Nov 12, 2024 SVD singular value decomposition

$$X_{N\times D} = U_{N\times N} \cdot S_{N\times D} \cdot V^T_{D\times D}$$

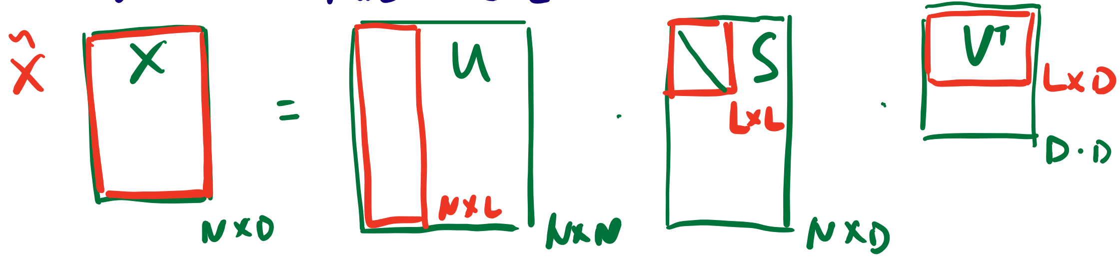
U : left singular matrix . $U^TU = I_N$

V : right singular matrix $V^TV = I_D$

S : $\begin{bmatrix} \boxed{\begin{matrix}\sigma_1 & \\ & \ddots \; \sigma_L\end{matrix}} & 0 \\ & \sigma_D \\ 0 & \end{bmatrix}_{N\times D}$ singular values: $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_D$

select $L < D$ singular values

$$\hat{X} = \hat{U}_{N\times L} \cdot \hat{S}_{L\times L} \cdot \hat{V}^T_{L\times D}$$



$N \cdot D$ original matrix

$N\cdot L + L + L\cdot D \ll N \cdot D$

$\hat{X}$: rank L approximation of the original matrix X

PCA using SVD

SVD: $X = U \cdot S \cdot V^T$

PCA : covariance matrix $S = X^TX$

$$S = (USV^T)^T (USV^T)$$
$$= VS^T \underbrace{U^T U}_{I} S V^T \qquad D = S^T S = \begin{bmatrix} \sigma_1^2 & & \\ & \ddots & \\ & & \sigma_D^2 \end{bmatrix}$$
$$= VS^T S V^T$$
$$= VDV^T$$
$$X^T X = V \cdot D \cdot V^T$$
$$X^T X \cdot V = V \cdot D \cdot \underbrace{V^T V}_{I} = V \cdot D$$ eigen decomposition

D : eigenvalues of $X^T X$

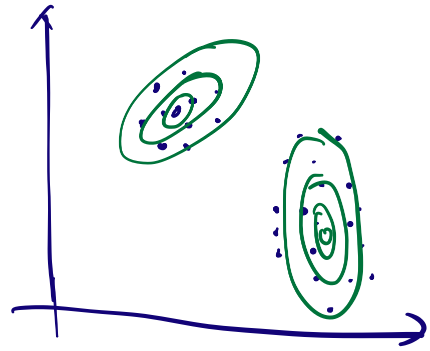
V : eigenvectors of $X^T X$, right singular matrix of X

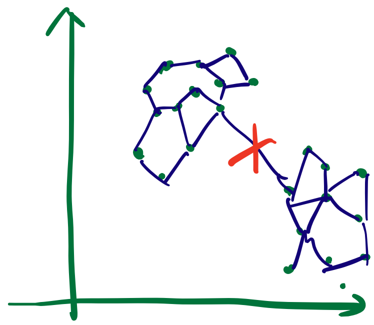
U : left singular matrix of X

eigenvectors of XX^T

$$Z = X \cdot W_{D \times L} \qquad W = \begin{bmatrix} | & & | \\ \cdots & & \cdots \\ | & & | \end{bmatrix}_{\lambda_1 \cdots \lambda_L} = V$$
$$= U \cdot S \underbrace{V^T \cdot V}_{I}$$
$$= U \cdot S$$
$$X = Z \cdot W^T$$
$$= U \cdot S \cdot V^T$$

Spectral Clustering





build a graph

graph cut