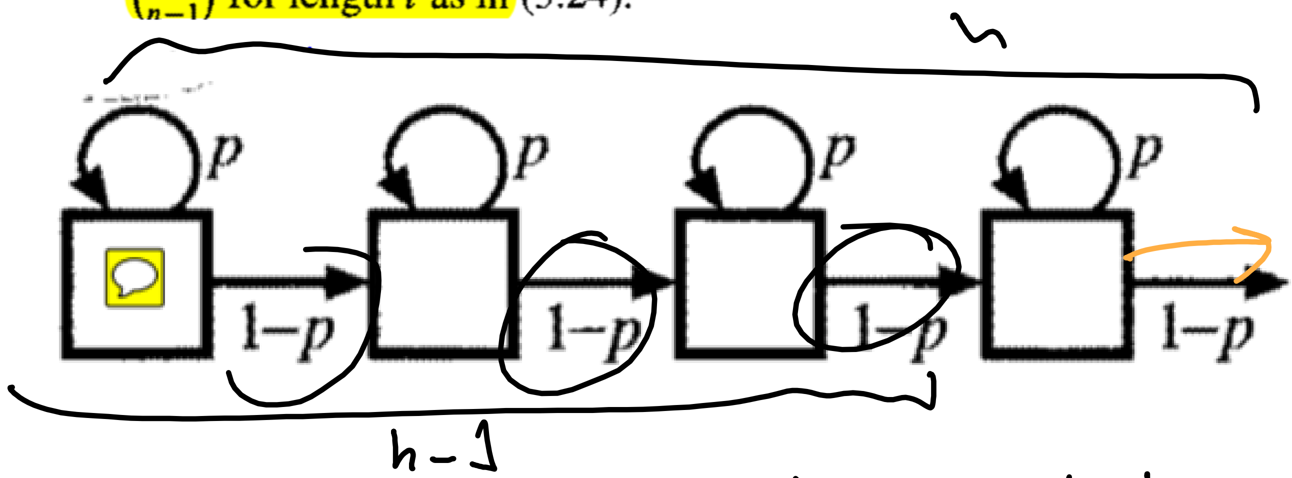
3.8 Show that the number of paths through an array of $n$ states is indeed $\binom{l-1}{n-1}$ for length $l$ as in (3.24).




model as negative binomial distribution:

# trials = length of sequence: $\ell$

# successes = transition to next state : $n-1+1$

forward transitions

except the last one which is necessarily a forward

$P(\text{success}) = 1-p$

# failures = staying in the same state: $\ell - n$

self-loops

$P(\text{failure}) = p$

$n :=$ # states = minimum length

a sequence with length $\ell$ and $\ell-1$ transitions

$n-1$ : forward transitions

$\ell-n$ : self transitions

number of trials (length of the sequence) at which the $n$th success occurs.

($n$: # forward transitions)

$$P(L = e \mid p, n) = \binom{e-1}{n-1} \times (1-P)^{n} \times P^{e-n}$$

The last transition must be a <u>forward one</u>. So, we need to choose

<u>$n-1$ success</u> (or $e-n$ failures)

<u>from $e-1$ trials.</u>