QUESTION NO:- 01

solve the initial value problem

$$(3x+8)(y^2+4)dx - 4y(u^2+5x+6)dy = 0$$

$$\therefore y(1) = 2.$$

$= (3x+8)(y^2+4)du - 4y(n^2+2u+3u+6)dy$

$= \qquad - 4y(n(n+2)+3(n+2))dy.$

$=$

$$(3u+8)(y^2+u)du = 4y(n+3)(u+2)dy.$$

$$\int \frac{3u+8}{(n+3)(x+2)} du. = \int \frac{4y}{y^2+4} dy.$$

$$= 2\int \frac{2y}{y^2+4} dy.$$

$$= \ln(y^2+4) + c$$

Let $\frac{3u+8}{(n+3)(n+2)} = \frac{A}{x+3} + \frac{B}{x+2}$

$$3n+8 = A(n+2) + B(u+3)$$

$x+3=0 \Rightarrow x=-3$

$\cancel{\frac{-9+8}{(\ )}} \cancel{=} \cancel{\frac{A}{-3+3}} \cancel{+} \cancel{\frac{B}{-3+2}}$

$$-9+8 = A(-3+2)$$

$$-1 = A(-1)$$

$$\boxed{A = 1}$$

let $x = -2$

$$3x+8 = A(x+2) + B(x+3)$$

$$3(-2)+8 = B(-2+3)$$

$$-6+8 = B$$

$$\boxed{B = 2}$$

$$\frac{3x+8}{(x+3)(x+2)} = \frac{1}{x+3} + \frac{2}{x+2}$$

$$\int \frac{1}{x+3} + \frac{2}{x+2} \, dx = 2\ln(y^2+4) + c$$

$$\int \frac{1}{x+3} \, dx + \int 2 \cdot \frac{1}{x+2} \, dx = 2 \ln(y^2+4) + c$$

$$\ln(x+3) + 2\ln(x+2) = 2\ln(y^2+4) + c$$

$$x = 1 \quad , \quad y = 2$$

$$\ln(1+3) + 2\ln(1+2) = 2\ln(4+4) + c$$

$$\ln 4 + 2\ln 3 = 2\ln 8 + c$$

$$\frac{3.58}{4.16} = c$$

$c = 0.860$

$\ln (x+2)(x+3) = \ln (y^2+4) + 0.86$

QUESTION NO :- 02

Solve

$$y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$$

$$\frac{y\sqrt{1+x^2}}{xy}\,dx + \frac{x\sqrt{1+y^2}}{xy}\,dy = 0$$

$$\frac{\sqrt{1+x^2}}{x}\,dx + \frac{\sqrt{1+y^2}}{y}\,dy = 0$$

$$\int \frac{\sqrt{1+x^2}}{x}\,dx = \int \frac{t}{x}\cdot\frac{t}{x}\,dt$$

let $\sqrt{1+x^2} = t$

$1+x^2 = t^2$

$x^2 = t^2-1$

$2x\,dx = 2t\,dt$

$dx = \frac{t}{x}\,dt$

$$= \int \frac{t^2}{x^2}\,dt$$

$$= \int \frac{t^2}{t^2-1}\,dt$$

$$= \int \left(1+\frac{1}{t^2-1}\right)dt$$

$$= \int dt + \int \frac{1}{t^2-1}\,dt$$

$$= t + \frac{1}{2}\ln\frac{t-1}{t+1}$$

$$= \sqrt{1+x^2} + \frac{1}{2}\ln\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}$$

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$$= \sqrt{1+x^2} + \frac{1}{2} \ln \frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1} \times \frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}-1}$$

$$= \sqrt{1+x^2} + \frac{1}{2} \ln \frac{(\sqrt{1+x^2}-1)^2}{\cancel{1}+x^2-\cancel{1}}$$

$$= \sqrt{1+x^2} + 1/2 \ln \frac{(\sqrt{1+x^2}-1)^2}{|x|}$$

Similarly;

$$\int \frac{\sqrt{1+y^2}}{y}\, dy = \sqrt{1+y^2} + 1/2 \ln \frac{\sqrt{1+y^2}-1}{|y|}$$

Solve

$$\frac{dy}{dx} = \frac{y-x+1+5-5}{y-x+5}$$

$$= \frac{\cancel{y-x+5}}{\cancel{y-x+5}}^{\,1} + \frac{y-x-4}{y-x+5}$$

QUESTION NO :- 03

$$\frac{dy}{dx} = \frac{y-x+1}{y-x+5} \quad \text{— (1)}$$

Put $y - x = z$ — (2)

$$\frac{dy}{dx} - 1 = \frac{dz}{dx}$$

$$\frac{dy}{dx} = \frac{dz}{dx} + 1 \quad \text{— (3)}$$

Using (3) & (2) ; equation (1) becomes,

$$1 + \frac{dz}{dx} = \frac{z+1}{z+5}$$

$$\frac{dz}{dx} = \frac{z+1}{z+5} - 1$$

$$\frac{dz}{dx} = \frac{z+1-z-5}{z+5} = {-4}/{z+5}$$

$$\int (z+5)\, dx = \int -4\, dx$$

$$\int (z+5)\, dy = -4 \int dx$$

$$\frac{z^2}{2} + 5z = -4x + c.$$

$$\frac{z^2 + 10z}{2} = -4x + c$$

$$z^2 + 10z = -8x + 2c$$

$$(y-x)^2 + 10(y-x) = -8x + c'$$

$$(y-x)^2 + 10(y-x) + 8x = c'$$

$$(y-x)^2 + 10y - 10x + 8x = c'$$

$$\boxed{(y-x)^2 + 10y - 2x = c'}$$

QUESTION NO :- 04

$\frac{dy}{dx} =$

$(\sqrt{x+y} + \sqrt{x-y})\,dx - (\sqrt{x+y} - \sqrt{x-y})\,dy = 0$

$(\sqrt{x+y} + \sqrt{x-y})\,dx = (\sqrt{x+y} - \sqrt{x-y})\,dy$

$$\frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} = \frac{dy}{dx}$$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}}\left[\frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}\right]$$

$$= \frac{(1+v) + (1-v) + 2(\sqrt{1+v}\,\sqrt{1-v})}{(1+v)(1-v)}$$

$$= \frac{2 + 2\sqrt{1-v^2}}{2v.}$$

$$= \int \frac{v}{(1+v^2) + \sqrt{1-v^2})} \, du = \int \frac{du}{u}$$

$$= \int \frac{v}{1-v^2+\sqrt{1-u^2}} \, dx \qquad \text{put } v = \sin u$$

$$dv = \cos u \, du.$$

$$= \int \frac{\sin u \cos u}{1-\sin^2 u + \sqrt{1-\sin^2 u}} \, du$$

$$= \int \frac{\sin u \cos u}{\cos^2 u + \cos u} \, du$$

$$= -\int \frac{-\sin u}{\cos u + 1} \, du.$$

$$= -\ln(\cos u + 1)$$

$$= -\ln(\cos u + 1) - \ln u + \ln c$$

$$= \ln(\sqrt{1-\sin^2 u} + 1) = \ln c u.$$

$$\ln\left(\frac{\sqrt{x^2-y^2}}{x} + x\right)^{-1} = \ln c x.$$

$$c = \sqrt{x^2-y^2} + x$$