curves

Higher degree approximations :-

① Explicit

$$y = f(x)$$

② Implicit

$$f(x,y) = 0$$

Defines curves implicitly as solution of equation system.

line : $ax + by + c = 0$

circle : $x^2 + y^2 - R^2 = 0$

③ Parametric

$$x = x(t), \; y = y(t)$$

Position on the curve is defined through a parameter.

Parametric curves form a rich variety of free form smooth curves.

"Splines"

Cubic Splines :-

$$P(t) = B_1 + B_2 t + B_3 t^2 + B_4 t^3$$

$$= \sum_{i=1}^{4} B_i t^{i-1} \qquad t_1 \le t \le t_2$$

is defined as a cubic polynomial of the parameter t.

$$\Rightarrow \quad X(t) = \sum_{i=1}^{4} B_n t^{i-1}$$

$$y(t) = \sum_{i=1}^{4} B_i \, t^{i-1}$$

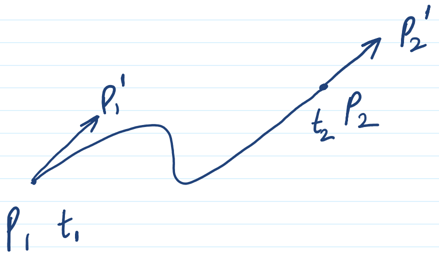
Vetors

$$P'(t) = B_2 + 2B_3 t' + 3B_4 t^2$$

Tangent vectors at some location.

Example:—

Given,



Two end points P_1 and P_2 of the cubic curve.
(position vectors)

P_1' and P_2' are the tangent vectors at
point P_1 and P_2.

These points are defined for the parameter values
t_1 and t_2.

To obtain the generalized curve,

Let $t_1 = 0$

$$P(0) = B_1 = P_1$$

$$P(t_2) = B_1 + B_2 t_2 + B_3 t_2^2 + B_4 t_2^3 = P_2$$

$$P'(0) = B_2 = P_1'$$

$$P'(t_2) = B_2 + 2B_3 t_2' + 3B_4 t_2^2 = P_2'$$

Solving for B_1, B_2, B_3 and B_4

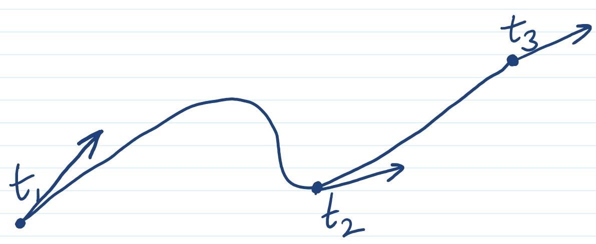
$$B_1 = P_1$$
$$B_2 = P_2$$
$$B_3 = \frac{3(P_2 - P_1)}{t_2^2} - \frac{2(P_1)}{t_2} - \frac{P_2}{t_2}$$

$$B_4 = \frac{2(P_1 - P_2)}{t_2^2} + \frac{P_1'}{t_2^2} + \frac{P_2'}{t_2^2}$$

where P_1 and P_2 gives the position of the endpoints.
and P_1' and P_2' gives the direction of the tangent vectors.

$$P(t) = P_1 + P_1' t + \left(\frac{3(P_2 - P_1)}{t_2} - \frac{2P'}{t_2} - \frac{P_2'}{t_2} \right) t^2$$
$$+ \left(\frac{2(P_1 - P_2)}{t_2^3} + \frac{P_1'}{t_2^2} + \frac{P_2'}{t_2^2} \right) t^3$$

Extending this idea to set of n points.



Joining of Segments

2 SEGMENTS: P_1 P_2 P_3 (Points)

P_1' P_2' P_3' (Tangents)

where P_2 and P_2' are the intermediate point and its tangent vector which is determined through some continuity constraint.

* Piecewise spline of degree k has continuity of order (k-1) at the internal points.

Thus cubic splines have second order continuity i.e. $P_2''(t)$ is continuous over the joint.

$$P''(t) = \sum_{i=1}^{4} (i-1)(i-2) B_i t^{i-3} \quad t_1 \leq t \leq t_2$$

at $t = t_2$

First Segment

$$P'' = 6 B_4 t_2 + 2 B_3$$

Second Segment

$$P'' = 2 B_3$$

So, $(6 B_4 t_2 + 2 B_3)_{seg\,1} = (2 B_3)_{seg\,2}$

Substitute the expressions for B_4 and B_3 and rearranging