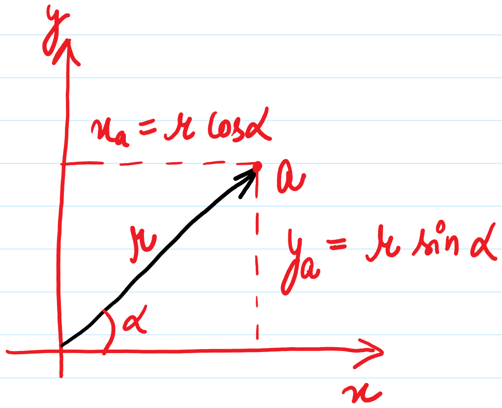
4. Linear Transformations-2 (23/1/24)

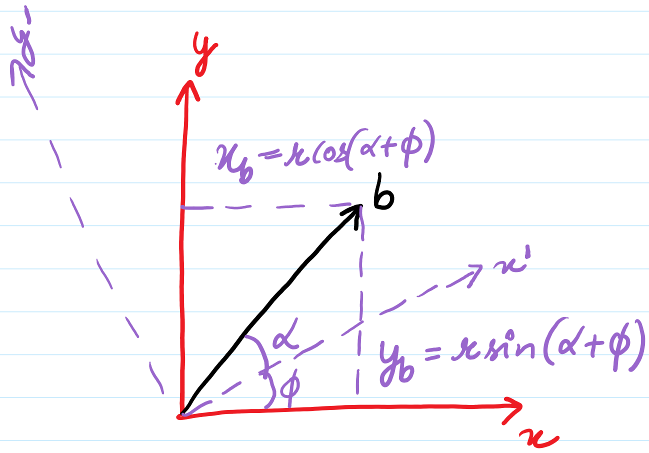
19 January 2024 06:09

Source: "Fundamentals of Computer Graphics, 4th Edition"
by Steve Marschner and Peter Shirley, A K Peters/CRC Press, 2015.

Rotation :-



$$x_a = r\cos\alpha \quad \text{— (1)}$$
$$y_a = r\sin\alpha$$



$$x_b = r\cos(\alpha+\phi) = r\cos\alpha\cos\phi - r\sin\alpha\sin\phi$$

$$y_b = r\sin(\alpha+\phi) = r\sin\alpha\cos\phi + r\cos\alpha\sin\phi$$

from eq (1)

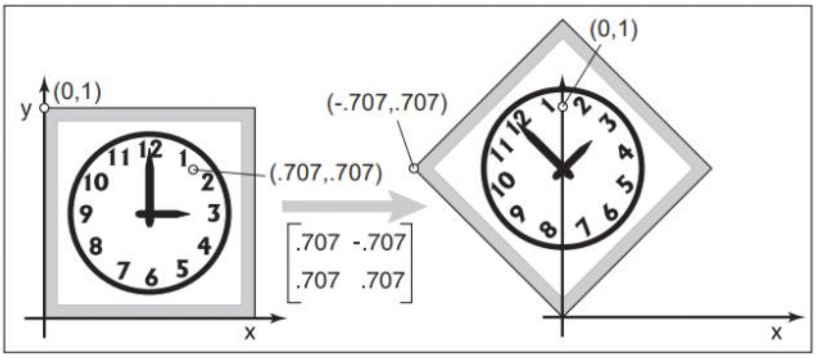
$$x_b = x_a\cos\phi - y_a\sin\phi$$

$$y_b = y_a\cos\phi + x_a\sin\phi$$

$$\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x_a \\ y_a \end{bmatrix} = \begin{bmatrix} x_b \\ y_b \end{bmatrix}$$

$\Rightarrow$ Let $\phi = +45^\circ$

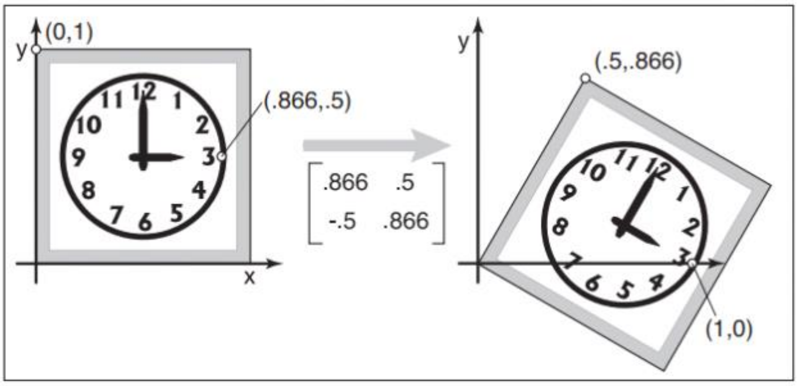
$$\begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix}$$



$\Rightarrow$ Let $\phi = -30^\circ$

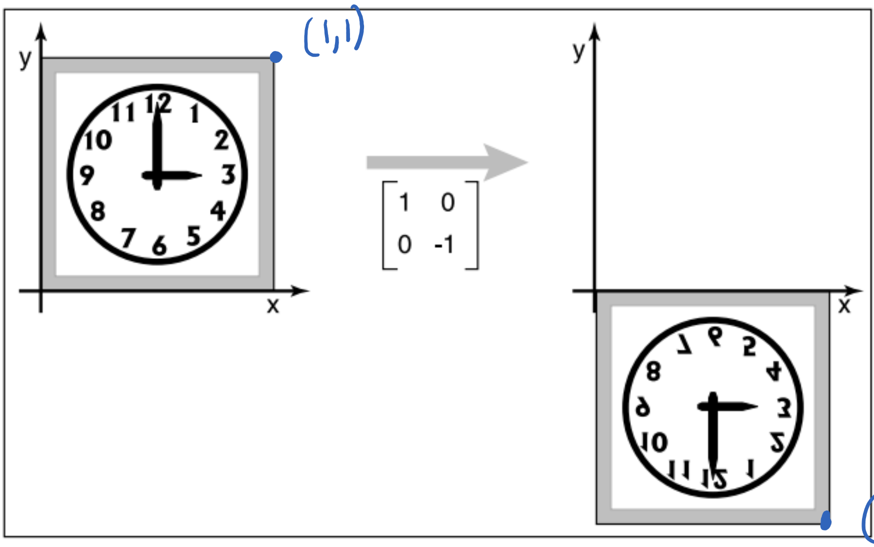
$$\begin{bmatrix} \cos(-30^\circ) & -\sin(-30^\circ) \\ \sin(-30^\circ) & \cos(-30^\circ) \end{bmatrix}$$

$$= \begin{bmatrix} \cos 30^\circ & \sin 30^\circ \\ -\sin 30^\circ & \cos 30^\circ \end{bmatrix}$$



Reflection :-

About x-axis, reflect-x $= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

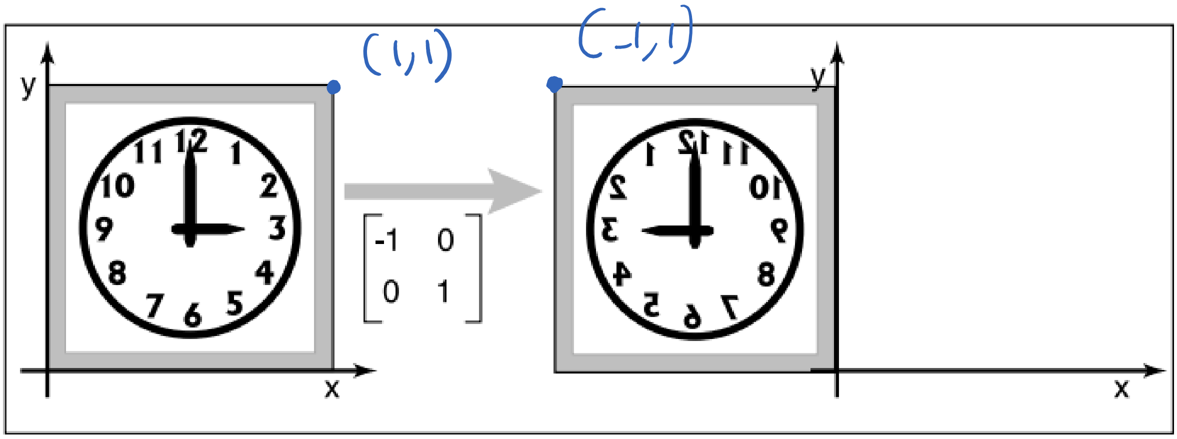


(1,1)

(1,-1)

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

About y-axis, reflect-y $= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$



(1,1)

(-1,1)

Compositions of Transformations :-

Applying more than one transformations.

Let R = Rotation Matrix

S = Scale Matrix

$v_1 \rightarrow Sv_1 \rightarrow Rv_2$

v_2 $\qquad$ v_3

$$\Rightarrow \quad V_3 = R(S v_1)$$

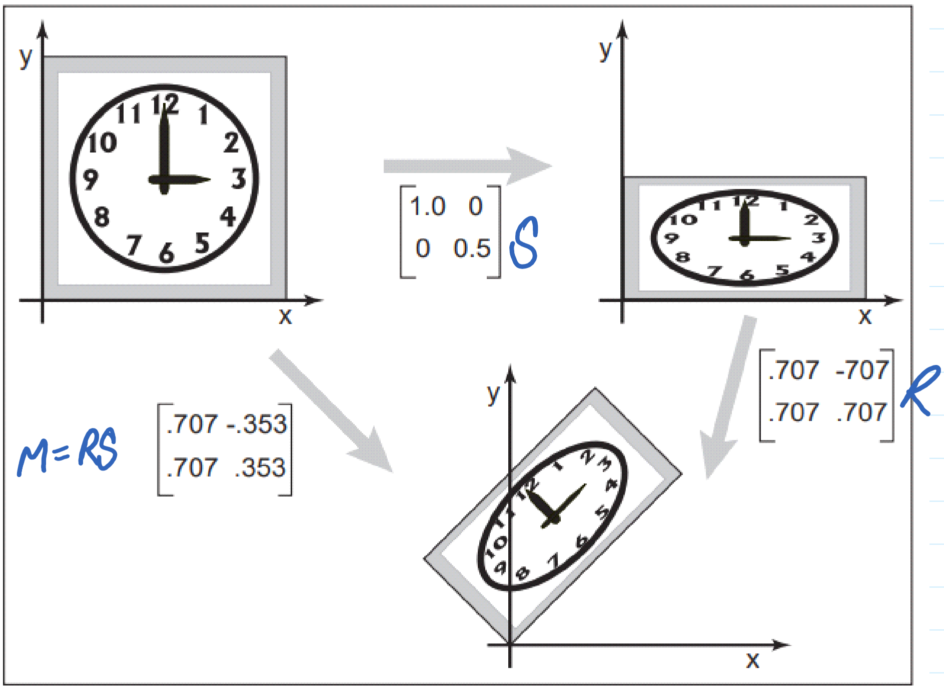
$$\boxed{V_3 = (RS)v_1}$$

$$\Rightarrow \quad \underset{\sim}{M} = RS$$

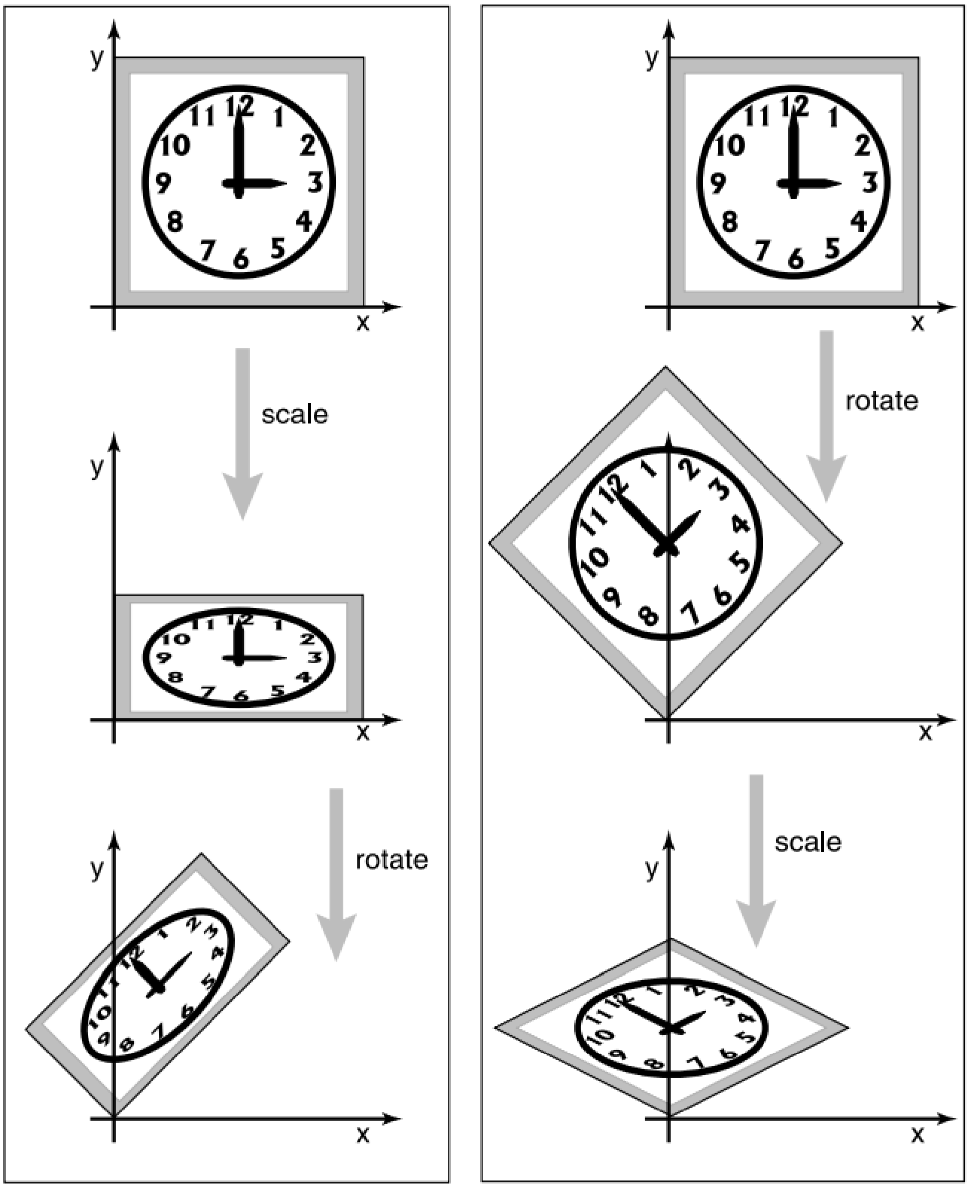
in terms of single matrix.

$$R = \begin{bmatrix} 1 & 0 \\ 0 & .5 \end{bmatrix}_{2\times2} \qquad S = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix}_{2\times2}$$

$$M = RS = \begin{bmatrix} & \\ & \end{bmatrix}_{2\times2}$$



$$RS = SR \,?$$



"Any linear transformation can be though of as a combination of rotation and scaling."