Errata, precisions

Section 3.2.1

- Observation 3.8 / Equation (3.11), p. 28: The elements in max/min of $\tilde{l}_i$ must be taken with the absolute value.

Section 3.2.2

- The subroutine computing angles must handle the case where subsequent control points are equal.
- For the simplified angle method it is not necessary to perform the complete affine transformation, as $-x|x|$ has the desired variations, but the complete transformation is in better sync with the syntax of the rest of the program.
- Default threshold values: The threshold value using the simplified method merits a few comments. Say we will have $\alpha_\tau \in [0, 10)$ measured in degrees. The simplified method returns angles

  $$\tilde{\alpha} = -\frac{\pi}{2} \cos\alpha |\cos\alpha| + \frac{\pi}{2}. \qquad (1)$$

  A regression fit between α and $\tilde{\alpha}$ on the interval specified gives an excellent ($r^2 = 1.000$) fit for a quadratic polynomial $f(x) = \tau_c x^2$, with $\tau_c = 1.543$. Se Figure 1. The correspondence is shown in red for angles $\alpha \in [0, 10)$ yielding angles $\tilde{\alpha} \in [0, 2.8)$. The regression fit is shown in black.

Section 3.2.3

- Baseline-method: default threshold value r is obtained from $\sin \frac{\pi - \alpha_{tau}}{2} = rl/l, \alpha_{tau} = \frac{\pi}{180}$ (2 degrees).
- Length-ratio method: threshold value must be linked to worst-case scenario: right angle (or flat angle, beyond the point?). Approximation: $R = 1 + \delta / l_{max}$.
- The length-ratio method offers some reuse of length-calculations from the length-method. The combination of these two methods merits more study, i.e., the application of the length-method in a prephase and a second phase of refinement using the length-ratio method.

Section 3.4

- Knot placement equation: From Böhms equation, setting $\omega_i = \frac{z - t_i}{t_{i+p} - t_i}$, we have

  $$\bar{t}_i^* = \omega_i t_i^* + (1 - \omega_i) t_{i-1}^*. \qquad (2)$$

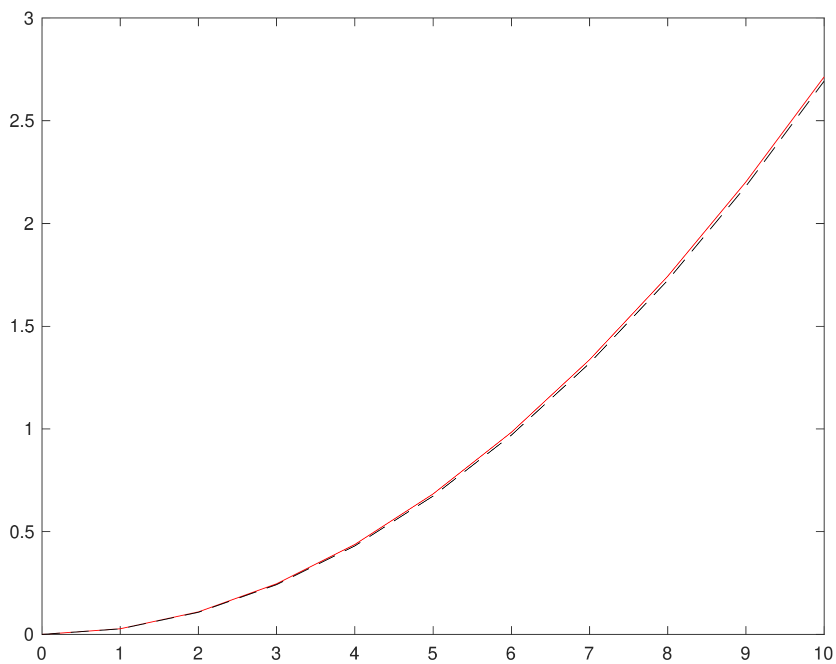

Figure 1: *Correspondance between the actual angle and the angle returned by the simplified method for small angles (degrees).*

If we want a new control point at a ratio $\omega_\mu/(1-\omega_\mu)$ between control points c_μ and $c_{\mu-1}$, then solving for z gives

$$z = \omega_\mu(t_{\mu+p} - t_\mu) + t_\mu. \tag{3}$$

This means, if we want a 'tighter' control point at c_μ, choose $\omega = 1$, i.e., set $z = t_{\mu+p}$. If we want a new control point midway between two old control points c_μ and $c_{\mu-1}$, choose $\omega = 1/2$, i.e., set $z = t_\mu + \frac{1}{2}(t_{\mu+p} - t_\mu)$.

- Knot placement: if we want to imitate Chaikin's algorithm, it can only be done one segment at a time. We cannot in general impose that the insertion of a single knot cuts all p segments in half.

- Stopping criterion: is it cheaper/better to check the p new control points using the distance threshold eps, or after all better to define absolute stopping criterions depending on C?

- A partitioning Σ can be kept or updated between iterations. If it refers to indices, the update is straigthforward. If it refers to original control point values, then a rule must be given for how to decide the new break point indices when some of these old break point values disappear.

- Notation 3.2: It is more clever to index while referencing 'to the left' in the control polygon, i.e., that the segment $[c_{\mu-1}, c_\mu)$ is associated with c_μ, to keep indexing closer to the indexing used in Böhms equation. We should have introduced the backwards difference $\Delta P_i = P_i - P_{i-1}$ to ease implementation of the methods.

Cosmetics

- It is better with figures numbered according to chapter.

Next steps

- Algorithm: general structure, implementing more of the auxiliary functions.

- Generalize and show feasibility of methods for coefficient dimension s=3.