Techniques of Integration

EXERCISE 4.2

Q No. 1 $I = \int \frac{dx}{\sqrt{a^2+x^2}}$

Put $x = a\tan\theta \Rightarrow dx = a\sec^2\theta d\theta$

$$I = \int \frac{a\sec^2\theta d\theta}{\sqrt{(a^2 + a^2\tan^2\theta)}}$$

$$= \int \frac{a\sec^2\theta d\theta}{\sqrt{a^2(1+\tan^2\theta)}} = \int \frac{a\sec^2\theta d\theta}{a\sqrt{(1+\tan^2\theta)}} =$$

$$\int \frac{\sec^2\theta d\theta}{\sqrt{(1+\tan^2\theta)}}$$

$$= \int \frac{\sec^2\theta d\theta}{\sec\theta} = \int \sec\theta d\theta = ln|\sec\theta + \tan\theta|$$

Now substitution returns:

$$I = ln\left|\sqrt{1+\tan^2\theta} + \tan\theta\right|$$

$$I = ln\left|\sqrt{1+\left(\frac{x}{a}\right)^2} + \frac{x}{a}\right| = ln\left|\frac{\sqrt{x^2+a^2}+x}{a}\right|$$

Q No. 2 $I = \int \frac{dx}{\sqrt{x^2-a^2}}$

Put $x = a\cosh\theta \Rightarrow dx = a\sinh\theta d\theta$

$$I = \int \frac{a\sinh\theta d\theta}{\sqrt{(a^2\cosh^2\theta - a^2)}}$$

$$= \int \frac{a\sinh\theta d\theta}{\sqrt{a^2(\cosh^2\theta - 1)}} = \int \frac{a\sinh\theta d\theta}{a\sqrt{(\cosh^2\theta - 1)}}$$

$$= \int \frac{\sinh\theta d\theta}{\sqrt{(\cosh^2\theta-1)}} = \int \frac{\sinh\theta d\theta}{\sinh\theta}$$

$$= \int d\theta = \theta$$

Now substitution returns:

$$= \cosh^{-1}\frac{x}{a}$$

Q No. 3 $I = \int \tan x dx$

$$\int \tan x dx = \int \frac{\sin x}{\cos x}dx$$

$$= -\int \frac{-\sin x}{\cos x}dx = -\ln(\cos x) = \ln(\sec x)$$

Q No. 4 $I = \int \cot x dx$

$$\int \cot x dx = \int \frac{\cos x}{\sin x}dx = \ln(\sin x)$$

Q No. 5 $I = \int \sec x dx$

$$\int \sec x dx = \int \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)}dx$$

$$= \int \frac{\sec^2 x + \sec x\tan x)}{(\sec x + \tan x)}dx = \ln(\sec x + \tan x)$$

Q No. 6 $I = \int \csc x dx$

$$\int \csc x dx = -\int \frac{-\csc x(\csc x + \cot x)}{(\csc x + \cot x)}dx$$

$$= -\int \frac{-\operatorname{cosec}^2 x - \csc x\cot x)}{(\csc x + \cot x)} = -\ln(\csc x + \cot x)$$

by rationalizing this answer we can get another result

i.e $\ln(\csc x - \cot x)$

Q No. 7 $I = \int (ax^2 + 2bx + c)^2(ax + b)dx$

$$I = \frac{1}{2}\int (ax^2 + 2bx + c)^2(2ax + 2b)dx$$

$$I = \frac{(ax^2 + 2bx + c)^{2+1}}{2+1}$$

Q No. 8 $I = \int \sqrt{\frac{1+x}{1-x}}dx$

By rationalizing we get, $\frac{1+x}{1-x} \times \frac{1+x}{1+x} = \frac{(1+x)^2}{1-x^2}$

So, $I = \int \frac{1+x}{\sqrt{1-x^2}}dx = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{xdx}{\sqrt{1-x^2}}$

$= sin^{-1}x + \int (1-x^2)^{-\frac{1}{2}}xdx$

$= sin^{-1}x - \frac{1}{2}\int (1-x^2)^{-\frac{1}{2}}(-2x)dx$

$= sin^{-1}x - \frac{1}{2}\frac{(1-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$

$= sin^{-1}x - \frac{1}{2}\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}}$

$= sin^{-1}x - \sqrt{1-x^2}$

Q No. 9 $\int \frac{dx}{a+\sqrt{bx+c}}$

(linear under square root)

Put $\sqrt{bx+c} = z$

$\Rightarrow \quad bx + c = z^2$

$\Rightarrow \quad b\,dx = 2zdz$

$I = \int \frac{2zdz/b}{a+z}$

$I = \frac{2}{b}\int \frac{zdz}{a+z}$

$I = \frac{2}{b}\int \left(1 - \frac{a}{a+z}\right)dz$

$I = \frac{2}{b}\int dz - \frac{2a}{b}\int \frac{dz}{a+z}$

$I = \frac{2}{b}z - \frac{2a}{b}ln(a+z)$

$I = \frac{2}{b}\sqrt{bx+c} - \frac{2a}{b}ln\,(a+\sqrt{bx+c})$

Q No. 10 $\int \frac{dx}{(1+x^2)tan^{-1}x}$

$I = \int \frac{1/(1+x^2)}{tan^{-1}x}dx \quad = \ln(tan^{-1}x)$

Q No. 11 $I = \int \frac{sinx+cosx}{sinx-cosx}dx$

$I = \int \frac{cosx-(-sinx)}{sinx-cosx}dx \;= \ln(sinx - cosx)$

Q No. 12 $I = \int \frac{sin\sqrt{x}}{\sqrt{x}}dx$

(Substitute the complicated angle)

Put $\sqrt{x} = z \;\Rightarrow\; x = z^2 \;\Rightarrow\; dx = 2zdz$

$I = \int \frac{sinz}{z}.2zdz \quad = 2\int sinzdz \quad = -2cosz$

$= -2cos\sqrt{x}$

Q No. 13 $I = \int \sqrt{e^{2x}+e^{3x}}dx$

$I = \int \sqrt{e^{2x}+e^{3x}}dx = \int \sqrt{e^{2x}(1+e^x)}dx$

$I = \int \sqrt{1+e^x}.e^x dx \quad = \frac{(1+e^x)^{\frac{1}{2}+1}}{\frac{1}{2}+1}$

Q No. 14 $I = \int \frac{dx}{e^x+e^{-x}}$

I

$= \int \frac{e^x dx}{e^x(e^x+e^{-x})}$ $\quad$ $(multiplied\ D^r and\ N^r by\ e^x)$

$I = \int \frac{e^x dx}{e^{2x}+1} \quad = \tan^{-1}(e^x)$

<u>Alternatively,</u>

Put $e^x = z \quad \Rightarrow e^x dx = dz$

So $I = \int \frac{e^x dx}{e^{2x}+1} = \int \frac{dz}{z^2+1} = \tan^{-1}z = \tan^{-1}(e^x)$

Q No. 15 $I = \int \frac{e^{2x}dx}{\sqrt{e^x-1}}$

$$Put\ e^x = z \quad \Rightarrow e^x dx = dz$$

$$I = \int \frac{e^x.e^x dx}{\sqrt{e^x-1}} = \int \frac{zdz}{\sqrt{z-1}} = \int \frac{(z-1+1)dz}{\sqrt{z-1}}$$

$$I = \int \frac{(z-1)dz}{\sqrt{z-1}} + \int \frac{dz}{\sqrt{z-1}}$$

$$I = \int (z-1)^{1-\frac{1}{2}}dz + \int (z-1)^{\frac{-1}{2}}dz$$

$$I = \int (z-1)^{\frac{1}{2}}dz + \int (z-1)^{\frac{-1}{2}}dz$$

$$I = \frac{(z-1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{(z-1)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}$$

$$I = \frac{(z-1)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{(z-1)^{\frac{1}{2}}}{\frac{1}{2}} = \frac{2}{3}(e^x-1)^{\frac{3}{2}} + 2\sqrt{e^x-1}$$

Q No. 16 $I = \int \frac{\cos(\ln x)}{x}dx$

(Substitute the complicated angle)

Put $\ln x = z \quad \Rightarrow \quad \frac{1}{x}dx = dz$

$$I = \int cosz dz = sinz = \sin(\ln x)$$

Q No. 17 $I = \int \frac{2x+5}{\sqrt{x^2+5x+7}}dx$

$$I = \int (x^2+5x+7)^{-\frac{1}{2}}.(2x+5)dx$$

$$I = \frac{(x^2+5x+7)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$$

Q No. 18 $I = \int \frac{x+2}{\sqrt{2x^2+8x+5}}dx$

$$I = \int (2x^2+8x+5)^{-\frac{1}{2}}.(x+2)dx$$

$$I = \frac{1}{4}\int (2x^2+8x+5)^{-\frac{1}{2}}.4(x+2)dx$$

$$I = \frac{1}{4}\int (2x^2+8x+5)^{-\frac{1}{2}}.(4x+8)dx$$

$$I = \frac{(2x^2+8x+5)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$$

Q No. 19 $I = \frac{\sqrt{x^2-a^2}}{x^4}dx$

$Put\ x = acosh\theta \quad \Rightarrow \quad dx = asinh\theta$

$$I = \int \frac{\sqrt{a^2\cosh^2\theta - a^2}}{a^4\cosh^4\theta} asinh\theta d\theta$$

$$I = \int \frac{a\sqrt{\cosh^2\theta - 1}}{a^4\cosh^4\theta} asinh\theta d\theta$$

$$I = \int \frac{\sqrt{\cosh^2\theta - 1}}{a^2\cosh^4\theta}\sqrt{\cosh^2\theta - 1}\, d\theta$$

$$I = \frac{1}{a^2}\int \frac{\cosh^2\theta - 1}{\cosh^4\theta}d\theta$$

$$I = \frac{1}{a^2}\int \frac{1}{\cosh^2\theta}d\theta - \frac{1}{a^2}\int \frac{1}{\cosh^4\theta}d\theta$$

$$I = \frac{1}{a^2}\int sech^2\theta d\theta - \frac{1}{a^2}\int sech^4\theta d\theta$$

$$I = \frac{1}{a^2}tanh\theta - \frac{1}{a^2}I_1 \text{------------}(1)$$

$$I_1 = \int sech^4\theta d\theta$$

$$I_1 = \int sech^2\theta.sech^2\theta d\theta$$

$$I_1 = \int (1-tanh^2\theta).sech^2\theta d\theta$$

$$I_1 = \int sech^2\theta d\theta - \int tanh^2\theta sech^2\theta d\theta$$

$$I_1 = tanh\theta - \frac{tanh^3\theta}{3}$$

Putting in eq(1) we get,

$$I = \frac{1}{a^2}tanh\theta - \frac{1}{a^2}tanh\theta + \frac{tanh^3\theta}{3a^2}$$

$$I = \frac{tanh^3\theta}{3} = \frac{1}{3}.\left(\frac{sinh\theta}{cosh\theta}\right)^3 = \frac{1}{3}.\left(\frac{\sqrt{cosh^2\theta - 1}}{cosh\theta}\right)^3$$

$$I = \frac{1}{3a^2}\left(\frac{\sqrt{\frac{x^2}{a^2}-1}}{\frac{x}{a}}\right)^3 = \frac{(x^2-a^2)^{\frac{3}{2}}}{3a^2x^3}$$

Prepared by: **Aqeel Nawaz** aqeel_755@yahoo.com 03349587500

Q No. 20 $I = \int cos^6 x sin^3 x dx$

$$I = \int cos^6 x . sin^2 x . sinx\, dx$$

$$= \int cos^6 x . (1 - cos^2 x) . sinx\, dx$$

$$= \int cos^6 x \;\; . sinx\, dx - \int cos^8 x \;\; . sinx\, dx$$

$$= -\int cos^6 x \;\; . (-sinx) dx + \int cos^8 x \;\; . (-sinx)\, dx$$

$$= -\frac{cos^7 x}{7} + \frac{cos^9 x}{9}$$

Q No. 21 $I = \int tan^3 x sec^3 x dx$

$$I = \int tan^2 x . sec^2 x . secxtanx\, dx$$

$$I = \int (sec^2 x - 1) . sec^2 x . secxtanx\, dx$$

$$I = \int sec^4 x . secxtanx\, dx - \int sec^2 x . secxtanx\, dx$$

$$I = \frac{sec^5 x}{5} - \frac{sec^3 x}{3}$$

Q No. 22 $I = \int cot^3 x csc^4 x dx$

$$I = \int cot^2 x csc^3 x . (cotxcscx) dx$$

$$I = -\int cot^2 x csc^3 x . (-cotxcscx) dx$$

$$I = -\int (csc^2 x - 1) csc^3 x . (-cotxcscx) dx$$

$$I = -\int csc^5 x\, (-cotxcscx) dx + \int csc^3 x (-cotxcscx) dx$$

$$I = \frac{-csc^6 x}{6} + \frac{csc^4 x}{4}$$

Alternatively,

$$I = \int cot^3 x csc^4 x dx$$

$$I = \int cot^3 x csc^2 x . (csc^2 x) dx$$

$$I = -\int cot^3 x csc^2 x . (-csc^2 x) dx$$

$$I = -\int cot^3 x (cot^2 x + 1) . (-csc^2 x) dx$$

$$I = -\int cot^5 x\, (-csc^2 x) dx - \int cot^3 x\, (-csc^2 x) dx$$

$$I = -\frac{1}{6} cot^6 x - \frac{1}{4} cot^4 x$$

Q No. 23 $I = \int \frac{dx}{\sqrt{2x^2+3x+4}}$

$$I = \int \frac{dx}{\sqrt{2(x^2 + \frac{3}{2}x + 2)}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x^2 + \frac{3}{2}x + 2)}}$$

Completing square of

$$x^2 + \frac{3}{2}x + 2$$

$$= (x)^2 + 2\left(\frac{3}{4}\right)(x) + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 + 2$$

$$= \left(x + \frac{3}{4}\right)^2 - \frac{9}{16} + 2$$

$$= \left(x + \frac{3}{4}\right)^2 - \frac{9}{16} + \frac{32}{16}$$

$$= \left(x + \frac{3}{4}\right)^2 + \frac{23}{16}$$

$$= \left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2$$

So,

$$I = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2}}$$

$$I = \frac{1}{\sqrt{2}} \sinh^{-1}\left(\frac{x + \frac{3}{4}}{\frac{\sqrt{23}}{4}}\right) = \frac{1}{\sqrt{2}} \sinh^{-1}\left(\frac{4x + 3}{\sqrt{23}}\right)$$

Q No. 24 $I = \sqrt{a^2 - x^2}dx$

Put $x = asin\theta \quad \Rightarrow dx = acos\theta d\theta$

$$I = \int \sqrt{a^2 - a^2sin^2\theta}\, acos\theta\, d\theta$$

$$I = \int a\sqrt{1 - sin^2\theta}\, acos\theta\, d\theta$$

$$I = a^2 \int cos^2\theta d\theta$$

$$I = \frac{a^2}{2}\int (1 + cos2\theta)d\theta \qquad as \;\; cos^2\theta = \frac{1 + cos2\theta}{2}$$

$$I = \frac{a^2}{2}(\theta + \frac{sin2\theta}{2})$$

$$I = \frac{a^2}{2}(\theta + \frac{2sin\theta cos\theta}{2})$$

$$I = \frac{a^2}{2}(\theta + sin\theta\sqrt{1 - sin^2\theta})$$

Substitution returned:

$$I = \frac{a^2}{2}\left(\sin^{-1}\frac{x}{a} + \frac{x}{a}\sqrt{1 - \frac{x^2}{a^2}}\right)$$

$$I = \frac{a^2}{2}\left(\sin^{-1}\frac{x}{a} + \frac{x}{a}\sqrt{\frac{a^2 - x^2}{a^2}}\right)$$

$$I = \frac{a^2}{2}\sin^{-1}\frac{x}{a} + \frac{a^2}{2}.\frac{x}{a}.\frac{\sqrt{a^2 - x^2}}{a}$$

$$I = \frac{x}{2}.\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$$

Q No. 25 $I = \int (2x + 3)\sqrt{2x + 1}\, dx$

$$I = \int (2x + 1 + 2)\sqrt{2x + 1}dx$$

$$I = \int (2x + 1)\sqrt{2x + 1}dx + 2\int \sqrt{2x + 1}\, dx$$

$$I = \int (2x + 1)^{1 + \frac{1}{2}}dx + 2\int (2x + 1)^{\frac{1}{2}}dx$$

$$I = \frac{1}{2}\int (2x + 1)^{\frac{3}{2}}.2dx + \int (2x + 1)^{\frac{1}{2}}2.dx$$

$$I = \frac{1}{2}.\frac{(2x + 1)^{\frac{3}{2} + 1}}{\frac{3}{2} + 1} + \frac{(2x + 1)^{\frac{1}{2} + 1}}{\frac{1}{2} + 1}$$

Q No. 26 $I = \int (1 + x^2)^{-\frac{3}{2}} dx$

$Put \;\; x = tan\theta \quad \Rightarrow \quad dx = sec^2\theta d\theta$

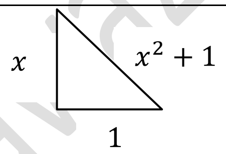
$$I = \int (1 + tan^2\theta)^{-\frac{3}{2}}.sec^2\theta\, d\theta$$

$$I = \int (sec^2\theta)^{-\frac{3}{2}}.sec^2\theta\, d\theta$$

$$I = \int \frac{sec^2\theta}{sec^3\theta}d\theta \quad = \int cos\theta d\theta \quad = sin\theta$$

In right triangle :

$tan\theta = \frac{x}{1}$



By Pythagorean's theorem we can find the Hyp. So

$sin\theta = \frac{x}{x^2+1}$

Hence $I = \frac{x}{x^2+1}$

Q No. 27 $I = \int \frac{x^2}{\sqrt{x^2+1}}dx$

$$I = \int \frac{x^2 + 1 - 1}{\sqrt{x^2 + 1}}dx \quad = \int \left(\frac{x^2 + 1}{\sqrt{x^2 + 1}} - \frac{1}{\sqrt{x^2 + 1}}\right)dx$$

$$I = \int \sqrt{x^2 + 1}dx - \int \frac{dx}{\sqrt{x^2 + 1}}$$

$$I = \left[\frac{x}{2}\sqrt{x^2 + 1} + \frac{1}{2}\sinh^{-1}x\right] - \sinh^{-1}x$$

$$I = \frac{x}{2}\sqrt{x^2 + 1} - \frac{1}{2}\sinh^{-1}x$$

Q No. 28 $I = \int (2x + 4)\sqrt{2x^2 + 3x + 1}dx$

$$I = \frac{1}{2}\int (2x^2 + 3x + 1)^{\frac{1}{2}}.(4x + 8)dx$$

$$I = \frac{1}{2}\int (2x^2 + 3x + 1)^{\frac{1}{2}}.(4x + 3 + 5)dx$$

$$I = \frac{1}{2}\int (2x^2 + 3x + 1)^{\frac{1}{2}}.(4x + 3)dx + \frac{1}{2}\int (2x^2 + 3x + 1)^{\frac{1}{2}}.5dx$$

$$I = \frac{1}{2}\frac{(2x^2 + 3x + 1)^{\frac{1}{2} + 1}}{\frac{1}{2} + 1} + \frac{5}{2}.\sqrt{2}\int \left(x^2 + \frac{3}{2}x + \frac{1}{2}\right)^{\frac{1}{2}}dx$$

$$I = \frac{1}{3}(2x^2 + 3x + 1)^{\frac{3}{2}} + \frac{5}{\sqrt{2}}\int \sqrt{\left(x + \frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2}\, dx$$

Completing square of

$$x^2+\frac{3}{2}x+\frac{1}{2}$$

$$=(x)^2+2\left(\frac{3}{4}\right)(x)+\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2+\frac{1}{2}$$

$$=\left(x+\frac{3}{4}\right)^2-\frac{9}{16}+\frac{1}{2}$$

$$=\left(x+\frac{3}{4}\right)^2-\frac{9}{16}+\frac{8}{16}$$

$$=\left(x+\frac{3}{4}\right)^2-\frac{1}{16}$$

$$=\left(x+\frac{3}{4}\right)^2-\left(\frac{1}{4}\right)^2$$

Hence,

$$I=\frac{1}{3}(2x^2+3x+1)^{\frac{3}{2}}$$

$$+\frac{5}{\sqrt{2}}\left[\frac{x+\frac{3}{4}}{2}\sqrt{\left(x+\frac{3}{4}\right)^2-\left(\frac{1}{4}\right)^2}-\frac{(1/4)^2}{2}\cosh^{-1}\frac{x+\frac{3}{4}}{\frac{1}{4}}\right]$$

$$I=\frac{1}{3}(2x^2+3x+1)^{\frac{3}{2}}$$

$$+\frac{5}{\sqrt{2}}\left[\frac{4x+3}{8}.\frac{\sqrt{(4x+3)^2-(1)^2}}{4}-\frac{1}{32}\cosh^{-1}(4x+3)\right]$$

$$I=\frac{1}{3}(2x^2+3x+1)^{\frac{3}{2}}$$

$$+\frac{5}{32\sqrt{2}}\left[4x+\sqrt{(4x+3)^2-1}-\cosh^{-1}(4x+3)\right]$$

Q No. 29 $I=\int\frac{dx}{3sinx+4cosx}$

Let $3=rsint$ and $4=rcost$

Squaring and adding, we get,	Dividing , we get
$3^2+4^2=r^2sin^2t+r^2cos^2t$ $25=r^2$ $r=5$	$\frac{rsint}{rcost}=\frac{3}{4}$ $t=\tan^{-1}(\frac{3}{4})$

$$I=\int\frac{dx}{rsintsinx+rcostcosx}$$

$$I=\frac{1}{r}\int\frac{dx}{\cos(x-t)}$$

$$I=\frac{1}{r}\int\sec(x-t)\,dx$$

$$I=\frac{1}{r}ln|\sec(x-t)+\tan(x-t)|$$

$$I=\frac{1}{5}ln\left|\sec\left(x-\tan^{-1}\frac{3}{4}\right)+tan\left(x-\tan^{-1}\frac{3}{4}\right)\right|$$

Q No. 30 $I=\int\frac{tanxdx}{cosx+secx}$

$$I=\int\frac{\frac{sinx}{cosx}}{cosx+\frac{1}{cosx}}dx$$

$$I=\int\frac{sinx}{\cos^2x+1}dx$$

$$I=-\int\frac{-sinx}{\cos^2x+1}dx$$

$$I=\tan^{-1}(cosx)$$

Q No. 31 $I=\int\frac{dx}{sin(x-a)sin(x-b)}$

$$1=\frac{\sin(a-b)}{\sin(a-b)}=\boxed{\frac{\sin(a-b+x-x)}{\sin(a-b)}}$$

$$=\frac{\sin(x-b-x+a)}{\sin(a-b)}$$

$$=\frac{\sin[(x-b)-(x-a)]}{\sin(a-b)}$$

$$=\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(a-b)}$$

So,

$$I=\frac{1}{\sin(a-b)}$$

$$\int\left(\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\right)$$

$$I=\frac{1}{\sin(a-b)}\int\left[\frac{\cos(x-a)}{\sin(x-a)}-\frac{\cos(x-b)}{\sin(x-b)}\right]dx$$

$$I=\frac{1}{\sin(a-b)}[ln\sin(x-a)-ln\sin(x-b)]$$

$$I=\frac{1}{\sin(a-b)}ln\frac{sin(x-a)}{sin(x-b)}$$

Q No. 32 $I = \int tanx\ln(secx)\,dx$

Put $lnsecx = z \Rightarrow dz = \frac{1}{secx}.secx.tanx.dx$

$\Rightarrow dz = tanxdx$

$I = zdz = \frac{z^2}{2} = \frac{(lnsecx)^2}{2}$

Q No. 33 $I = \int \frac{dx}{(3tanx+1)cos^2x}$

$I = \boxed{\int \frac{\sec^2 xdx}{3tanx+1}} = \frac{1}{3}\int \frac{3\sec^2 xdx}{3tanx+1} = \frac{1}{3}\ln(3tanx+1)$

Q No. 34 $I = \int e^{sinx} cosx\, dx$

Put $sinx = z \Rightarrow cosxdx = dz$

$I = \int e^z dz = e^z = e^{sinx}$

Q No. 35 $I = \int \sqrt{1 + 3cos^2x}\, sin2xdx$

Put $\cos^2 x = z \Rightarrow 2cosx(-sinx)dx = dz$

$\Rightarrow -2sinx.cosxdx = dz$ or $sin2xdx = -dz$

$I = -\int (1+3z)^{\frac{1}{2}} dz$

$I = -\frac{1}{3}\int (1+3z)^{\frac{1}{2}}.3dz$

$I = -\frac{1}{3}.\frac{(1+3z)^{\frac{1}{2}+1}}{\frac{1}{2}+1}$

$I = -\frac{2}{9}(1+3z)^{\frac{3}{2}}$

$I = -\frac{2}{9}(1+3\cos^2 x)^{\frac{3}{2}}$

Q No. 36 $I = \int \frac{sin2xdx}{\sqrt{1+cos^2x}}$

Put $\underline{\cos^2 x = z} \Rightarrow 2cosx(-sinx)dx = dz$

$\Rightarrow -2sinx.cosxdx = dz$ or $sin2xdx = -dz$

$I = -\int (1+z)^{\frac{-1}{2}} dz$

$I = -\frac{(1+z)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}$

$I = -2\sqrt{1+\cos^2 x}$

Q No. 37 $I = \int \frac{dx}{2sin^2x+3cos^2x}$

Divide N^r *and* D^r *by* $\cos^2 x$

$I = \int \frac{\sec^2 xdx}{2\tan^2 x + 3}$

Put $\tan x = z$ $\sec^2 xdx = dz$

$I = \int \frac{dz}{2z^2+3} = \frac{1}{2}\int \frac{dz}{z^2 + \frac{3}{2}}$

$I = \frac{1}{2}.\sqrt{\frac{2}{3}}\tan^{-1}\frac{\sqrt{2}z}{\sqrt{3}}$ $\quad as\ (\frac{1}{a}\tan^{-1}\frac{x}{a})$

Q No. 38 $I = \int \frac{1}{\sqrt{x}} sec\sqrt{x}tan\sqrt{x}\, dx$

Put $\sqrt{x} = z \Rightarrow \frac{1}{2\sqrt{x}}dx = dz \Rightarrow \frac{dx}{\sqrt{x}} = 2dz$

$I = \int secztanz.2dz = 2secz = 2sec\sqrt{x}$

Q No. 39 $I = \int [\pi^{sinx} + (sinx)^{\pi}]cosxdx$

Put $sinx = z \Rightarrow cosxdx = dz$

$I = \int \pi^z dz + \int z^{\pi} dz$

$I = \boxed{\frac{\pi^z}{ln\pi}} + \frac{z^{\pi+1}}{\pi+1}$

$I = \frac{\pi^{sinx}}{ln\pi} + \frac{(sinx)^{\pi+1}}{\pi+1}$

Q No. 40 $I = \int \frac{cosxdx}{3sinx+4\sqrt{sinx}}$

Put $\sqrt{sinx} = z \Rightarrow sinx = z^2 \Rightarrow cosxdx = 2zdz$

$I = \int \frac{2zdz}{3z^2+4z} = 2\int \frac{dz}{3z+4} = \frac{2}{3}\int \frac{3dz}{3z+4}$

$I = \frac{2}{3}\ln(3z+4)$

$I = \frac{2}{3}\ln(3\sqrt{sinx}+4)$

Prepared by: **Aqeel Nawaz** aqeel_755@yahoo.com 03349587500