Factor Analysis

$p(z) \sim N(\mu_0, \Sigma_0)$ z ~ Hidden variable

$p(x|z,\theta) \sim N(Wz + \mu, \Psi)$

$z \in \mathbb{R}^L$ $x \in \mathbb{R}^D$ $D \gg L$

$\mu_{0\,L\times 1}$ $\Sigma_{0\,L\times L}$ $W_{D\times L}$ $\mu_{D\times 1}$ $\Psi_{D\times D}$

Loading matrix

Often assume $\mu_0 = 0$, $\Sigma_0 = I$

$p(x|z,\theta) \sim N(\mu, \Psi + WW^T)$

$\theta = \{W, \Psi, \mu\}$ Ψ. diagonal matrix

x domain. $\Sigma_{D\times D}$ D^2

$p(x|z,\theta) \sim N(\mu, \Psi + \underline{WW^T})$ $\hat{W}\hat{W}^T = WW^T$

Ψ diagonal, D

W, $D \times L$

μ, D

\# parameters $D + D\times L + D \ll D^2$

① less memory. less parameters

② Dimention Reduction

$P(z_i | x_i, \theta) \sim N(m_i, \Sigma)$

$x_i \longleftrightarrow z_i$

D dimention space   L dimention space

③ Factor Analysis

Hidden factors in low dimension space z.

issue: unidentifiability, W is not unique

Consider $RR^T = I$

$\hat{W} = W \cdot R$

$\hat{W}\hat{W}^T = W \cdot R \cdot R^T \cdot W^T = W \cdot W^T$

X space

$e_i = [1, 0\ 0 \ \cdots \ 0]_{D\times 1}$ each feacher

$p(z_i | e_i, \theta)$ $\underset{D\times1}{e_i} \longleftrightarrow z_{i\,L\times 1}$