CSE 431/531: Algorithm Analysis and Design (Fall 2024)
Dynamic Programming

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Outline

Subsequence

- $A = bacdca$
- $C = adca$

Subsequence

- $A = b\textcolor{red}{a}c\textcolor{red}{dca}$
- $C = adca$
- C is a subsequence of A

Subsequence

- $A = bacdca$
- $C = adca$
- C is a subsequence of A

Def. Given two sequences $A[1 \,..\, n]$ and $C[1 \,..\, t]$ of letters, C is called a subsequence of A if there exists integers $1 \leq i_1 < i_2 < i_3 < \ldots < i_t \leq n$ such that $A[i_j] = C[j]$ for every $j = 1, 2, 3, \cdots, t$.

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- Exercise: how to check if sequence C is a subsequence of A?

Common subsequence

Def. Given two sequences $A[1 \mathrel{..} n]$ and $B[1 \mathrel{..} m]$ of letters, C is called a common subsequence of A and B if C is a subsequence of A and also a subsequence of B.

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- Example: $A = adecadf$ and $B = caefcad$

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- Example: $A = adecadf$ and $B = caefcad$
- Common subsequence: $C = adcaf \quad ?$

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- Example: $A = adecadf$ and $B = caefcad$
- Common subsequence: $C = adcaf \quad ?$
- Common subsequence: $C = aead \quad ?$

Common subsequence

Def. Given two sequences $A[1 \,..\, n]$ and $B[1 \,..\, m]$ of letters, C is called a common subsequence of A and B if C is a subsequence of A and also a subsequence of B.

- Example: $A = adecadf$ and $B = caefcad$
- Common subsequence: $C = adcaf \quad ?$
- Common subsequence: $C = aead \quad ?$
- Common subsequence: $C = acad \quad ?$

Edit distance with two operations (insertions and deletions)

Def. Given two sequences $A[1 \mathinner{..} n]$ and $B[1 \mathinner{..} m]$ of letters, $d(A, B)$ is called a edit distance with insert and delete operations of A and B if $d(A, B)$ is the minimum number of edit operations needed to transform A into B, where possible operations are:

- insert a character
- delete a character

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- Example: $A = abc$ and $B = adef$

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- insert a character
- delete a character

- Example: $A = abc$ and $B = adef$
- Distance $d(A, B) = 5$: delete b, delete c, insert d, insert e, and insert f.

Edit distance with three operations (insertions, deletions and replacing)

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- insert a character
- delete a character
- modify (or replace) a character

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Edit distance with three operations (insertions, deletions and replacing)

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- insert a character
- delete a character
- modify (or replace) a character

- Example: $A = abc$ and $B = adef$
- Distance $d(A, B) = 3$: replace b to d, replace c to e, and insert character f.

Longest Common Subsequence

Input: $A[1 \,..\, n]$ and $B[1 \,..\, m]$

Output: the longest common subsequence of A and B

Example:

- $A = `bacdca'$
- $B = `adbcda'$

Longest Common Subsequence

Input: $A[1 \mathrel{..} n]$ and $B[1 \mathrel{..} m]$

Output: the longest common subsequence of A and B

Example:

- $A = `bacdca'$
- $B = `adbcda'$
- $\mathsf{LCS}(A, B) = `adca'$

Longest Common Subsequence

Input: $A[1\,..\,n]$ and $B[1\,..\,m]$

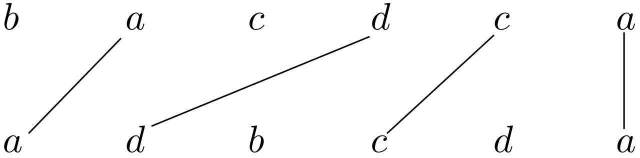
Output: the longest common subsequence of A and B

Example:

- $A = `bacdca'$
- $B = `adbcda'$
- $\mathrm{LCS}(A, B) = `adca'$

- Applications: edit distance (diff), similarity of DNAs

Matching View of LCS



- Goal of LCS: find a maximum-size non-crossing matching between letters in A and letters in B.

Reduce to Subproblems

- $A = `bacdca'$
- $B = `adbcda'$

Reduce to Subproblems

- $A = `bacdca'$
- $B = `adbcda'$

Reduce to Subproblems

- $A = `bacdc'$
- $B = `adbcd'$

Reduce to Subproblems

- $A = `bacdc'$
- $B = `adbcd'$

- either the last letter of A is not matched:

- or the last letter of B is not matched:

Reduce to Subproblems

- $A = `bacdc'$
- $B = `adbcd'$

- either the last letter of A is not matched:
- need to compute LCS($`bacd', `adbcd'$)
- or the last letter of B is not matched:

Reduce to Subproblems

- $A = `bacdc'$
- $B = `adbcd'$

- either the last letter of A is not matched:
- need to compute LCS($`bacd', `adbcd'$)
- or the last letter of B is not matched:
- need to compute LCS($`bacdc', `adbc'$)

Dynamic Programming for LCS

- $opt[i, j], 0 \le i \le n, 0 \le j \le m$: length of longest common sub-sequence of $A[1 .. i]$ and $B[1 .. j]$.

Dynamic Programming for LCS

- $opt[i, j], 0 \le i \le n, 0 \le j \le m$: length of longest common sub-sequence of $A[1 \,..\, i]$ and $B[1 \,..\, j]$.
- if $i = 0$ or $j = 0$, then $opt[i, j] = 0$.

Dynamic Programming for LCS

- $opt[i, j], 0 \le i \le n, 0 \le j \le m$: length of longest common sub-sequence of $A[1 .. i]$ and $B[1 .. j]$.
- if $i = 0$ or $j = 0$, then $opt[i, j] = 0$.
- if $i > 0, j > 0$, then

$$opt[i, j] = \begin{cases} & \text{if } A[i] = B[j] \\ & \text{if } A[i] \neq B[j] \end{cases}$$

Dynamic Programming for LCS

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- if $i > 0, j > 0$, then

$$opt[i,j] = \begin{cases} opt[i-1, j-1] + 1 & \text{if } A[i] = B[j] \\ & \text{if } A[i] \neq B[j] \end{cases}$$

Dynamic Programming for LCS

- $opt[i,j], 0 \le i \le n, 0 \le j \le m$: length of longest common sub-sequence of $A[1 .. i]$ and $B[1 .. j]$.
- if $i = 0$ or $j = 0$, then $opt[i,j] = 0$.
- if $i > 0, j > 0$, then

$$opt[i,j] = \begin{cases} opt[i-1, j-1] + 1 & \text{if } A[i] = B[j] \\ \max \begin{cases} opt[i-1, j] \\ opt[i, j-1] \end{cases} & \text{if } A[i] \neq B[j] \end{cases}$$

Dynamic Programming for LCS

for $j \leftarrow 0$ to m **do**
    $opt[0, j] \leftarrow 0$
for $i \leftarrow 1$ to n **do**
    $opt[i, 0] \leftarrow 0$
    for $j \leftarrow 1$ to m **do**
        if $A[i] = B[j]$ **then**
            $opt[i, j] \leftarrow opt[i-1, j-1] + 1$
        else if $opt[i, j-1] \geq opt[i-1, j]$ **then**
            $opt[i, j] \leftarrow opt[i, j-1]$
        else
            $opt[i, j] \leftarrow opt[i-1, j]$

Dynamic Programming for LCS

for $j \leftarrow 0$ to m **do**
  $opt[0, j] \leftarrow 0$
for $i \leftarrow 1$ to n **do**
  $opt[i, 0] \leftarrow 0$
  for $j \leftarrow 1$ to m **do**
    if $A[i] = B[j]$ **then**
      $opt[i, j] \leftarrow opt[i-1, j-1] + 1$, $\pi[i, j] \leftarrow$ "$\nwarrow$"
    else if $opt[i, j-1] \geq opt[i-1, j]$ **then**
      $opt[i, j] \leftarrow opt[i, j-1]$, $\pi[i, j] \leftarrow$ "$\leftarrow$"
    else
      $opt[i, j] \leftarrow opt[i-1, j]$, $\pi[i, j] \leftarrow$ "$\uparrow$"

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$
1	0 $\perp$						
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1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$						
5	$0 \perp$						
6	$0 \perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$					
5	$0 \perp$						
6	$0 \perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$				
5	$0 \perp$						
6	$0 \perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$			
5	$0 \perp$						
6	$0 \perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$
1	0 $\perp$	0 $\leftarrow$	0 $\leftarrow$	1 $\nwarrow$	1 $\leftarrow$	1 $\leftarrow$	1 $\leftarrow$
2	0 $\perp$	1 $\nwarrow$	1 $\leftarrow$	1 $\leftarrow$	1 $\leftarrow$	1 $\leftarrow$	2 $\nwarrow$
3	0 $\perp$	1 $\uparrow$	1 $\leftarrow$	1 $\leftarrow$	2 $\nwarrow$	2 $\leftarrow$	2 $\leftarrow$
4	0 $\perp$	1 $\uparrow$	2 $\nwarrow$	2 $\leftarrow$	2 $\leftarrow$		
5	0 $\perp$						
6	0 $\perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	
5	$0 \perp$						
6	$0 \perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0 \perp$						
6	$0 \perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0 \perp$						
6	$0 \perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$
1	0 $\perp$	0 $\leftarrow$	0 $\leftarrow$	1 $\nwarrow$	1 $\leftarrow$	1 $\leftarrow$	1 $\leftarrow$
2	0 $\perp$	1 $\nwarrow$	1 $\leftarrow$	1 $\leftarrow$	1 $\leftarrow$	1 $\leftarrow$	2 $\nwarrow$
3	0 $\perp$	1 $\uparrow$	1 $\leftarrow$	1 $\leftarrow$	2 $\nwarrow$	2 $\leftarrow$	2 $\leftarrow$
4	0 $\perp$	1 $\uparrow$	2 $\nwarrow$	2 $\leftarrow$	2 $\leftarrow$	3 $\nwarrow$	3 $\leftarrow$
5	0 $\perp$	1 $\uparrow$					
6	0 $\perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0\ \perp$	$0\ \perp$	$0\ \perp$	$0\ \perp$	$0\ \perp$	$0\ \perp$	$0\ \perp$
1	$0\ \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0\ \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0\ \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0\ \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0\ \perp$	$1 \uparrow$	$2 \uparrow$				
6	$0\ \perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0 \perp$	$1 \uparrow$	$2 \uparrow$	$2 \leftarrow$			
6	$0 \perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0 \perp$	$1 \uparrow$	$2 \uparrow$	$2 \leftarrow$	$3 \nwarrow$		
6	$0 \perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0 \perp$	$1 \uparrow$	$2 \uparrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$	
6	$0 \perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0 \perp$	$1 \uparrow$	$2 \uparrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$	$3 \leftarrow$
6	$0 \perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0 \perp$	$1 \uparrow$	$2 \uparrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$	$3 \leftarrow$
6	$0 \perp$						

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0 \perp$	$1 \uparrow$	$2 \uparrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$	$3 \leftarrow$
6	$0 \perp$	$1 \nwarrow$					

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0 \perp$	$1 \uparrow$	$2 \uparrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$	$3 \leftarrow$
6	$0 \perp$	$1 \nwarrow$	$2 \uparrow$				

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0 \perp$	$1 \uparrow$	$2 \uparrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$	$3 \leftarrow$
6	$0 \perp$	$1 \nwarrow$	$2 \uparrow$	$2 \leftarrow$			

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0 \perp$	$1 \uparrow$	$2 \uparrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$	$3 \leftarrow$
6	$0 \perp$	$1 \nwarrow$	$2 \uparrow$	$2 \leftarrow$	$3 \uparrow$		

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$
1	0 $\perp$	0 $\leftarrow$	0 $\leftarrow$	1 $\nwarrow$	1 $\leftarrow$	1 $\leftarrow$	1 $\leftarrow$
2	0 $\perp$	1 $\nwarrow$	1 $\leftarrow$	1 $\leftarrow$	1 $\leftarrow$	1 $\leftarrow$	2 $\nwarrow$
3	0 $\perp$	1 $\uparrow$	1 $\leftarrow$	1 $\leftarrow$	2 $\nwarrow$	2 $\leftarrow$	2 $\leftarrow$
4	0 $\perp$	1 $\uparrow$	2 $\nwarrow$	2 $\leftarrow$	2 $\leftarrow$	3 $\nwarrow$	3 $\leftarrow$
5	0 $\perp$	1 $\uparrow$	2 $\uparrow$	2 $\leftarrow$	3 $\nwarrow$	3 $\leftarrow$	3 $\leftarrow$
6	0 $\perp$	1 $\nwarrow$	2 $\uparrow$	2 $\leftarrow$	3 $\uparrow$	3 $\leftarrow$	

Example

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0 \perp$	$1 \uparrow$	$2 \uparrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$	$3 \leftarrow$
6	$0 \perp$	$1 \nwarrow$	$2 \uparrow$	$2 \leftarrow$	$3 \uparrow$	$3 \leftarrow$	$4 \nwarrow$

Example: Find Common Subsequence

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$	0 $\perp$
1	0 $\perp$	0 $\leftarrow$	0 $\leftarrow$	1 $\nwarrow$	1 $\leftarrow$	1 $\leftarrow$	1 $\leftarrow$
2	0 $\perp$	1 $\nwarrow$	1 $\leftarrow$	1 $\leftarrow$	1 $\leftarrow$	1 $\leftarrow$	2 $\nwarrow$
3	0 $\perp$	1 $\uparrow$	1 $\leftarrow$	1 $\leftarrow$	2 $\nwarrow$	2 $\leftarrow$	2 $\leftarrow$
4	0 $\perp$	1 $\uparrow$	2 $\nwarrow$	2 $\leftarrow$	2 $\leftarrow$	3 $\nwarrow$	3 $\leftarrow$
5	0 $\perp$	1 $\uparrow$	2 $\uparrow$	2 $\leftarrow$	3 $\nwarrow$	3 $\leftarrow$	3 $\leftarrow$
6	0 $\perp$	1 $\nwarrow$	2 $\uparrow$	2 $\leftarrow$	3 $\uparrow$	3 $\leftarrow$	4 $\nwarrow$

Example: Find Common Subsequence

	1	2	3	4	5	6
A	b	a	c	d	c	a
B	a	d	b	c	d	a

	0	1	2	3	4	5	6
0	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$	$0 \perp$
1	$0 \perp$	$0 \leftarrow$	$0 \leftarrow$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$
2	$0 \perp$	$1 \nwarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$
3	$0 \perp$	$1 \uparrow$	$1 \leftarrow$	$1 \leftarrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$
4	$0 \perp$	$1 \uparrow$	$2 \nwarrow$	$2 \leftarrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$
5	$0 \perp$	$1 \uparrow$	$2 \uparrow$	$2 \leftarrow$	$3 \nwarrow$	$3 \leftarrow$	$3 \leftarrow$
6	$0 \perp$	$1 \nwarrow$	$2 \uparrow$	$2 \leftarrow$	$3 \uparrow$	$3 \leftarrow$	$4 \nwarrow$