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II. Introduction

Text of the introduction.

- minimum effort control - minimizing the control effort, where the effort is defined as maximum amplitude of the control [1]–[3]
- minimum effort control in robotics (kinematically redundant manipulators) [4]–[7],
- traditional control methods of power converters, carrier-based PWM techniques (sinusoidal, TIPWM) and space vector-based techniques (SVPWM).

III. Motivation and principles

A. Redundant Voltage Source Network

The problem of redundant voltage source network, degrees of freedom. Analogy to voltage source converters. Clarke's transform, A matrix, vector x as a vector of final voltages with minimum amplitudes.

$$\begin{aligned} x &= [\cdots]^T, \\ A &= [\cdots], \end{aligned} \tag{1}$$

Motivation -¿ minimize voltage needed.

B. Optimization problem definition

Linear system, minimum infinity norm, solution vector x. Primal problem definition...

$$\min_{\mathbf{Ax}=\mathbf{y}} ||\mathbf{x}||_\infty, \tag{2}$$

C. Solution using linear programming???

Note sure if this is to include in the paper......Solution using linear programming - linprog() in Matlab environment, suitable for mathematical modeling and rapid method development for any type of converter. Not effective for real-time application...

IV. Proposed Solution

A. Solution way for $m \times (m+1)$ matrices

If matrix A is of size $m \times (m+1)$ and if its rank is m (its rows are independent), then we have

$$\{x \mid Ax = y\} = \{x^0 + sa \mid s \in \mathbb{R}\},$$

where x^0 is an arbitrary point which satisfies $Ax^0 = y$ and a is a nonzero element of kernel of A (it satisfies $Aa = 0$). Then (7) is equivalent to

$$\begin{aligned} &\underset{x,s}{\text{minimize}}\ \|x\|_\infty \\ &\text{subject to } x = x^0 + sa, \end{aligned}$$

which amounts solving an optimization problem in one dimension

$$\underset{s}{\text{minimize}}\ \|x^0 + sa\|_\infty. \tag{3}$$

For the the optimal solution of (3), there will be some indices $i \neq j$ such that the components satisfy

$$(x^0 + sa)_i = \pm(x^0 + sa)_j.$$

This have two solutions (if properly defined)

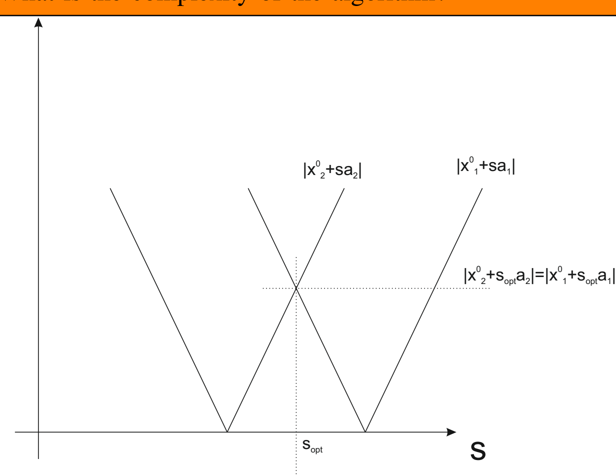
$$\begin{aligned} s^1 &= \frac{-x_i^0 + x_j^0}{a_i - a_j}, \\ s^2 &= \frac{-x_i^0 - x_j^0}{a_i + a_j}. \end{aligned}$$

Then among all these s, we select the one with minimal $\|x^0 + sa\|_\infty$. We summarize the procedure in Algorithm IV.1.

Algorithm IV.1 For solving (7) with one degree of freedom

Find some nonzero $\boldsymbol{a}$ such that $A\boldsymbol{a} = 0$
Find some $\boldsymbol{x}^0$ such that $A\boldsymbol{x}^0 = \boldsymbol{y}$
$f_{\min} \leftarrow \infty$
for i in $1 : m+1$ **do**
  for j in $i+1 : m+1$ **do**
    if $a_i \neq a_j$ **then**
      $s \leftarrow \frac{-x_i^0 + x_j^0}{a_i - a_j}$
      if $\|\boldsymbol{x}^0 + s\boldsymbol{a}\|_\infty \leq f_{\min}$ **then**
        $f_{\min} \leftarrow \|\boldsymbol{x}^0 + s\boldsymbol{a}\|_\infty$
        $s_{\text{opt}} \leftarrow s$
      end if
    end if
    if $a_i \neq -a_j$ **then**
      $s \leftarrow \frac{-x_i^0 - x_j^0}{a_i + a_j}$
      if $\|\boldsymbol{x}^0 + s\boldsymbol{a}\|_\infty \leq f_{\min}$ **then**
        $f_{\min} \leftarrow \|\boldsymbol{x}^0 + s\boldsymbol{a}\|_\infty$
        $s_{\text{opt}} \leftarrow s$
      end if
    end if
  end for
end for

What is the complexity of the algorithm?



B. Solution way for a general matrix A

... Duality of ℓ_1 and ℓ_∞ norm (D. G. Luenberger) [8]. Dual problem defined by Cadzow:

The Cadzow algorithm [9], [10] is based on a solution search of the associated dual problem

$$\max_{\|\mathbf{A}^T\mathbf{u}\|_1 \leqq 1} \mathbf{y}^T\mathbf{u} = \min_{\mathbf{Ax}=\mathbf{y}} ||\mathbf{x}||_\infty, \tag{4}$$

using the alignment property between final vectors $\mathbf{A}^T\mathbf{u}^0$ and $\mathbf{x}^0$ to evaluate $\mathbf{x}^0$:

$$\left[\mathbf{A}^T\mathbf{u}^0\right]^T\mathbf{x}^0 = \left\|\mathbf{A}^T\mathbf{u}^0\right\|_1 \left\|\mathbf{x}^0\right\|_\infty, \tag{5}$$

i.e.

$$\left\|\mathbf{x}^0\right\|_\infty = \mathbf{y}^T\mathbf{u}^0 = \left[\mathbf{A}^T\mathbf{u}^0\right]^T\mathbf{x}^0. \tag{6}$$

C. ???

Primal problem

$$\begin{aligned} \underset{x}{\text{minimize}} \qquad & \|x\|_\infty \\ \text{subject to} \qquad & Ax = y. \end{aligned} \tag{7}$$

Dual problem

$$\begin{aligned} \underset{u}{\text{maximize}} \qquad & y^\top u \\ \text{subject to} \qquad & \|A^\top u\|_1 \le 1. \end{aligned} \tag{8}$$

Dual problem enhanced

$$\begin{aligned} \underset{u^+,u^-,z^+,z^-,w}{\text{maximize}} \qquad & y^\top u^+ - y^\top u^- \\ \text{subject to} \qquad & A^\top u^+ - A^\top u^- - z^+ + z^- = 0, \\ & \sum (z_i^+ + z_i^-) + w = 1, \\ & u^+, u^-, z^+, z^-, w \ge 0. \end{aligned} \tag{9}$$

Comments

- All three problems are equivalent and they are linear problems.
- Each linear problem has a solution in an extremal point (corner) of its feasible set. The number of extremal points is finite.
- The solution set of (8) does not depend on y. Denote the finite set of extremal points by U.
- Previous two bullets imply the following: For every y, there is always some $u \in U$ which solves (8).
- I do not know how to compute the extremal points of (8) but there is a formula for computing the extremal points of (9).

This suggests that the way to go is to compute the extremal points of (9). Since they are in an enhanced space (u^+, u^-, z^+, z^-, w), we reduce them into the original space u corresponding to problem (8). This reduction will create a superset of the extremal points of (8). But there is a way of obtaining the set of extremal points of (8) from this superset.

Theorem IV.1. *There is a finite set U such that for any y, there exists some $u \in U$ such that u is the solution of* (8) *for y. This U equals to the set of extremal points of $\{u \mid \|A^\top u\|_1 \le 1\}$.*

Offline: Compute the set U with finite number of elements
Input: y for which we need to solve (7)
Select $\bar{u} \in \operatorname{argmax}_{u\in U} y^\top u$. It solves (8)
Based on complementarity find the solution of (7)

V. ??? Possibly appendix

Computing directly the set of extremal points of the feasible set of (8) is complicated. However, there is a connected set, for which the computation is possible. We describe it in the following statement.

Algorithm V.1 For finding extremal points of the set $\{v \mid Bv = b,\ v \ge 0\}$

Input: Matrix B of size (m, n) with $\operatorname{rank} B = m$
$V \leftarrow \emptyset$
Denote by $\mathcal{I}$ the set of all tuples of length m selected from $1, \dots, n$ (without replacement).
for $I \in \mathcal{I}$ **do**
  Let $B_{\text{sub}} := B_{\cdot,I}$ be the (m, m) submatrix of B with columns indexed by I
  if $\operatorname{rank} B_{\text{sub}} = m$ **then**
    Compute the unique solution z_{sub} of $B_{\text{sub}} z_{\text{sub}} = b$
    if $z_{\text{sub}} \ge 0$ **then**
      Define the vector z with 0 everywhere and z_{sub} on I
      $V \leftarrow V \cup \{z\}$
    end if
  end if
end for
Output: V as the set of extremal points of (10)

Proposition V.1. *Consider matrix B of size (m, n) with* $\operatorname{rank} B = m$ *and any vector b. Then Algorithm V.1 computes the set of all extremal points of*

$$\{v \mid Bv = b,\ v \ge 0\}. \tag{10}$$

Lemma V.2. *Dual problem is equivalent to* (9). *The solution u of* (8) *can be recovered as $u = u^+ - u^-$.*

Proof. The constraint $\|A^\top u\|_1 \le 1$ can be equivalently written by

$$\sum_i |A^\top u|_i \le 1$$

Now we use the standard trick and write the absolute value as the difference of its positive and negative part $z = z^+ - z^-$ with its absolute value $|z| = z^+ + z^-$. This gives

$$\begin{aligned} A^\top u &= z^+ - z^-, \\ z^+, z^- &\ge 0, \\ \sum_i (z_i^+ + z_i^-) &\le 1. \end{aligned}$$

Since there is no non-negativity constraint on u, we use the same trick to write $u = u^+ - u^-$ with $u^+, u^- \ge 0$. Finally, we change the inequality to inequality constraint by adding a slack variable $w \ge 0$. This gives (9). ☐

Now we can use Proposition V.1 to find the extremal points of the feasible set of (9). Due obtain the form necessary for the proposition, we set

$$\begin{aligned} v &= \begin{pmatrix} u^+ & u^- & z^+ & z^- & w \end{pmatrix}^\top, \\ B &= \begin{pmatrix} A^\top & -A^\top & -I & I & 0 \\ 0 & 0 & 1^\top & 1^\top & 1 \end{pmatrix}, \\ b &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \end{aligned} \tag{11}$$

Using Proposition V.1 we can compute the extramal points of the feasible set of (9). This is done in the first two steps of Algorithm V.2. These points have components $\begin{pmatrix} u^+ & u^- & z^+ & z^- & w \end{pmatrix}$ while the feasible set of (8) is only in the u-space. Proposition V.1 also says that to get to the u-space, we need to set $u = u^+ - u^-$. This is done in the next two steps of Algorithm V.2. However, this procedure is likely to create a superset of the set of extremal points of (8). For this reason, we remove the points which lie in the interior (and thus, they cannot be extremal).

The next lemma proves that this procedure indeed finds all extremal points of (8).

Lemma V.3. *If A has full row rank, then the set U found by Algorithm V.2 is the set of extremal points of* (8).

Proof.

Difficult.

$\square$

Algorithm V.2 For finding U

Input: Matrix A of size (k, l) with rank $A = k$

Set B and b as in (11)

Use Algorithm V.1 to compute V

Enumerate $V = \{\begin{pmatrix} u^{+,i} & u^{-,i} & z^{+,i} & z^{-,i} & w^i \end{pmatrix}\}_{i=1}^I$

Set $U = \{u^{+,i} - u^{-,i}\}_{i=1}^I$

Remove all points from U which are in the interior of its convex hull

Output: U as the set of extremal points of (8)

VI. Application/Control of Power Converters)

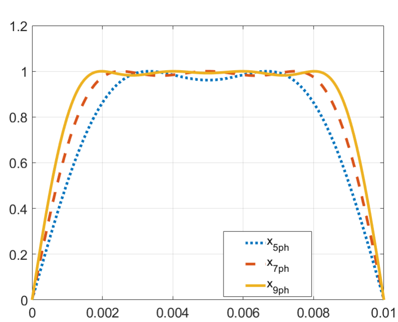


Fig. 1. Figure example

A. Traditional three-phase converters

Conventional three-phase converters, correlation to SVPWM and

B. Three-phase four-leg converters

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C. Multilevel converters

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D. Multiphase converters

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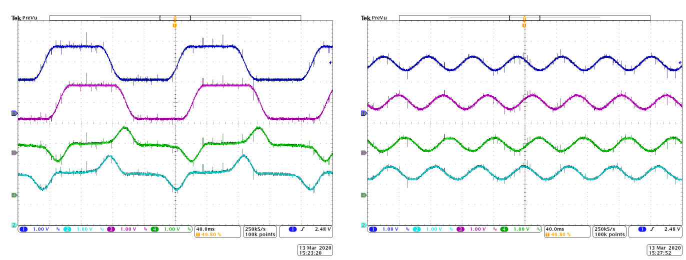
VII. Experimental Results

Some experimental results.



(a) Fundamental with 3rd , 5th , and 7th harmonic component.

(b) 3rd harmonic component only.

Fig. 2. Example of experimental results

VIII. Conclusion

Conclusion Text text text. Text text text. Text text text. Text text text. Text text text. Text text text. Text text text. Text text text. Text text text. Text text text..

Appendix

???

Consider the interval

$$U_1 := \{u \mid -1 \leq u \leq 1\}.$$

Writing $u = u^+ - u^-$ as the difference of its positive $u^+ \geq 0$ and negative part $u^- \geq 0$, the previous set is connected with

$$U_2 := \{(u^+, u^-) \mid -1 \leq u^+ - u^- \leq 1, u^+, u^- \geq 0\}.$$

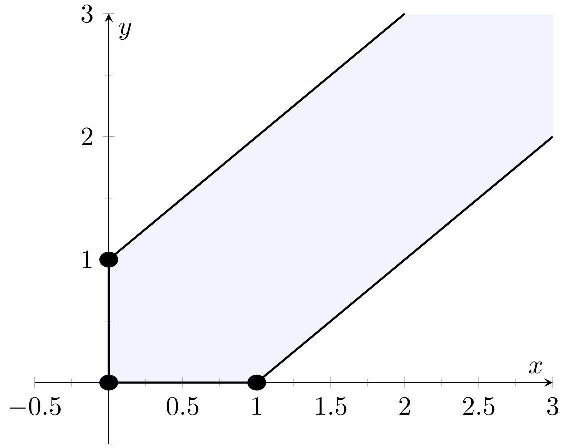


Fig. 3. ???

We depict U_2 in Figure 3.

While U_1 has two extremal points, U_2 has three extremal points. Even though these sets describe the same feasible set for optimization.

Acknowledgment

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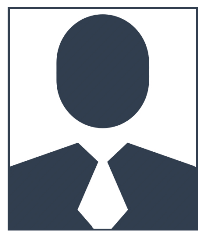

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