Introduction to Machine Learning

Optimization

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Outline

Machine Learning as Optimization
Convex Optimization
Gradient Descent
Issues with Gradient Descent
Stochastic Gradient Descent

Machine Learning as Optimization Problem[1]

- Learning is optimization
- Faster optimization methods for faster learning
- Let $w \in \mathbb{R}^d$ and $f_0(w), f_1(w), \ldots, f_m(w)$ be real-valued functions.
- Standard optimization formulation is:

$$\begin{aligned} \underset{w}{\text{minimize}} \quad & f_0(w) \\ \text{subject to} \quad & f_i(w) \leq 0, \ i = 1, \ldots, m. \end{aligned}$$

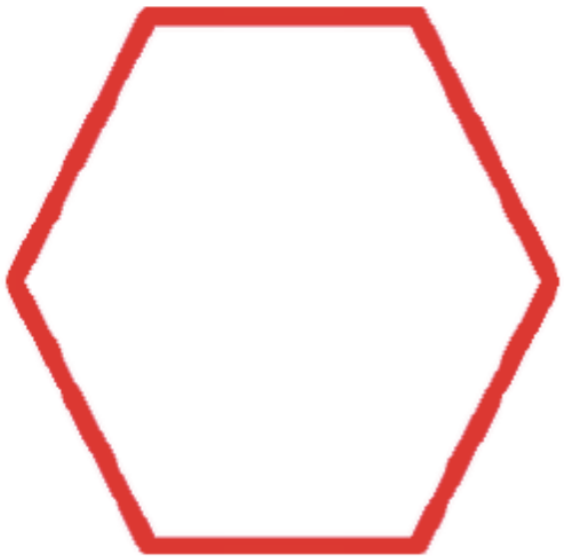
[1] Adapted from `http://ttic.uchicago.edu/~gregory/courses/ml2012/lectures/tutorial_optimization.pdf`. Also see, `http://www.stanford.edu/~boyd/cvxbook/` and `http://scipy-lectures.github.io/advanced/mathematical_optimization/`.

Solving Optimization Problems

- Methods for **general optimization problems**
  - Simulated annealing, genetic algorithms
- Exploiting *structure* in the optimization problem
  - **Convexity**, Lipschitz continuity, smoothness

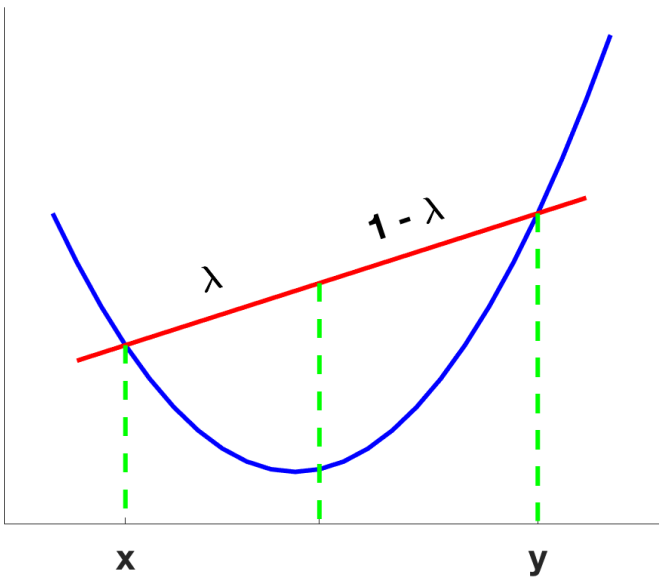
Convexity

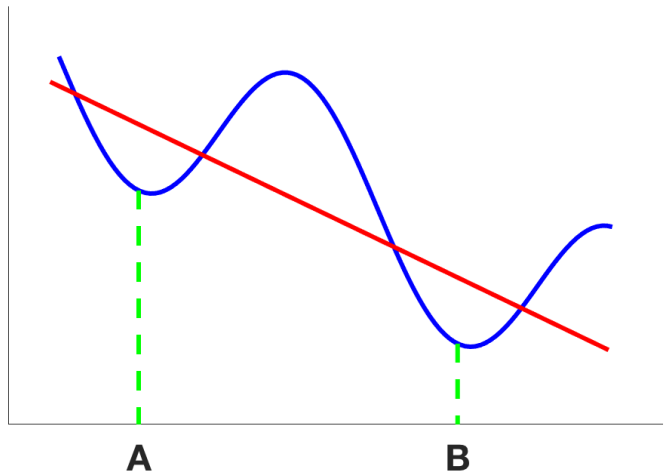
- Convex Sets



Convex Function

- Convex function





Convex Optimization

- Optimality Criterion

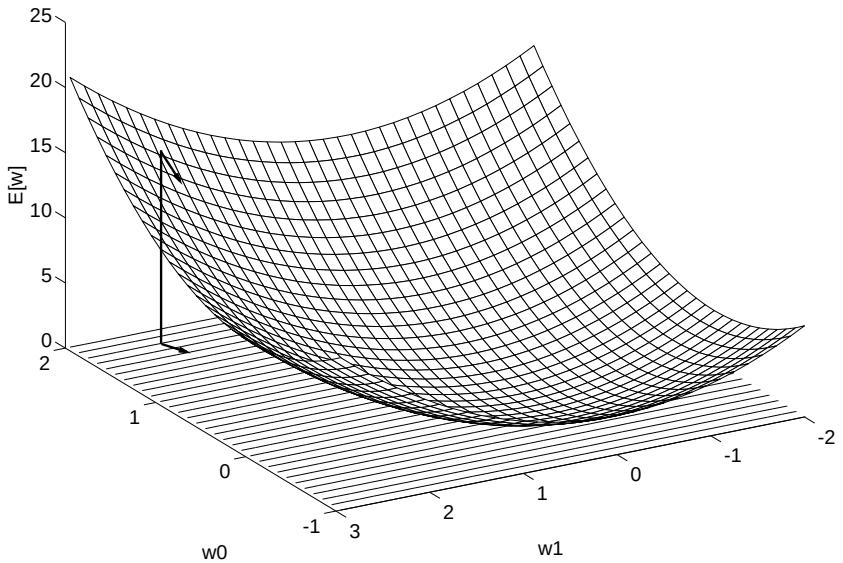
$$\begin{aligned} \underset{w}{\text{minimize}} \quad & f_0(w) \\ \text{subject to} \quad & f_i(w) \leq 0,\ i = 1, \ldots, m. \end{aligned}$$

  where all $f_i(w)$ are **convex functions**.
- w_0 is feasible if $w_0 \in Dom\ f_0$ and all constraints are satisfied
- A feasible w^* is optimal if $f_0(w^*) \leq f_0(w)$ for all w satisfying the constraints

Gradient of a Function

- Denotes the direction of steepest ascent

$$\nabla E(\mathbf{w}) = \begin{bmatrix} \frac{\partial E}{\partial w_0} \\ \frac{\partial E}{\partial w_1} \\ \vdots \\ \frac{\partial E}{\partial w_d} \end{bmatrix}$$



Finding Extremes of a Single Variable Function

- Set derivative to 0
- Second derivative for minima or maxima

Finding Extremes of a Multiple Variable Function - Gradient Descent

1. Start from any point in variable space
2. Move along the direction of the steepest descent (or ascent)
   - By how much?
   - A learning rate (η)
   - What is the direction of steepest descent?
     - Gradient of E at $\mathbf{w}$

Training Rule for Gradient Descent

$$\mathbf{w} = \mathbf{w} - \eta \nabla E(\mathbf{w})$$

For each weight component:

$$w_j = w_j - \eta \frac{\partial E}{\partial w_j}$$

Convergence Guaranteed?

- Error surface contains only one global minimum
- Algorithm *will* converge
  - Examples need not be linearly separable
- η should be *small enough*
- Impact of too large η?
- Too small η?

Issues with Gradient Descent

- Slow convergence
- Stuck in local minima

Stochastic Gradient Descent [1]

- **Update weights after every training example or a batch of examples.**
- For sufficiently small η, closely approximates Gradient Descent.

Gradient Descent	**Stochastic Gradient Descent**
Weights updated after summing error over all examples	Weights updated after examining a batch of examples
More computations per weight update step	Significantly lesser computations
Risk of local minima	Reduce the risk of local minima but can't avoid it

References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