21. Bspline Curves

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Source: 1. Computer Graphics by Donald Hearn and M. Pauline Baker

Introduction to Computer Graphics, NPTEL Course by Prof. Prem Kalra

2. **Computer Graphics** by Donald Hearn and M. Pauline Baker

$$P(t) = \sum_{i=0}^{n} b_i J_i^n(t)$$

$$\boxed{J_i^n(t) = {}^nC_i t^i (1-t)^{n-i}}$$

$$J_0^n(0) = {}^nC_0\, 0^0 (1-0)^n$$
$$= 1 \cdot 1 \cdot 1$$
$$= 1$$

$$\frac{n!}{0!\,n!}$$

$$\boxed{\begin{array}{l} {}^nC_0 = 1 \text{ because } 0! = 1 \\ x^0 = 1 \end{array}}$$

Properties :-

① End point Interpolation

At $t=0$

$$i=0, \; J_0^n(0) = \overbrace{{}^nC_0}^{1}\,\overbrace{(0)^0}^{1}\,\overbrace{(1-0)^n}^{1} = 1$$

$$i \neq 0, \; J_i^n(0) = {}^nC_i\,(0)^i(1-0)^n = 0$$

$$P(0) = b_0 J_0^n(0) = b_0$$

At $t=1$

$$i=n, \; J_n^n(1) = \frac{n!}{n!(1)}(1)^n(0)^{n-n} = 1$$

$$i \neq n, \; J_i^n(1) = \frac{n!}{(n-i)!\,i!}(1)^n(1-1)^{n-i} = 0$$

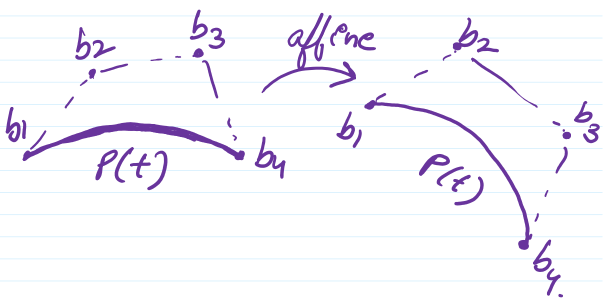
$$P(1) = b_n J_n^n(1) = b_n$$

$$P(t) = \sum_{i=0}^{n} b_i J_i^n(t)$$

$$\boxed{J_i^n(t) = {}^nC_i t^i (1-t)^{n-i}}$$

② Affine Invariance

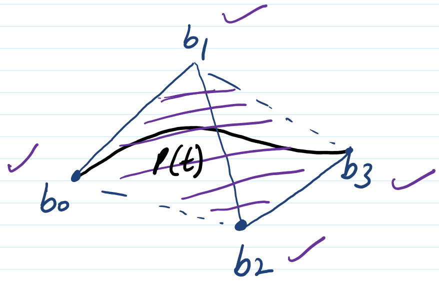
Applying an affice transformation to the curve is equivalent to applying the transformation to the control points.



③ Convex hull

Curve lies in the convex hull of the

control points



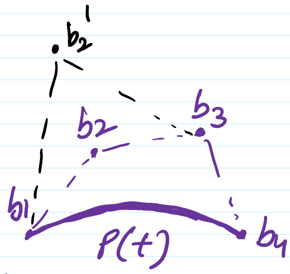
$$\sum_{i=0}^{n} J_i^n(t) = 1$$

$J_i^n(t)$: non-negative for $t \in [0,1]$

④ Symmetry

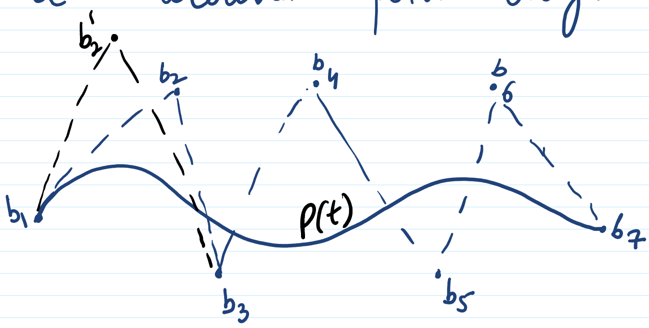
$P(t)$ defined by $b_0, b_1, b_2, \ldots, b_n$ is equal to $P(1-t)$ defined by $b_n, b_{n-1}, \ldots, b_0$

$$\sum_{i=0}^{n} b_i J_i^n(t) = \sum_{i=0}^{n} b_{n-i} J_i^n(1-t)$$



⑤ Pseudo Local Control

Moving a Bezier point the whole curve will change but the maximum influence will be around point only.

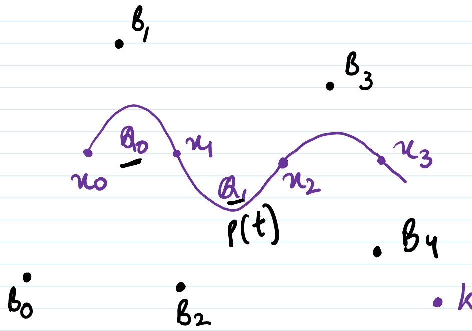


B_Splines

Each control point is associated with a unique basis / blending function.

Each point affects the shape of the curve over a range of parameter values where the basis function is non-zero.

$\Rightarrow$ "LOCAL CONTROL"



$Q_0 : B_0 B_1 B_2 B_3$
$Q_1 : B_1 B_2 B_3 B_4$

Parameter t is defined as

$$x_i \le t \le x_{i+1}$$

$x_0\ x_1\ x_2\ x_3$: knot values.

Mathematically,

polynomial spline function,

$$P(t) = \sum_{i=0}^{n} B_i \underbrace{N_{i,k}(t)}_{\uparrow} \qquad t_{min} \le t \le t_{max}$$

$$2 \le k \le n+1$$

where

$N_{i,k}$ is blending / basis function

$$N_{i,1}(t) = \begin{cases} 1 & x_i \le t < x_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$\underline{N_{i,k}}(t) = \frac{(t - x_i)\, N_{i,k-1}(t)}{\underline{x_{i+k-1} - x_i}} + \frac{(x_{i+k} - t)\, N_{i+1,k-1}(t)}{\underline{x_{i+k} - x_{i+1}}}$$

Properties :-

① Local control

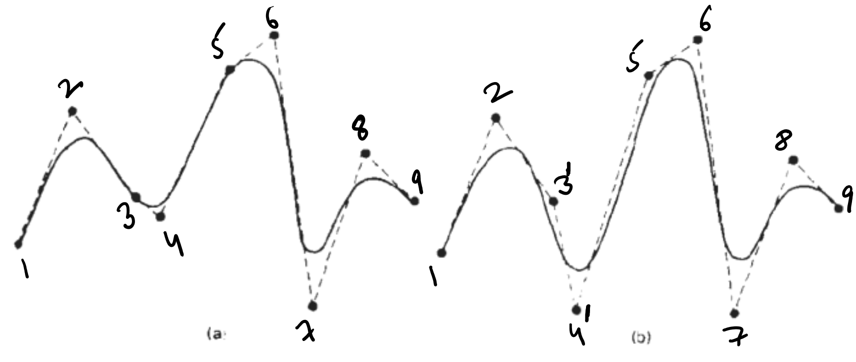


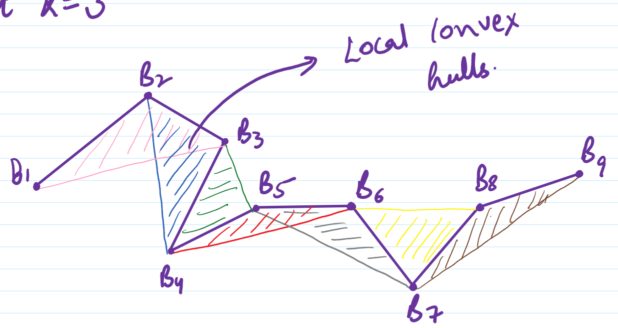
Figure 10-41
Local modification of a B-spline curve. Changing one of the control points in (a) produces curve (b), which is modified only in the neighborhood of the altered control point.

② Convex Hull

For a B-Spline curve of order K a point on the curve lies within the convex hull of K neighboring points.

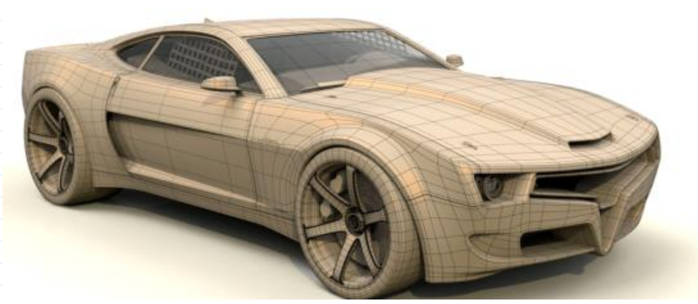
All points of B-Spline curve must lie within the union of all such convex hulls.

Let $K=3$



BSpline surfaces :-

$$P(u,v) = \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} B_{i_1,i_2} N_{i_1,K_1}(u) N_{i_2,K_2}(v)$$



Summary :-

	Cubic Splines	Hermite Splines	Bezier Curves	B-Spline
Interpolation	All control points	End Point	End Point	No
Local Control	No	Yes	No	Yes