28. Ray Tracing (cont.)

31 March 2024 19:10

Source: 1. Ray Tracing Essentials, Part-1 to 7
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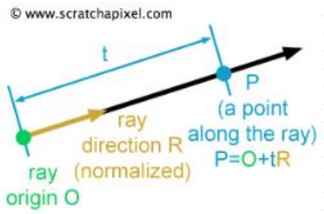
2. 3D Computer Graphics Primer: Ray-Tracing as an Example

From <https://www.scratchapixel.com/lessons/3d-basic-rendering/introduction-to-ray-tracing/implementing-the-raytracing-algorithm.html>

Ray can be mathematically defined as a point (origin) and a direction.

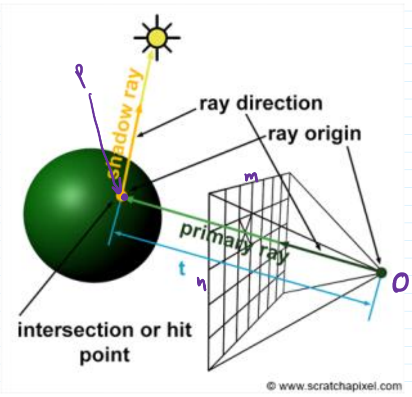


Any point on this ray (P),

$$\boxed{P = O + tR}$$

'O' is ray origin.
't' is distance from origin to the point P.

Ray-tracing is to find mathematical solutions to compute the intersection of this ray with various types of geometry.



$P = O + tR$

Ray-Sphere Intersection :-

Sphere of radius 'r' centered at origin,

$$x^2 + y^2 + z^2 = r^2$$

Let (x, y, z) is on the surface then, $x^2 + y^2 + z^2 = r^2$

If (x, y, z) is inside the surface, $x^2 + y^2 + z^2 < r^2$

If (x, y, z) is outside the surface, $x^2 + y^2 + z^2 > r^2$

If the sphere is not in origin but an arbitrary point (C_x, C_y, C_z), then equation of sphere becomes,

$$(C_x - x)^2 + (C_y - y)^2 + (C_z - z)^2 = r^2 \quad \text{—— (1)}$$

$(C_x - x)^2 + (C_y - y)^2 + (C_z - z)^2 - r^2 = 0$

$\Rightarrow$ Any point P that satisfies this equation is on the sphere.

We want to know if our ray $P = O + tR$ hits the sphere.

$\Rightarrow$ point $P = O + tR$ on the ray should satisfy the equation ①

Let (P_x, P_y, P_z) be the coordinates of the point on the ray being tested.

$$(C_x - P_x)^2 + (C_y - P_y) + (C_z - P_z)^2 = r^2$$

$$\boxed{\left(C_x - (O_x + tR_x)\right)^2 + \left(C_y - (O_y + tR_y)\right)^2 + \left(C_z - (O_z + tR_z)\right)^2 - r^2 = 0}$$

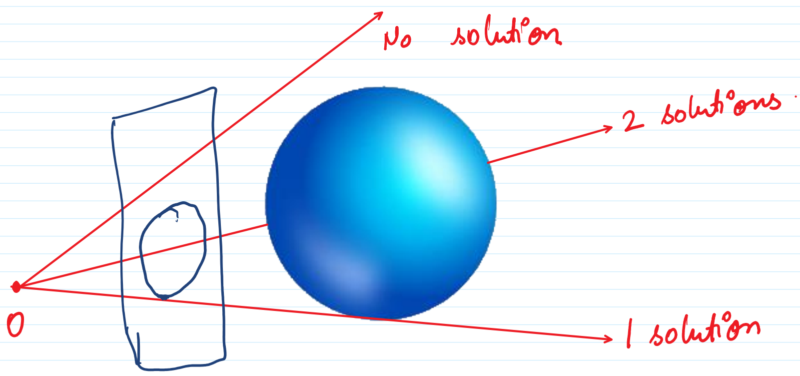
which is a quadratic equation with, $at^2 + bt + c = 0$

known:
- $C = [C_x, C_y, C_z]$
- $P = [P_x, P_y, P_z]$
- $r =$ radius of circle

unknown:
- $t =$ distance of ray for the point being tested.

Solving for t,

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Q:- Consider a ray originating from point $O(-3, -3)$ with direction $R(1, 1)$.

Let P be a point on the ray at a distant 't'.

Find out whether the ray will hit / miss the circle with equation,

$$x^2+y^2-4=0$$

If hit also find out the point(s) of intersection.

$P = O + Rt$

$P_x = -3+t$

$P_y = -3+t$

$(-3+t)^2 + (-3+t)^2 - 4 = 0$

$2(-3+t)^2 - 4$

$2(9 + t^2 + 2x - 3xt) - 4 = 0$

$18 + 2t^2 - 12t - 4 = 0$

$2t^2 - 12t + 14 = 0$

$$\boxed{t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

$(b^2 - 4ac) = (-12)^2 - 4 \times 2 \times 14$

$= 32$

$> 0 \Rightarrow$ 2 solutions

$(-1.415, -1.415)$

$(1.415, 1.415)$