Asymptotic Analysis with Limit Analysis

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The limit analysis of the asymptotic relationship between $f(n)$ and $g(n)$:

- If $\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq \infty$, then $f(n) = O(g(n))$.
- If $\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq 0$, then $f(n) = \Omega(g(n))$.
- If $\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq 0$ and $\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq \infty$, then $f(n) = \Theta(g(n))$.)
- If $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ does not exist, then $f(n)$ and $g(n)$ have no asymptotic relationships.

Example 1: $f(n) = \log_2 n$**,** $g(n) = \log_8 n$

Approach (by Limit analysis): We could check the $\lim_{n\to\infty} f(n)/g(n)$ with the following argument.

Because $\lim_{n\to\infty} f(n)/g(n) = 3 \neq \infty$, we know that f(n) is in $O(g(n))$.

Because $\lim_{n\to\infty} f(n)/g(n) = 3 \neq 0$, we know that f(n) is in $\Omega(g(n))$.

Since $f(n) \in O(g(n))$ and $f(n) \in \Theta(g(n))$, f(n) is in $\Theta(g(n))$.

Please prove that $\log_a n = \Theta(\log_b n)$ **for any constant** $a, b > 0$**.**

Example 2: $f(n) = n^2$**,** $g(n) = 1$ **if** n **is odd and** $g(n) = n^3$ **if** n **is even.**

Approach (by Limit analysis): We could check the $\lim_{n\to\infty} f(n)/g(n)$ with the following argument.

Because $\lim_{n\to\infty, n \text{ is odd}} f(n)/g(n) = \infty$, we know that f(n) is not in $O(g(n))$.

Because $\lim_{n\to\infty, n \text{ is even}} f(n)/g(n) = 0$, we know that f(n) is not in $\Omega(g(n))$.

Since $f(n) \notin \Theta(g(n))$, $f(n)$ is not in $\Theta(g(n))$.