Introduction to Machine Learning

General Note About Linear Classifiers

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Outline

Loss Function for Linear Classification

- Linear binary classification can be written as a general optimization problem:

$$\min_{\mathbf{w},b} L(\mathbf{w}, b) = \min_{\mathbf{w},b} \sum_{n=1}^{N} \mathbb{I}(y_n(\mathbf{w}^\top \mathbf{x}_n + b) < 0) + \lambda R(\mathbf{w}, b)$$

- $\mathbb{I}$ is an **indicator function** (1 if (.) is negative, 0 otherwise)
- Objective function = **Loss function** + λ**Regularizer**
- Objective function wants to **fit training data well** and **have simpler solution**

0-1 Loss is Hard to Optimize

- Combinatorial optimization problem
- NP-hard
- No polynomial time algorithm
- Loss function is non-smooth, non-convex
- Small changes in $\mathbf{w}$, b can change the loss by lot

Approximations to 0-1 Loss

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  - Also known as *surrogate loss functions*

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- **Hinge Loss**

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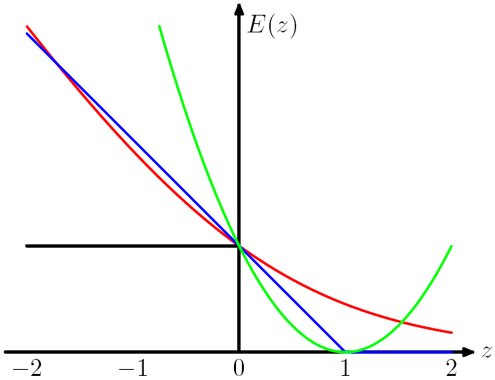
Logistic Regression

- **Log Loss**

Plot of Loss Functions

- black, indicator loss
- green, squared loss
- red, log loss
- blue, hinge loss



Role of Regularizers

- Recall the optimization problem for linear classification

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- What is the role of the regularizer term?

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- What is the role of the regularizer term?
  - Ensure simplicity
- Ideally we want most entries of **w** to be zero
- Why?
- Desired minimization

$$R(\mathbf{w}, b) = \sum_{d=1}^{D} \mathbb{I}(w_d \neq 0)$$

- NP Hard

Approximate Regularization

- **Norm based regularization**
  - l_2 squared norm

$$\|\mathbf{w}\|_2^2 = \sum_{d=1}^{D} w_d^2$$

  - l_1 norm

$$\|\mathbf{w}\|_1 = \sum_{d=1}^{D} |w_d|$$

  - l_p norm

$$\|\mathbf{w}\|_p = (\sum_{d=1}^{D} w_d^p)^{1/p}$$

  - Norm becomes non-convex for $p < 1$
  - l_1 norm gives best results
  - l_2 norm is easiest to deal with