

Yield Criteria for Ductile Materials under Plane Stresses

Aim:

1. To investigate the principal stresses σ_a and σ_b at any given point of a structural element or machine component when it is in a state of plane stress.
2. To evaluate the two different yield criteria (*Tresca* and *Von Mises*) for ductile materials under plane stresses.

Introduction

Structural elements and machine components made of a ductile material are usually designed so that the material will not yield under the expected loading conditions. When the element or component is under uniaxial stress (Fig. 1), the value of the normal stress σ_x , which will cause the material to yield, may be obtained readily from a tensile test conducted on a specimen of the same material, since the test specimen and the structural element or machine component are in the same state of stress. Thus, regardless of the actual mechanism which causes the material to yield, we may state that the element or component will be safe as long as $\sigma_x < \sigma_Y$, where σ_Y is the yield strength of the test specimen.

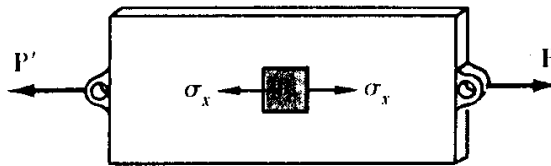


Fig. 1 Tensile yield test under uniaxial stress

On the other hand, when a structural element or machine component is in a state of plane stress (Fig. 2a), the principal stresses σ_a and σ_b at any given point can be determined (Fig. 2b). The material may then be regarded as being in a state of biaxial stress at that point. Since this state is different from the state of uniaxial stress found in a specimen subjected to a tensile test, it is clearly not possible to predict directly from such a test whether or not the structural element or machine component under investigation will fail. Some criteria regarding the actual mechanism of failure of the material must be established.

Maximum-Shearing-Stress Criterion.

According to this criterion, a given structural component is safe as long as the maximum value τ_{\max} of the shearing stress in that component remains smaller than the corresponding value of the shearing stress in a tensile-test specimen of the same material as the specimen starts to yield.

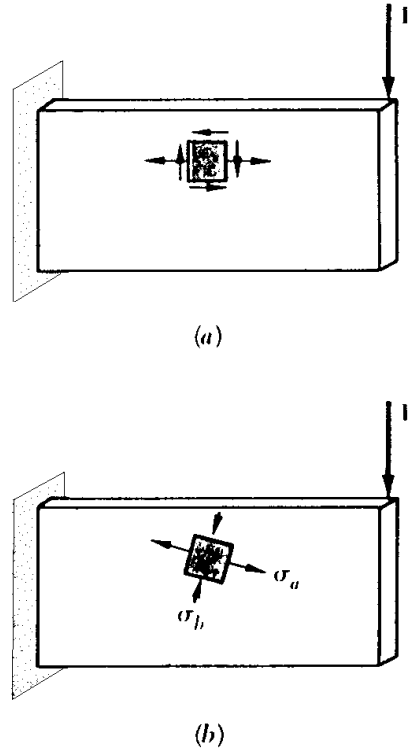


Fig. 2 A structural element in a state of plane stress

The maximum value of the shearing stress under a centric axial load is equal to half the value of the corresponding normal axial stress, hence the maximum shearing stress in a tensile-test specimen is $\frac{1}{2}\sigma_Y$ as the specimen starts to yield. On the other hand, for plane stress, the maximum value τ_{\max} of the shearing stress is equal to $\frac{1}{2}|\sigma_{\max}|$ if the principal stresses are either both positive or both negative, and to $\frac{1}{2}|\sigma_{\max} - \sigma_{\min}|$ if the maximum stress is positive and the minimum stress negative. Thus, if the principal stresses σ_a and σ_b have the same sign, the maximum-shearing-stress criterion gives

$$|\sigma_a| < \sigma_Y \quad |\sigma_b| < \sigma_Y$$

If the principal stresses σ_a and σ_b have opposite signs, the maximum-shearing-stress criterion yields

$$|\sigma_a - \sigma_b| < \sigma_Y$$

The relations obtained have been represented graphically in Fig. 3. Any given state of stress will be represented in that figure by a point of coordinates σ_a and σ_b , where σ_a and σ_b are the two principal stresses. If this point falls within the area shown in the

figure, the structural component is safe. If it falls outside this area, the component will fail as a result of yield in the material. The hexagon associated with the initiation of yield in the material is known as ***Tresca's hexagon***.

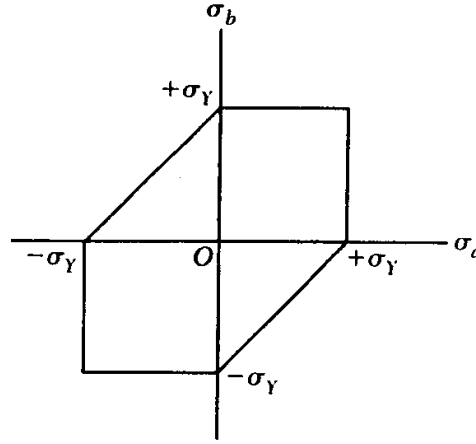


Fig. 3 The Tresca's hexagon

Maximum-Distortion-Energy Criterion

According to this criterion, known as the ***von Mises criterion***, a given structural component is safe as long as the maximum value of the distortion energy per unit volume in that material remains smaller than $(u_d)_Y$, the distortion energy per unit volume required to cause yield in a tensile-test specimen of the same material. If the distortion energy per unit volume in an isotropic material under plane stress is u_d , the maximum-distortion-energy criterion indicates that the structural component is safe as long as $u_d < (u_d)_Y$, i.e., as long as the point of coordinates σ_a and σ_b falls within the area shown in Fig. 4. This area is bounded by the ellipse of equation

$$\sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2 = \sigma_Y^2$$

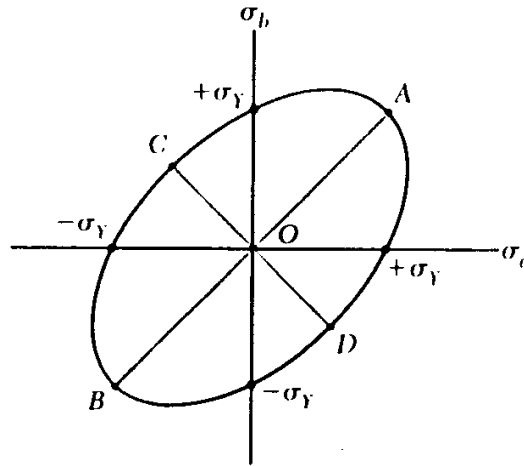


Fig. 4 The von Mises' ellipse

Apparatus

Tensile tester, Fixture for performing the shear yield test, polypropylene (pp) strip specimens for shear yield tests, pp dumbbell specimens for tensile yield tests.

Procedure

1. Perform the tensile yield test with the pp dumbbell specimens and evaluate the yield point from the resulting stress-strain curve.
2. Mount the fixture on the tensile tester and perform the shear yield test with the pp strip specimens.
3. Construct the corresponding *Tresca's hexagon* and the *von Mises' ellipse* and demonstrate the validity of these yield criteria.

Discussion

Explain the principle of deriving the two yield criteria (*Tresca* and *von Mises*) and their main differences, and how they can be applied in the design of structural components.