UNDERSTANDING ADVERTISING ADSTOCK TRANSFORMATIONS

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ABSTRACT

Advertising effectiveness and Return on Investment (ROI) are typically measured through econometric models that measure the impact of varying levels of advertising Gross Ratings Points (GRPs) on sales or on purchase decision and choice. TV advertising has both dynamic and diminishing returns effects on sales, different models capture these dynamic and nonlinear effects differently. This paper focuses on reviewing the econometric rationale behind the popularized Adstock transformation model that allows the inclusion of lagged and non-linear effects in linear models based on aggregate data.

1. Introduction

This paper is intended as a review of existing models of Television Adstock transformations that enable the inclusion of dynamic and nonlinear effects of Television advertising within linear sales response models.

Television advertising is one of the largest investments for consumer marketing and companies invest a lot of effort in measuring the impact and ROI of TV advertising. This is typically done using either individual response (e.g. Discrete Choice models) aggregate response (e.g. Marketing-mix models). For the purposes of this paper, we will restrict our inquiry to aggregate response models. These models are linear in parameters but can account for non-linearity through variable transformations.

It is well known that TV advertising has both dynamic and diminishing returns effects on sales. Television advertising has an effect extending several periods after the original exposure, which is generally referred to as advertising carry-over or 'Adstock' (Broadbent, 1979).

The underlying theory of Adstock is that exposure to Television Advertising builds awareness in consumer markets, resulting in sales. Each new exposure to advertising increases awareness to a new level and this awareness will be higher if there have been recent exposures and lower if there have not been. This is the decay effect of Adstock and this decay eventually reduces awareness to its base level, unless or until this decay is reduced by new exposures.

1.1. Decay Effect

This decay effect can be mathematically modelled and is usually expressed in terms of the 'half-life' of the advertising. A 'two-week half-life' means that it takes two weeks for the awareness of an advertising to decay to half its present level. Every Ad copy is assumed to have a unique half-life. Some academic studies have suggested half-life range around 7-12 weeks (Leone 1995), while industry practitioners typically report half-lives between 2-5 weeks, with the average for Fast Moving Consumer Goods (FMCG) Brands at 2.5 weeks.

Adstock half-life can be estimated through a distributed lag model response with lags of the TV Gross Ratings Point (GRP) variable, using Least Squares, or from the lag parameter in the Adstock formulation (geometric lag), or indirectly using a 't-ratio' method by recursively testing different values for the decay parameter through an iterative process with sales panel data or awareness/image tracking data against the corresponding advertising schedules (Fry, Broadbent, Dixon 1999).

1.2. Diminishing Returns Effect

Advertising can also have diminishing returns to scale or in other words the relationship between advertising and demand can be nonlinear. For example, the effect of 200 GRPs of advertising in a week on demand for a brand maybe less than twice that achieved with 100 GRPs of advertising. Typically, each incremental amount of advertising causes a progressively lesser effect on demand increase. This is a result of advertising saturation.

The usual approach to account for saturation is to transform the advertising variable to a non-linear scale for example log or negative exponential transformations. It is this transformed variable that is used in the sales response models.

T_t can be transformed to an appropriate nonlinear form like the logistic or negative exponential distribution, depending upon the type of diminishing returns or 'saturation' effect the response function is believed to follow.

For example if advertising awareness followed a logarithmic distribution, then in a linear sales response model we would have:

$$S_T = \beta Log\ (T_T) + \varepsilon_T \qquad\qquad (1)$$

Where S_T is sales at time T, T_T is the level of advertising GRPs at time T and ε_T is the random error component. In this case for 100 GRPs and assuming $\beta = 1$, the sales effect of advertising would be 4.6 units and for 200 GRPs the sales effect would be 5.3 units. Therefore, for a 100% increase in advertising we would only have a 15% increase in sales. Advertising typically has a lower elasticity than other elements of the marketing-mix. This is considered acceptable by Brand Managers since advertising is also believed to have a long-term positive effect on Brand Equity, which is usually not captured by most econometric models.

Several versions of Adstock transformation are applied in the industry, we will examine some popular models in the following sections.

2. Adstock Models

2.1. Simple Decay-Effect Model
Below is a simple formulation of the basic Adstock model of Broadbent (1979):

$$A_t = T_t + \lambda A_{t-1} \qquad t=1,\ldots, n \qquad\qquad (2)$$

Where A_t is the Adstock at time t, T_t is the value of the advertising variable at time t and λ is the 'decay' or lag weight parameter. Inclusion of the A_{t-1} term,

imparts an infinite lag structure to this model, with the effect of the first Adstock term, approaching *0,* as *t* tends to ∞*.*

This is a simple decay model, because it captures only the dynamic effect of advertising, not the diminishing returns effect.

This model is also approximately equivalent to an infinite distributed lag model as shown below:

From (1) we have, $A_t = T_t + \lambda A_{t-1}$

Recursively substituting and expanding we have,

$$A_t = T_t + \lambda T_{t-1} + \lambda^2 T_{t-2} + \dots \lambda^n T_{t-n} \qquad (3)$$

Since *λ* is normally less than 1, λ^n will tend to *0* in limit as *n* tends to ∞. Therefore, this infinite polynomial distributed lag can be approximated by imposing a finite lag structure within an Almon distributed lag model (Almon 1965).

2.2. Log Decay Model

The Log Decay model applies a straightforward logarithmic distribution to the advertising variable

$$A_t = LogT_t + \lambda A_{t-1} \qquad (4)$$

This is a relatively inflexible non-linear specification of the Adstock model, as it doesn't allow for varying saturation levels.

2.3. Negative Exponential Decay Model

The below formulation applies a negative exponential distribution to the basic Adstock formula, using two parameters, the 'decay' or lag weight parameter *λ* and the learning rate or saturation parameter v.

$$A_t = 1\text{-}e^{(-v\ Tt)} + \lambda A_{t-1} \qquad (5)$$

This model is comparatively more flexible as different values of the parameter *v* can be empirically tested in a response model to correctly measure the level of current advertising saturation.

2.4. Logistic (S-Curve) Decay Model

Using a logistic distribution instead of negative exponential will impart an S-shape to the Adstock variable, implying an inflexion point or 'threshold' level

of GRPs before diminishing returns set in. Below this threshold, the logistic function imparts exponential returns.

$$A_t = 1/(1+e^{(-v\ Tt)}) + \lambda A_{t-1} \qquad (6)$$

As in the negative exponential model, the parameter v can be used to model different saturation levels.

3. Half-Life Estimation

Advertising half-life, η is calculated in the same manner as estimating decay half-life for radioactive substances:

Assume that in time-period $t+n$, A_t would have decayed to $A_t/2$. Therefore using equation (1) and assuming that no new advertising is present (so the first term in the equation equals zero),

$$A_{t+1} = \lambda A_t \qquad (7)$$

In time period $t+n$, where n is the value of the half-life η, we have,

$$A_{t+n} = A_t/2 \qquad (8)$$

Therefore from (3) and (4), and recursively substituting,

$$A_t/2 = \lambda^n A_t \qquad (9)$$

And,

$$\lambda^n = 1/2 \qquad (10)$$

Finally taking n as the value of the Half-Life η, we have,

$$\eta = Log\ (0.5)/Log\ (\lambda) \qquad (11)$$

Different values for λ can be empirically tested in an econometric model to estimate the half-life for an advertising program.

For example, a λ of 0.25 implies a half-life of 2.4 weeks.

4. Conclusion

Adstock transformation is an efficient and effective technique to incorporate nonlinear and dynamic advertising effects in sales response models. The alternative option of building a dynamic nonlinear model is both computationally expensive and complex to estimate.

The industry popularity of Adstock transformations is due to its relatively simple nature and direct applicability to measuring Advertising Saturation. It helps Brand Managers determine if more or less investment is needed to make advertising more effective and yield a better ROI. One point that should be kept in mind is that the minimum threshold ROI acceptable is typically a break-even ROI, since in the short-term advertising pays for itself and there is a longer-term benefit over and above the short-term return in the form of Brand Equity improvement.

Measuring the Advertising Half-Life enables Brand Managers to also efficiently space advertising schedules to maximize the effect of each advertising exposure. Depending on whether the half-life of a copy is long or short exposures can be efficiently spaced to maximize sales response.

References

Almon, S. (1965) "The distributed lag between capital appropriations and expenditures", *Econometrica*, 33, 178-196.

Broadbent, S. (1979) "One Way TV Advertisements Work", *Journal of the Market Research Society* Vol. 23 no.3.

Dubé, J., Hitsch, G. J., Manchanda, P. (2004) "An Empirical Model of Advertising Dynamics", *Quantitative Marketing and Economics*, 3 (2005) 2 Pg. 107-144

Fry, T.R.L., Broadbent, S. and Dixon, J.M. (2000), "Estimating Advertising Half-life and the Data Interval Bias", *Journal of Targeting, Measurement & Analysis in Marketing*, 8, 314-334.

Leone, R.P. (1995) "Generalizing what is known about temporal aggregation and advertising carry-over", *Marketing Science*, 14, G141-G150.