CSE 531: Algorithm Analysis and Design | Fall 2024

Homework 4

Instructor: Kelin Luo | **Deadline: Dec/02/2024**

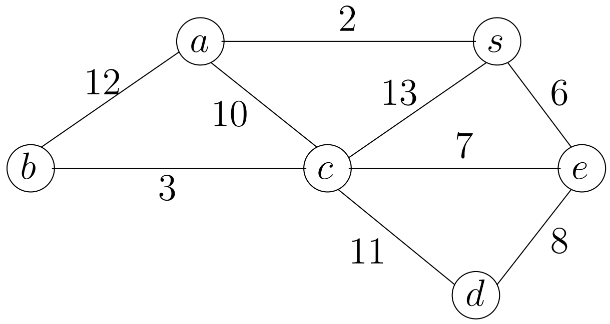
Your Name: ______________________ Your Student ID: ________________

Problems	1	2	3	4	5	Total
Max. Score	12	8	12	8	10	40
Your Score						

Requirement:

- Save and submit your HW 4 submission to Brightspace as a single typed PDF file. Name your file: Your Last Name Your First Name YourStudents ID Number Assignment Number. Example: **Doe_John_55552222_HW4**
- You should view your submission after you upload it to make sure that it is not corrupted or malformed. Submissions that are rotated, upside down, or that do not load will not receive credit. Illegible submissions may also lose credit depending on what can be read. You are responsible for making sure your submission went through successfully.
- The HW 4 deadline is 02 Dec, 11:59PM EST. Late submissions (within 24 hours with 25 % penalty or 48 hours with a 50% penalty) should be submitted via Email to Instructor and Head TAs. Submissions will close 48 hours after the deadline.
- Only the most recent submission is kept on Brightspace.
- Note that your total points for HW4 will not exceed 40. Suppose you receive $P1$, $P2$, $P3$, $P4$ and $P5$ points for each of the questions. Your HW4 grade, as counted towards your final grade, will be calculated as $5*\min\{P1+P2+P3+P4+P5, 40\}/40$.
- (Only) For HW4 Problem 5, you are allowed (and encouraged) to utilize Generative Artificial Intelligence (GenAI) tools such as ChatGPT, Google Gemini, Meta.AI, Claude.AI, GitHub CoPilot, or others. Proper documentation, citation, and acknowledgment are required when using these tools. Guidelines for citing GenAI in student work at the University at Buffalo can be found through the University Libraries-Citing Generative AI. Please note that generative AI, such as large language models, can produce content that may falsify references, fabricate facts, or inaccurately represent ideas. This content is derived from input provided by human authors and may contain inaccuracies or exhibit biases. Additionally, the legal and copyright status of inputs and outputs from generative AI is currently uncertain. For more information, refer the University Libraries-AI, Authorship and Copyright. You are responsible for the content and accuracy of all work submitted in this class, including any material supported by generative AI.

Problem 1 (12 points). Consider the following graph G with non-negative edge weights.



(a) **(6 points)** Use Prim's algorithm to compute the minimum spanning tree of G. You can use the following table to describe the execution of the algorithm. The algorithm maintains a set S of vertices. The d value of a vertex $v \notin S$ is $\min_{u \in S:(u,v)\in E} w(u,v)$. The π value of a vertex v is the vertex $u \in S$ such that $d(v) = w(u,v)$; if $d(v) = \infty$, then $\pi(v) =$ " $\perp$ ". You can try to complete the value of the table following the Prim algorithm in our lecture notes, the edges in the MST are __________________, and the total weight is _________.

iteration	vertex added to S	a		b		c		d		e	
		d	π	d	π	d	π	d	π	d	π
1	s	2	s	∞	$\perp$	13	s	∞	$\perp$	6	s
2											
3											
4											
5											
6											

Table 1: Prim's Algorithm for Minimum Spanning Tree

(b) **(6 points)** Use Dijkstra's algorithm to compute the shortest paths from s to all other vertices in G. You can use the following table to describe the execution of the algorithm on the instance. The algorithm maintains a set S of vertices. The d value of a vertex $v \notin S$ is $\min_{u \in S:(u,v)\in E}(d(u) + w(u,v))$. The π value of a vertex v is the vertex $u \in S$ such that $d(v) = d(u) + w(u,v)$; if $d(v) = \infty$, then $\pi(v) =$ " $\perp$ ".

The parents of vertices are:

	a	b	c	d	e
parent					

You can try to complete the value of the above two tables following the Dijkstra's algorithm in our lecture notes. The shortest paths to a is _________, and the length is _________; The shortest paths to b is _________, and the length is _________; The shortest paths to c is _________, and the length is _________; The shortest paths to d is _________, and the length is _________; The shortest paths to e is _________, and the length is _________.

iteration	vertex added to S	a		b		c		d		e	
		d	π	d	π	d	π	d	π	d	π
1	s	2	s	∞	$\perp$	13	s	∞	$\perp$	6	s
2											
3											
4											
5											
6											

Table 2: Dijkstra's algorithm for Shortest Paths

Problem 2 (8 points). We are given a directed graph $G = (V, E)$ with positive weight function: $w : E \to R_{>0}$, and two vertices $s, t \in V$. Suppose we have already computed the d and π array using the Dijkastra's algorithm: $d[v]$ is the length of the shortest path from s to v, and $\pi[v]$ is the vertex before v in the path.

Please design a dynamic programming algorithm (i.e., Define sub-problems, establish recurrence relations between sub-problems, compute the base case, write the dynamic programming algorithm pseudo-code) to count the number of shortest paths from s to t in $O(n \log n + m)$ time where $n = |V|$ and $m = |E|$.

Hint: use the d and π array to help solve the counting problem. You do not need to worry about the integer overflow issue. That means, you assume a word can hold a very big integer, and basic operations over these big integers take $O(1)$ time.

Problem 3 (12 points). Please define the complexity classes P, NP, and Co-NP.

Reflecting on the Hamiltonian Cycle Problem (HC) and its complement $\overline{\text{HC}}$ Problem as discussed in our lecture, please complete the table below by indicating "Yes" or "No" for each item. Additionally, please provide a brief justification (i.e., applying the certificate and certifier arguments, or applying the notations) for each of your responses.

	Belongs to **P**	Belongs to **NP**	Belongs to **Co-NP**
HC			
$\overline{\text{HC}}$			

Problem 4 (8 points). Consider the Hamiltonian Path Problem we discussed in our lecture notes. You are required to provide a detailed argument demonstrating that the problem is NP-complete. This should include showing that the Hamiltonian Path Problem is in NP and providing a polynomial reduction from a known NP-hard problem.

Hints: Your argument should focus on reducing a known NP-hard problem (refer to one of the problems discussed in our lecture notes) to the problem you have selected.

(a) **(2 points)** Please write the definition of the Hamiltonian Path Problem along with the definition of a known NP-hard problem.

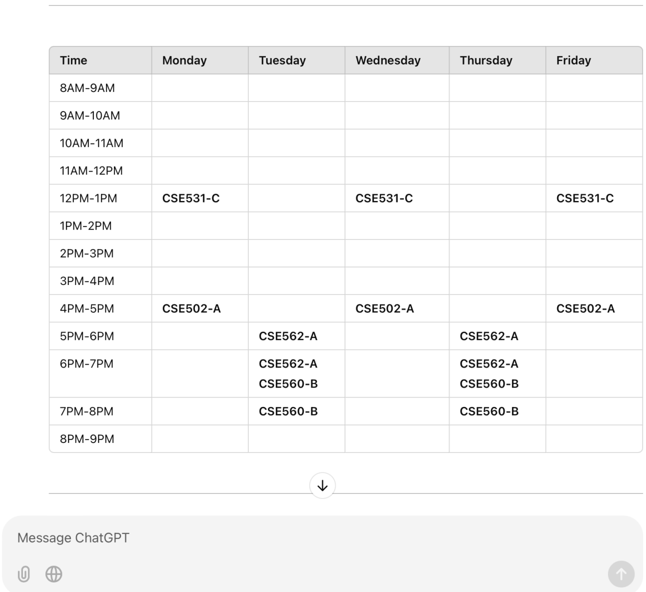
(b) **(2 points)** Please show that the Hamiltonian Path Problem is in NP.

(c) **(4 points)** Please write the polynomial reduction process.

Problem 5: Bonus Question (10 points). Consider courses offered in Fall 2024 from the remaining options, listed as Course-Section: CSE502-A, CSE505-A, CSE526-A, CSE531-C, CSE535-A, CSE545-A, CSE546-A, CSE547-A, CSE548-A, CSE555-A, CSE560-B, CSE562-A, CSE565-C, CSE568-A, CSE570-A, CSE573-B, CSE574-E, CSE587-B, CSE611-A, CSE616-A, CSE676-B. Please determine, step by step, the maximum number of courses that can be taken without scheduling conflicts (time conflicts) and provide a visual representation of the optimal course selection. Please note that your answers, including written explanations and screenshot results, should not exceed 2 A4 pages.

Hints: The courses listed above for Fall 2024 are scheduled either on TR (Tuesday and Thursday) or MWF (Monday, Wednesday, and Friday). Please remove any courses that are not being offered. For detailed class time slots (including days: Monday (M), Tuesday (T), Wednesday (W), Thursday (R), Friday (F) and times), please refer to the class schedule on our website. UBCSE-Class Schedule.

Note: You are encouraged to use GenAI tools at any step of solving this problem, such as extracting course schedule information from the course website, outlining algorithm steps to determine the maximum number of courses, creating visualizations. However, be aware that GenAI tools may generate content that includes incorrect references, fabricated facts, or misrepresentations of ideas. You may need to adjust or refine the generated solutions to ensure accuracy. Additionally, make sure to document the entire problem-solving process. See an example reference for figure illustration[1].



Time	Monday	Tuesday	Wednesday	Thursday	Friday
8AM-9AM					
9AM-10AM					
10AM-11AM					
11AM-12PM					
12PM-1PM	CSE531-C		CSE531-C		CSE531-C
1PM-2PM					
2PM-3PM					
3PM-4PM					
4PM-5PM	CSE502-A		CSE502-A		CSE502-A
5PM-6PM		CSE562-A		CSE562-A	
6PM-7PM		CSE562-A CSE560-B		CSE562-A CSE560-B	
7PM-8PM		CSE560-B		CSE560-B	
8PM-9PM					

Figure 1: This class schedule was created by OpenAI, incorporating course information:
CSE531-C: Mon, Wed, Fri at 12:00 PM - 12:50 PM
CSE502-A: Mon, Wed, Fri at 4:00 PM - 4:50 PM
CSE562-A: Tue, Thu at 5:00 PM - 6:20 PM
CSE560-B: Tue, Thu at 6:30 PM - 7:50 PM

[1] OpenAI. Text and Table generated by ChatGPT, Version GPT-4.0. Accessed November 10, 2024. https://chat.openai.com/.