Q:- Quiz (11)

$$(y-xy^2)dx+(x+x^2y^2)dy=0$$

$$y\,dx - xy^2dx + x\,dy + x^2y^2dy = 0$$

$$y\,dx + x\,dy - x^2y^2\left(\frac{dx}{x} - dy\right)=0$$

$$\frac{y\,dx + x\,dy}{x^2y^2} - \frac{dx}{x} + dy = 0.$$

$$d\left(\frac{-1}{xy}\right) - \frac{dx}{x} + dy = 0$$

Integrate

$$-\frac{1}{xy} - \ln x + y = c.$$

$$y = c + \ln x + \frac{1}{xy}$$

Q:-

Sol

$$(3x^2y^4+2xy)dx + (2x^3y^3 - x^2)dy = 0$$

$$M = 3x^2y^4+2xy$$

$$N = 2x^3y^3 - x^3$$

$$\frac{\partial M}{\partial y} = 12x^2y^3+2x$$

$$\frac{\partial N}{\partial x} = 6x^2y^3 - 3x^2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\Rightarrow \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}$$

$$\Rightarrow \frac{-6x^2y^3-4x}{y(3x^2y^3+2x)} \Rightarrow \frac{-2\cancel{(3x^2y^3+2x)}}{y\cancel{(3x^2y^3+2x)}}$$

Scanned with CamScanner

$$= -2/y$$

$$I.F = e^{-2\int 1/y \, dy} \Rightarrow e^{-2\ln y} = \frac{1}{y^2}$$

x given eq by I.F

$$\frac{3x^2y^4+2xy}{y^2}dx + \frac{2x^3y^3-x^2}{y^2}dy = 0$$

$$\frac{\cancel{y}(3x^2y^3+2x)}{y^{\cancel{2}}}dx + \frac{2x^3y^3-x^2}{y^2}dy = 0$$

$$M = \frac{3x^2y^3+2x}{y} \qquad N = \frac{2x^3y^3-x^2}{y^2}$$

$$\frac{\partial M}{\partial y} = \frac{y(9x^2y^2)-(3x^2y^3+2x)(1)}{y^2}$$

$$= \frac{9x^2y^3-3x^2y^3-2x}{y^2}$$

$$= \frac{6x^2y^3-2x}{y^2}$$

$$\frac{\partial N}{\partial x} = \frac{1}{y^2}\left[6x^2y^3-2x\right]$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

eq is exact

$$\int M dx + \int (\text{term of } N \text{ free of } x)\, dy = 0$$

$$\frac{1}{y}\int (3x^2y^3+2x)\, dx + 0 = 0$$

$$\frac{1}{y}\left(\frac{\cancel{3}x^3y^3}{\cancel{3}} + \frac{\cancel{2}x^2}{\cancel{2}}\right) = C$$

Scanned with CamScanner

$$x^3y^2 + \frac{x^2}{y} = c.$$

Q:-

$$y - x\frac{dy}{dx} = x + y\frac{dy}{dx}.$$

$$\frac{y\,dx - x\,dy}{\cancel{dx}} = \frac{x\,dx + y\,dy}{\cancel{dx}}$$

$$y\,dx - x\,dy = x\,dx + y\,dy$$

$$(y-x)\,dx - (x+y)\,dy = 0 \quad \text{--- (1)}$$

$M = y - x$ $\qquad\qquad$ $N = -(x+y)$

$\frac{\partial M}{\partial y} = 1$ $\qquad\qquad$ $\frac{\partial N}{\partial x} = -1$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$$

$$I.F = \frac{1}{xM + yN}$$

$$x(y-x) + y(-x-y)$$

$$= \cancel{xy} - x^2 - \cancel{xy} - y^2$$

$$= -x^2 - y^2$$

$$I.F = \frac{-1}{x^2+y^2}$$

$\times$ eq (1) by I.F

$$\frac{x-y}{x^2+y^2}\,dx + \frac{x+y}{x^2+y^2}\,dy = 0$$

$$M = \frac{x-y}{x^2+y^2} \qquad N = \frac{x+y}{x^2+y^2}$$

$$\frac{\partial M}{\partial y} = \frac{(x^2+y^2)(-1)-(x-y)(2y)}{(x^2+y^2)^2} \qquad \frac{\partial N}{\partial x} = \frac{(x^2+y^2)(1)-(x+y)(2x)}{(x^2+y^2)^2}$$

$$= \frac{-x^2-y^2-2xy+2y^2}{(x^2+y^2)^2} \qquad = \frac{x^2+y^2-2x^2-2xy}{(x^2+y^2)^2}$$

$$= \frac{y^2-x^2-2xy}{(x^2+y^2)^2} \qquad = \frac{-x^2+y^2-2xy}{(x^2+y^2)^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

eq is exact

$$\int M\,dx + \int (\text{term of } N \text{ free of } x)\,dy = 0$$

$$\int \frac{x-y}{x^2+y^2}\,dx + 0 = 0$$

$$\int \frac{x}{x^2+y^2}\,dx - y\int \frac{1}{1+(y/x)^2}\,dx.$$

$$\frac{1}{2}\int \frac{2x}{x^2+y^2}\,dx + \tan^{-1}(y/x) = c.$$

$$\frac{1}{2}\ln(x^2+y^2) + \tan^{-1}(y/x) = c$$

Q. $\frac{dy}{dx} = e^{2x} + y - 1$

$dy = (e^{2x} + y - 1)\,dx$

$(e^{2x} + y - 1)dx - dy = 0$ — (1)

$M = e^{2x} + y - 1$ $\quad N = -1$

$\frac{\partial m}{\partial y} = 1$ $\quad \frac{\partial N}{\partial x} = 0$

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} \Rightarrow \frac{1 - 0}{-1} = -1$

$I.F = e^{-\int dx} \Rightarrow e^{-x}$

X eq (1) by I.F

$e^{-x}(e^{2x} + y - 1)dx - e^{-x}dy = 0$

$M = e^{x} + e^{-x}y - e^{-x}$ $\quad N = -e^{-x}$

$\frac{\partial m}{\partial y} = +e^{-x}$ $\quad \frac{\partial N}{\partial x} = e^{-x}$

$\frac{\partial m}{\partial y} = \frac{\partial N}{\partial x}$

eq is exact

$\int M dx + \int (\text{term of } N \text{ free of } x)dy = 0$

$\int (e^{x} + e^{-x}y - e^{-x})dx + \int (0)dy = 0$

$\cancel{e^{2x} + xy - x - y} = c$

$e^{x} - ye^{-x} + e^{-x} = c$

Q. $(3xy+y^2)\,dx + (x^2+xy)\,dy = 0$

$M = 3xy + y^2$ $\quad N = x^2 + xy$

$\frac{\partial M}{\partial y} = 3x + 2y$ $\quad \frac{\partial N}{\partial x} = 2x + y$.

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$$

eq is not exact

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$$

$$\Rightarrow \frac{3x+2y-2x-y}{x(x+y)}.$$

$$\Rightarrow \frac{\cancel{(x+y)}}{x\cancel{(x+y)}} \Rightarrow 1/x.$$

$$I.F = e^{\int 1/x\, dx} \Rightarrow e^{\ln x} = x.$$

X given eq by I.F.

$(3x^2y + xy^2)\,dx + (x^3 + x^2y)\,dy = 0$

$M = 3x^2y + xy^2$ $\quad N = x^3 + x^2y$

$\frac{\partial M}{\partial y} = 3x^2 + 2xy$ $\quad \frac{\partial N}{\partial x} = 3x^2 + 2xy$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

eq is exact

$\int M\,dx + \int(\text{terms of } N \text{ free of } x)\,dy = 0$

$\int(3x^2y + xy^2)\,dx + 0 = 0$

$\frac{\cancel{3}x^3y}{\cancel{3}} + \frac{x^2y^2}{2} = c$

$x^3y + \frac{x^2y^2}{2} = c.$