9. Perspective Projection

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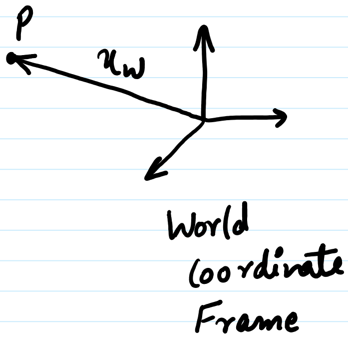
Source: First Principles of Computer Vision, Prof. Shree Nayyar

From <https://fpcv.cs.columbia.edu/>



Camera lies on the world coordinate frame.

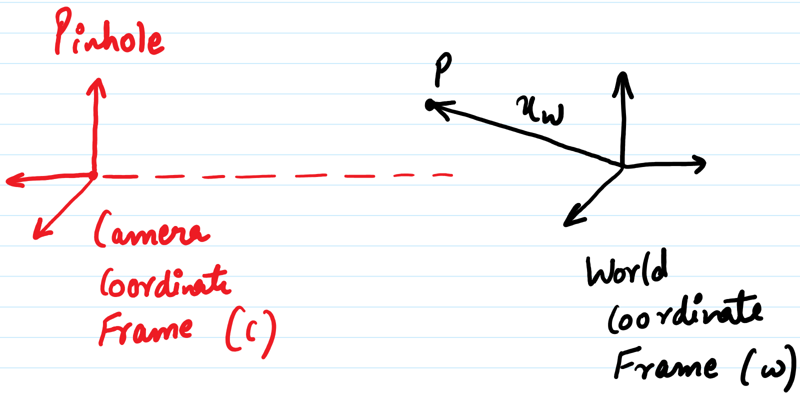
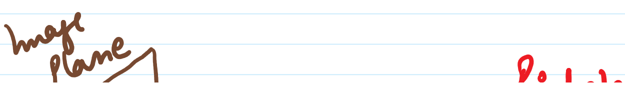
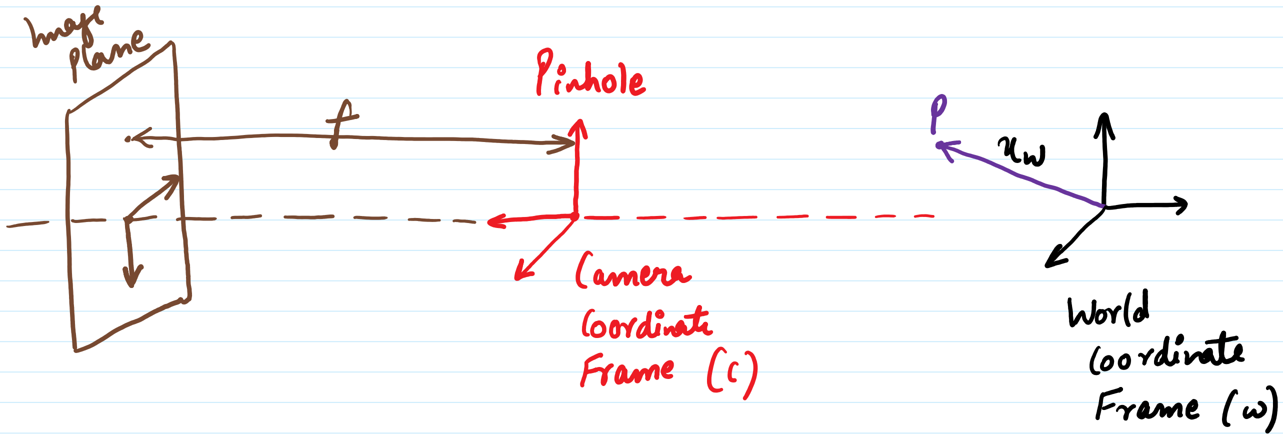
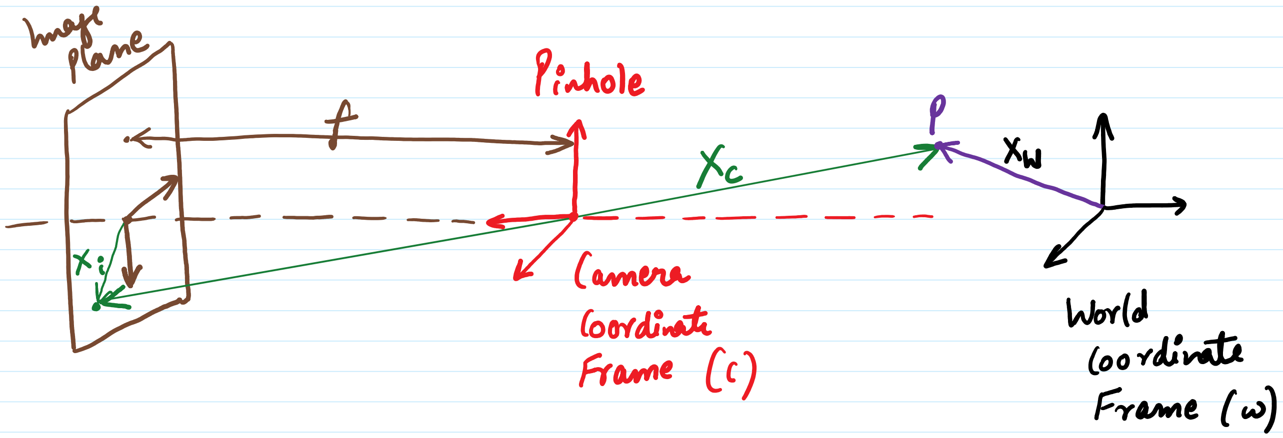


Image Plane is at a distance of 'f' from the camera frame 'c'
This distance is called as "Focal Length (f)"





The goal is to know the relative position of 'C' wrt 'W' to take from point P in 'W' to point x_i in the image plane.



$$X_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \qquad X_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} \qquad X_W = \begin{bmatrix} x_W \\ y_W \\ z_W \end{bmatrix}$$

Image Coordinates — Camera Coordinates — World Coordinates

Steps in 3D to 2D Imaging Model:-

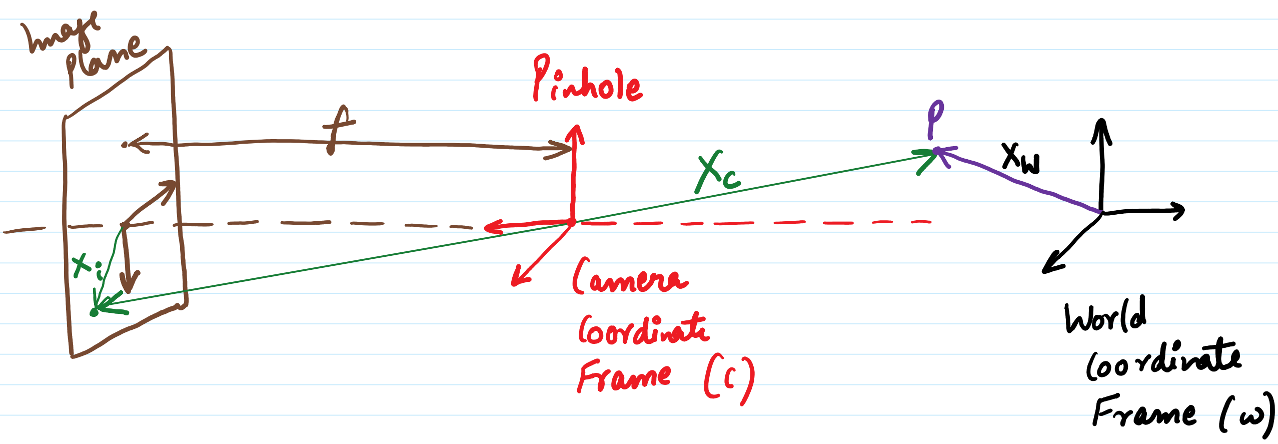
① Coordinate Transformation

World Coordinates $\longrightarrow$ Camera Coordinates

② Perspective Projection

Camera Coordinates $\longrightarrow$ Image Coordinates

Perspective Projection :-



$$X_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \qquad X_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

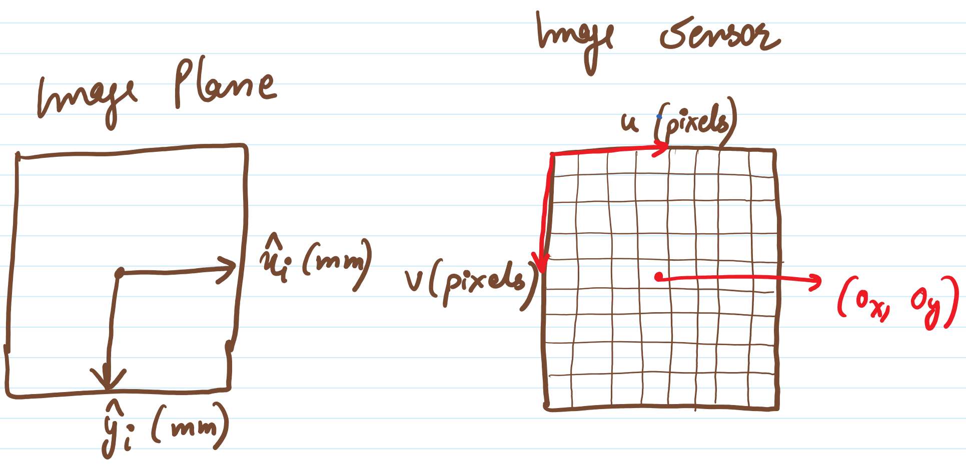
Image Coordinates   Camera Coordinates

From diagram,

$$\frac{x_i}{f} = \frac{x_c}{z_c} \quad \text{and} \quad \frac{y_i}{f} = \frac{y_c}{z_c}$$

$$\Rightarrow \quad x_i = f\frac{x_c}{z_c} \quad \text{and} \quad y_i = f\frac{y_c}{z_c} \quad \text{——— (1)}$$

where (x_i, y_i) are the coordinates of points on the image.



If m_x and m_y are the pixel densities (pixels/mm) in x and y directions,

$\Rightarrow$ Top-left corner is origin.

$\Rightarrow$ (o_x, o_y) is the principle point where optical axis pierces.

Then Pixel coordinates becomes:

from equation (1),

$$u = m_x x_i = m_x f\frac{x_c}{z_c} + o_x$$

$$u = m_x x_i = m_x f \frac{x_c}{z_c} + o_x$$

$$v = m_y y_i = m_y f \frac{y_c}{z_c} + o_y$$

— (2)

pixel density

focal length

Pixel density and focal length are unknown.
are properties of the camera.

Let $f_x = m_x f$

$f_y = m_y f$

$\Rightarrow$ Put in equation (2)

$$\boxed{u = f_x \frac{x_c}{z_c} + O_x, \quad v = f_y \frac{y_c}{z_c} + O_y}$$

$$\boxed{u = f_x \frac{x_c}{z_c} + O_x, \quad v = f_y \frac{y_c}{z_c} + O_y}$$

4 unknowns

$(f_x, f_y) \rightarrow$ focal length in x and y direction.

$(o_x, o_y) \rightarrow$ Principle point.

$(f_x, f_y, o_x, o_y) \rightarrow$ Intrinsic Parameters of the camera.

"Camera's Internal Geometry"