

Sep 5, 2024

$$\underline{P(y=1|x^*)} = \frac{P(x|y=1) \cdot P(y=1)}{P(x|y=1)P(y=1) + P(x|y=0) \cdot P(y=0)}$$

$$P(x|y=1)$$

$$P(y=1)$$

$$x = [x_1, x_j, x_d]$$

$$x_j \in [0, 1]$$

$$y \in [0, 1]$$

$$d=1$$

$$P(x|y=1) \sim \text{Ber}(\theta_1) \rightarrow P(x|y=1)$$

$$P(x|y=0) \sim \text{Ber}(\theta_0)$$

$$2$$

$$d=2$$

0	0	$P_{00}$
0	1	$P_{01}$
1	0	$P_{10}$
1	1	$P_{11}$

$$\underline{3}$$

$$\rightarrow P(x|y=0)$$

$$\underline{3} = 2^2 - 1$$

$$3 \times 2 \uparrow$$

$$d=3, n$$

$$2 \cdot (2^3 - 1)$$

Assumption,  $x_1, \dots, x_d$  independent given  $y$

$$P(x|y=1) = \prod_{i=1}^d \underline{P(x_i|y=1)}$$

Naive Bayes

1 parameter for each  $\underline{P(x_j|y=1)}$

$D$

$\underline{P(x|y=1)}$

$D$

$P(x|y=0)$

2D parameters

$$2(2^d - 1) \Rightarrow 2D$$

Likelihood

$$L(D|\theta)$$

$$= \prod_{i=1}^N P(y_i) \cdot \prod_{j=1}^D (x_{ij} | y_i)$$

$\uparrow$   $\theta$   $\uparrow$   $\theta_{ij}$   
 $\theta_{ij}$

Example:

$$P(y=1) = \frac{5}{10}$$

$$P(y=0) = \frac{5}{10}$$

$$P(\text{civ} | y=1) = \frac{3}{5}$$

$$P(\text{civ} | y=0) = \frac{2}{5}$$

$$P(\text{small} | y=1) = \frac{1}{5}$$

$$P(\text{small} | y=0) = \frac{3}{5}$$

$$P(\text{light} | y=1) = \frac{2}{5}$$

$$P(\text{light} | y=0) = \frac{3}{5}$$

$$P(\text{oval} | y=1) = 1 - P(\text{civ} | y=1) = \frac{2}{5}$$

$$P(y=1 | \text{civ}, \text{small}, \text{light})$$

$$\begin{aligned} &\propto P(\text{civ}, \text{small}, \text{light} | y=1) \cdot P(y=1) \quad \text{Bayes rule} \\ &= \frac{P(\text{civ} | y=1) P(\text{small} | y=1) P(\text{light} | y=1) P(y=1)}{\quad \text{Naive Assumption}} \\ &= \frac{3}{5} \cdot \frac{1}{5} \cdot \frac{2}{5} \cdot \frac{5}{125} \\ &= \frac{3}{125} \end{aligned}$$

$$P(y=0 | \text{civ}, \text{small}, \text{light}) = \frac{9}{125}$$

$$P(y=0 | x^*) > P(y=1 | x^*)$$

$$3 \text{ features. } \begin{cases} 2 \cdot (2^3 - 1) = 14 \text{ parameters} \\ 2 \cdot 3 = 6 \text{ parameters} \end{cases}$$

$$P(y=1 | x^*) = \frac{\frac{3}{125}}{\frac{3}{125} + \frac{9}{125}} = 0.25 \quad \text{normalization}$$

$$P(y=0 | x^*) = 0.75$$