Quantum Battery

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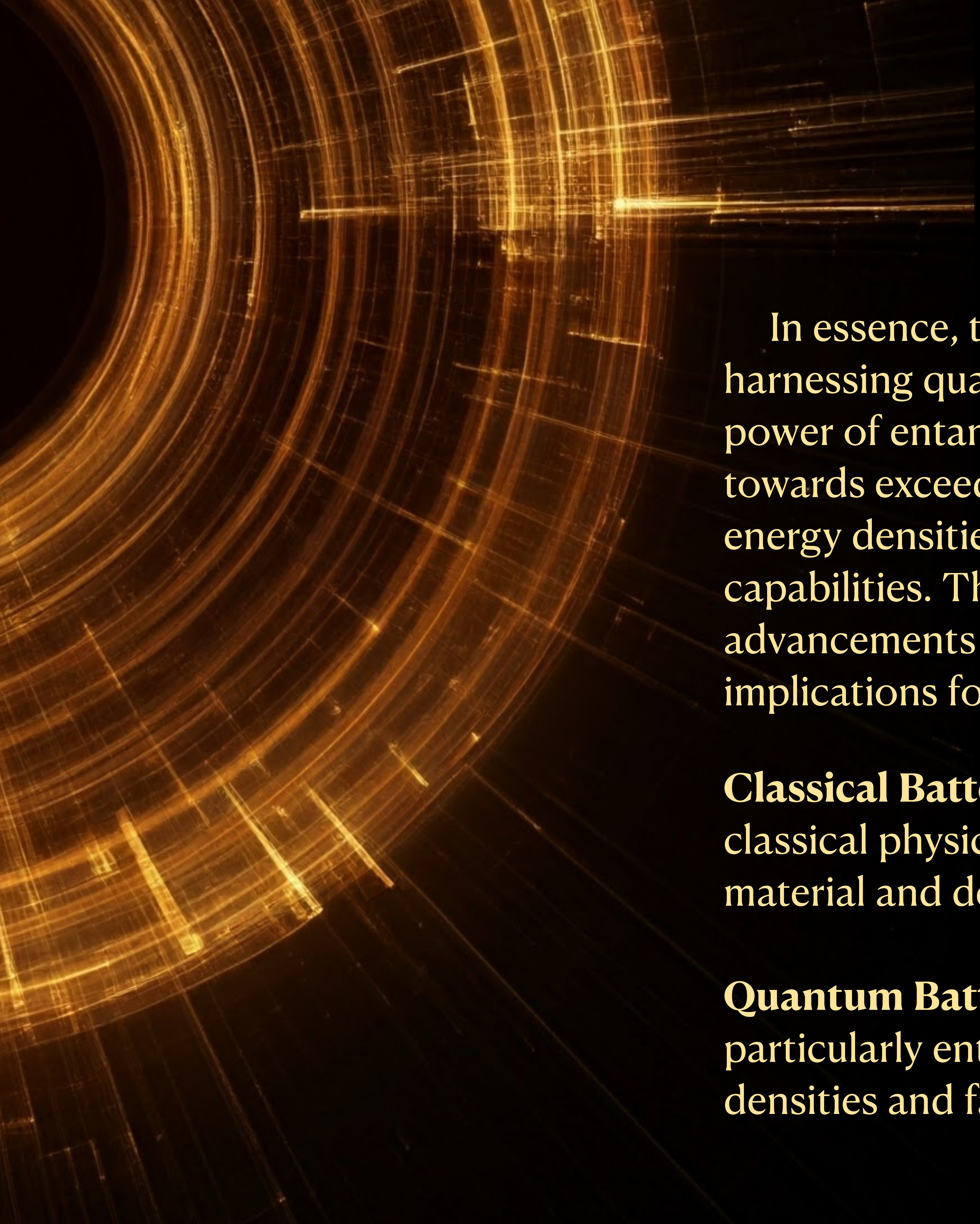
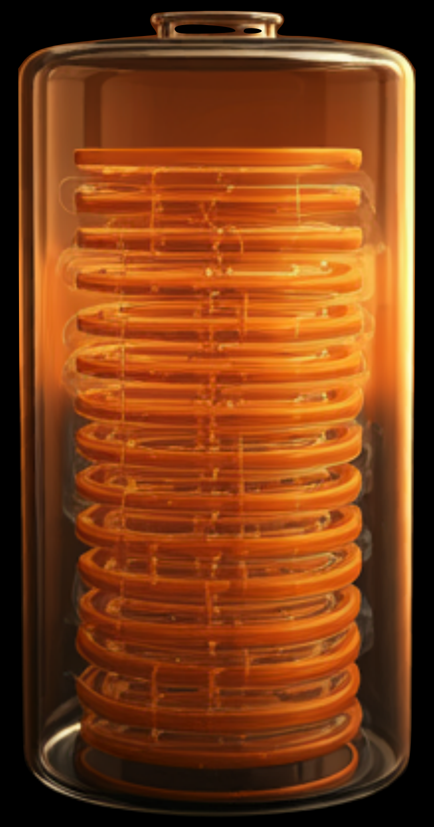


Introduction

In essence, this paper introduces a groundbreaking approach to harnessing quantum phenomena for energy storage. By leveraging the power of entanglement and QET, quantum batteries offer a path towards exceeding classical limitations, achieving unprecedented energy densities, and enabling near-instantaneous charging capabilities. This research paves the way for transformative advancements in energy storage and transfer technologies, with implications for quantum computing and beyond

Classical Battery Limitations: Traditional batteries are limited by classical physics, restricting their energy density (EC) based on material and design constraints.

Quantum Batteries: Quantum batteries use quantum mechanics, particularly entanglement, to potentially achieve higher energy densities and faster charging rates.



The Quantum Energy Teleportation (QET) was first proposed by **Masahiro Hotta,** a method to redistribute energy using only LOCC (local operations and classical communication). This bypasses the need for direct energy transport. Negative energy density is achieved via ground-state quantum interference, enabling local suppression of quantum fluctuations. In the spin chain model, energy interactions occur in a 1D lattice of spins, with the Hamiltonian written as:

$$H = \sum_{n} T_n$$

where T_n represents local interaction terms, and the ground state satisfies $H|g\rangle = 0.$

Steps of the QET Protocol

Alice's Local Measurement: Alice measures a local observable ($\sigma_A = \mathbf{u}_A \cdot \boldsymbol{\sigma}_{n_A}$) at site n_A, injecting energy:

$$E_A = \sum_{\mu} \langle g|P_A(\mu) H P_A(\mu)|g\rangle$$

Here, $P_A(\mu)$ projects onto the eigenstates of σ_A, and $|g\rangle$ is the ground state.

Classical Communication: Alice announces her measurement result μ to Bob using a classical channel, ensuring no energy flow between them.

Bob's Local Operation: Bob applies a unitary operation $V_B(\mu)$ at site n_B, defined as:

$$V_B(\mu) = \cos\theta \, I + i(-1)^{\mu} \sin\theta \, \sigma_B$$

where θ depends on parameters ξ and η related to the ground state: $\cos(2\theta) = \frac{\xi}{\sqrt{\xi^2 + \eta^2}}, \ \sin(2\theta) = -\frac{\eta}{\sqrt{\xi^2 + \eta^2}}$

Energy Extraction by Bob

Bob extracts energy E_B from the spin chain:

$$E_B = \frac{1}{2}\left(\sqrt{\xi^2 + \eta^2} - \xi\right)$$

where:

- $\xi = \langle g|\sigma_B H \sigma_B|g\rangle$, the energy density near Bob.
- $\eta = \langle g|\sigma_A [H, \sigma_B]|g\rangle$, capturing correlations between Alice and Bob's sites.

Conservation and Negative Energy

Local energy conservation ensures:

$$E_B + \mathrm{Tr}[\rho H_{n_B}] = 0$$

After Bob's operation, the localized energy density $\mathrm{Tr}[\rho H_{n_B}]$, becomes negative, allowing energy release to Bob's device.

Significance

The protocol exploits entanglement and negative energy to redistribute energy effectively without physical transfer. Energy extraction is non-trivial because Bob relies on Alice's measurement to unlock local quantum fluctuations. The protocol respects causality and local energy conservation while achieving effective energy transfer.

Quantum Battery Charging

Local Charging

In **local charging**, each quantum subsystem, like a qubit, is charged independently. The Hamiltonian for local interactions is written as:

$$H_{\|}(N) = \sum_{i=1}^{N} V_i$$

where V_i, operates on each subsystem individually. While local charging is scalable, it is slower because the charging time increases linearly with the number of units, N.

Global Charging

Global charging involves coupling all subsystems simultaneously using many-body interactions. The Hamiltonian for global charging is:

$$H_{\#}(N) = -H_0(N) + \alpha_N |E\rangle\langle G| + \alpha_N^* |G\rangle\langle E|$$

Here, $|G\rangle$ and $|E\rangle$ are the ground and excited states of the system, while α_N represents the interaction strength. Global charging allows for faster charging, as the time required scales inversely with N, leading to a quadratic enhancement in charging power compared to local charging.

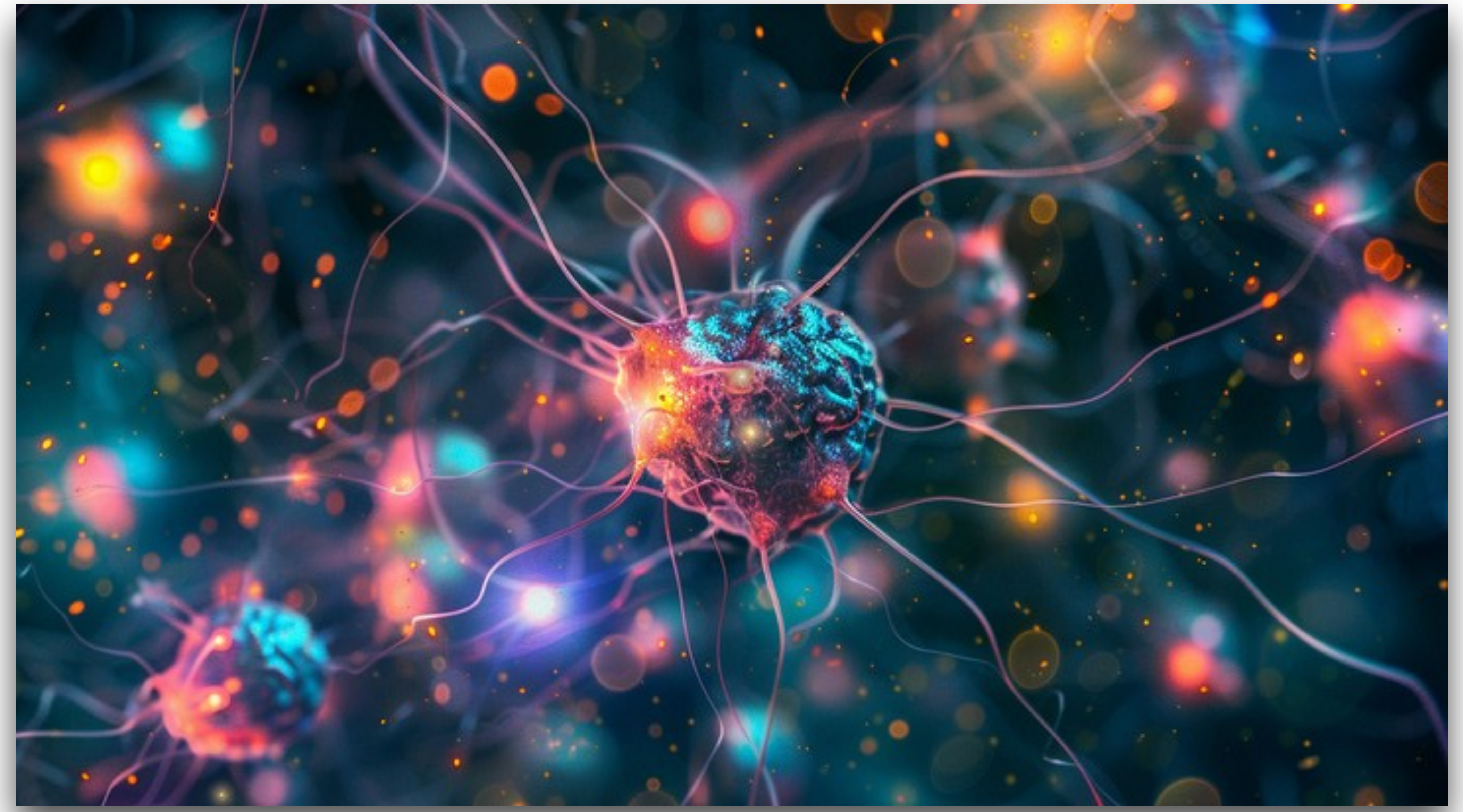
Quantum Battery Model

The **Dicke model** involves a group of two-level systems (qubits) that interact with a single mode of a cavity field. It initially appeared to show a quantum advantage in energy charging, but this advantage actually arises from classical collective behavior rather than quantum entanglement, so it doesn't provide a true quantum benefit.

The **SYK model**, on the other hand, is based on many-body fermionic systems with random interactions. It uses quantum entanglement and chaotic dynamics to achieve a genuine quantum advantage, making it more efficient for energy storage and charging, especially as the system size increases.

Now we will start to review the new protocol proposed by **Hotta**

Key Components of the Protocol:



1. **System Description:**
   - The system consists of a **spin chain** of locally interacting spin-$\frac{1}{2}$ particles (qubits).
   - The Hamiltonian H is decomposed into local Hamiltonians:

$$H = \sum_{n=1}^{N} H_n.$$

   - The highest energy eigenstate $|E_{\max}\rangle$ is **entangled** and non-degenerate, ensuring uniqueness.

2. **Participants:**
   - **Alice** and **Bob** share the entangled highest energy state $|E_{\max}\rangle$.

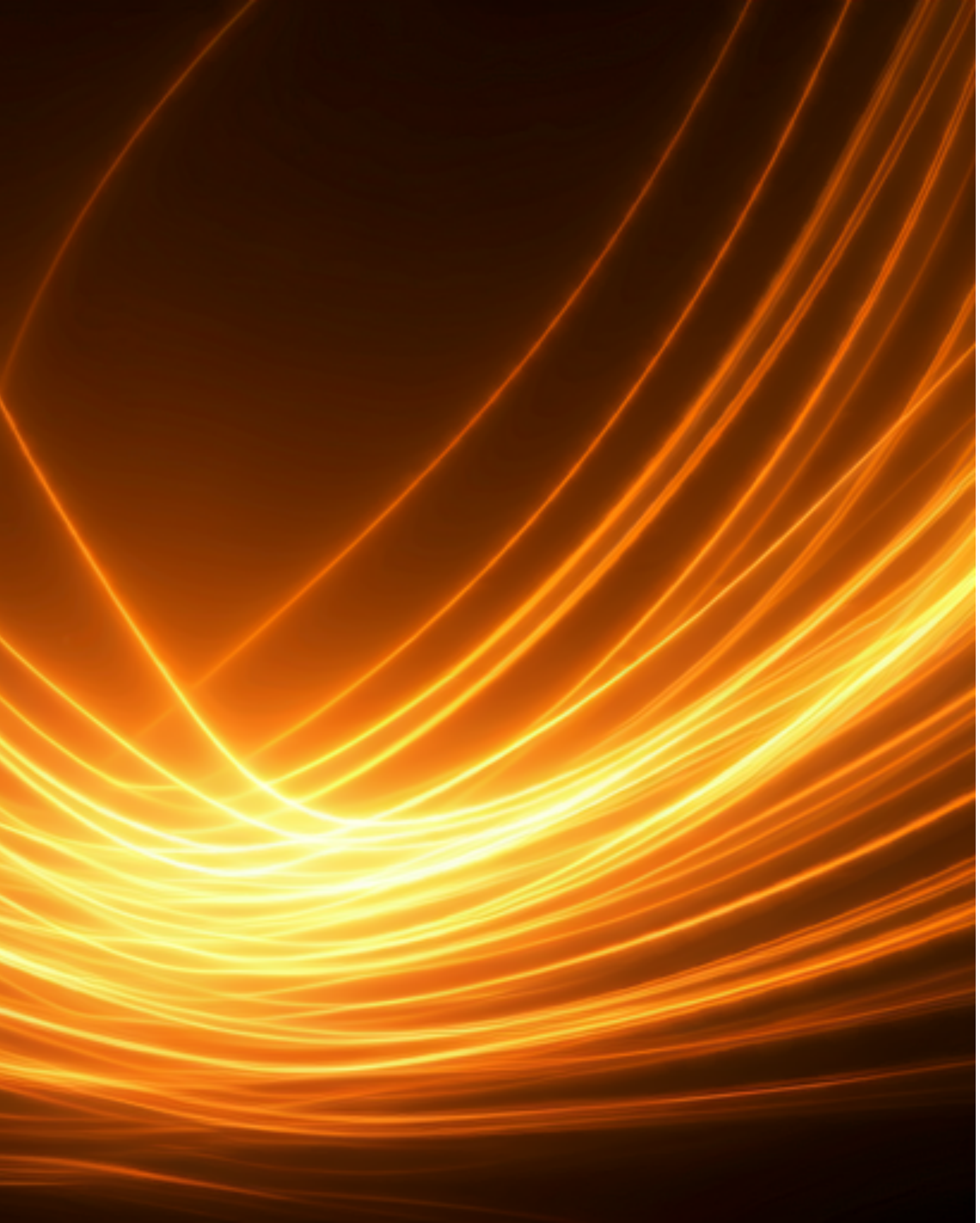
3. **Protocol Description:**
   - **Alice's Measurement:**
     - Alice measures her subsystem using a projective operator $P_n^A(\mu)$:

$$P_n^A(\mu) = \frac{1 + \mu\sigma_n^A}{2}, \quad \mu \in \{-1, +1\}.$$

     - This measurement extracts energy from her local subsystem.
   - **Communication:**
     - Alice sends the result μ to Bob through a classical channel.
   - **Bob's Operations:**
     - Based on μ, Bob performs a unitary operation $U_n^B(\mu)$ on his subsystem.
     - Bob then measures his local observables (X_n^B, Y_n^B, Z_n^B) to compute the localized energy.

4. **State Evolution and Density Matrix:**
   - After Alice's measurement and Bob's operation, the state evolves as:

$$\rho = \sum_{\mu \in \{-1,1\}} U_n^B(\mu) P_n^A(\mu) |E_{\max}\rangle\langle E_{\max}| P_n^A(\mu) U_n^{B\dagger}(\mu).$$



- The initial energy of Bob's localized subsystem is determined by the expectation value of H_n^B in the initial state $|E_{\max}\rangle$:

$$E_{\text{initial}}^B = \mathrm{Tr}(|E_{\max}\rangle\langle E_{\max}|H_n^B).$$

- Since $|E_{\max}\rangle$ is the highest energy eigenstate of H, the localized subsystem begins with its energy contributions determined by this entangled state.

- Bob's local energy after the process is:

$$\langle E_n^B\rangle = \mathrm{Tr}(H_n^B \rho),$$

. **The Inequality:**

$$\langle E_n^B\rangle = \mathrm{Tr}(H_n^B \rho) \geq \mathrm{Tr}(|E_{\max}\rangle\langle E_{\max}|H_n^B).$$

- **Meaning:**
  - Bob's final local energy ($\langle E_n^B\rangle$) is **greater than or equal to** his initial local energy.
  - This shows that the protocol has successfully **increased or conserved Bob's local energy**, making it particularly useful for charging quantum batteries.

Energy and operator at Bob's End

1. The Unitary Operation $U_n(\mu)$:

Bob's operation is defined as:

$$U_n(\mu) = \cos\theta\, I - i\mu \sin\theta\, \sigma_n,$$

where:

- I: Identity operator, ensuring the operation is unitary.
- σ_n: A Pauli operator acting on Bob's local subsystem.
- $\mu \in \{-1, +1\}$: The measurement outcome received from Alice.
- θ: A parameter determined by the energy optimization, as defined below.

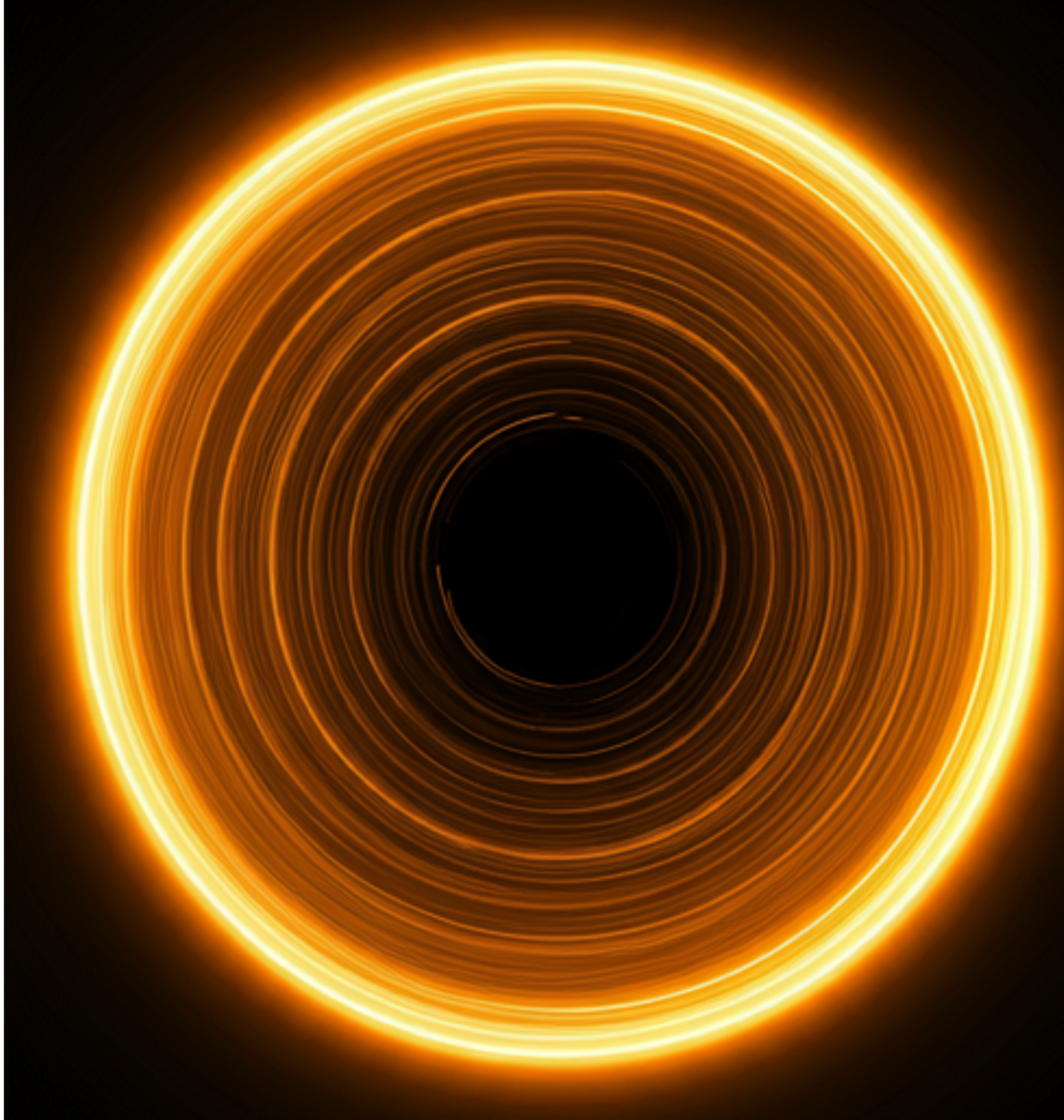
2. Optimizing θ:

The parameter θ is chosen to maximize the energy Bob can store. The relationships for $\cos(2\theta)$ and $\sin(2\theta)$ are:

$$\cos(2\theta) = \frac{\xi}{\sqrt{\xi^2 + \eta^2}}, \quad \sin(2\theta) = \frac{\eta}{\sqrt{\xi^2 + \eta^2}},$$

where:

- $\xi = \langle E_{\max} | \sigma_n H \sigma_n | E_{\max} \rangle$: Represents the overlap of the energy-shifted state.
- $\eta = \langle E_{\max} | \sigma_n \dot{\sigma}_n | E_{\max} \rangle$: Encodes the contribution of the commutator of H and σ_n.



3. Constraint on the Local Hamiltonian:

To ensure energy optimization works effectively, the local Hamiltonian H_n must satisfy:

$$[H, \sigma_n] = [H_n, \sigma_n].$$

This requirement ensures that the commutator remains confined to the local subsystem, simplifying the calculation of $\dot{\sigma}_n = i[H, \sigma_n]$.

Average Energy Stored in Bob's Local System

The final expression for the average energy stored by Bob is:

$$\langle E_n \rangle = \mathrm{Tr}[\rho H_n] = \frac{1}{2}(\xi^2 + \eta^2) - \xi.$$

Key Features:

1. **Positivity of Energy:**
   - $\langle E_n \rangle > 0$ if $\eta \neq 0$, meaning Bob can charge energy effectively as long as η (the contribution from the commutator) is non-zero.
   - This contrasts with QET, where $\langle E_n \rangle$ can be **negative**, reflecting a loss in local energy.
2. **Dependence on ξ and η:**
   - $\xi^2 + \eta^2$: Represents the total contribution from energy-shifting and commutator-induced dynamics.
   - $-\xi$: Reflects a correction due to the initial state properties.

Distinguishing Features of This Protocol

1. Accessibility and Experimentation:

- **Hardware Compatibility:**
  - The protocol builds on the foundations of QET, which has already been experimentally realized in quantum hardware setups like superconducting qubits and trapped ions.
  - This makes transitioning to this protocol more feasible.
- **Simplification:**
  - By using LOCC (Local Operations and Classical Communication), the protocol avoids complex global operations, reducing experimental challenges.

2. Instantaneous Energy Charging:

- The use of LOCC ensures **instantaneous energy transfer and charging** at Bob's location once Alice communicates the measurement result.

3. Comparison to Other Studies:

- **Conventional Quantum Batteries:**
  - Often rely on highly non-local operations or complex entanglement protocols, making them challenging to implement.
  - This protocol simplifies the requirements while retaining high efficiency.
- **QET Protocols:**
  - Unlike QET, where local energy can be depleted at Bob's location, this protocol ensures **positive energy storage**, making it directly applicable to battery-like systems.

Model, Results and Future

1. Hamiltonian Setup

Definition:

The Hamiltonian is the governing operator for the system's energy. Here, the total Hamiltonian is decomposed as:

$$H = H_0 + H_1 + V$$

- H_0: Represents Alice's local Hamiltonian.
- H_1: Represents Bob's local Hamiltonian.
- V: Describes the interaction between Alice's and Bob's systems.

Components:

1. **Alice's Hamiltonian (H_0):**

$$H_0 = -hZ_0 - \frac{h^2}{\sqrt{h^2 + k^2}}$$

   - Z_0: Pauli-Z operator on Alice's qubit.
   - h: A real parameter dictating the strength of Z_0.
   - The constant term ensures the Hamiltonian is properly normalized for the protocol.

2. **Bob's Hamiltonian (H_1):**

$$H_1 = -hZ_1 - \frac{h^2}{\sqrt{h^2 + k^2}}$$

   - Z_1: Pauli-Z operator on Bob's qubit.
   - Shares the same structure as H_0.

3. **Interaction Term (V):**

$$V = -2kX_0X_1 - \frac{2k^2}{\sqrt{h^2 + k^2}}$$

   - X_0 and X_1: Pauli-X operators on Alice's and Bob's qubits.
   - k: A real parameter describing the interaction strength between the two systems.

2. Maximal Energy State ($|E_{max}\rangle$)

Definition:

The state $|E_{max}\rangle$ represents the system's maximum energy eigenstate under the Hamiltonian H. It is given by:

$$|E_{max}\rangle = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{h}{\sqrt{h^2+k^2}}}|00\rangle - \frac{1}{\sqrt{2}}\sqrt{1 + \frac{h}{\sqrt{h^2+k^2}}}|11\rangle$$

Key Features:

1. **Superposition State:**
   - Combines $|00\rangle$ and $|11\rangle$ with specific coefficients dependent on h and k.
   - The coefficients reflect the balance between local terms (h) and interaction strength (k).
2. **Energy Properties:**
   - The state is constructed to satisfy: $\langle E_{max}|H|E_{max}\rangle = \langle E_{max}|H_0|E_{max}\rangle = \langle E_{max}|H_1|E_{max}\rangle = \langle E_{max}|V|E_{max}\rangle = 0$
   - This means $|E_{max}\rangle$ is at a zero-energy baseline and no unitary operation can increase its energy.

3. Protocol Overview

Alice and Bob interact via the system and exchange classical information to extract energy. The steps are:

Step 1: Alice's Measurement

- Alice measures her qubit in the $\sigma_A = X_0$ basis.
- The outcome $\mu \in \{-1, 1\}$ corresponds to the eigenvalues of X_0.

Step 2: Classical Communication

- Alice communicates the measurement result (μ) to Bob via a classical channel.

Step 3: Bob's Operation

- Bob applies a unitary transformation $U_B(\mu)$ on his qubit, defined as:

$$U_B(\mu) = \cos\phi\, I - i\mu \sin\phi\, Y_B$$

  - Y_B: Pauli-Y operator on Bob's qubit.
  - ϕ: A parameter optimized for energy extraction.

4. Energy Extraction and Optimization

Optimization of ϕ:

To maximize the energy extracted, ϕ is chosen based on the parameters h and k:

$$\cos(2\phi) = \frac{h^2 + 2k^2}{\sqrt{(h^2 + 2k^2)^2 + h^2k^2}}$$

$$\sin(2\phi) = \frac{hk}{\sqrt{(h^2 + 2k^2)^2 + h^2k^2}}$$

Energy Gained by Bob:

The average energy extracted by Bob is:

$$\langle E_B \rangle = \sum_{\mu \in \{-1,1\}} \langle E_{max} | P_A(\mu) U_B(\mu) H_B U_B(\mu) P_A(\mu) | E_{max} \rangle$$

Breaking it down:

- $P_A(\mu)$: Projector for Alice's measurement outcome.
- The terms combine to give:

$$\langle E_B \rangle = \frac{1}{\sqrt{h^2 + k^2}} \left[hk \sin(2\phi) - (h^2 + 2k^2)(1 - \cos(2\phi)) \right]$$

5. Advantages of the Protocol

1. **Instantaneous Charging**:
   - Energy transfer occurs immediately after Alice's measurement and communication, without needing to wait.
2. **Experimental Simplicity**:
   - The protocol uses only local operations and classical communication (LOCC), simplifying implementation.
3. **Compatibility with Quantum Computers**:
   - It is practical for current quantum devices, enabling experimental validation.
4. **Comparison with Quantum Energy Teleportation (QET)**:
   - Unlike QET, where Bob accumulates energy via unitary evolution, this protocol limits Bob's energy extraction using LOCC. Alice's measurement disrupts unitarity, allowing Bob to surpass his local energy threshold, a process not possible in QET by unitary evolution alone.

6. Broader Implications

Quantum Batteries:

- This protocol demonstrates how quantum entanglement and measurement can be harnessed for energy storage and transfer, paving the way for practical quantum batteries.

Future Directions:

1. **Scalability:**
   - Extending the protocol to larger spin chains and multi-party systems to test robustness and scalability.
2. **Experimental Validation:**
   - Real-world implementation using quantum computers or other quantum devices to validate the protocol's practical effectiveness.
3. **Applications:**
   - Exploring applications in quantum networks, quantum thermodynamics, or energy-efficient quantum devices.

End

Thank You