Quiz 6 Solutions

CSE 4/574

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Question 1

The following diagram shows a training set with four negative points (green circles) and four positive points (purple squares). It has no useful linear separating boundary, but the

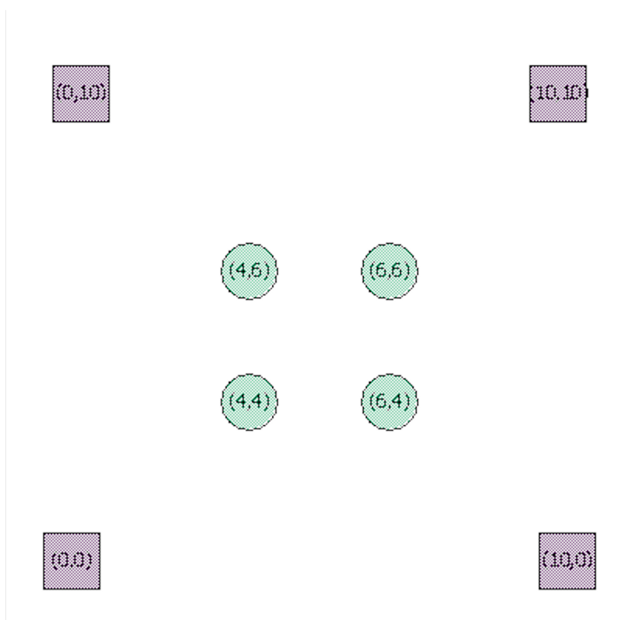

Figure 1: Enter Caption

following nonlinear transformation turns points (x_1, x_2) in the space shown to points in another space, which we shall call (y_1, y_2). The transformation is: $y_1 = (x_1 - 5)^2$, $y_2 = (x_2 - 5)^2$. In the $(y1, y2)$ space, something very convenient happens. All the negative points are mapped to the point $(1, 1)$, and all the positive points are mapped to the point $(25, 25)$. Your task is to find the maximum-margin separator in the (y_1, y_2) space (the transformed space). Then, determine which new points in the original (x_1, x_2) space would be classified as positive, and which would be classified as negative. That is, what curve in the original space transforms to the straight-line boundary in the transformed space? Identify the true statement in the list of choices below.

Correct Choice

$(0, 4)$ is on the boundary.

Problem Explanation:

First, the maximum-margin separator between the points $(1, 1)$ and $(25, 25)$ is the perpendicular line between them. The midpoint of the line between them is evidently $(13, 13)$. Since the line from $(1, 1)$ to $(25, 25)$ has a slope of 1, its perpendicular line has a slope of -1. The line with slope -1 going through $(13, 13)$ is $y_2 = 26 - y_1$, or $y_1 + y_2 = 26$. If we substitute $(x1-5)^2$ for $y1$ and $(x2-5)^2$ for $y2$, the equation becomes $(x1-5)^2 + (x2-5)^2 = 26$. This curve is the boundary; it is a circle with center at $(5, 5)$ and radius $\sqrt{26}$. Any point $(x1, x2)$ with $(x1-5)^2 + (x2-5)^2 < 26$ is classified as negative, and points with $(x1-5)^2 + (x2-5)^2 > 26$ are classified as positive.

Question 2

Suppose we use the transformation $\phi((x, y)) = (x^2, y^2, xy)$. That is, the two-dimensional vector (x, y) is turned into a three-dimensional vector (x^2, y^2, xy). There is a kernel function $K(u, v)$ for this transformation; the function such that $K(u, v) = \phi(u) \cdot \phi(v)$. The kernel function measures the similarity, in the transformed space, between the vectors u and v.
Let us consider the eight vectors from the original space: $(1, 2)$, $(1, -2)$, $(-1, 2)$, $(-1, -2)$, $(2, 1)$, $(2, -1)$, $(-2, 1)$, and $(-2, -1)$.
Your task is to compute the kernel function for each pair of these vectors. Note that the task is simpler than it looks, because pairs of original vectors transform to the same vector, so there are only four different vectors in the transformed space.
Now, find in the list below the pair of vectors that are MOST similar.

Correct Choice

$(1, -2)$ and $(1, 2)$

Problem Explanation:

$$
\begin{aligned}
&K((1, -2), (1, 2)) \\
&= \phi((1, -2)) \cdot \phi((1, 2)) \\
&= (1, 4, -2) \cdot (1, 4, 2) \\
&= 1 + 16 - 4 = 13
\end{aligned}
$$

Incorrect Choice 1

$(2, -1)$ and $(1, 2)$

Problem Explanation:

$$
\begin{aligned}
&K((2,-1),(1,2)) \\
&= \phi((2,-1)) \cdot \phi((1,2)) \\
&= (4,1,-2) \cdot (1,4,2) \\
&= 4+4-4 = 4
\end{aligned}
$$

Incorrect Choice 2

$(-1,2)$ and $(-2,-1)$

Problem Explanation:

$$
\begin{aligned}
&K((-1,2),(-2,-1)) \\
&= \phi((-1,2)) \cdot \phi((-2,-1)) \\
&= (1,4,-2) \cdot (4,1,2) \\
&= 4+4-4 = 4
\end{aligned}
$$

Incorrect Choice 3

$(1,2)$ and $(-2,1)$

Problem Explanation:

$$
\begin{aligned}
&K((1,2),(-2,1)) \\
&= \phi((1,2)) \cdot \phi((-2,1)) \\
&= (1,4,2) \cdot (4,1,-2) \\
&= 4+4-4 = 4
\end{aligned}
$$