DIFFERENTIAL EQUATIONS

QUIZ # 01

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QUESTION # 01

Solve the initial value problem

$$(3x+8)(y^2+4)\,dx - 4y(x^2+5x+6)\,dy = 0$$

$$y(1) = 2$$

$$(3x+8)(y^2+4)\,dx = 4y(x^2+5x+6)\,dy$$

$$\frac{3x+8}{x^2+5x+6}\,dx = \frac{4y}{y^2+4}\,dy$$

Apply integration :-

$$\int \frac{3x+8}{x^2+3x+2x+6}\,dx = 2\int \frac{2y}{y^2+4}\,dx$$

$$\int \frac{3x+8}{x(x+3)+2(x+3)}\,dx = 2\int \frac{2y}{y^2+4}\,dx$$

$$\int \frac{3x+8}{(x+3)(x+2)}\,dx = 2\ln(y^2+4) + C \quad (a)$$

let :-

$$\frac{3x+8}{(x+3)(x+2)} = \frac{A}{x+2} + \frac{B}{x+3} \quad \text{—(1)}$$

by Multiplying both sides with $(x+2)(x+3)$,
we get

$$3x+8 = A(x+3) + B(x+2) \quad \text{—(2)}$$

$x+2=0$; $x+3=0$

$x=-2$; $x=-3$

put $x=-2$ in eq (2)

$$3(-2)+8 = A(-2+3) + B(-2+2)$$
$$-6+8 = A(1) + B(0)$$
$$2 = A$$
$$\boxed{A = 2}$$

Now put $x=-3$ in (2)

$$3(-3)+8 = A(-3+3) + B(-3+2)$$
$$-9+8 = A(0) + B(-1)$$
$$-1 = -B$$
$$\boxed{B = 1}$$

put values in eq (i)

$$\frac{3x+8}{(x+3)(x+2)} = \frac{2}{x+2} + \frac{1}{x+3}$$

$$\Rightarrow \int \frac{3x+8}{(x+3)(x+2)}\,dx = 2\int \frac{1}{x+2}\,dx + \int \frac{1}{x+3}\,dx$$

$$= 2\ln(x+2) + \ln(x+3)$$

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put in (a)

$$2\ln(x+2)+\ln(x+3) = 2\ln(y^2+4)+c$$

By putting: $x=1$, $y=2$

$$\boxed{c = 9/16}$$

$$\ln(x+2)^2(x+3) = \ln(y^2+4)^2(9/16)$$

QUESTION # 2

Solve :-

$$y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$$

$$y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$$

Divide by xy

$$\frac{y\sqrt{1+x^2}}{xy}\,dx + \frac{x\sqrt{1+y^2}}{xy} = 0/xy$$

$$\frac{\sqrt{1+x^2}}{xy}\,dx + \frac{\sqrt{1+y^2}}{y}\,dy = 0$$

let $\sqrt{1+x^2} = t$

$1+x^2 = t^2$

$x^2 = t^2-1$

$2x\,dx = 2t\,dt$

$dx = \frac{t}{x}\,dt$

integ rate:-

$$\int \frac{\sqrt{1+x^2}}{x}\,dx = \int \frac{t}{x}\cdot\frac{t}{x}\,dt$$

$$= \int \frac{t^2}{x^2}\,dt$$

$$= \int \frac{t^2}{t^2-1}\,dt$$

$$= \int \left(1+\frac{1}{t^2-1}\right)dt$$

$$= \int dt + \int \frac{1}{t^2-1}\,dt$$

$$= t + \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right)$$

$$= \sqrt{1+x^2} + \frac{1}{2}\ln\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}$$

$$= \sqrt{1+x^2} + \frac{1}{2}\ln\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\times\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}-1}$$

$$= \sqrt{1+x^2} + \frac{1}{2}\ln\frac{(\sqrt{1+x^2}-1)^2}{1-x^2-1}$$

$$= \sqrt{1+x^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{x^2}\right)^2$$

$$= \sqrt{1+x^2} + \frac{1}{2}\ln\frac{(+\sqrt{1+x^2}-1)^2}{|x|}$$

So

$$\int \frac{\sqrt{1+y^2}}{y}\,dy = \sqrt{1+y^2} + \frac{1}{2}\ln\frac{\sqrt{1+y^2}-1}{|y|}$$

General solution is:

$$\sqrt{1+x^2} + \sqrt{1+y^2} + \frac{1}{2}\ln\frac{(\sqrt{1+x^2}-1)^2}{|x|} + \frac{1}{2}\ln\frac{(\sqrt{1+y^2}-1)^2}{|y|}$$

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QUES #03

$$\frac{dy}{dx} = \frac{y-x+1}{y-x+5}$$

$$\frac{dy}{dx} = \frac{y-x+1}{y-x+5} \quad \text{—— (1)}$$

put $y - x = z$ in (1)

$$\frac{dy}{dx} - 1 = \cancel{1+}\frac{dz}{dx}$$

$$\frac{dy}{dx} = 1 + \frac{dz}{dx}$$

$$1 + \frac{dz}{dx} = \frac{z+1}{z+5}$$

$$\frac{dz}{dx} = \frac{z+1}{z+5}$$

$$\frac{dz}{dx} = \frac{z+1}{z+5} - 1$$

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$$= \frac{z+1-z-5}{z+5}$$

$$\frac{dz}{dx} = \frac{-4}{z+5}$$

Apply integration

$$\int (z+5)\,dz = -4\int dx$$

$$\frac{z^2}{2} + 5z = -4x + c$$

$$z^2 + 10z = -8x + 2c$$

$$(y-x)^2 + 10(y-x) = -8x + c'$$

$$(y^2-x)^2 + 10(y-x) + 8x = c'$$

$$y - x^2 + 10y - 2x = c'$$

QUESTION #04

$$(\sqrt{x+y} + \sqrt{x-y})\,dx - (\sqrt{x+y} - \sqrt{x-y})\,dy$$

$$(\sqrt{x+y} + \sqrt{x-y})\,dx = (\sqrt{x+y} - \sqrt{x-y})\,dy$$

$$\frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} = \frac{dy}{dx}$$

Put

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{x}\,(\sqrt{1+v} + \sqrt{1-v})}{\sqrt{x}\,(\sqrt{1+v} - \sqrt{1-v})}$$

Rationalizing

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$= \frac{(1+v) + (1-v) + 2(\sqrt{1+v}\cdot\sqrt{1-v})}{(1+v) - (1-v)}$$

$$= \frac{2 + 2\sqrt{1-v^2}}{2v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$x\frac{dx}{dv} = \frac{1+\sqrt{1-v^2}-v^2}{v}$$

apply integration :-

$$\int \frac{v}{(1-v^2)+\sqrt{1-v^2}}\,dv = \int \frac{dx}{x} \quad \text{— (A)}$$

$$\int \frac{v}{(1-v^2)+\sqrt{1-v^2}}\,dv$$

put $v = \sin x$

$dv = \cos x\,dx$

$$= \int \frac{\sin x \cos x\,dx}{(1-\sin^2 x)+\sqrt{1-\sin^2 x}}$$

$$= \int \frac{\sin x \cos x\,dx}{\cos^2 x + \cos x}$$

$$= \int \frac{\sin x\,\cancel{\cos x}}{\cancel{\cos x}(\cos x+1)}\,dx$$

$$= -\int \frac{-\sin x}{\cos x+1}\,dx$$

$$= -\ln(\cos x+1) \quad \text{put in (A)}$$

$$-\ln(\cos x+1) = \ln x + \ln c$$

$$-\ln(\sqrt{1-\sin^2 x}+1) = \ln cx$$

$$-\ln\left(\sqrt{1-\frac{y^2}{x^2}}+1\right) = \ln cx$$

$$\ln\left(\frac{\sqrt{x^2-y^2}+x}{x}\right)^{-1} = \ln cx$$

$$c' = \sqrt{x^2-y^2}+x$$