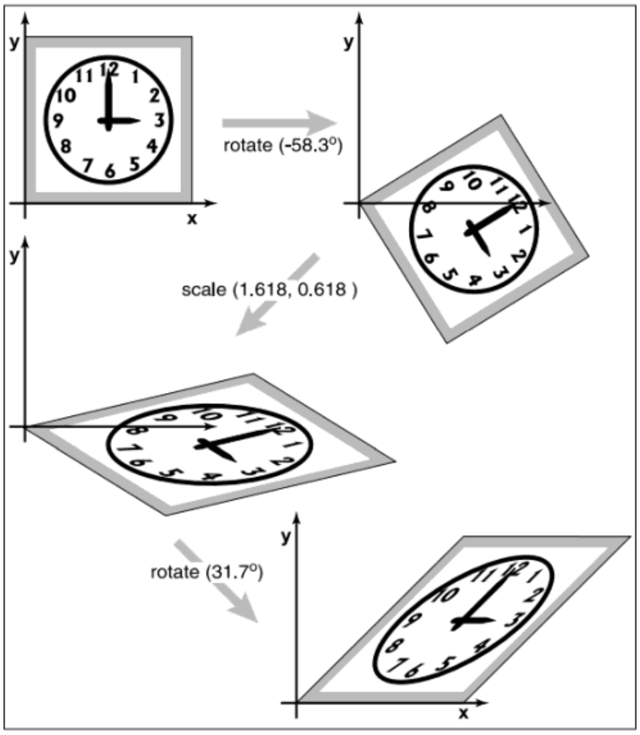
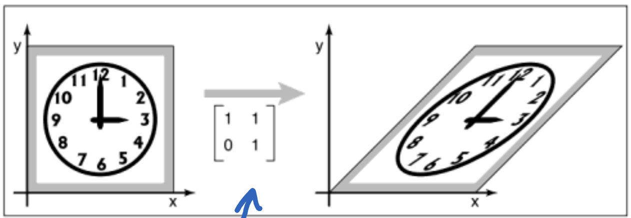
Linear Transformations-3 (23/1/24)

23 January 2024 12:08

Decomposition of Transformations :-

"undo" the compositions.





Shearing

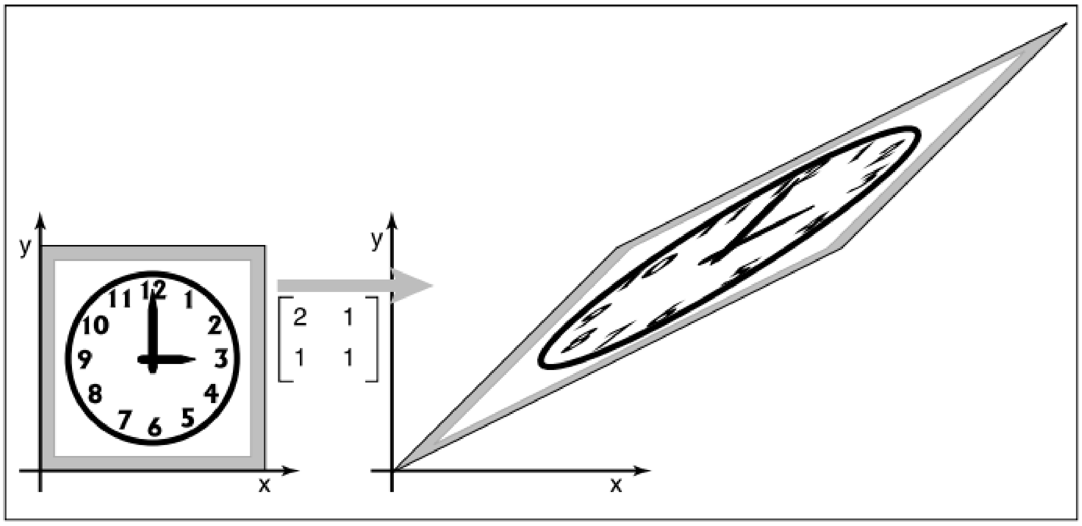
How? Using Symmetric Eigenvalue Decomposition.

$$A = RSR^T$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \mathbf{R} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \mathbf{R}^{\mathrm{T}}$$

$$= \begin{bmatrix} 0.8507 & -0.5257 \\ 0.5257 & 0.8507 \end{bmatrix} \begin{bmatrix} 2.618 & 0 \\ 0 & 0.382 \end{bmatrix} \begin{bmatrix} 0.8507 & 0.5257 \\ -0.5257 & 0.8507 \end{bmatrix}$$

$$= \text{rotate}\,(31.7^\circ)\ \text{scale}\,(2.618, 0.382)\ \text{rotate}\,(-31.7^\circ).$$

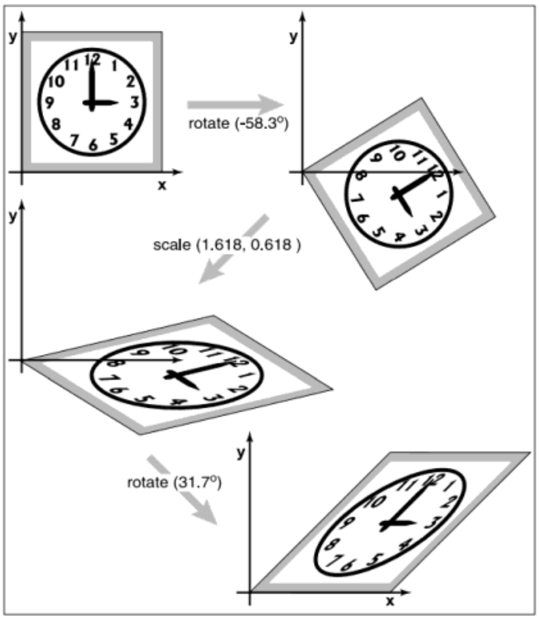


Singular Value Decomposition

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \mathbf{R}_2 \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \mathbf{R}_1$$

$$= \begin{bmatrix} 0.8507 & -0.5257 \\ 0.5257 & 0.8507 \end{bmatrix} \begin{bmatrix} 1.618 & 0 \\ 0 & 0.618 \end{bmatrix} \begin{bmatrix} 0.5257 & 0.8507 \\ -0.8507 & 0.5257 \end{bmatrix}$$

$$= \text{rotate}\,(31.7^\circ)\ \text{scale}\,(1.618, 0.618)\ \text{rotate}\,(-58.3^\circ).$$

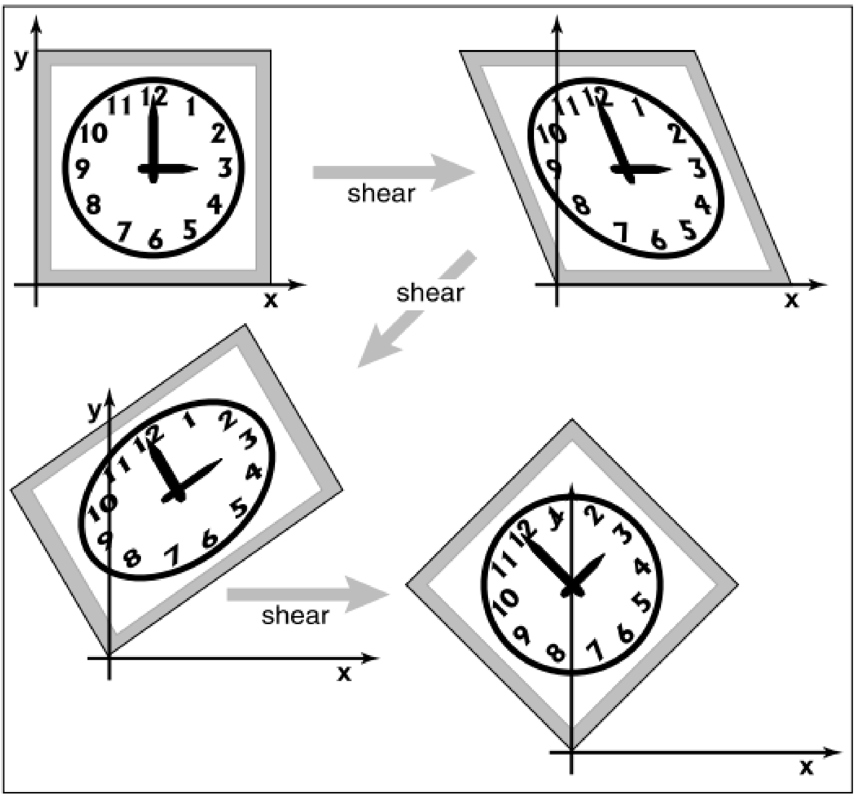


Paeth Decomposition of Rotation (Paeth 1990)

Paeth Decomposition of Rotation (Paeth, 1990)

$$\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} = \begin{bmatrix} 1 & \frac{\cos\phi - 1}{\sin\phi} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \sin\phi & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{\cos\phi - 1}{\sin\phi} \\ 0 & 1 \end{bmatrix}$$

Example: Rotate(45°) $= \begin{bmatrix} 1 & 1-\sqrt{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \sqrt{2}/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1-\sqrt{2} \\ 0 & 1 \end{bmatrix}$



3D Linear Transformations

① scale (s_x, s_y, s_z) $= \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}$

② Rotation about x-axis :- $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$

② Rotation about x-axis :- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$

y-axis :- $\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$

z-axis :- $\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

③ Shear-x $(d_y, d_z) = \begin{bmatrix} 1 & d_y & d_z \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$