Interpolation

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Direct Method of Interpolation

What is Interpolation ?

Given (x_0,y_0), (x_1,y_1), ... (x_n,y_n), find the value of 'y' at a value of 'x' that is not given.

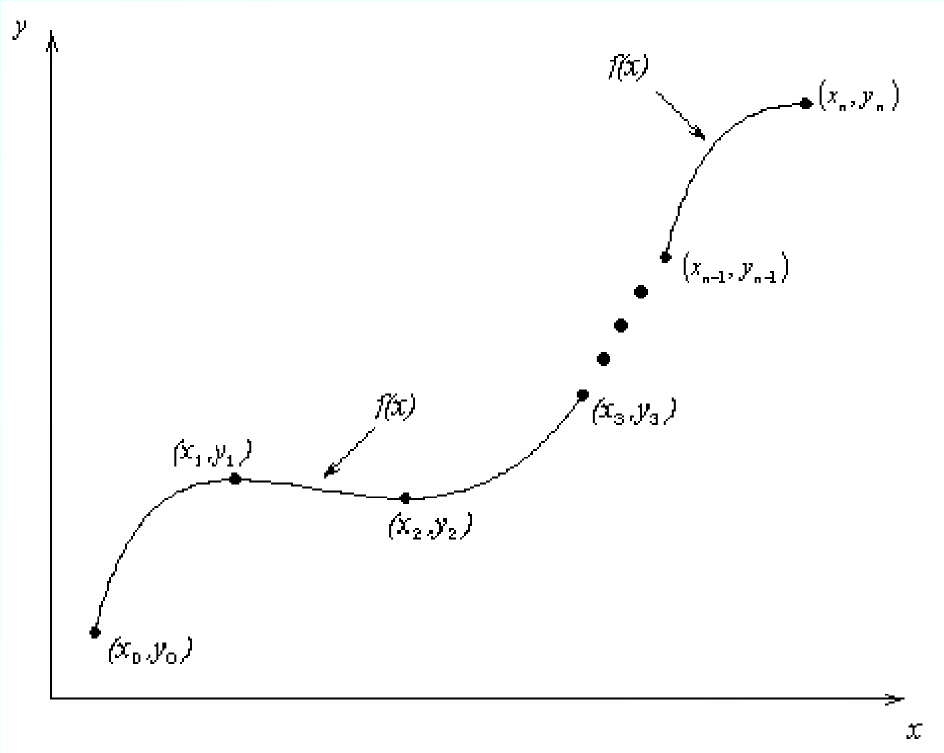


Figure 1 Interpolation of discrete.

Interpolants

Polynomials are the most common choice of interpolants because they are easy to:

- Evaluate
- Differentiate, and
- Integrate

Direct Method

Given 'n+1' data points (x_0,y_0), (x_1,y_1),............. (x_n,y_n), pass a polynomial of order 'n' through the data as given below:

$$y = a_0 + a_1 x + ... + a_n x^n.$$

where a_0, a_1,.................. a_n are real constants.

- Set up 'n+1' equations to find 'n+1' constants.
- To find the value 'y' at a given value of 'x', simply substitute the value of 'x' in the above polynomial.

Example 1

The upward velocity of a rocket is given as a function of time in Table 1.

Find the velocity at t=16 seconds using the direct method for linear interpolation.

Table 1 Velocity as a function of time.

$t,(\mathrm{s})$	$v(t),(\mathrm{m/s})$
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

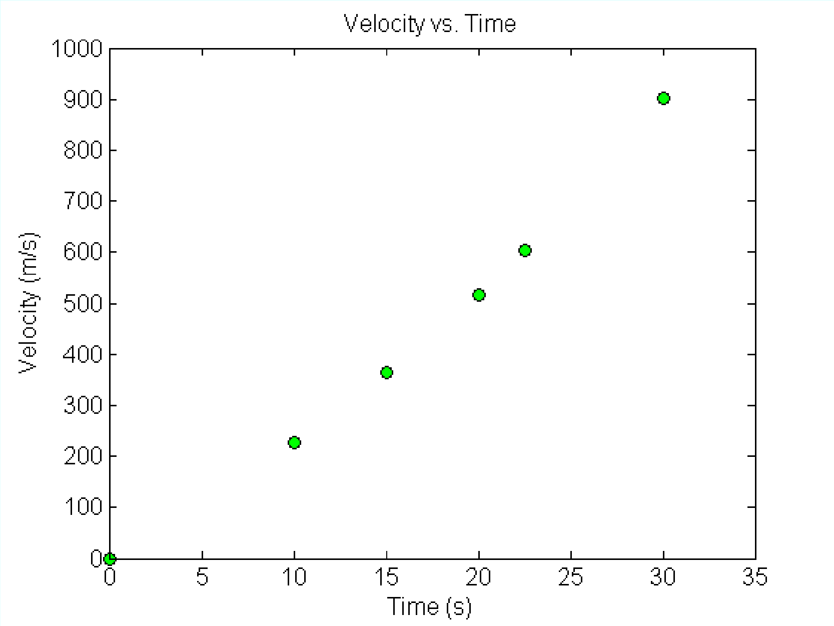


Figure 2 Velocity vs. time data for the rocket example

Linear Interpolation

$$v(t) = a_0 + a_1 t$$

$$v(15) = a_0 + a_1(15) = 362.78$$

$$v(20) = a_0 + a_1(20) = 517.35$$

Solving the above two equations gives,

$$a_0 = -100.93 \qquad a_1 = 30.914$$

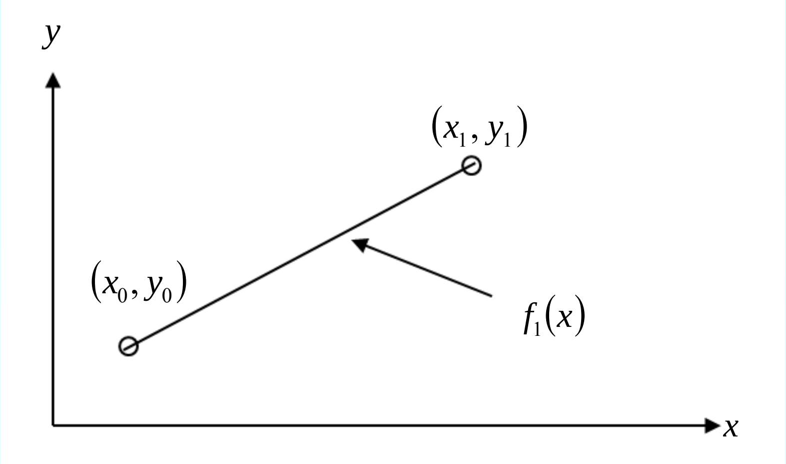


Figure 3 Linear interpolation.

Hence

$$v(t) = -100.93 + 30.914t, \; 15 \le t \le 20.$$

$$v(16) = -100.93 + 30.914(16) = 393.7 \text{ m/s}$$

Example 2

The upward velocity of a rocket is given as a function of time in Table 2.

Find the velocity at t=16 seconds using the direct method for quadratic interpolation.

Table 2 Velocity as a function of time.

$t, (\text{s})$	$v(t), (\text{m/s})$
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

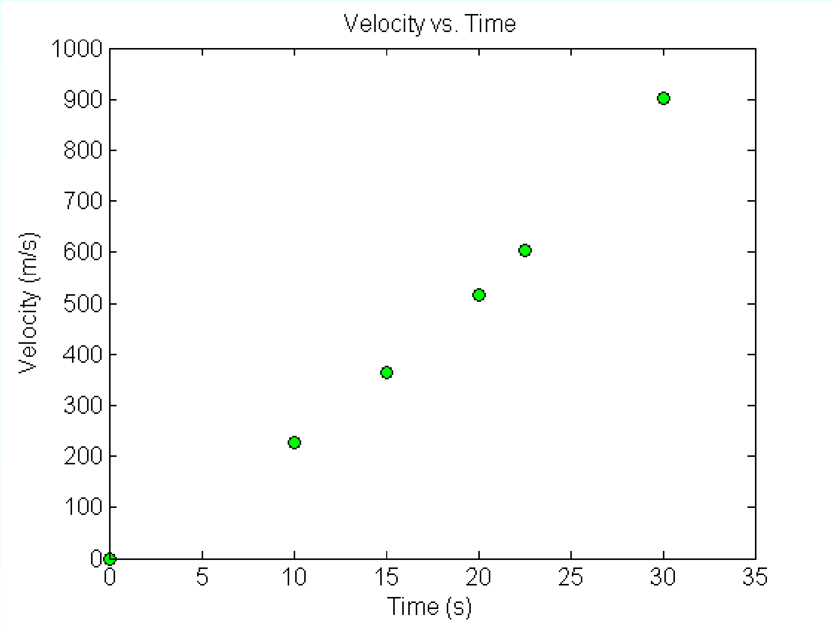


Figure 5 Velocity vs. time data for the rocket example

Quadratic Interpolation

$$v(t) = a_0 + a_1 t + a_2 t^2$$

$$v(10) = a_0 + a_1(10) + a_2(10)^2 = 227.04$$

$$v(15) = a_0 + a_1(15) + a_2(15)^2 = 362.78$$

$$v(20) = a_0 + a_1(20) + a_2(20)^2 = 517.35$$

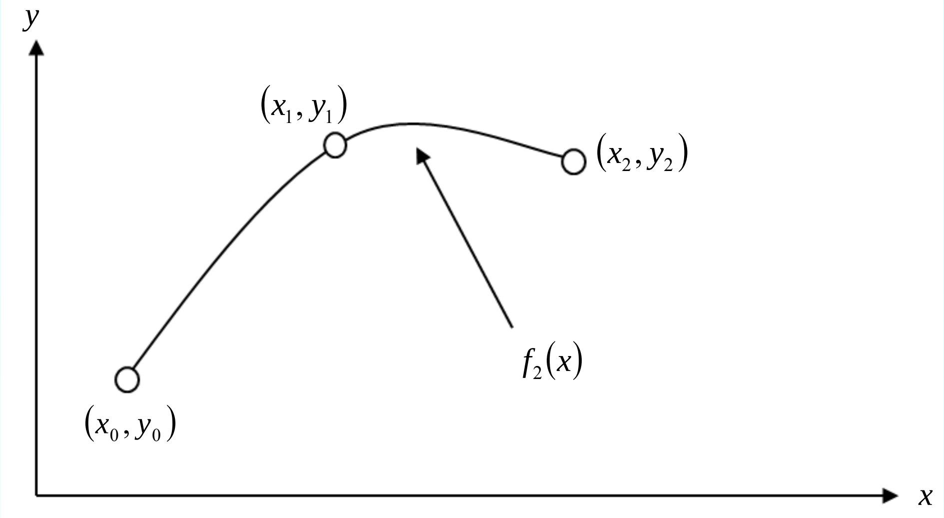


Figure 6 Quadratic interpolation.

Solving the above three equations gives

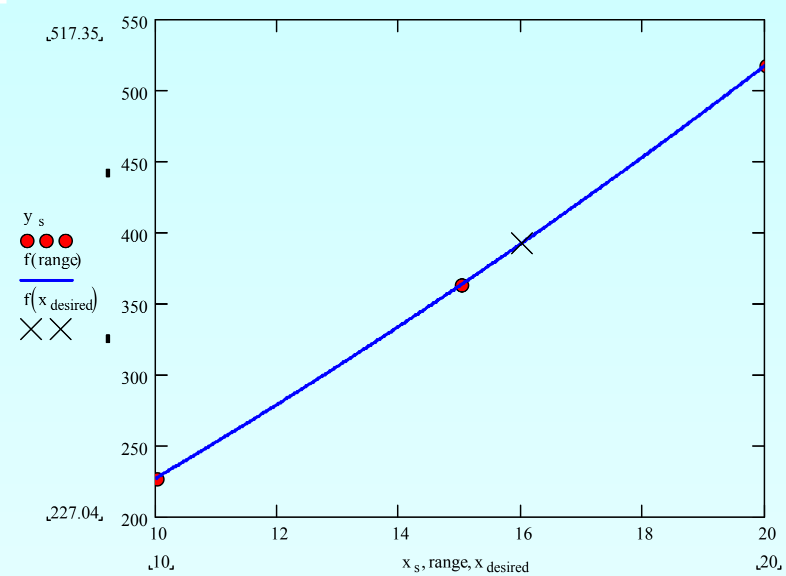
$$a_0 = 12.05 \quad a_1 = 17.733 \quad a_2 = 0.3766$$

Quadratic Interpolation (cont.)

$$v(t) = 12.05 + 17.733t + 0.3766t^2, \; 10 \le t \le 20$$

$$v(16) = 12.05 + 17.733(16) + 0.3766(16)^2$$

$$= 392.19 \text{ m/s}$$



The absolute relative approximate error $|\in_a|$ obtained between the results from the first and second order polynomial is

$$|\in_a| = \left|\frac{392.19 - 393.70}{392.19}\right| \times 100$$

$$= 0.38410\%$$

Example 3

The upward velocity of a rocket is given as a function of time in Table 3.

Find the velocity at t=16 seconds using the direct method for cubic interpolation.

Table 3 Velocity as a function of time.

$t,(\mathrm{s})$	$v(t),(\mathrm{m/s})$
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

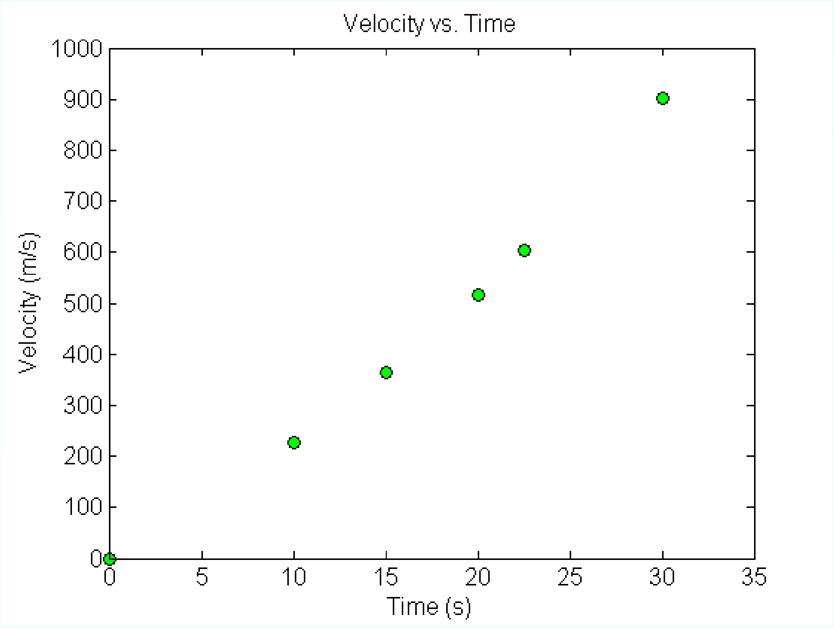


Figure 6 Velocity vs. time data for the rocket example

Cubic Interpolation

$$v(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$v(10) = 227.04 = a_0 + a_1(10) + a_2(10)^2 + a_3(10)^3$$

$$v(15) = 362.78 = a_0 + a_1(15) + a_2(15)^2 + a_3(15)^3$$

$$v(20) = 517.35 = a_0 + a_1(20) + a_2(20)^2 + a_3(20)^3$$

$$v(22.5) = 602.97 = a_0 + a_1(22.5) + a_2(22.5)^2 + a_3(22.5)^3$$

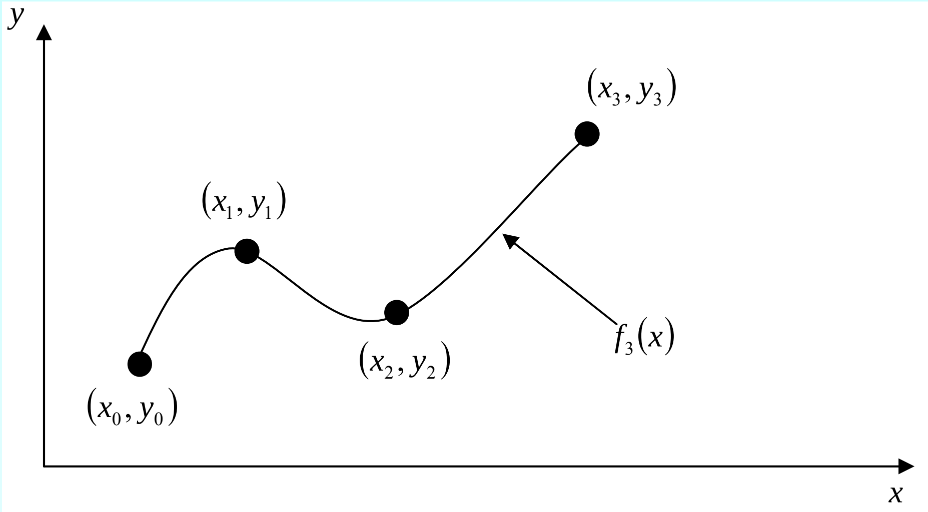


Figure 7 Cubic interpolation.

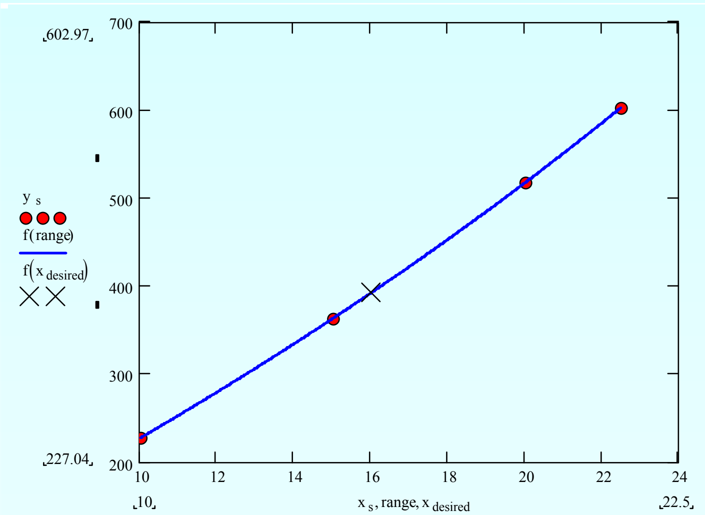
$$a_0 = -4.2540 \quad a_1 = 21.266 \quad a_2 = 0.13204 \quad a_3 = 0.0054347$$

Cubic Interpolation (contd)

$$v(t) = -4.2540 + 21.266t + 0.13204t^2 + 0.0054347t^3, \quad 10 \le t \le 22.5$$

$$v(16) = -4.2540 + 21.266(16) + 0.13204(16)^2 + 0.0054347(16)^3$$

$$= 392.06 \text{ m/s}$$



The absolute percentage relative approximate error $|\in_a|$ between second and third order polynomial is

$$|\in_a| = \left|\frac{392.06 - 392.19}{392.06}\right| \times 100$$

$$= 0.033269\,\%$$

Comparison Table

Table 4 Comparison of different orders of the polynomial.

Order of Polynomial	1	2	3
$v(t=16)$ m/s	393.7	392.19	392.06
Absolute Relative Approximate Error	----------	0.38410 %	0.033269 %

Distance from Velocity Profile

Find the distance covered by the rocket from t=11s to t=16s ?

$$v(t) = -4.3810 + 21.289t + 0.13064t^2 + 0.0054606t^3, \quad 10 \le t \le 22.5$$

$$s(16) - s(11) = \int_{11}^{16} v(t)dt$$

$$= \int_{11}^{16} \left(-4.2540 + 21.266t + 0.13204t^2 + 0.0054347t^3\right)dt$$

$$= \left[-4.2540t + 21.266\frac{t^2}{2} + 0.13204\frac{t^3}{3} + 0.0054347\frac{t^4}{4}\right]_{11}^{16}$$

$$= 1605 \text{ m}$$

Acceleration from Velocity Profile

Find the acceleration of the rocket at t=16s given that

$$v(t) = -4.2540 + 21.266t + 0.13204^2 + 0.0054347t^3, 10 \le t \le 22.5$$

$$a(t) = \frac{d}{dt} v(t)$$

$$= \frac{d}{dt}\left(-4.2540 + 21.266t + 0.13204t^2 + 0.0054347t^3\right)$$

$$= 21.289 + 0.26130t + 0.016382t^2, \quad 10 \le t \le 22.5$$

$$a(16) = 21.266 + 0.26408(16) + 0.016304(16)^2$$

$$= 29.665 \text{ m/s}^2$$

Lagrange Method of Interpolation

Lagrangian Interpolation

Lagrangian interpolating polynomial is given by

$$f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

where 'n' in $f_n(x)$ stands for the n^{th} order polynomial that approximates the function $y = f(x)$ given at $(n+1)$ data points as $(x_0, y_0), (x_1, y_1), \ldots\ldots, (x_{n-1}, y_{n-1}), (x_n, y_n)$, and

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

$L_i(x)$ is a weighting function that includes a product of $(n-1)$ terms with terms of $j = i$ omitted.

Example

The upward velocity of a rocket is given as a function of time in Table 1. Find the velocity at t=16 seconds using the Lagrangian method for linear interpolation.

Table Velocity as a function of time

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

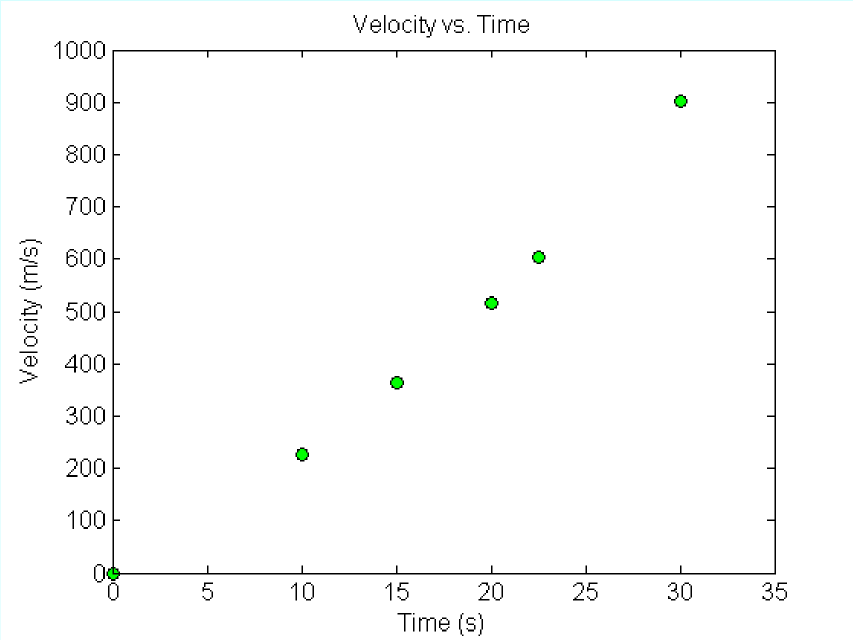


Figure. Velocity vs. time data for the rocket example

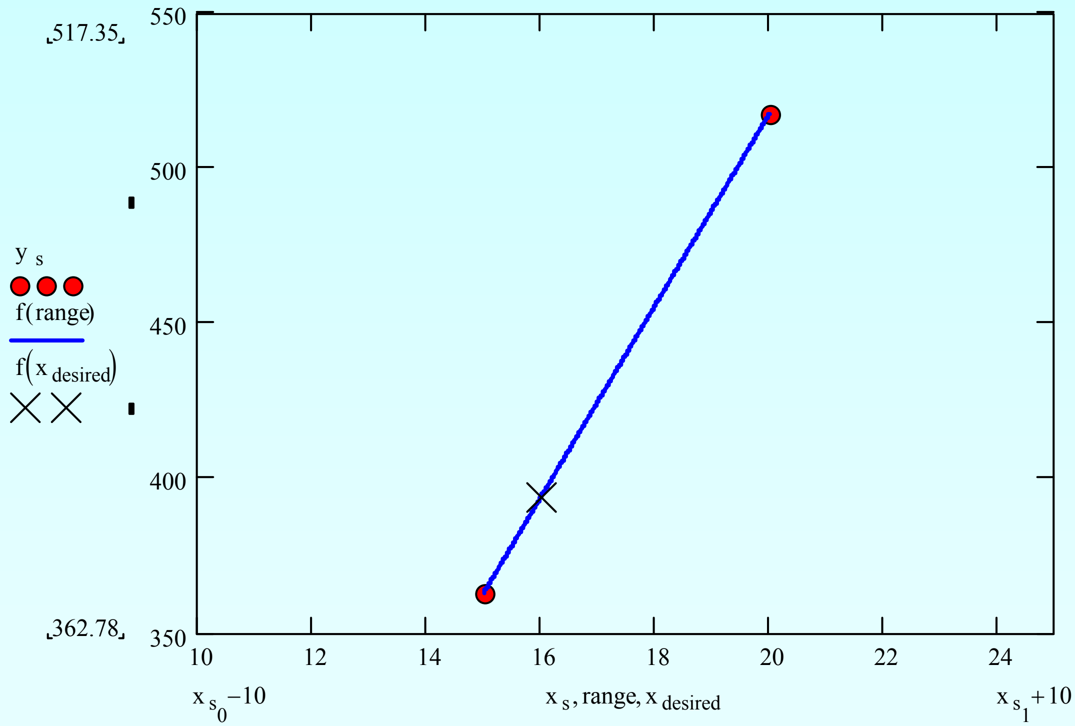
Linear Interpolation

$$v(t) = \sum_{i=0}^{1} L_i(t)v(t_i)$$

$$= L_0(t)v(t_0) + L_1(t)v(t_1)$$

$$t_0 = 15, v(t_0) = 362.78$$

$$t_1 = 20, v(t_1) = 517.35$$



Linear Interpolation (contd)

$$L_0(t) = \prod_{\substack{j=0 \\ j\neq 0}}^{1} \frac{t - t_j}{t_0 - t_j} \quad = \frac{t - t_1}{t_0 - t_1}$$

$$L_1(t) = \prod_{\substack{j=0 \\ j\neq 1}}^{1} \frac{t - t_j}{t_1 - t_j} \quad = \frac{t - t_0}{t_1 - t_0}$$

$$v(t) = \frac{t - t_1}{t_0 - t_1} v(t_0) + \frac{t - t_0}{t_1 - t_0} v(t_1) \quad = \frac{t - 20}{15 - 20}(362.78) + \frac{t - 15}{20 - 15}(517.35)$$

$$v(16) = \frac{16 - 20}{15 - 20}(362.78) + \frac{16 - 15}{20 - 15}(517.35)$$

$$= 0.8(362.78) + 0.2(517.35)$$

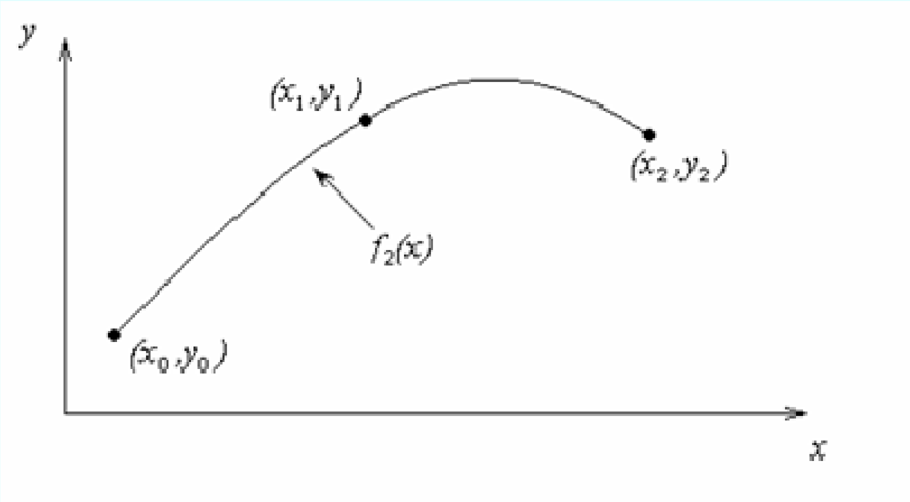
$$= 393.7 \text{ m/s.}$$

Quadratic Interpolation

For the second order polynomial interpolation (also called quadratic interpolation), we choose the velocity given by

$$v(t) = \sum_{i=0}^{2} L_i(t)v(t_i)$$

$$= L_0(t)v(t_0) + L_1(t)v(t_1) + L_2(t)v(t_2)$$



Example

The upward velocity of a rocket is given as a function of time in Table 1. Find the velocity at t=16 seconds using the Lagrangian method for quadratic interpolation.

Table Velocity as a function of time

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

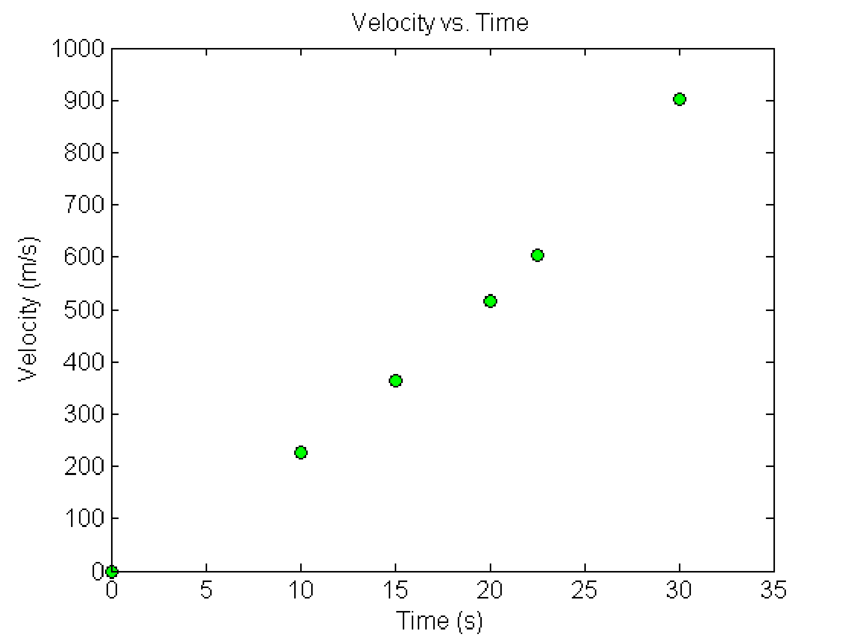


Figure. Velocity vs. time data for the rocket example

Quadratic Interpolation (contd)

$$t_0 = 10,\ v(t_0) = 227.04$$

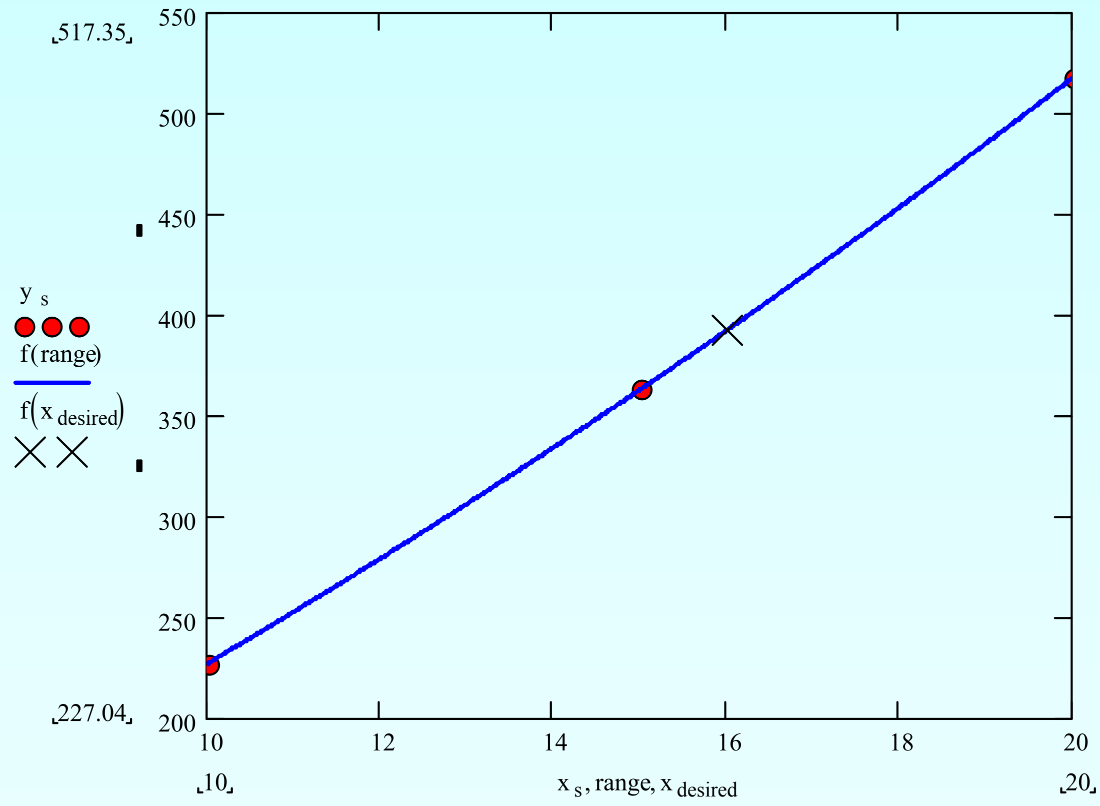
$$t_1 = 15,\ v(t_1) = 362.78$$

$$t_2 = 20,\ v(t_2) = 517.35$$

$$L_0(t) = \prod_{\substack{j=0\\ j\neq 0}}^{2} \frac{t - t_j}{t_0 - t_j} = \left(\frac{t - t_1}{t_0 - t_1}\right)\left(\frac{t - t_2}{t_0 - t_2}\right)$$

$$L_1(t) = \prod_{\substack{j=0\\ j\neq 1}}^{2} \frac{t - t_j}{t_1 - t_j} = \left(\frac{t - t_0}{t_1 - t_0}\right)\left(\frac{t - t_2}{t_1 - t_2}\right)$$

$$L_2(t) = \prod_{\substack{j=0\\ j\neq 2}}^{2} \frac{t - t_j}{t_2 - t_j} = \left(\frac{t - t_0}{t_2 - t_0}\right)\left(\frac{t - t_1}{t_2 - t_1}\right)$$



Quadratic Interpolation (contd)

$$v(t)=\left(\frac{t-t_1}{t_0-t_1}\right)\left(\frac{t-t_2}{t_0-t_2}\right)v(t_0)+\left(\frac{t-t_0}{t_1-t_0}\right)\left(\frac{t-t_2}{t_1-t_2}\right)v(t_1)+\left(\frac{t-t_0}{t_2-t_0}\right)\left(\frac{t-t_1}{t_2-t_1}\right)v(t_2)$$

$$v(16)=\left(\frac{16-15}{10-15}\right)\left(\frac{16-20}{10-20}\right)(227.04)+\left(\frac{16-10}{15-10}\right)\left(\frac{16-20}{15-20}\right)(362.78)+\left(\frac{16-10}{20-10}\right)\left(\frac{16-15}{20-15}\right)(517.35)$$

$$=(-0.08)(227.04)+(0.96)(362.78)+(0.12)(527.35)$$

$$=392.19\text{ m/s}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the first and second order polynomial is

$$|\in_a|=\left|\frac{392.19-393.70}{392.19}\right|\times 100$$

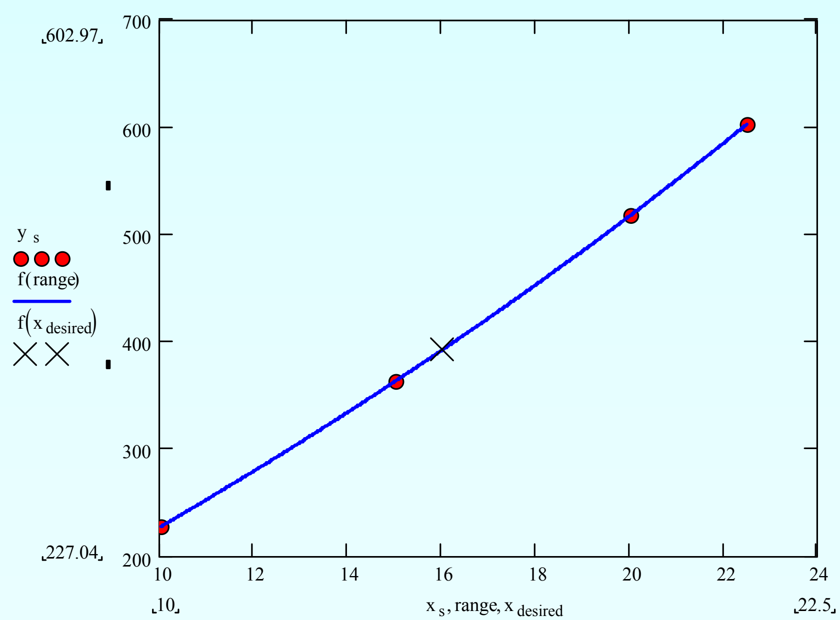
$$=0.38410\%$$

Cubic Interpolation

For the third order polynomial (also called cubic interpolation), we choose the velocity given by

$$v(t) = \sum_{i=0}^{3} L_i(t)v(t_i)$$

$$= L_0(t)v(t_0) + L_1(t)v(t_1) + L_2(t)v(t_2) + L_3(t)v(t_3)$$



Example

The upward velocity of a rocket is given as a function of time in Table 1. Find the velocity at t=16 seconds using the Lagrangian method for cubic interpolation.

Table Velocity as a function of time

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

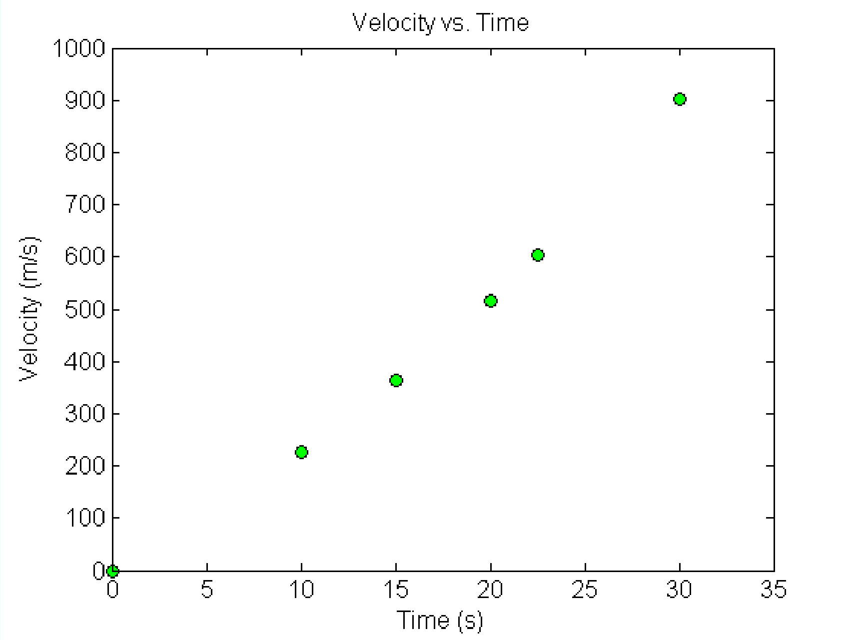


Figure. Velocity vs. time data for the rocket example

Cubic Interpolation (contd)

$$t_o = 10, \; v(t_o) = 227.04 \qquad t_1 = 15, \; v(t_1) = 362.78$$

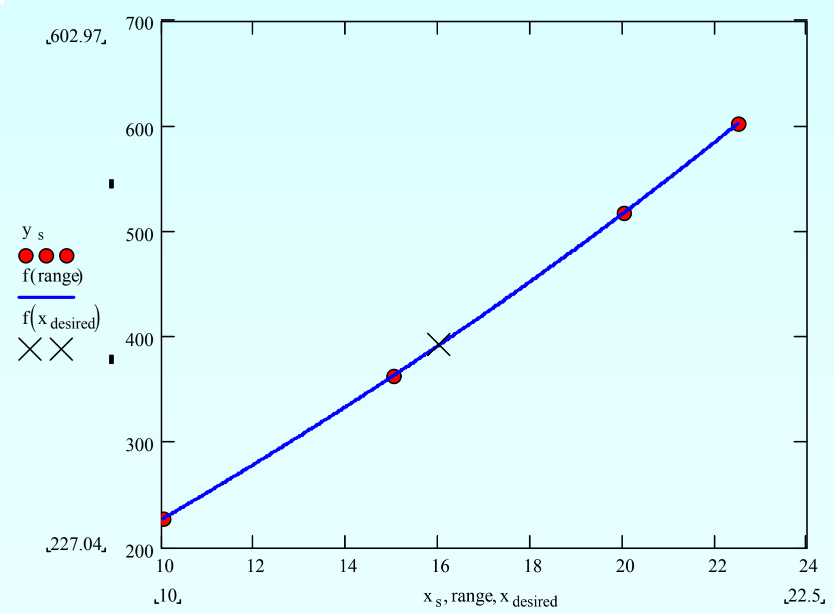
$$t_2 = 20, \; v(t_2) = 517.35 \qquad t_3 = 22.5, \; v(t_3) = 602.97$$

$$L_0(t) = \prod_{\substack{j=0 \\ j\neq 0}}^{3} \frac{t - t_j}{t_0 - t_j} = \left(\frac{t - t_1}{t_0 - t_1}\right)\left(\frac{t - t_2}{t_0 - t_2}\right)\left(\frac{t - t_3}{t_0 - t_3}\right);$$

$$L_1(t) = \prod_{\substack{j=0 \\ j\neq 1}}^{3} \frac{t - t_j}{t_1 - t_j} = \left(\frac{t - t_0}{t_1 - t_0}\right)\left(\frac{t - t_2}{t_1 - t_2}\right)\left(\frac{t - t_3}{t_1 - t_3}\right)$$

$$L_2(t) = \prod_{\substack{j=0 \\ j\neq 2}}^{3} \frac{t - t_j}{t_2 - t_j} = \left(\frac{t - t_0}{t_2 - t_0}\right)\left(\frac{t - t_1}{t_2 - t_1}\right)\left(\frac{t - t_3}{t_2 - t_3}\right);$$

$$L_3(t) = \prod_{\substack{j=0 \\ j\neq 3}}^{3} \frac{t - t_j}{t_3 - t_j} = \left(\frac{t - t_0}{t_3 - t_0}\right)\left(\frac{t - t_1}{t_3 - t_1}\right)\left(\frac{t - t_2}{t_3 - t_2}\right)$$



Cubic Interpolation (contd)

$$v(t)=\left(\frac{t-t_1}{t_0-t_1}\right)\left(\frac{t-t_2}{t_0-t_2}\right)\left(\frac{t-t_3}{t_0-t_3}\right)v(t_1)+\left(\frac{t-t_0}{t_1-t_0}\right)\left(\frac{t-t_2}{t_1-t_2}\right)\left(\frac{t-t_3}{t_1-t_3}\right)v(t_2)$$

$$+\left(\frac{t-t_0}{t_2-t_0}\right)\left(\frac{t-t_1}{t_2-t_1}\right)\left(\frac{t-t_3}{t_2-t_3}\right)v(t_2)+\left(\frac{t-t_1}{t_3-t_1}\right)\left(\frac{t-t_1}{t_3-t_1}\right)\left(\frac{t-t_2}{t_3-t_2}\right)v(t_3)$$

$$v(16)=\left(\frac{16-15}{10-15}\right)\left(\frac{16-20}{10-20}\right)\left(\frac{16-22.5}{10-22.5}\right)(227.04)+\left(\frac{16-10}{15-10}\right)\left(\frac{16-20}{15-20}\right)\left(\frac{16-22.5}{15-22.5}\right)(362.78)$$

$$+\left(\frac{16-10}{20-10}\right)\left(\frac{16-15}{20-15}\right)\left(\frac{16-22.5}{20-22.5}\right)(517.35)+\left(\frac{16-10}{22.5-10}\right)\left(\frac{16-15}{22.5-15}\right)\left(\frac{16-20}{22.5-20}\right)(602.97)$$

$$=(-0.0416)(227.04)+(0.832)(362.78)+(0.312)(517.35)+(-0.1024)(602.97)$$

$$=392.06\,\text{m/s}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the first and second order polynomial is

$$|\in_a|=\left|\frac{392.06-392.19}{392.06}\right|\times 100$$

$$=0.033269\%$$

Comparison Table

Order of Polynomial	**1**	**2**	**3**
v(t=16) m/s	393.69	392.19	392.06
Absolute Relative Approximate Error	--------	0.38410%	0.033269%

Distance from Velocity Profile

Find the distance covered by the rocket from t=11s to t=16s ?

$$v(t) = (t^3 - 57.5t^2 + 1087.5t - 6750)(-0.36326) + (t^3 - 52.5t^2 + 875t - 4500)(1.9348)$$
$$+ (t^3 - 47.5t^2 + 712.5t - 3375)(-4.1388) + (t^3 - 45t^2 + 650t - 3000)(2.5727)$$

$$v(t) = -4.245 + 21.265t + 0.13195t^2 + 0.00544t^3, \quad 10 \le t \le 22.5$$

$$s(16) - s(11) = \int_{11}^{16} v(t)dt$$

$$\approx \int_{11}^{16} (-4.245 + 21.265t + 0.13195t^2 + 0.00544t^3)dt$$

$$= [-4.245t + 21.265\frac{t^2}{2} + 0.13195\frac{t^3}{3} + 0.00544\frac{t^4}{4}]_{11}^{16}$$

$$= 1605 \text{ m}$$

Acceleration from Velocity Profile

Find the acceleration of the rocket at t=16s given that

$$v(t) = -4.245 + 21.265t + 0.13195t^2 + 0.00544t^3, \qquad 10 \le t \le 22.5$$

$$a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}\left(-4.245 + 21.265t + 0.13195t^2 + 0.00544t^3\right)$$

$$= 21.265 + 0.26390t + 0.01632t^2$$

$$a(16) = 21.265 + 0.26390(16) + 0.01632(16)^2$$

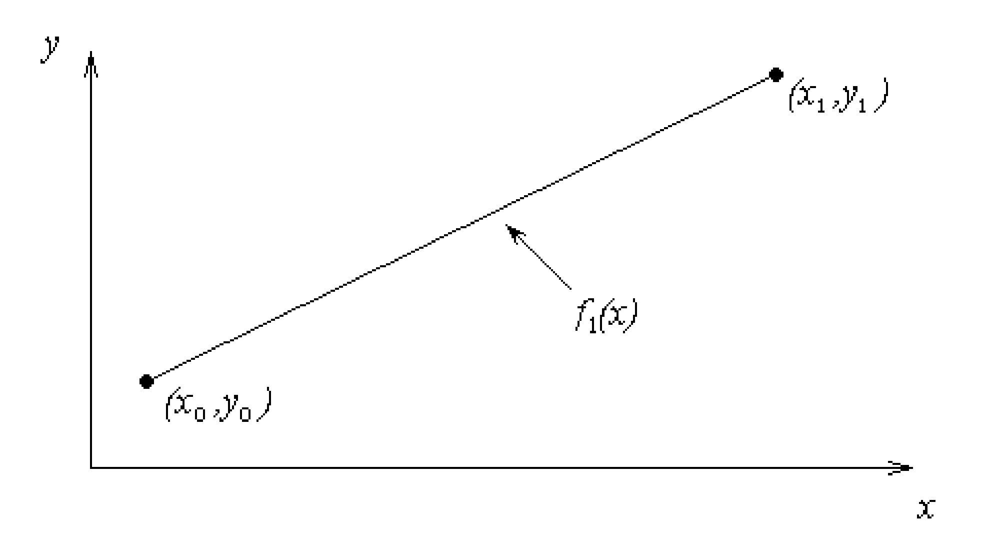
$$= 29.665 \;\; m/s^2$$

Newton’s Divided Difference Method of Interpolation

Newton's Divided Difference Method

Linear interpolation: Given $(x_0, y_0), (x_1, y_1),$ pass a linear interpolant through the data

$$f_1(x) = b_0 + b_1(x - x_0)$$



where

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Example

The upward velocity of a rocket is given as a function of time in Table 1. Find the velocity at t=16 seconds using the Newton Divided Difference method for linear interpolation.

Table. Velocity as a function of time

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

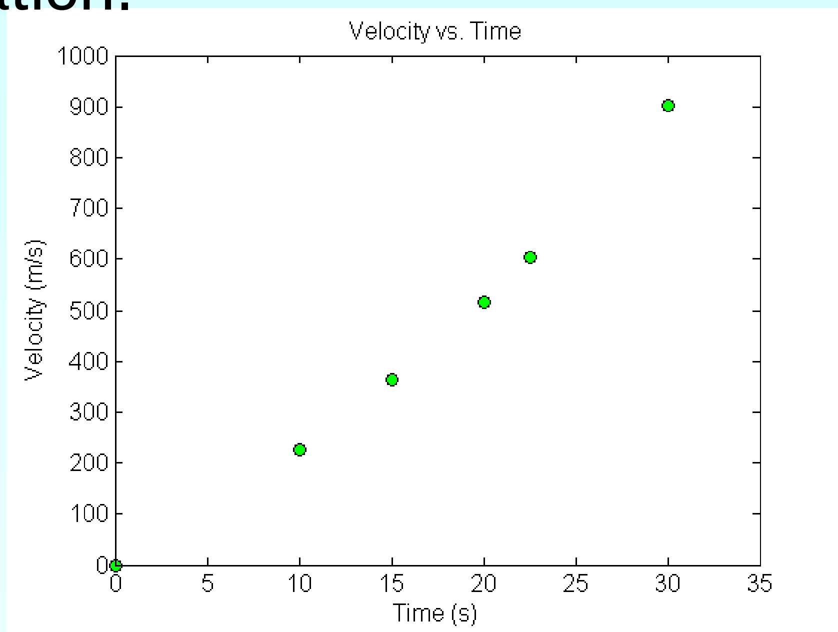


Figure. Velocity vs. time data for the rocket example

Linear Interpolation

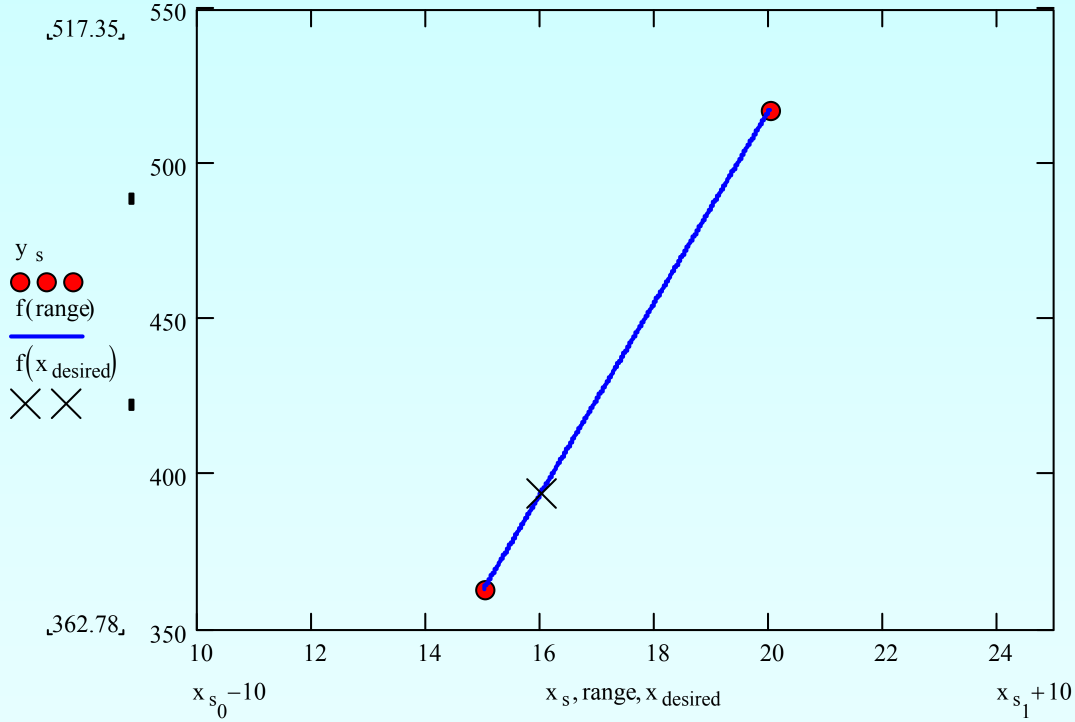
$$v(t) = b_0 + b_1(t - t_0)$$

$$t_0 = 15,\ v(t_0) = 362.78$$

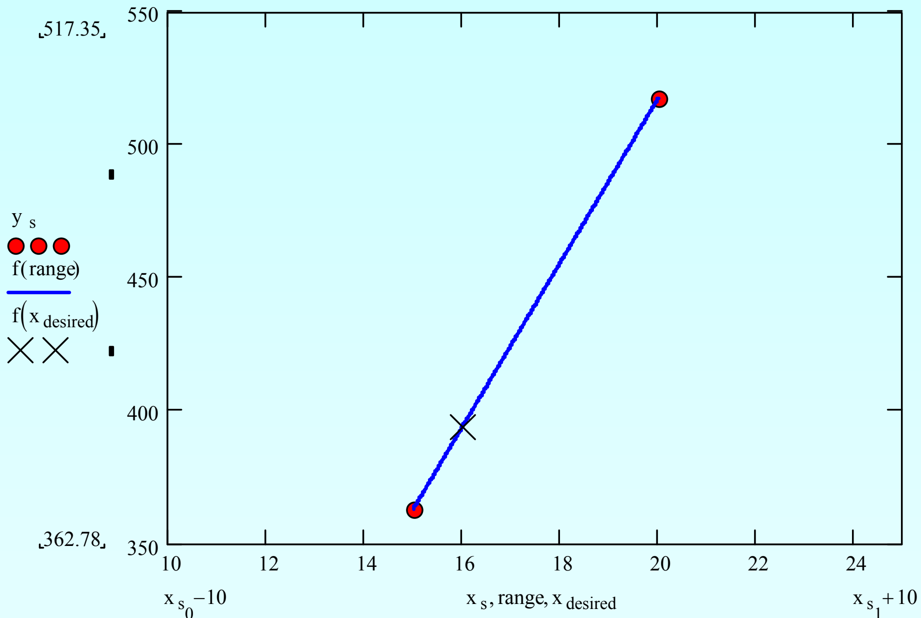
$$t_1 = 20,\ v(t_1) = 517.35$$

$$b_0 = v(t_0) \ = 362.78$$

$$b_1 = \frac{v(t_1) - v(t_0)}{t_1 - t_0} = 30.914$$



Linear Interpolation (contd)



$$v(t) = b_0 + b_1(t - t_0)$$

$$= 362.78 + 30.914(t - 15),\ 15 \le t \le 20$$

At $t = 16$

$$v(16) = 362.78 + 30.914(16 - 15)$$

$$= 393.69 \text{ m/s}$$

Quadratic Interpolation

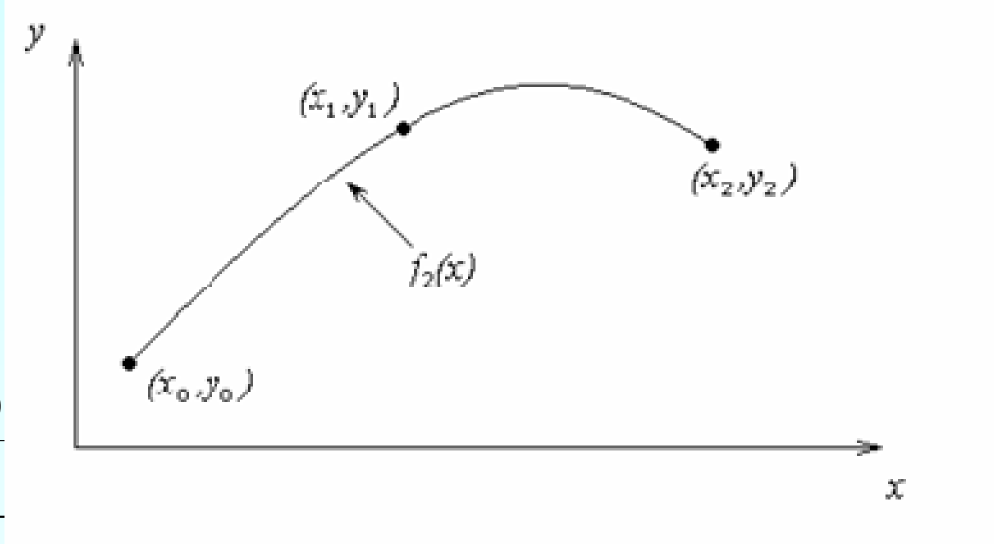
Given (x_0, y_0), (x_1, y_1), and (x_2, y_2), fit a quadratic interpolant through the data.

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$



Example

The upward velocity of a rocket is given as a function of time in Table 1. Find the velocity at t=16 seconds using the Newton Divided Difference method for quadratic interpolation.

Table. Velocity as a function of time

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

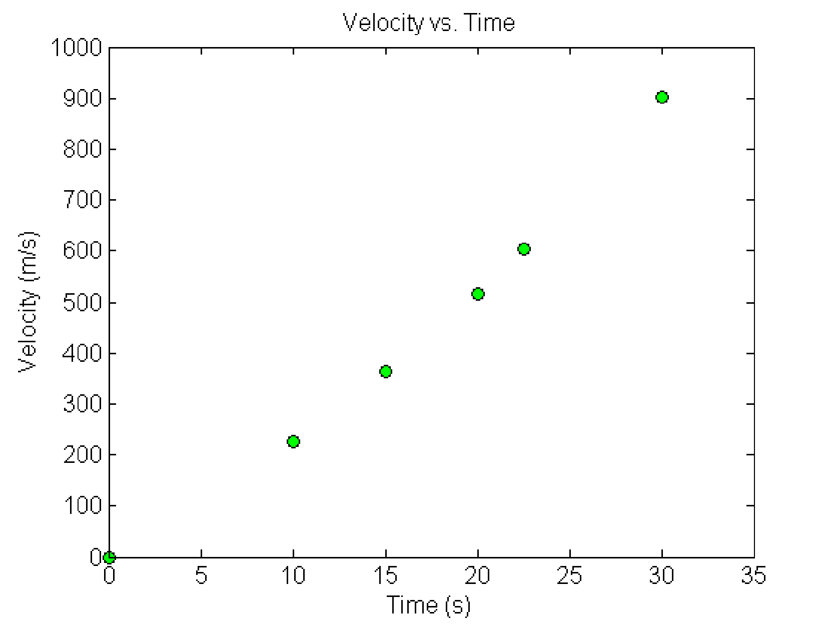
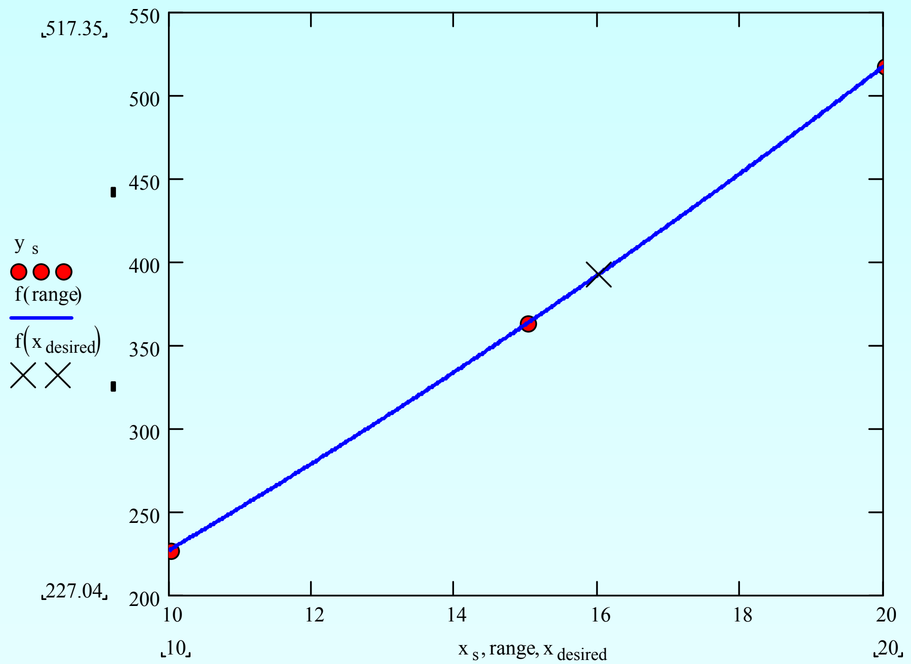


Figure. Velocity vs. time data for the rocket example

Quadratic Interpolation (contd)



$$t_0 = 10,\ v(t_0) = 227.04$$

$$t_1 = 15,\ v(t_1) = 362.78$$

$$t_2 = 20,\ v(t_2) = 517.35$$

Quadratic Interpolation (contd)

$$b_0 = v(t_0)$$

$$= 227.04$$

$$b_1 = \frac{v(t_1) - v(t_0)}{t_1 - t_0} = \frac{362.78 - 227.04}{15 - 10}$$

$$= 27.148$$

$$b_2 = \frac{\dfrac{v(t_2) - v(t_1)}{t_2 - t_1} - \dfrac{v(t_1) - v(t_0)}{t_1 - t_0}}{t_2 - t_0} = \frac{\dfrac{517.35 - 362.78}{20 - 15} - \dfrac{362.78 - 227.04}{15 - 10}}{20 - 10}$$

$$= \frac{30.914 - 27.148}{10}$$

$$= 0.37660$$

Quadratic Interpolation (contd)

$$v(t) = b_0 + b_1(t - t_0) + b_2(t - t_0)(t - t_1)$$

$$= 227.04 + 27.148(t - 10) + 0.37660(t - 10)(t - 15), \qquad 10 \le t \le 20$$

At $t = 16,$

$$v(16) = 227.04 + 27.148(16 - 10) + 0.37660(16 - 10)(16 - 15) \; = 392.19 \text{ m/s}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the first order and second order polynomial is

$$|\in_a| \; = \left|\frac{392.19 - 393.69}{392.19}\right| \text{x}100$$

$$= 0.38502\ \%$$

General Form

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

where

$$b_0 = f[x_0] = f(x_0)$$

$$b_1 = f[x_1, x_0] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = f[x_2, x_1, x_0] = \frac{f[x_2, x_1] - f[x_1, x_0]}{x_2 - x_0} = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

Rewriting

$$f_2(x) = f[x_0] + f[x_1, x_0](x - x_0) + f[x_2, x_1, x_0](x - x_0)(x - x_1)$$

General Form

Given $(n+1)$ data points, $(x_0, y_0), (x_1, y_1), \ldots\ldots, (x_{n-1}, y_{n-1}), (x_n, y_n)$ as

$$f_n(x) = b_0 + b_1(x - x_0) + \ldots. + b_n(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

where

$$b_0 = f[x_0]$$

$$b_1 = f[x_1, x_0]$$

$$b_2 = f[x_2, x_1, x_0]$$

$$\vdots$$

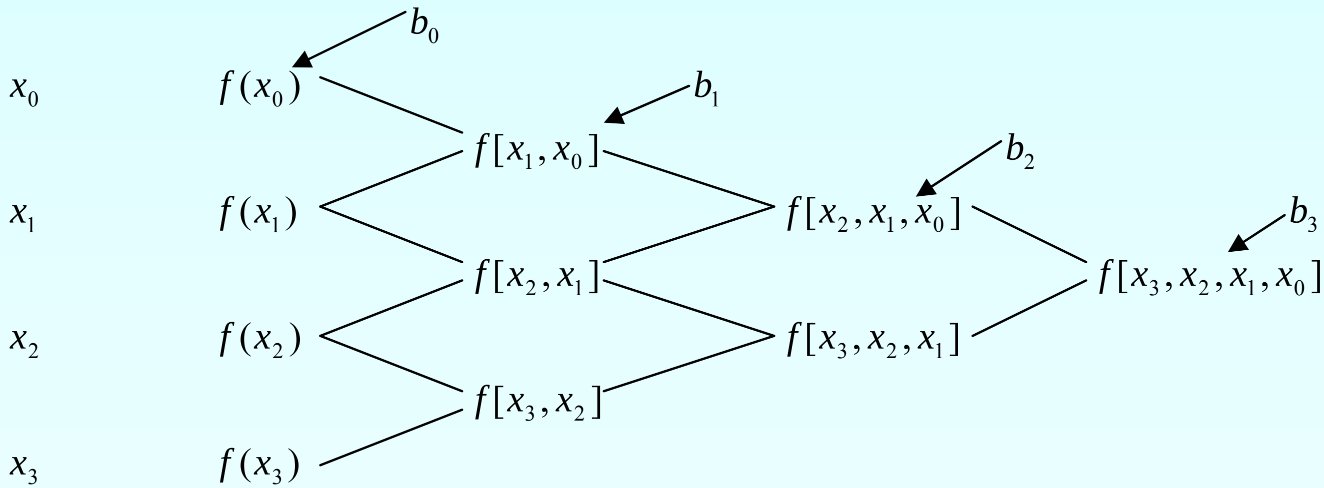
$$b_{n-1} = f[x_{n-1}, x_{n-2}, \ldots., x_0]$$

$$b_n = f[x_n, x_{n-1}, \ldots., x_0]$$

General form

The third order polynomial, given (x_0, y_0), (x_1, y_1), (x_2, y_2), and (x_3, y_3), is

$$
\begin{aligned}
f_3(x) = f[x_0] + f[x_1, x_0](x - x_0) + f[x_2, x_1, x_0](x - x_0)(x - x_1) \\
+ f[x_3, x_2, x_1, x_0](x - x_0)(x - x_1)(x - x_2)
\end{aligned}
$$



Example

The upward velocity of a rocket is given as a function of time in Table 1. Find the velocity at t=16 seconds using the Newton Divided Difference method for cubic interpolation.

Table. Velocity as a function of time

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

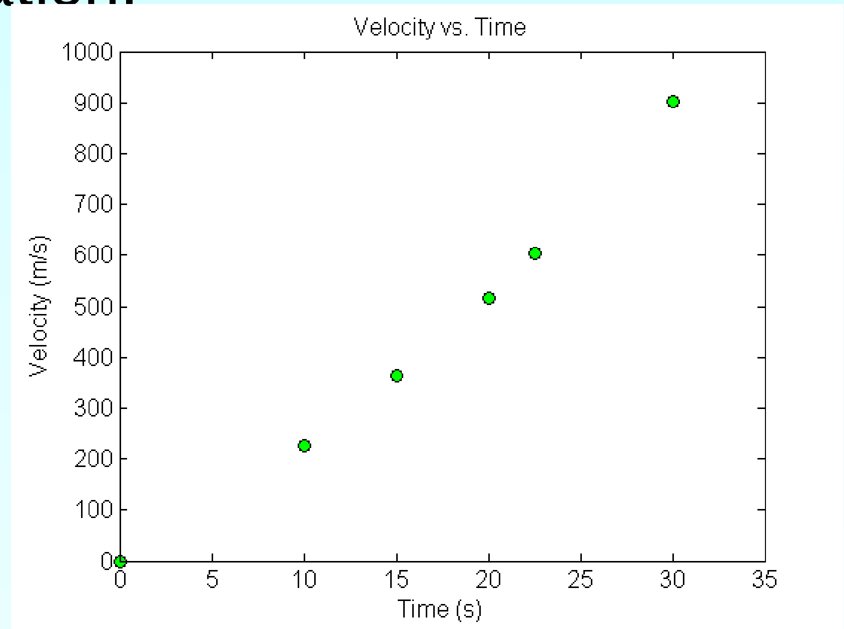


Figure. Velocity vs. time data for the rocket example

Example

The velocity profile is chosen as

$$v(t) = b_0 + b_1(t - t_0) + b_2(t - t_0)(t - t_1) + b_3(t - t_0)(t - t_1)(t - t_2)$$

we need to choose four data points that are closest to $t = 16$

$$t_0 = 10, \quad v(t_0) = 227.04$$

$$t_1 = 15, \quad v(t_1) = 362.78$$

$$t_2 = 20, \quad v(t_2) = 517.35$$

$$t_3 = 22.5, \quad v(t_3) = 602.97$$

The values of the constants are found as:

$$b_0 = 227.04; \; b_1 = 27.148; \; b_2 = 0.37660; \; b_3 = 5.4347 \times 10^{-3}$$

Example

	b_0	b_1	b_2	b_3
$t_0 = 10$	227.04			
		27.148		
$t_1 = 15,$	362.78		0.37660	
		30.914		5.4347×10^{-3}
$t_2 = 20,$	517.35		0.44453	
		34.248		
$t_3 = 22.5,$	602.97			

$$b_0 = 227.04; \quad b_1 = 27.148; \; b_2 = 0.37660; \quad b_3 = 5.4347 \times 10^{-3}$$

Example

Hence

$$v(t) = b_0 + b_1(t - t_0) + b_2(t - t_0)(t - t_1) + b_3(t - t_0)(t - t_1)(t - t_2)$$

$$= 227.04 + 27.148(t - 10) + 0.37660(t - 10)(t - 15)$$

$$+ 5.4347 * 10^{-3}(t - 10)(t - 15)(t - 20)$$

At $t = 16$,

$$v(16) = 227.04 + 27.148(16 - 10) + 0.37660(16 - 10)(16 - 15)$$

$$+ 5.4347 * 10^{-3}(16 - 10)(16 - 15)(16 - 20)$$

$$= 392.06 \text{ m/s}$$

The absolute relative approximate error $|\in_a|$ obtained is

$$|\in_a| = \left|\frac{392.06 - 392.19}{392.06}\right| \text{x}100$$

$$= 0.033427\ \%$$

Comparison Table

Order of Polynomial	1	2	3
v(t=16) m/s	393.69	392.19	392.06
Absolute Relative Approximate Error	----------	0.38502 %	0.033427 %

Distance from Velocity Profile

Find the distance covered by the rocket from t=11s to t=16s ?

$$v(t) = 227.04 + 27.148(t-10) + 0.37660(t-10)(t-15) + 5.4347*10^{-3}(t-10)(t-15)(t-20) \qquad 10 \le t \le 22.5$$

$$= -4.2541 + 21.265t + 0.13204t^2 + 0.0054347t^3 \qquad 10 \le t \le 22.5$$

So

$$s(16) - s(11) = \int_{11}^{16} v(t)dt$$

$$= \int_{11}^{16} (-4.2541 + 21.265t + 0.13204t^2 + 0.0054347t^3)dt$$

$$= \left[-4.2541t + 21.265\frac{t^2}{2} + 0.13204\frac{t^3}{3} + 0.0054347\frac{t^4}{4}\right]_{11}^{16}$$

$$= 1605\ m$$

Acceleration from Velocity Profile

Find the acceleration of the rocket at t=16s given that

$$v(t) = -4.2541 + 21.265t + 0.13204t^2 + 0.0054347t^3$$

$$a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}\left(-4.2541 + 21.265t + 0.13204t^2 + 0.0054347t^3\right)$$

$$= 21.265 + 0.26408t + 0.016304t^2$$

$$a(16) = 21.265 + 0.26408(16) + 0.016304(16)^2$$

$$= 29.664\ m/s^2$$