Quiz Solutions

CSE 4/574

Fall 2024

Question 1

Suppose you go for a medical checkup. Doctor administers a test to you for a serious disease. The test is 95% accurate (when tested on 100 patients with the disease, it identifies 95 of them as positive). It also is 99% accurate in identifying patients who do not have a disease (when tested on 100 disease free people, it only calls 1 out of 100 as positive).

Assuming that you tested positive, which of the following are true?

Correct Choice

If it is a moderately rare disease (10 in 100 chance), there is close to 90% chance of actually having the disease.

Choice Explanation:
Let $y = 1$ means that the patient have the disease, $x = 1$ means that the patient have tested positive for the disease.

$$
\begin{aligned}
P(x = 1 \mid y = 1) &= 0.95 \\
P(x = 1 \mid y = 0) &= 0.01 \\
P(y = 1 \mid x = 1) &= \frac{P(x = 1 \mid y = 1) \cdot P(y = 1)}{P(x = 1 \mid y = 1) \cdot P(y = 1) + P(x = 1 \mid y = 0) \cdot P(y = 0)} \\
&= \frac{0.95 \cdot P(y = 1)}{0.95 \cdot P(y = 1) + 0.01 \cdot P(y = 0)}
\end{aligned}
$$

(1)

Given prior information $P(y = 1) = 0.1$,

$$
P(y = 1 \mid x = 1) = \frac{0.95 \cdot 0.1}{0.95 \cdot 0.1 + 0.01 \cdot 0.9} \approx 0.91
$$

Incorrect Choice 1

If it is a moderately rare disease (10 in 100 chance), there is close to 10% chance of actually having the disease.

Choice Explanation:
Refer to the explanation above.

Incorrect Choice 2

If it is a rare disease (1 in 100 chance), there is close to 1% chance of actually having the disease.

Choice Explanation:

$$P(y = 1) = 0.01, P(y = 0) = 0.99$$
$$P(y = 1 \mid x = 1) = \frac{0.95 \cdot P(y = 1)}{0.95 \cdot P(y = 1) + 0.01 \cdot P(y = 0)}$$
$$P(y = 1 \mid x = 1) = \frac{0.95 \cdot 0.01}{0.95 \cdot 0.01 + 0.01 \cdot 0.99}$$
$$P(y = 1 \mid x = 1) = \frac{0.95 \cdot 0.01}{0.95 \cdot 0.01 + 0.01 \cdot 0.99} \approx 0.49$$

Incorrect Choice 3

If it is a rare disease (1 in 100 chance), there is close to 10% chance of actually having the disease.

Choice Explanation:
Refer to the explanation above.