Day: Sat Sun Mon Tue Wed Thu Fri

Subject: .... Date: ..../..../.... Time: ....

Soln: first do a scatter plot & decide the function you want to fit:

$y = mx + c$

$y = a_1 + a_2 x + a_3 x^2$

Problem 1 | Original image I:

$$I = \begin{bmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{bmatrix}$$

Zero-padded image $\tilde{I}$:

$$\tilde{I} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 & 0 \\ 0 & 8 & 5 & 2 & 0 \\ 0 & 9 & 6 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

filters F, dimensions: 3×3

$$G(i,j) = \sum_{u=0}^{2} \sum_{v=0}^{2} F(u,v) \cdot \tilde{I}(i+u, j+v)$$

Day : Sat Sun Mon Tue Wed Thu Fri

Subject: filter

Date : ....../....../...... Time : ....

a) $F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$G(0,0) = 0\cdot\bar{I}(0,0) + 0\cdot\bar{I}(0,1) + 0\cdot\bar{I}(0,2) + 0\cdot\bar{I}(1,0) + 1\cdot\bar{I}(1,1) + 0\cdot\bar{I}(1,2) + 0\cdot\bar{I}(2,0) + 0\cdot\bar{I}(2,1) + 0\cdot\bar{I}(2,2)$$

$$G(0,0) = 7$$

$$\bar{I} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 & 0 \\ 0 & 8 & 5 & 2 & 0 \\ 0 & 9 & 6 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G(0,0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 4 \\ 0 & 8 & 5 \end{bmatrix}$$

b) $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

~~G(0,0)~~

$$G(0,0) = 1\cdot\bar{I}(0,0) + 0\cdot\bar{I}(0,1) + 0\cdot\bar{I}(0,2) + 0\cdot\bar{I}(1,0) + 0\cdot\bar{I}(1,1) + 0\cdot\bar{I}(1,2) + 0\cdot\bar{I}(2,0) + 0\cdot\bar{I}(2,0) + 0\cdot\bar{I}(2,1) + 0\cdot\bar{I}(2,2)$$

$$G(0,0) = 0$$

$$G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 4 \\ 0 & 8 & 5 \end{bmatrix}$$

c) $F = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

Region of $I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 4 \\ 0 & 8 & 5 \end{bmatrix}$

$$G(0,0) = (1\cdot 0 + 1\cdot 0 + 1\cdot 0) + (0\cdot 0 + 0\cdot 7 + 0\cdot 4) + (-1\cdot 0 - 1\cdot 8 - 1\cdot 5)$$

$$G(0,0) = 0 + 0 - 13 = -13$$

$$G(0,1) = (1\cdot 0 + 1\cdot 0 + 1\cdot 0) + (0\cdot 7 + 0\cdot 4 + 0\cdot 1) + (-1\cdot 8 - 1\cdot 5 - 1\cdot 2)$$

$$G(0,1) = 0 + 0 - 15 = -15$$

$G(0,2)$, $I = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 2 & 0 \end{bmatrix}$

$$G(0,2) = (1\cdot 0 + 1\cdot 0 + 1\cdot 0) + (0\cdot 4 + 0\cdot 1 + 0\cdot 0) + (-1\cdot 5 - 1\cdot 2 - 1\cdot 0)$$

$$G(0,2) = 0 + 0 - 7 = -7$$

So on, $G = \begin{bmatrix} -13 & -15 & -7 \\ -14 & -18 & -8 \\ -9 & -11 & -3 \end{bmatrix}$

Day: Sat Sun Mon Tue Wed Thu Fri

Subject :

Date : / / Time :

The filter detects horizontal edges by calculating the difference between the sum of the upper row and the lower row. Positive values indicate a transition from darker to lighter pixels, while negative values indicate a transition from lighter to dark pixels. Flat regions with no horizontal gradient produce values near zero.

d) $F' = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

$G(0,0) = (-1\cdot0 + 0\cdot0 + 1\cdot0) + (-1\cdot7 + 0\cdot4 + 1\cdot1) + (-1\cdot8 + 0\cdot5 + 1\cdot2)$

$G(0,0) = -15.$

for all i & j $\quad G = \begin{bmatrix} -15 & -12 & -9 \\ -15 & -12 & -9 \\ -15 & -12 & -9 \end{bmatrix}$

This ~~filt~~ filter emphasis verticle transitions, while (c) emphasis horizontal transition.

e) $G(0,0) = (1\cdot0 + 2\cdot0 + 1\cdot0) + (2\cdot0 + 4\cdot7 + 2\cdot4) + (1\cdot0 + 2\cdot8 + 1\cdot5)$

$G(0,0) = 0 + (0 + 28 + 8) + (0 + 16 + 5) = 57$

CamScanner

Day: Sat Sun Mon Tue Wed Thu Fri

Subject: ....

Date: ....../....../...... Time: ....

$G(0,1) = (1.0 + 2.0 + 1.0) + (2.7 + 4.4 + 2.1) + (1.8 + 2.5 + 1.2)$

$G(0,1) = 0 + (14 + 16 + 2) + (8 + 10 + 2)$

$= 52$

$$G(i,j) = \begin{bmatrix} 57 & 52 & 39 \\ 78 & 72 & 54 \\ 93 & 86 & 63 \end{bmatrix}$$

The filter Fe is a Gaussian blur filter that smooths the image, reducing noise and minor details while preserving general structures. The weights in Fe give higher importance to the center pixel and its immediate neighbors.

f) $F = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

This filter is a box blur or mean filter that averages all the pixels in the region equally.

$G(0,0) = (1.0 + 1.0 + 1.0) + (1.0 + 1.7 + 1.4) + (1.0 + 1.8 + 1.5) = 24$

Day: Sat Sun Mon Tue Wed Thu Fri

Subject :

Date : / / Time :

final output $G_f = \begin{bmatrix} 29 & 27 & 15 \\ 36 & 39 & 21 \\ 42 & 45 & 29 \end{bmatrix}$

The filter f applies a box blur that averages all pixels values within the filter region equally, creating a smoothing effect but without weighting the center pixel more than others. This is a simpler smoothing filter compared to the Gaussian blur.

Difference between e & f

Filter e: Gaussian blur gives more weight to the center and its immediate neighbors, making it more effective at preserving details while reducing noise.

Filter f: Box blur treats all pixels equally, which can result in a more uniform smoothing effect but may lose finer details in the image.

Day: Sat Sun Mon Tue Wed Thu Fri

Subject: Date: / / Time:

i) a) Show how correlation can be written as a vector dot product

let's proceed & relate it to the given equation $G(i,j) = f^T t_{ij}$.

Correlation operation at pixel (i,j) is:

$$G(i,j) = \sum_{u=0}^{k-1} \sum_{v=0}^{k-1} f(u,v) \cdot \tilde{I}(i+u, j+v)$$

$f(u,v)$ are the filter weights.

$\tilde{I}(i+u, j+v)$ are the pixel values from the padded image at the $(i+u, j+v)$ positions.

Let the filter f be of size $k \times k$.

Flatten the matrix f into a single column vector f' of size $k^2 \times 1$

For example, if f is,

$$f = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) \\ f(1,0) & f(1,1) & f(1,2) \\ f(2,0) & f(2,1) & f(2,2) \end{bmatrix}$$

Then, $f = \begin{bmatrix} f(0,0) \\ f(0,1) \\ f(0,2) \\ f(1,0) \\ f(1,1) \\ f(1,2) \\ f(2,0) \\ f(2,1) \\ f(2,2) \end{bmatrix}$

Similarly, extract $k \times k$ patch of the padded image $\tilde{I}$ around (i,j). Flatten it into a column vector t_{ij} of size $k^2 \times 1$. If the patch T_{ij} is:

$$T_{ij} = \begin{bmatrix} \tilde{I}(i,j) & \tilde{I}(i,j+1) & \tilde{I}(i,j+2) \\ \tilde{I}(i+1,j) & \tilde{I}(i+1,j+1) & \tilde{I}(i+1,j+2) \\ \tilde{I}(i+2,j) & \tilde{I}(i+2,j+1) & \tilde{I}(i+2,j+2) \end{bmatrix}$$

Then $t_{ij} = \begin{bmatrix} \tilde{I}(i,j) \\ \tilde{I}(i,j+1) \\ \tilde{I}(i,j+2) \\ \tilde{I}(i+1,j) \\ \tilde{I}(i+1,j+1) \\ \tilde{I}(i+1,j+2) \\ \tilde{I}(i+2,j) \\ \tilde{I}(i+2,j+1) \\ \tilde{I}(i+2,j+2) \end{bmatrix}$

Day: Sat Sun Mon Tue Wed Thu Fri

Subject :... Date : ....../....../...... Time :..........

The correlation value $G(i,j)$ now written dot product of vectorized filter f and vectorized image patch t_{ij}!

$$G(i,j) = f^T t_{ij}$$

f^T is the transpose filter vector f

t_{ij} is vectorized patch image.

The dot product $f^T t_{ij}$ essentially performs an element-wise multiplication between image patch values. filter values: image patch values.