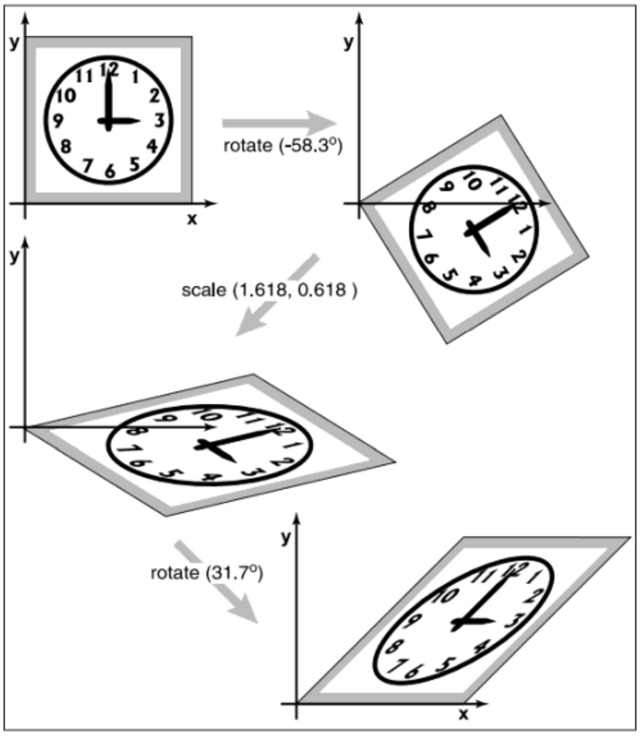
5. Linear Transformations - 3 (24/1/24)

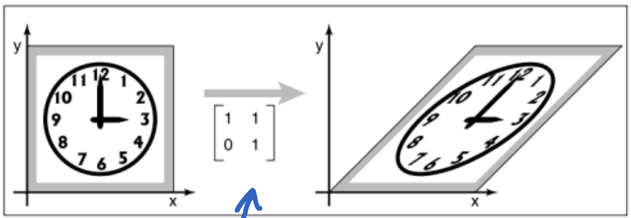
23 January 2024 12:08

Source: "Fundamentals of Computer Graphics, 4th Edition"
by Steve Marschner and Peter Shirley, A K Peters/CRC Press, 2015.

Decomposition of Transformations :-

"undo" the compositions.





shearing

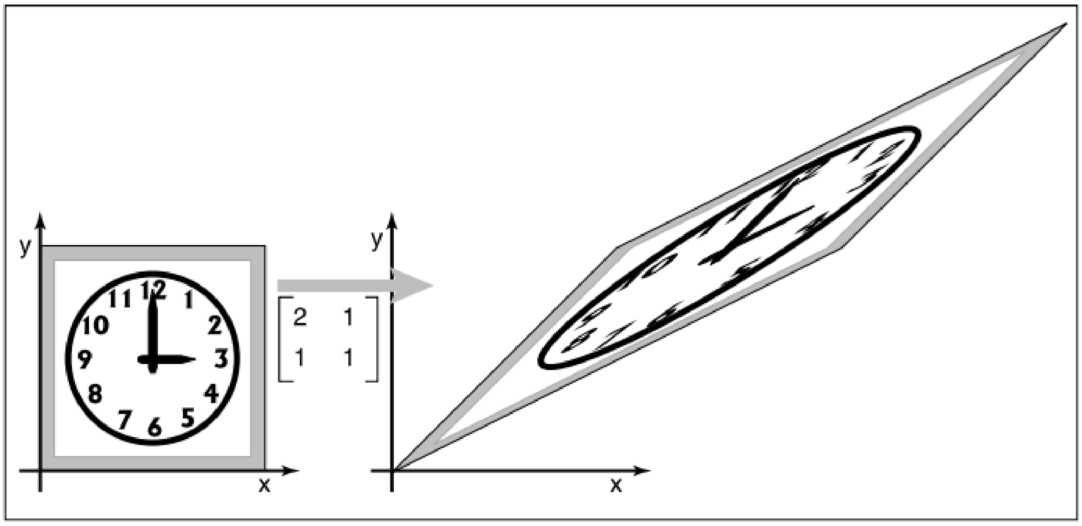
How? Using Symmetric Eigenvalue Decomposition.

$$A = RSR^T$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \mathbf{R} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \mathbf{R}^T$$

$$= \begin{bmatrix} 0.8507 & -0.5257 \\ 0.5257 & 0.8507 \end{bmatrix} \begin{bmatrix} 2.618 & 0 \\ 0 & 0.382 \end{bmatrix} \begin{bmatrix} 0.8507 & 0.5257 \\ -0.5257 & 0.8507 \end{bmatrix}$$

$$= \text{rotate}\,(31.7^\circ)\ \text{scale}\,(2.618, 0.382)\ \text{rotate}\,(-31.7^\circ).$$

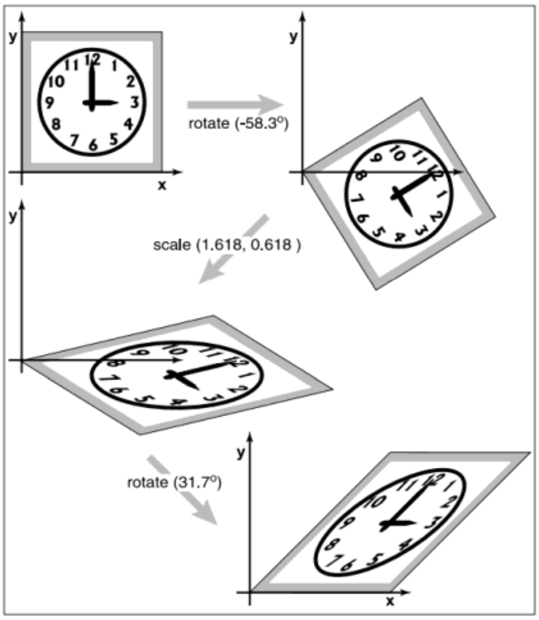


Singular Value Decomposition

$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} = \mathbf{R}_2 \begin{bmatrix}\sigma_1 & 0\\0 & \sigma_2\end{bmatrix} \mathbf{R}_1$$

$$= \begin{bmatrix}0.8507 & -0.5257\\0.5257 & 0.8507\end{bmatrix} \begin{bmatrix}1.618 & 0\\0 & 0.618\end{bmatrix} \begin{bmatrix}0.5257 & 0.8507\\-0.8507 & 0.5257\end{bmatrix}$$

$$= \text{rotate}\,(31.7^\circ)\ \text{scale}\,(1.618, 0.618)\ \text{rotate}\,(-58.3^\circ).$$

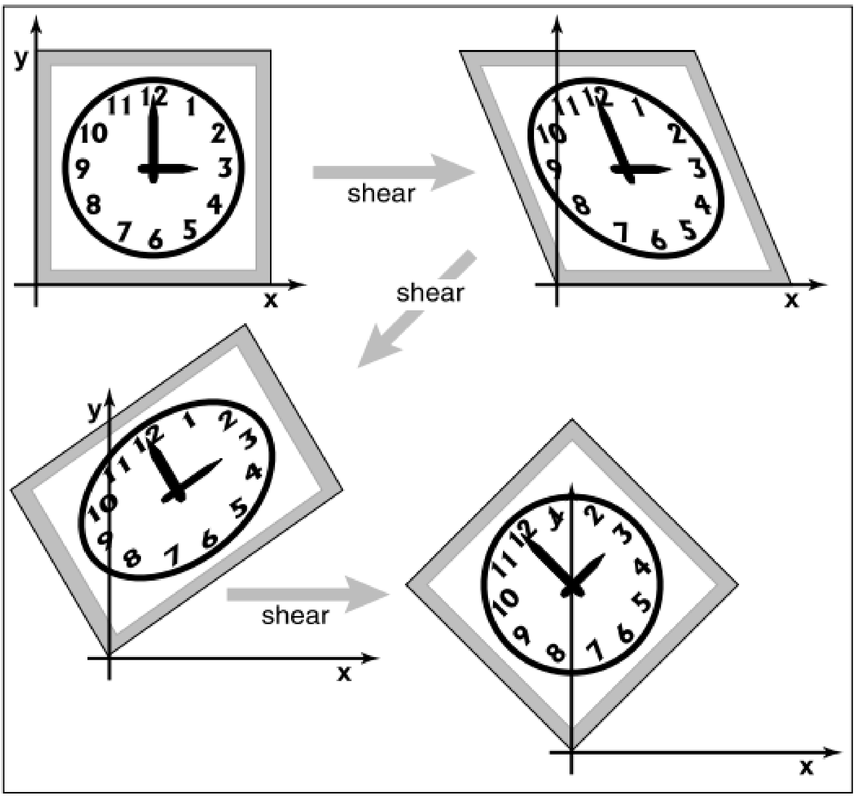


Paeth Decomposition of Rotation (Paeth 1990)

Paeth Decomposition of Rotation (Paeth, 1990)

$$\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} = \begin{bmatrix} 1 & \frac{\cos\phi - 1}{\sin\phi} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \sin\phi & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{\cos\phi - 1}{\sin\phi} \\ 0 & 1 \end{bmatrix}$$

Example: Rotate $(45°)$ $= \begin{bmatrix} 1 & 1-\sqrt{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \sqrt{2}/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1-\sqrt{2} \\ 0 & 1 \end{bmatrix}$



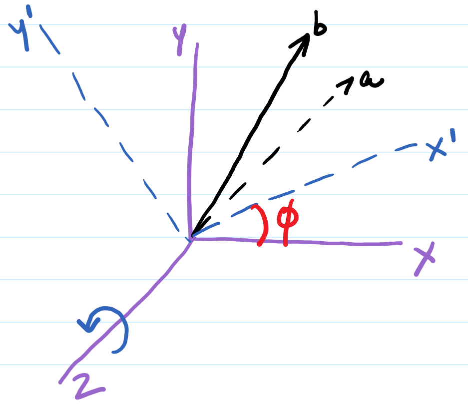
3D Linear Transformations

① $\text{scale}(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}$

② Shear $-x\,(d_y, d_z) = \begin{bmatrix} 1 & d_y & d_z \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

③ 3D-Rotation

① Rotation about z-axis.



$$\left.\begin{aligned} x_b &= x_a \cos\phi - y_a \sin\phi \\ y_b &= y_a \cos\phi + x_a \sin\phi \end{aligned}\right\} \text{from 2D rotation}$$

$$z_b = z_a$$

In matrix form (about z-axis)

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}$$

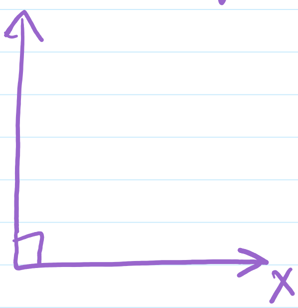
② Rotation about x-axis.

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}$$

(3) Rotation about y-axis.

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}$$

Canonical Coordinate System / World Coordinate System :-



The two vectors are orthogonal.
(at right angle)

Each of them are of unit length.

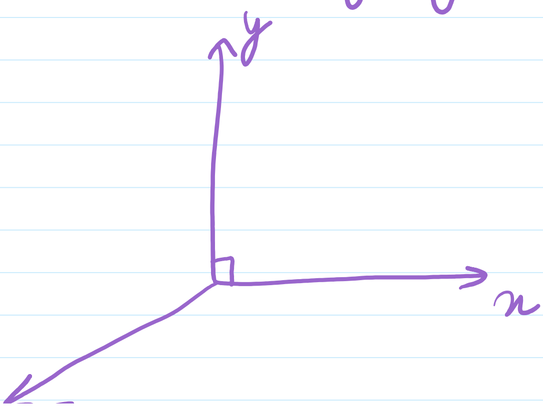
$\Rightarrow \|x\| = \|y\| = 1$

and $x.y = 0$

In 3D, three vectors x, y and z

$\|x\| = \|y\| = \|z\| = 1$

and $x.y = y.z = z.\omega = 0$



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