Computational Physics Project Proposal
Title: The Study of Dynamics of the Lorenz Equations

Importance

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = rx - y - xz$$
$$\dot{z} = xy - bz$$

The Lorenz Equations provide a simple and yet significant area of study as it provides scientists a chaotic dynamical outlook of a system. First derived by Edward Lorenz in 1963, the three-dimensional system came from a simplified model of convection system of the atmosphere. The trajectories of the system also converge into stable cycles which are now called strange attractors relating to fractals; understanding these natural phenomenon can unlock some of the mysteries the universe has to offer.

In addition, most of the problems we encounter in the real world tend to be non-linear and exhibit extreme dynamical changes which are more complicated and tough to analyse. Lorenz equation provides us with simple deterministic equations with jaw dropping dynamics when we change certain parameters and variables computationally. This ground-breaking study formulated by Lorenz made popular such that it birthed the "Butterfly" effect, which inspired many thinkers, philosophers and film makers along the way.

Deep diving into the Lorenz Equation is an important study to be made, and breaking barriers surrounding the topic might change the perception of what is conventional and what used to be imaginations. With the aid of computational physics, the Lorenz system can be solved numerically using RK4

Numerical Method

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = rx - y - xz$$
$$\dot{z} = xy - bz$$

First we will obtain the fixed points of the Lorenz Equations by setting $\dot{x}, \dot{y}, \dot{z} =$ (0,0,0) and realising there will be a pair of fixed points which Lorenz called C^+ and C^- which represent the left or right convection rolls, this then coalesce with the origin as a pitchfork bifurcation as we have r > 1.0.

The three-dimensional system can then be solved using the fourth order Runge Method to make an accurate approximation. The three system is placed through the RK4 function until we get strong data set of points, then plot them into a three-dimensional graph and two-dimensional using Axes3D from mpl_toolkits.mplot3d and plotting them in respective axis.

The RK4 runs the data value to produce the ODE estimate and append it to a temporary list.

Numerical Experiments

The initial conditions of the Lorenz Equations is important as the trajectories are highly dependent in its initial condition, so I will analyse the behaviour for different values ranging from 0 to 1 and observe the trajectories.

Next we will individually do numerical experiments for different values of *b*, sigma and r whilst keeping the rest constant (initial condition, r value and sigma value).

Further study will vary the r value such that we obtain stable limit cycles, unstable limit cycle, supercritical Hopf Bifurcations, and obtain aperiodic motion that relates to the strange attractor. Also, the study of Lyapunov Exponent functions may allow us to asses the behaviour of the system whether it is chaotic or not. Although Lyapunov exponent functions are tough to solve analytically, computational approach is simple.

Reference Material:

Non-linear Dynamics and Chaos with applications to Physics, Biology, Chemistry and Engineering by Steven H. Strogatz