1 st year 2nd semester Exam 2013.(2012-13)

Subject: Mathematics

MAT-103BM, Credit: 03 Time .03 Hours

Calculus and Differential Equations

Answer any five Questions.

1	(a)	How do you define the limit of a function f(x) at a point x_0? Using the $(\delta,\in)$ definition of limit, prove that $\underset{x\to 3}{Lt}\left(2x^3-3x^2-18x+29\right)=2$.	5
	(b)	What condition must be satisfied by a function if it is to be continuous at a point? Graph the function f(x)= $\begin{cases} x;0\le x<\frac{1}{2} \\ 1-x;\frac{1}{2}\le x<1 \end{cases}$ Examine the continuity of f(x) at x=$\frac{1}{2}$.	5
	©	What does it mean for a function to be differentiable? Let f(x)= $\begin{cases} 5x-4;0<x\le 1 \\ 4x^2-3x;1<x<2 \\ 3x+4;x\ge 2 \end{cases}$. Discuss $f'(x)$ at x=1 and x=2.	4
2	(a)	Differentiate $x^{\sin x}$ w.r. to $(\sin x)^x$.	5
	(b)	State Leibnitz's theorem. If $y=\left(x+\sqrt{x^2+1}\right)^m$, show that $\left(1+x^2\right)y_{n+2}+(2n+1)xy_{n+1}+\left(n^2-m^2\right)y_n=0$	5
	©	State Rolle's theorem. Show that the conclusion to the Rolle's theorem satisfy for the case f(x)=cos x on $[\pi,5\pi]$	4
3	(a)	Find the nth derivative of the function y=$e^x\sin^2 x$	4
	(b)	Find the open intervals on which f(x)=5+$12x-x^3$, is increasing, decreasing, concave up and concave down. Also find their relative extrema.	4
	©	If V is a function of x and y, prove that $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}+\frac{1}{r^2}\frac{\partial^2 v}{\partial r^2}$, where x = r $\cos\theta$ and y= $r\sin\theta$.	6
4		Answer any four of the followings: Evaluate (i) $\int\frac{dx}{\sqrt{2x^2+3x+4}}$ (ii) $\int\frac{(2x-3)}{3x^2+4x-7}dx$ (iii) $\int(4x+15)\sqrt{x^2+6x+10}dx$ (iv) $\int\frac{dx}{\left(1+x^2\right)\sqrt{1-x^2}}$	14

		(v) $\int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x}\,dx$	
5	(a)	Define definite integral as a limit of a sum. Evaluate from first principle, $\int_a^b e^x dx.$	4
	(b)	Show that $\int_0^1 \frac{\log(1+x)}{1+x^2}dx = \frac{\pi}{8}\log 2.$	5
	©	If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x\, dx$, then show that $I_n + I_{n-2} = \frac{1}{(n-1)}$ and hence obtain the value of I_5.	5
6	(a)	How do you calculate thr area of the region between the graphs of two continuous functions? Find the area bounded on the right by the line y = x- 2 , on the left by the parabola $x = y^2$ and below by the x- axis.	5
	(b)	Define improper integral. Evaluate $\int_1^4 \frac{1}{(x-2)^{\frac{2}{3}}}dx$	4
	©	Evaluate $\iint \sqrt{4x^2 - y^2}\,dx\,dy$ over the triangle formed by the straight lines y = 0, x = 1, y = x.	5
7	(a)	What do you mean by order and degree of a differential equation? Write down the general form of 1st order and 1st degree ODE. Solve the IVP: $4xydx + (x^2+1)dy = 0; y(0) = 1)$ by the method of variables separable.	5
	(b)	Test the exactness of ODE: $(y^2e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$ and find its general solution.	5
	©	Find the integrator factor of the linear differential equation $\cos^2 x\frac{dy}{dx} + y = \tan x$ and hence solve it.	4
8	(a)	Solve the differential equation $y'' + 4y = 12x^2 - 16x\cos 2x$ by the method of undetermined coefficients.	7
	(b)	Solve the differential equation $y'' - 2y' + y = e^x \sin^{-1} x$ by the method of variation of parameters.	7

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