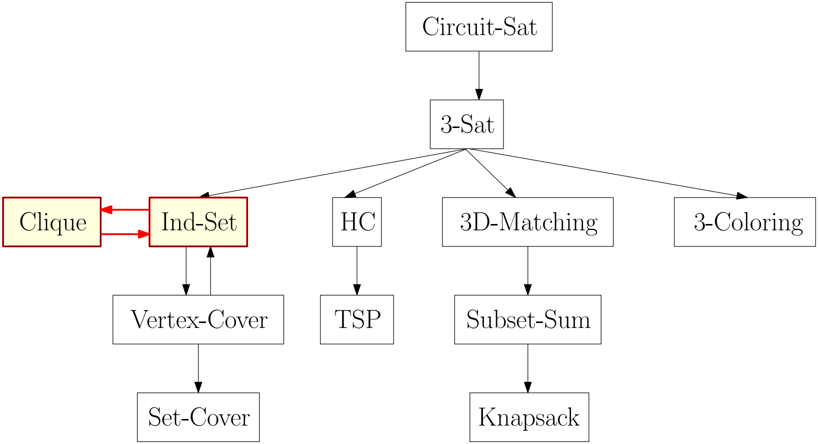
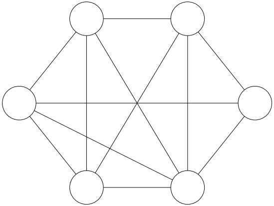
Reductions of NP-Complete Problems



Def. A clique in an undirected graph $G = (V, E)$ is a subset $S \subseteq V$ such that $\forall u, v \in S$ we have $(u, v) \in E$



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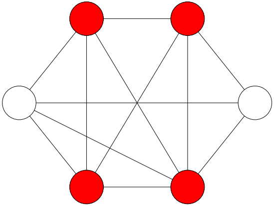

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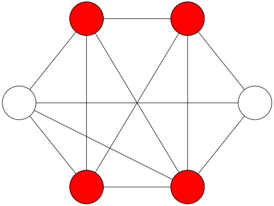

Clique Problem

Input: $G = (V, E)$ and integer $k > 0$,

Output: whether there exists a clique of size k in G

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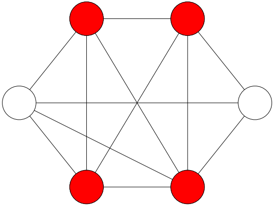

Clique Problem

Input: $G = (V, E)$ and integer $k > 0$,

Output: whether there exists a clique of size k in G

- What is the relationship between Clique and Ind-Set?

Clique $=_P$ Ind-Set

Def. Given a graph $G = (V, E)$, define $\overline{G} = (V, \overline{E})$ be the graph such that $(u, v) \in \overline{E}$ if and only if $(u, v) \notin E$.

Obs. S is an independent set in G if and only if S is a clique in $\overline{G}$.

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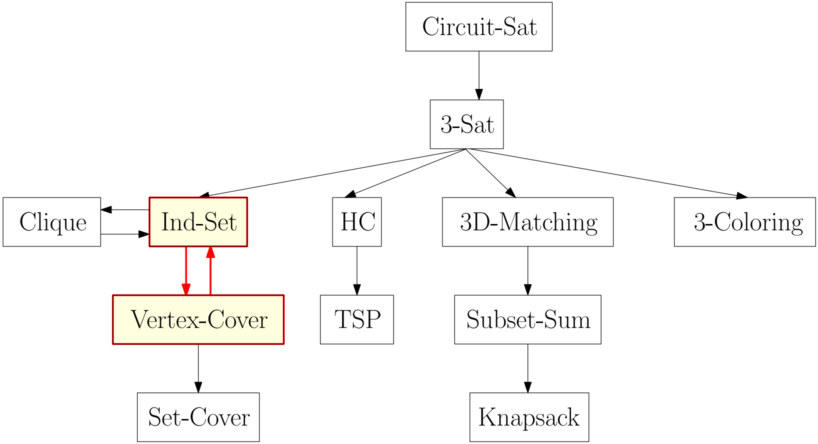
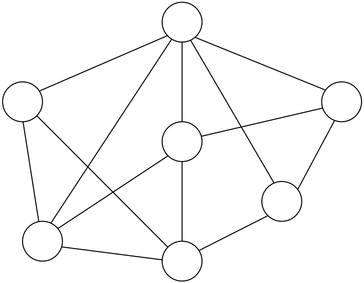


Vertex-Cover

Def. Given a graph $G = (V, E)$, a vertex cover of G is a subset $S \subseteq V$ such that for every $(u, v) \in E$ then $u \in S$ or $v \in S$.



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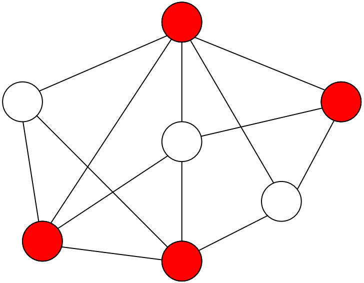

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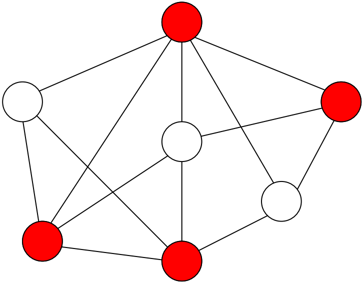

Vertex-Cover Problem

Input: $G = (V, E)$ and integer k

Output: whether there is a vertex cover of G of size at most k

Vertex-Cover $=_P$ Ind-Set

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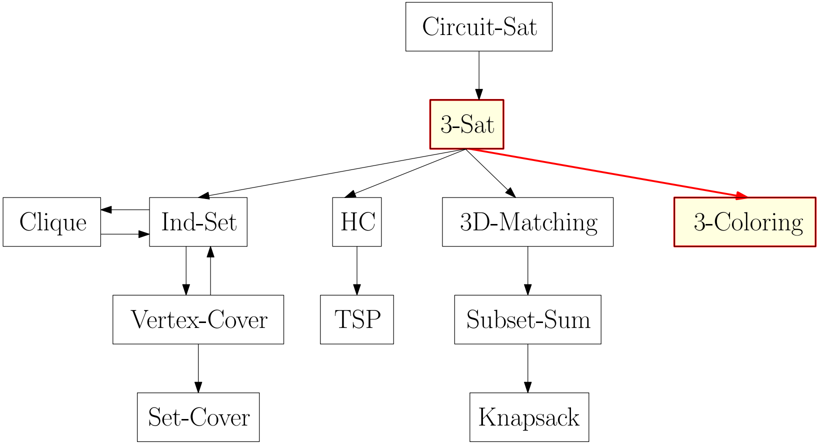
Q: What is the relationship between Vertex-Cover and Ind-Set?

Vertex-Cover $=_P$ Ind-Set

Q: What is the relationship between Vertex-Cover and Ind-Set?

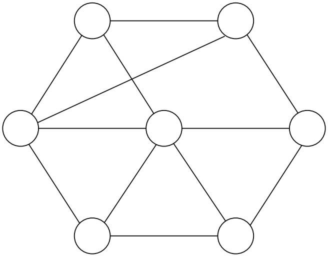
A: S is a vertex-cover of $G = (V, E)$ if and only if $V \setminus S$ is an independent set of G.

Reductions of NP-Complete Problems



k-coloring problem

Def. A k-coloring of $G = (V, E)$ is a function $f : V \to \{1, 2, 3, \cdots, k\}$ so that for every edge $(u, v) \in E$, we have $f(u) \neq f(v)$. G is k-colorable if there is a k-coloring of G.



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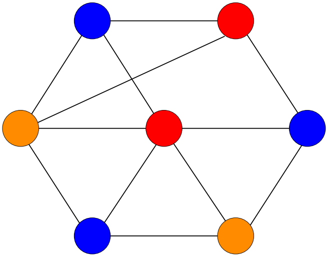

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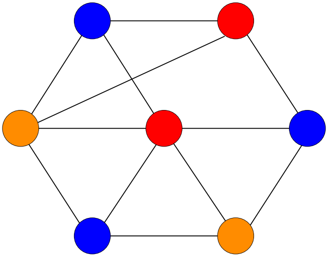

k-coloring problem

Input: a graph $G = (V, E)$

Output: whether G is k-colorable or not

2-Coloring Problem

Obs. A graph G is 2-colorable if and only if it is bipartite.

Q: How do we check if a graph G is 2-colorable?

2-Coloring Problem

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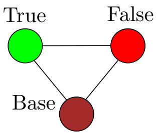
Q: How do we check if a graph G is 2-colorable?

A: We check if G is bipartite.

3-SAT $\leq_P$ 3-Coloring

- Construct the base graph

Base Graph



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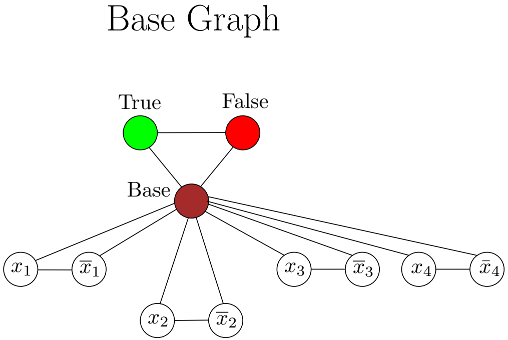
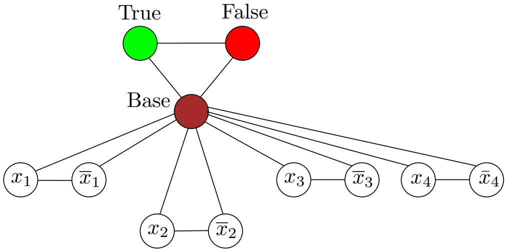


3-SAT $\leq_P$ 3-Coloring

- Construct the base graph
- Construct a gadget from each clause: gadget is 3-colorable if and only if the clause is satisfied.

Base Graph

$$x_1 \vee \neg x_2 \vee x_3$$



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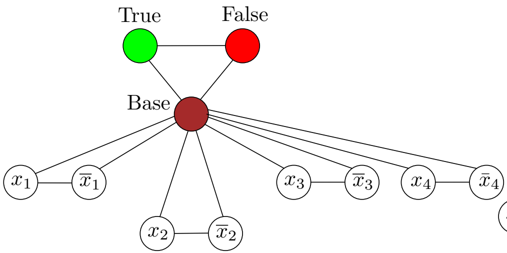

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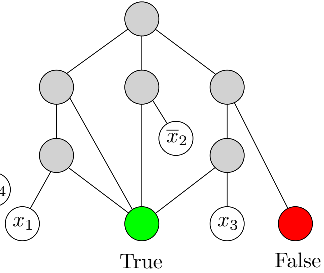

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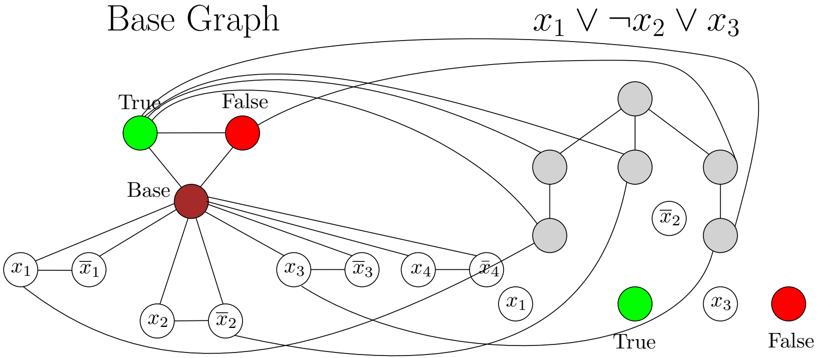

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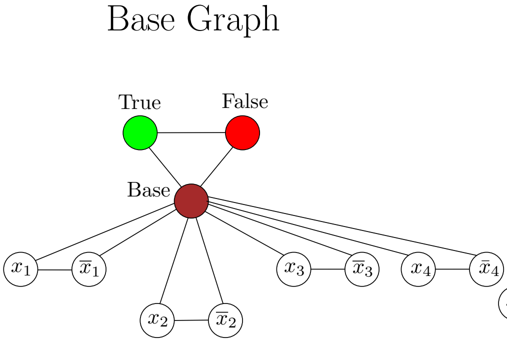

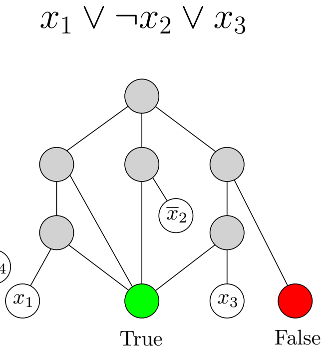

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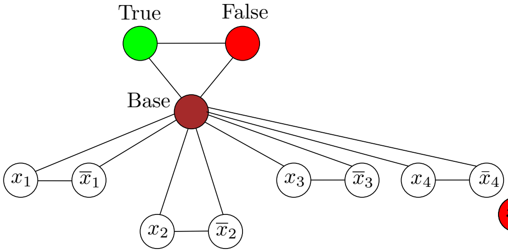

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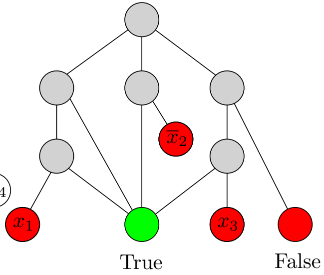

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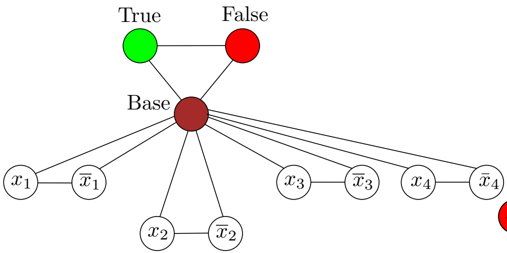

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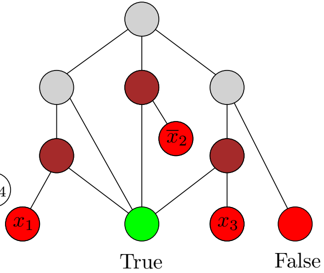

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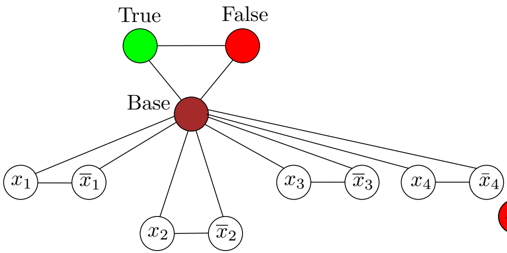

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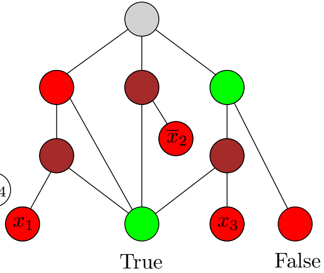

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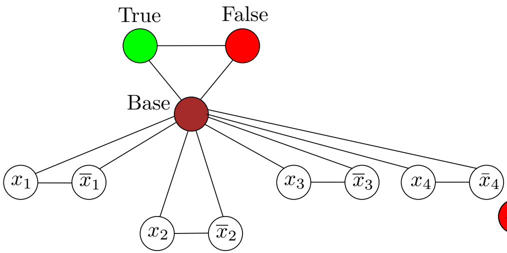

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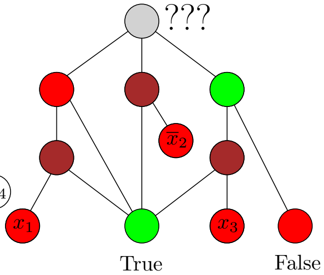

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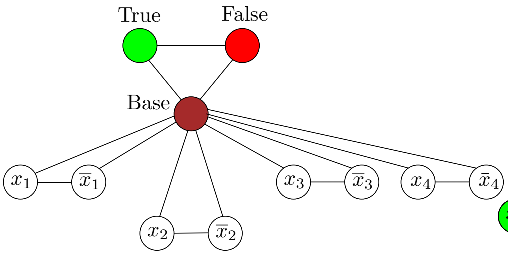

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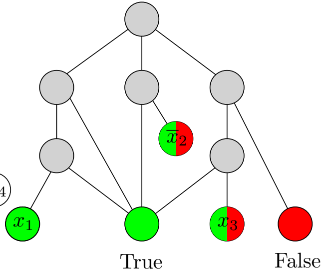

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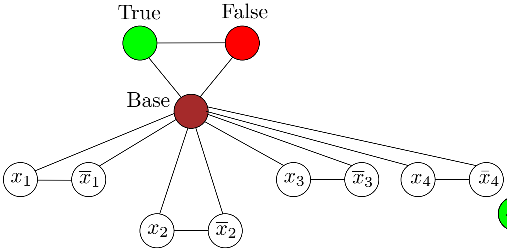

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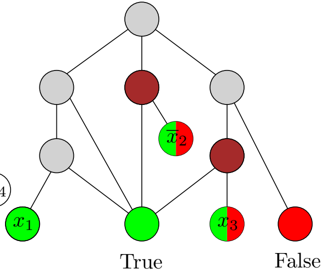

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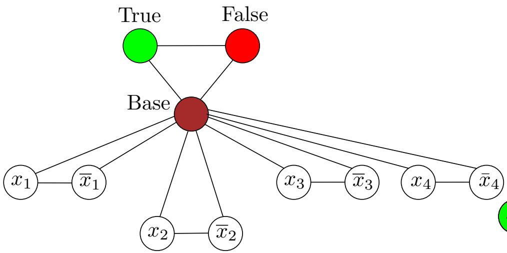

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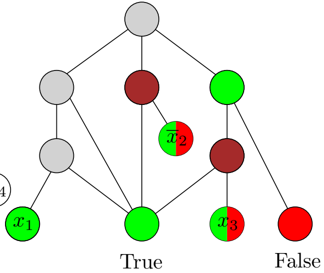

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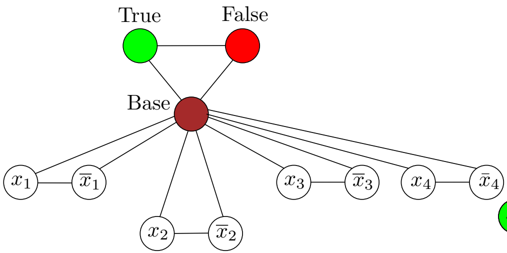

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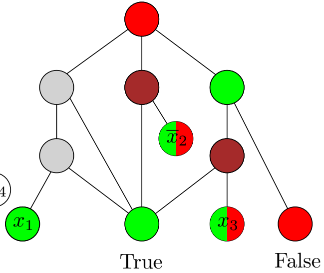

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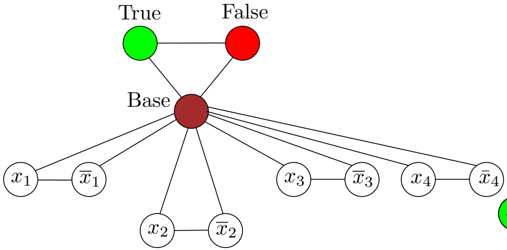

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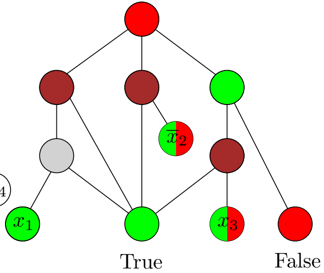

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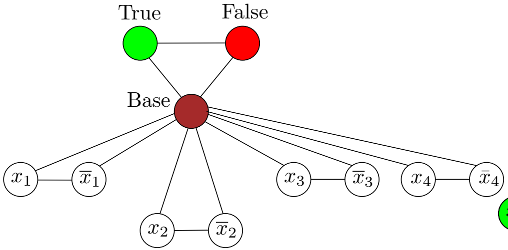

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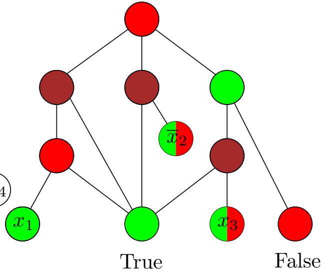

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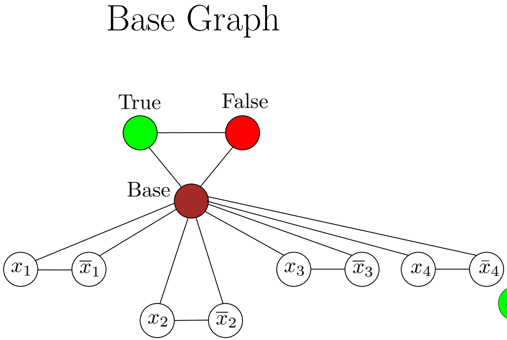

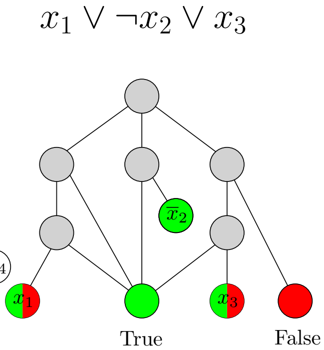

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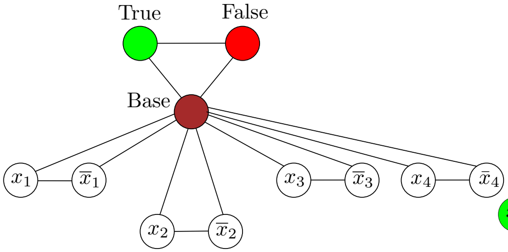

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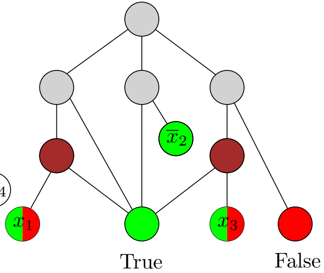

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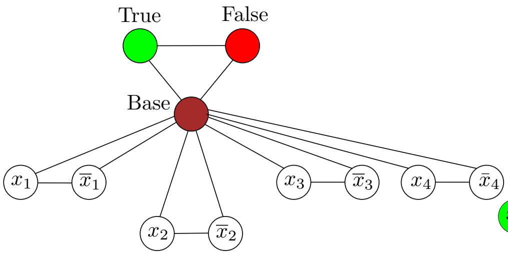

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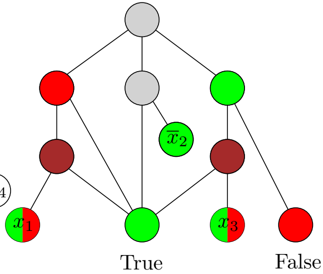

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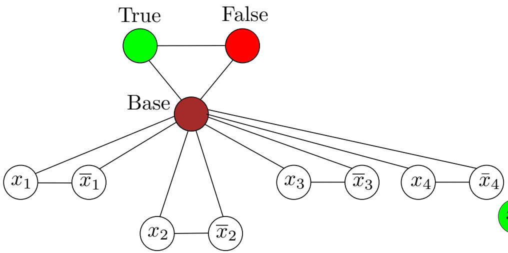

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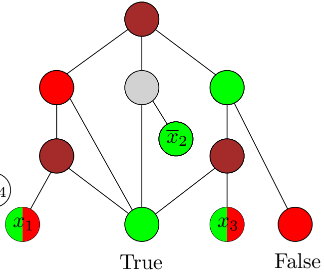

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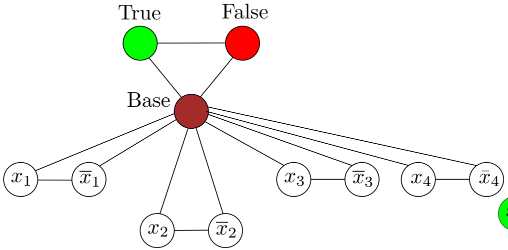

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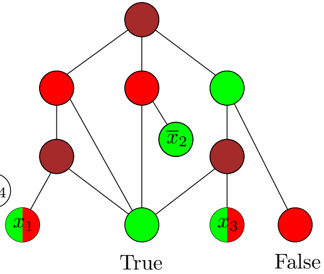

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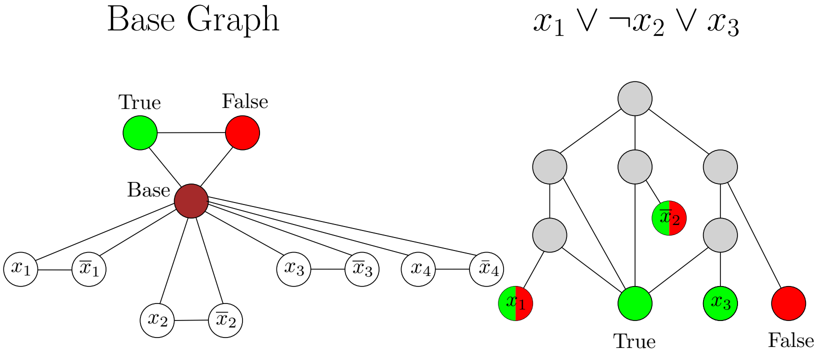

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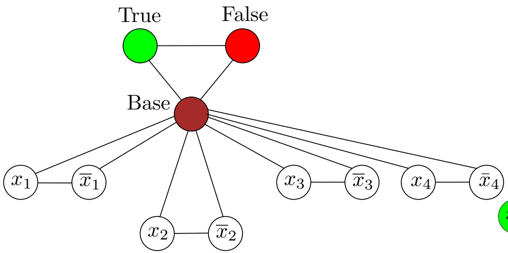

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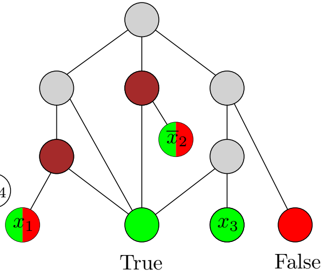
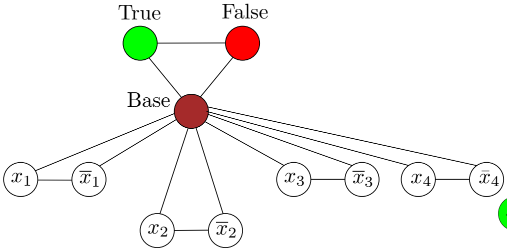


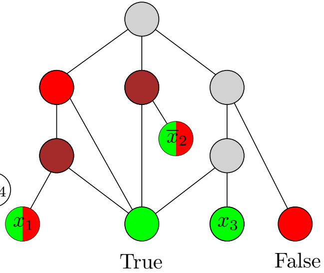
3-SAT $\leq_P$ 3-Coloring

- Construct the base graph
- Construct a gadget from each clause: gadget is 3-colorable if and only if the clause is satisfied.

Base Graph



$x_1 \vee \neg x_2 \vee x_3$



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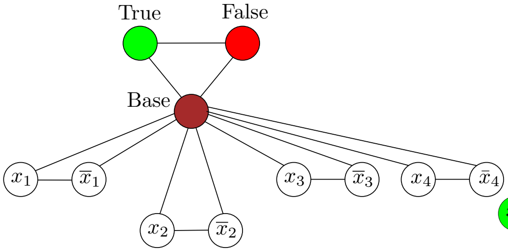

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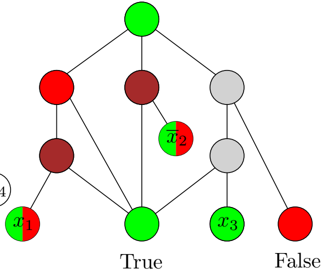

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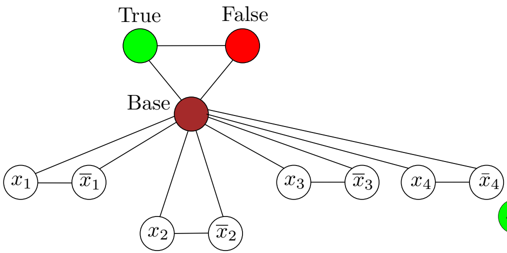

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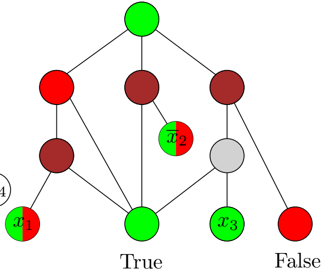

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