BEHAVIOUR OF CONTINOUS BEAMS

INTRODUCTION:

Continuous beams, which are beams with more than two supports and covering more than one span, are not statically determinate using the static equilibrium laws

e = strain
σ = stress (N/m^2)
E = Young's Modulus = σ /e (N/m^2)
y = distance of surface from neutral surface (m).
R = Radius of neutral axis (m).
I = Moment of Inertia (m^4 - more normally cm^4)
Z = section modulus = I/y_{max}(m^3 - more normally cm^3)
M = Moment (Nm)
w = Distributed load on beam (kg/m) or (N/m as force units)
W = total load on beam (kg) or (N as force units)
F= Concentrated force on beam (N)
L = length of beam (m)
x = distance along beam (m)

OBJECTIVE:

To find the shear force diagram and bending moment diagram for a given continuous beam.

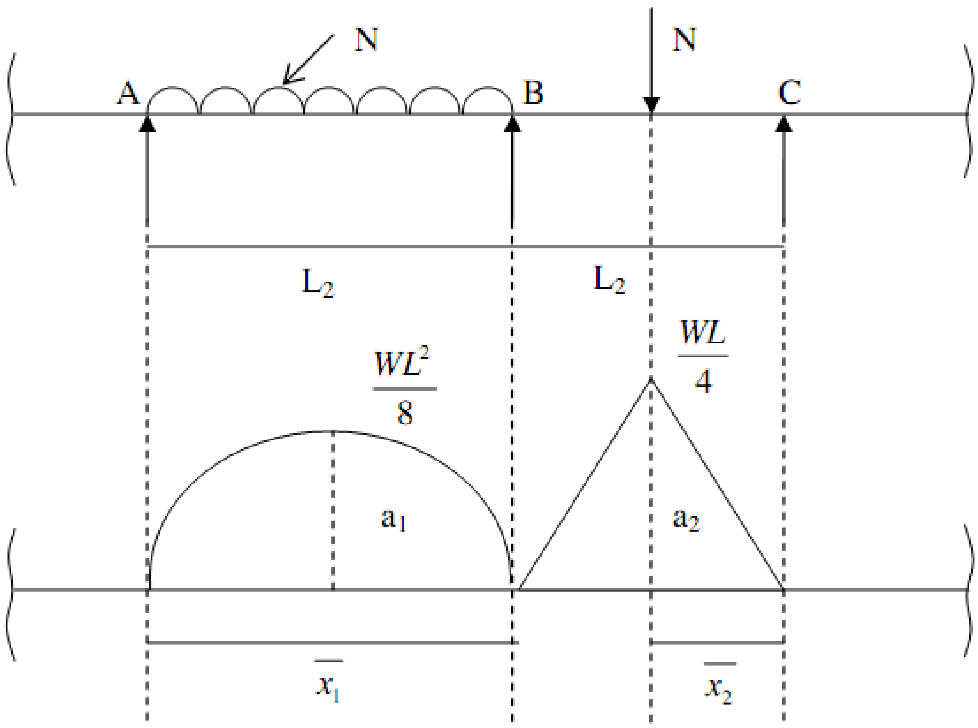
THEORY:

Beams placed on more than 2 supports are called continuous beams. Continuous beams are used when the span of the beam is very large, deflection under each rigid support will be equal zero.

BMD for Continuous beams:

BMD for continuous beams can be obtained by superimposing the fixed end moments diagram over the free bending moment diagram.

Three - moment Equation for continuous beams THREE MOMENT EQUATION



FREE B.M.

$$M_A\left(\frac{L_1}{E_1 I_1}\right)+2M_B\left(\frac{L_1}{E_1 I_1}+\frac{L_2}{E_2 I_2}\right)+M_C\left(\frac{L_2}{E_2 I_2}\right)$$

$$=\frac{-6a_1\overline{x_1}}{E_1 I_1 L_1}-\frac{6a_2\overline{x_2}}{E_2 I_2 L_2}-6\left[\frac{\delta_A-\delta_B}{L_1}+\frac{\delta_C-\delta_B}{L_2}\right]$$

The above equation is called generalized 3-moments Equation.
M_A, M_B and M_C are support moments E_1, E_2 Young's modulus of Elasticity of 2 Spans.

I_1, I_2 M O I of 2 spans,

a_1, a_2 → Areas of free B.M.D.

$\bar{x}_1$ and $\bar{x}_2$ → Distance of free B.M.D. from the end supports, or outer supports. (A and C)

δ_A, δ_B and δ_C → are sinking or settlements of support from their initial position. Normally Young's modulus of Elasticity will be same throughout than the Equation reduces to

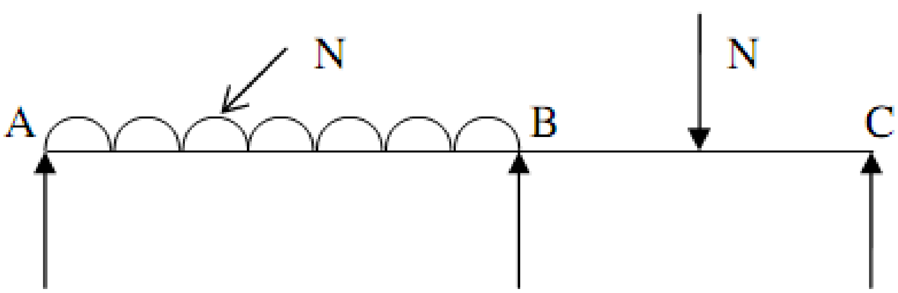
$$M_A\left(\frac{L_1}{I_1}\right)+2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right)+M_C\left(\frac{L_2}{I_2}\right)$$

$$=\frac{-6a_1\overline{x_1}}{I_1L_1}-\frac{6a_2\overline{x_2}}{I_2L_2}-6\left[\frac{\delta_A-\delta_B}{L_1}+\frac{\delta_C-\delta_B}{L_2}\right]$$

If the supports are rigid then $\delta_A = \delta_B = \delta_C = 0$

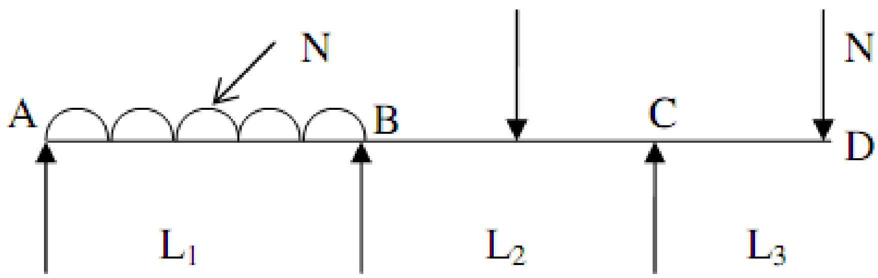
$$M_AL_1+2M_B(L_1+L_2)+M_CL_2=\frac{-6a_1\overline{x_1}}{L_1}-\frac{6a_2\overline{x_2}}{L_2}$$

Note:

1. 

   If the end supports are simple supports then $M_A = M_C = 0$.

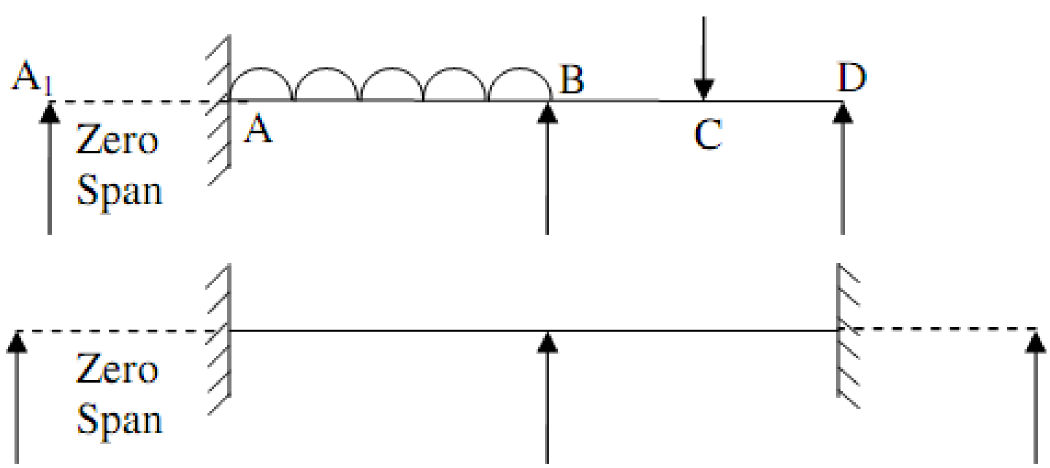
2.



$$M_C = -WL_3$$

If three is overhang portion then support moment near the overhang can be Computed directly.

3.

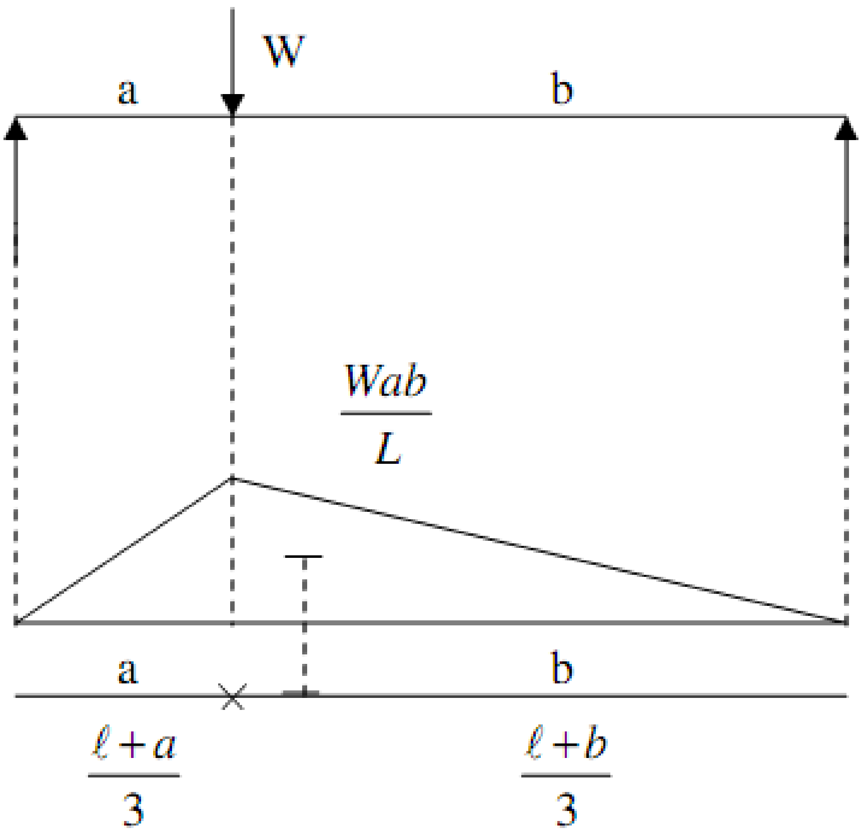


If the end supports are fixed assume an extended span of zero length and apply

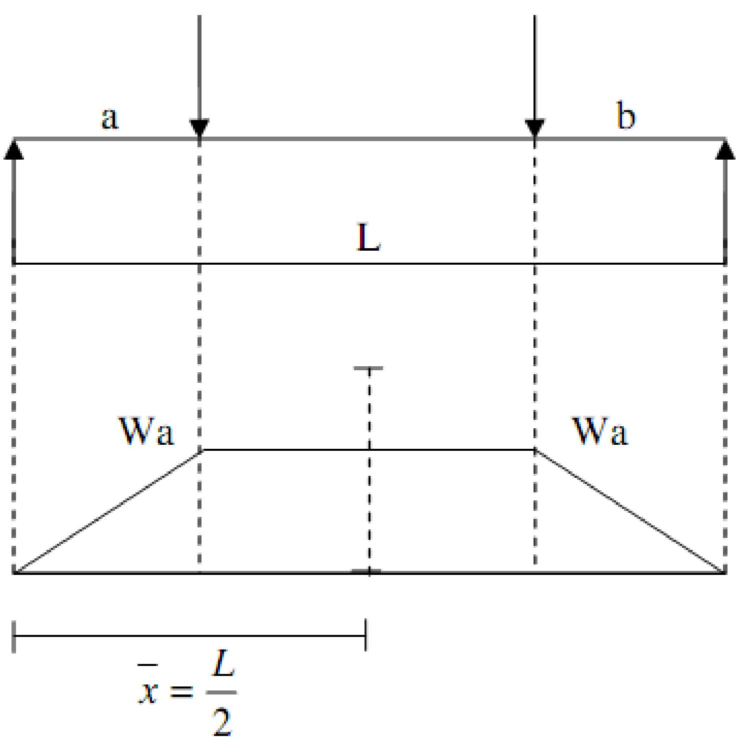
3- Moment equation.

NOTE:

i)



In this case centroid lies as shown in the figure.



Observation Table:

Section type	Types of loads	Length of member (L)	Breadth(b)	Depth (d)	Weight (W)	At a distance from section 'X'	Bending Moment (Knm)	S.F (Kn)	Deflection (Delta)
continuo's beams	Two Equal Spans – Uniform Load on One Span								
	Two Equal Spans – Concentrated Load at Center of One Span								
	Two Equal Spans – Concentrated Load at Any Point								
	Two Equal Spans – Uniformly Distributed Load								
	Two Equal Spans – Two Equal Concentrated Loads Symmetrically Placed								
	Two Unequal Spans – Uniformly Distributed Load								

	Two Unequal Spans – Concentrated Load on Each Span Symmetrically Placed								

Output:

1. Bending moment _________ (Knm)

2. Shear Force __________ (KN)

3 Deflections __________ (Yc)

References:

1. Theory of Structures volume: 1 by S.P.Guptha and G.S.Pandit

2. Reference taken from N.D.S.