InfiniteSeries

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1 Trigometric Series

1.1 partial sum of $\sin k$ is bounded, while for $\sin k^2$ it is not

For the first series, note that

$$2 \sin k \sin 0.5 = \cos(k - 0.5) \cos(k + 0.5).$$

. But this trick does't work since k^2 is not a arithmetic progression.

1.1.1 Tao's answer on the second part

No. If one selects a number k at random from 1 to a large number n, then for any fixed h, the random variables sin((k+1)2),. . . ,sin((k+h)2) asymptotically have mean zero, variance 1/2, and covariances 0, from standard Weyl sum estimates. Hence the variance of hi=1sin((k+i)2) is asymptotically h/2, which goes to infinity as h→. On the other hand, if the partial sums of sin(k2) were bounded, then this variance would have to be bounded also. [Exercise: what part of the above argument breaks down when working with sin(k) instead of sin(k2)?]

It may be possible to push this argument to show that the partial sums have to fluctuate by n infinitely often, but I haven't checked this (certainly a lower bound of n for some small >0 should be possible from the above argument, perhaps contingent on some conjecture about the irrationality measure of). Heuristically, the law of the iterated logarithm suggests that the sum can occasionally get as large as nloglogn, but no larger.