Primal problem

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \|x\|_\infty \\ \text{subject to} \quad & Ax = y. \end{aligned} \tag{1}$$

Dual problem

$$\begin{aligned} \underset{u}{\text{maximize}} \quad & y^\top u \\ \text{subject to} \quad & \|A^\top u\|_1 \le 1. \end{aligned} \tag{2}$$

Dual problem enhanced

$$\begin{aligned} \underset{u^+,u^-,z^+,z^-,w}{\text{maximize}} \quad & y^\top u^+ - y^\top u^- \\ \text{subject to} \quad & A^\top u^+ - A^\top u^- - z^+ + z^- = 0, \\ & \sum (z_i^+ + z_i^-) + w = 1, \\ & u^+, u^-, z^+, z^-, w \ge 0. \end{aligned} \tag{3}$$

Comments

- All three problems are equivalent and they are linear problems.
- Each linear problem has a solution in an extremal point (corner) of its feasible set. The number of extremal points is finite.
- The solution set of (2) does not depend on y. Denote the finite set of extremal points by U.
- Previous two bullets imply the following: For every y, there is always some $u \in U$ which solves (2).
- I do not know how to compute the extremal points of (2) but there is a formula for computing the extremal points of (3).

This suggests that the way to go is to compute the extremal points of (3). Since they are in an enhanced space (u^+, u^-, z^+, z^-, w), we reduce them into the original space u corresponding to problem (2). This reduction will create a superset of the extremal points of (2). But there is a way of obtaining the set of extremal points of (2) from this superset.