \frac{1+\sqrt{5}}{2}, \quad \frac{1-\sqrt{5}}{2}$$

$$y_c = C_1 + C_2 e^{x} + C_3 e^{\frac{1+\sqrt{5}}{2}x} + C_4 e^{\frac{1-\sqrt{5}}{2}x}$$

$$y_p = \frac{x^4+3x+1}{D\left[1-(2D^2-D^3)\right]}$$

$$= \frac{1}{D}\left[1-(2D^2-D^3)\right]^{-1}(x^4+3x+1)$$

$$= \frac{1}{D}\left[1+(2D^2-D^3)+(2D^2-D^3)^2+\cdots\right](x^4+3x+1)$$

$$= \frac{1}{D}\left[1+2D^2-D^3+4D^4+\cdots\right](x^4+3x+1)$$

$$= \frac{1}{D}\left[x^4+3x+1+2(12x^2)-2x+4(24)\right]$$

$$= \frac{1}{D}(x^4+24x^2-21x+97)$$

$$= \frac{x^5}{5}+\frac{24x^3}{3}-21\frac{x^2}{2}+97x$$

$$y = y_c + y_p$$

$\boxed{\text{Re } e^{ix} = \cos x}$ $\boxed{\text{Im } e^{ix} = \sin}$

Question:- $(D^3 - D^2 + D - 1)y = 4\sin x.$

Sol:- The C.E is

$D^3 - D^2 + D - 1 = 0$

$D^2(D-1) + 1(D-1) = 0$

$(D^2+1)(D-1) = 0$

$D - 1 = 0 \Rightarrow D = 1$

$D^2 + 1 = 0 \Rightarrow D^2 = -1 \Rightarrow D = \pm i$

$y_c = c_1 e^x + c_2 \cos x + c_3 \sin x$

$= -x(-\cos x + \sin x)$

$y = y_c + y_p$

$y_p = \frac{1}{(D-1)(D^2+1)} \cdot 4\sin$

$= \text{Im } \frac{1}{(D-1)(D+i)(D-i)} 4e^{ix}$

$= \text{Im} \frac{x}{(i-1)(2i)} 4e^{ix}$

$1-2i^2 = 2$

$2i^2 - 2i$

$-2-2i$

$-2(1+i)$

$= \frac{4x}{2} \text{Im} \frac{e^{ix}}{(-1-2i)}$

$= -2x \text{ Im } \frac{\cos x + i\sin x}{1+i} \times \frac{1-i}{1-i}$

$= -\cancel{2}x \text{ Im } \frac{(\cos x - i\cos x + i\sin x + \sin x)}{\cancel{2}}$

Question: $(D^3 - 2D^2 - 3D + 10)y = 40\cos x.$

Sol:

The C.E is.

$$D^3 - 2D^2 - 3D + 10 = 0$$

$$\begin{array}{r|rrr|r}
-2 & 1 & -2 & -3 & 10 \\
 & & -2 & +8 & -10 \\
\hline
 & 1 & -4 & 5 & 0
\end{array}$$

$D = -2$

$$D^2 - 4D + 5 = 0$$

$$a = 1,\ b = -4,\ c = 5$$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{4 \pm \sqrt{-4}}{2}$$

$$= \frac{4 \pm 2i}{2}$$

$$= 2 \pm i$$

$$y_c = c_1 e^{-2x} + (c_2 \cos x + c_3 \sin x)e^{2x}.$$

$$y_p = \frac{40\cos x}{D^3 - 2D^2 - 3D + 10}$$

$$= \frac{40}{D^2 \cdot D - 2D^2 - 3D + 10} \cos x$$

$$= \cancel{40\cos x} \frac{1}{-D + 2 - 3D + 10} 40\cos x.$$

$$= \frac{1}{-4D + 12} \cdot 40\cos x$$

$$= \frac{1}{-\cancel{4}(D-3)} \cdot \overset{10}{\cancel{40}} \cos x$$

$$= -10 \frac{1}{D-3} \cos x.$$

$$= -10 \frac{\cos x}{(D-3)(D+3)} (D+3)$$

$$= \frac{-10(D+3)\cos x}{D^2 - 9}$$

$$= \frac{\cancel{-10}(D+3)\cos x}{\cancel{-10}}.$$

$$= (D+3)\cos x.$$

$$= -\sin x + 3\cos x.$$

$$y = y_c + y_p$$

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Question: $(D^3+D)y = 2x^2+3\sin x$.

Sol:

The C-E is

$D^3 + D = 0$

$D(D^2+1) = 0$

$D = 0, \quad D^2+1 = 0 \Rightarrow D = \pm i$

$y_c = c_1 + c_2\cos x + c_3 \sin x$.

$$y_p = \frac{1}{D^3+D} \cdot 2x^2+3\sin x.$$

$$= \frac{2x^2 \, 1}{D(D^2+1)} \; 2x^2 + \frac{3\sin x \, 1}{D(D^2+1)}$$

$(1+x)^n = 1+nx$

$$= \frac{2}{D}(1+D^2)^{-1}x^2 + 3\frac{\text{Im}}{D(D+i)(D-i)} e^{ix}$$

$$= \frac{2}{D}(1-D^2)x^2 + 3\frac{x.}{i(2i)} \text{Im}\, e^{ix}.$$

$$= \frac{2}{D}(x^2-2) - \frac{3x}{2} \text{Im}(\cos x + i\sin x)$$

$$= 2\left(\frac{x^3}{3} - 2x\right) - \frac{3}{2}(x \sin)$$

Question:- $(D^4+D^2)y = 3x^2+6\sin x-2\cos x.$

Sol.. The C.E is.

$$D^4+D^2=0$$

$$D^2(D^2+1)=0$$

$$D=0,0,\pm i$$

$$y_c = c_1+c_2x+c_3\cos x+c_4\sin x.$$

$$y_p = \frac{1}{D^2(1+D^2)}3x^2+\frac{1}{D^2(1+D^2)}6\sin x-\frac{2}{D^2(1+D^2)}$$

Now $\frac{1}{D^2(1+D^2)}3x^2$

$$= \frac{1}{D^2}(1+D^2)^{-1}3x^2$$

$$= \frac{1}{D^2}(1-D^2)3x^2$$

$$= \frac{3x^2-6}{D^2}.$$

$$= \frac{1}{D}\int(3x^2-6)\,dx.$$

$$= \frac{1}{D}\left(\frac{\cancel{3}x^3}{\cancel{3}}-6x\right)$$

$$= \frac{x^4}{4}-\frac{6x^2}{2}.$$

$$= \frac{x^4}{4}-3x^2$$

دریان پر x لگادیں۔

$$\frac{6\sin x}{D^2(1+D^2)} = 6\frac{\operatorname{Im} e^{ix}}{D^2(D+i)(D-i)}$$

$$= \frac{6 \operatorname{Im}}{D^2(D+i)(D-i)} \cdot e^{ix}$$

$$= \frac{6 \operatorname{Im}}{(-1)(2i)} \cdot x e^{ix}$$

$$= -\frac{6x \operatorname{Im} e^{ix}}{2i}$$

$$= +3x \operatorname{Im} i(\cos x + i\sin x)$$

$$= 3x \cos x$$

$$\frac{-2\cos x}{D^2(1+D^2)} \Rightarrow \frac{-2 \operatorname{Re} e^{ix}}{D^2(D+i)(D-i)}$$

$$= \frac{-2 \operatorname{Re}}{(-1)(2i)} x e^{ix}$$

$$= \operatorname{Re} i(-x e^{ix})$$

$$= x \operatorname{Re} i(\cos + i\sin x)$$

$$= -x\sin x$$

$$y_P = \frac{x^4}{4} - 3x^2 + 3x\cos x - x\sin x.$$

Question $(D^2-2D+4)y = e^x \cos x$

Sol The C.F is.

$D^2-2D+4=0$

$a=1,\ b=-2,\ c=4$

$$D = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$D = \frac{2 \pm \sqrt{4-16}}{2}$$

$$D = \frac{2 \pm \sqrt{-12}}{2} \Rightarrow 1 \pm \sqrt{3}\, i$$

$y_c = (c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x)\, e^x$

for particular sol.

$$y_p = \frac{1}{D^2-2D+4} \cdot e^x \cos x$$

$$= \frac{1}{D^2-2D+4} \cdot e^x \,\mathrm{Re}\, e^{ix}$$

$$= \mathrm{Re} \frac{1}{D^2-2D+4} \cdot e^{(i+1)x}$$

$$= \mathrm{Re} \frac{1}{(1+i)^2-2(1+i)+4}\, e^{(1+i)x}$$

$$= \mathrm{Re} \frac{1}{1+i^2+\cancel{2i}-2-\cancel{2i}+4}\, e^x \cdot e^{ix}$$

$$= \mathrm{Re} \frac{1}{1-1+2} \cdot e^x (\cos x + i \sin x)$$

$$= \frac{1}{2} Re \frac{e^{x}\cos x + i e^{x}\sin x}{2}$$

$$= \frac{1}{2} \cdot e^{x}\cos x$$

↳ S is:

$$y = y_c + y_p$$

$$y = (C_1 \cos\sqrt{3}x + C_2 \sin\sqrt{3}x)e^{x} + \frac{e^{x}\cos x}{2}$$

Question:

$$(D^3 - D^2 + 3D + 5)y = e^{x}\sin 2x.$$

Sol: The C.E is:

$$D^3 - D^2 + 3D + 5 = 0$$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 3 & 5 \\ & & -1 & 2 & -5 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$

$D = -1$

$$D^2 - 2D + 5 = 0$$

$a = 1,\ b = -2,\ c = 5$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{2 \pm \sqrt{4 - 20}}{2}$$

$$D = \frac{2 \pm \sqrt{-16}}{2}$$

$$D = 1 \pm 2i$$

$$y_c = c_1 e^{-x} + (c_2 \cos 2x + c_3 \sin 2x) e^{x}$$

$$y_p = \frac{e^{x} \sin 2x}{(D+1)(D^2-2D+5)}$$

$$= Im \frac{e^{x} \cdot e^{i2x}}{(D+1)(D-1-2i)(D-1+2i)}$$

$$= Im \frac{e^{(1+2i)x}}{(D+1)(D-1-2i)(D-1+2i)}$$

$$= Im \frac{e^{(1+2i)x}}{(1+2i+1)(1+\cancel{2i}-1-\cancel{2i})(\cancel{1}+2i-\cancel{1}+2i)}$$

$(i-1)(i+1)$

i^2-1

$-1-1$

$$= Im \frac{x \cdot e^{(1+2i)x}}{(2+2i)(4i)}$$

$$= Im \frac{x e^{(1+2i)x}}{8(i+i^2)}$$

$$= Im \frac{x e^{(1+2i)x}}{8(i-1)}$$

$\frac{(1-i)(1+i)}{(1+i)}$

$1-i^2$

$$= -\frac{1}{8} Im \frac{x e^{x}(\cos 2x + i \sin 2x)}{(1-i)}$$

$$= -\frac{1}{16} Im (1+i) x e^{x}(\cos 2x + i \sin 2x)$$

$$= -\frac{1}{16} x e^{x}(\cos 2x + \sin 2x)$$