Modelling and Control of Underwater Vehicle: Sparus

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Introduction to Underwater Robotics, Modelling and Control

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Modelling of a hexapod leg

Problem 1

With the figure 2, compute all the dimension of the different bodies.

Problem 2

Considering the given global real mass matrix explains all the terms: from which part the various terms in the matrix originate.

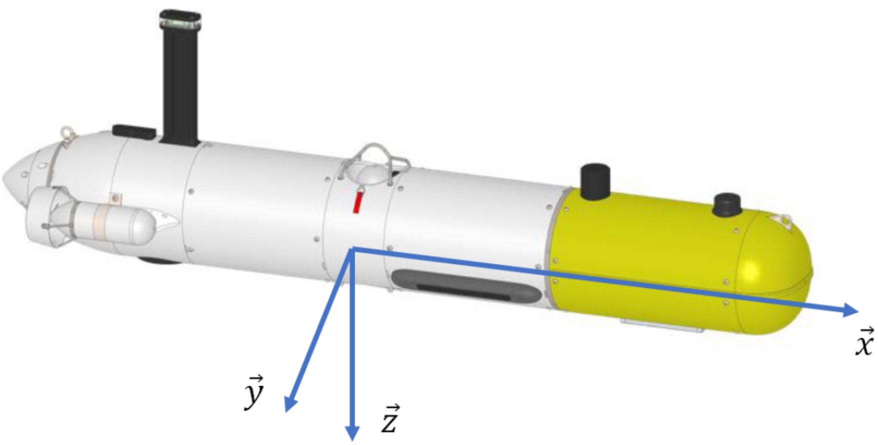


Figure 1: 3D view of the Sparus

Looking at the 3D representation of the AUV we can interpret the terms of the mass matrix below:

$$M_{RB}^{CO} = \begin{bmatrix} 52 & 0 & 0 & 0 & -0.1 & 0 \\ 0 & 52 & 0 & 0.1 & 0 & -1.3 \\ 0 & 0 & 52 & 0 & 1.3 & 0 \\ 0 & 0.1 & 0 & 0.5 & 0 & 0 \\ -0.1 & 0 & 1.3 & 0 & 9.4 & 0 \\ 0 & -1.3 & 0 & 0 & 0 & 9.5 \end{bmatrix} = \begin{bmatrix} m\boldsymbol{I_{3\times3}} & -m\boldsymbol{S}(\boldsymbol{r_g^b}) \\ m\boldsymbol{S}(\boldsymbol{r_g^b}) & \boldsymbol{I_b} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

$$M_{RB}^{CO} = \begin{bmatrix} m & 0 & 0 & 0 & mz_g & -my_g \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & my_g & -mx_g & 0 \\ 0 & -mz_g & my_g & I_x & -I_{xy} & -I_{xz} \\ mz_g & 0 & -mx_g & -I_{yx} & I_y & -I_{yz} \\ -my_g & mx_g & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix}$$

For a mass matrix composed of terms m_{ij}, each term represents how an acceleration on j direction contributes to a force or moment in the i direction.

To interpret the mass matrix, we can decompose it into four 3×3 matrices.

For the first component, we have:

$$M_{11} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} = \begin{bmatrix} 52 & 0 & 0 \\ 0 & 52 & 0 \\ 0 & 0 & 52 \end{bmatrix},$$

where each term on the diagonal associates a force in each direction due to a linear acceleration in the same direction. As expected, a force applied to some direction will only generate linear acceleration along the same direction.

The matrices M_{21} and M_{12} are responsible for generating moments in the body due to acceleration. The terms in these matrices can be explained by the break of symmetry of the body caused by the smaller parts, such as the antenna, or the difference in shape of the two ends of the vehicle, which shift the center of gravity of the AUV.

$$M_{21} = m\boldsymbol{S}(\boldsymbol{r_g^b}) = \begin{bmatrix} 0 & -mz_g & my_g \\ mz_g & 0 & -mx_g \\ -my_g & mx_g & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0.1 & 0 \\ -0.1 & 0 & 1.3 \\ 0 & -1.3 & 0 \end{bmatrix}$$

Analyzing the matrix M_{21}, we can better understand the distribution of mass in the body. Looking at each term in the matrix, we find that

$$\boldsymbol{r_g^b} = \begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix} = \begin{bmatrix} -0,025 \text{ m} \\ 0 \text{ m} \\ -0,0019 \text{ m} \end{bmatrix}$$

That means that there is actually a small displacement between the center of gravity CG and the center of the body CO, and CG is positioned slightly above CO, which results in the presence of the coupling terms in the mass matrix. The shift of the center of gravity along the x and z axes is probably due to the presence of the antenna and the thursters, that add up to the weight of the AUV.

Finally, we have the component corresponding to the inertia matrix.

$$M_{22} = \boldsymbol{I_g} = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 9.4 & 0 \\ 0 & 0 & 9.5 \end{bmatrix}$$

The off-diagonal terms represent the products of inertia. Since they are zero, it means that the distribution of mass is symmetric about all the planes of symmetry, which was probably an approximation when calculating the mass matrix, due to the small contribution to the innertia due to the smaller parts of the vehicle.

Problem 3

Compute each added mass matrix at the buoyancy center of the sparus. Excepted the main body, the CG and CB of the other bodies are at the same point.

For 1 plane of symmetry (XZ-symmetry) we have:

$$M_a = \begin{bmatrix} m_{a11} & 0 & m_{a13} & 0 & m_{a15} & 0 \\ 0 & m_{a22} & 0 & m_{a24} & 0 & m_{a26} \\ m_{a31} & 0 & m_{a33} & 0 & m_{a35} & 0 \\ 0 & m_{a42} & 0 & m_{a44} & 0 & m_{a46} \\ m_{a51} & 0 & m_{a53} & 0 & m_{a55} & 0 \\ 0 & m_{a62} & 0 & m_{a64} & 0 & m_{a66} \end{bmatrix}$$

where

$$m_{aij} = \sum [a_{ij}(x) \text{ contributions}]$$

Main hull

We will start calculating the added mass for the main hull, which is composed of a long cylinder and two smaller shapes attached to each end of the cylinder. These shapes can be approximated as two half-ellipsoids.

The hull has rotational symmetry about the x-axis, so, according to Slender Body theory its added mass matrix will look like:

$$M_a = \begin{bmatrix} m_{a11} & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{a22} & 0 & 0 & 0 & m_{a26} \\ 0 & 0 & m_{a33} & 0 & m_{a35} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{a53} & 0 & m_{a55} & 0 \\ 0 & m_{a62} & 0 & 0 & 0 & m_{a66} \end{bmatrix}$$

$$M_a = \begin{bmatrix} m_{a11} & 0 & 0 & 0 & 0 & 0 \\ 0 & \int_L a_{22}\, dx & 0 & 0 & 0 & \int_L x a_{22}\, dx \\ 0 & 0 & \int_L a_{33}\, dx & 0 & -\int_L x a_{33}\, dx & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\int_L x a_{33}\, dx & 0 & \int_L x^2 a_{33}\, dx & 0 \\ 0 & \int_L x a_{22}\, dx & 0 & 0 & 0 & \int_L x^2 a_{22}\, dx \end{bmatrix}$$

From Newman (1977, p. 152), we have:

$$\begin{aligned} a_{22} &= \pi \rho r^2 \\ a_{33} &= \pi \rho r^2 \\ a_{44} &= 0 \end{aligned}$$

If the x-axis is the longitudinal axis of the slender body, then the 3D added mass coefficients m_{aij} are calculated by summing the added mass coefficients of all the thin slices which are perpendicular to the 1-axis. This means that forces in x-direction cannot be obtained by slender body theory. Instead, to calculate the term m_{a11} we are going to use Lamb's K-factors, approximating the AUV shape as an ellipsoid with dimensions $a = 0.8$ m and $b = 0.115$ m. With the help of MATLAB, we find $m_{a11} = 1.6038$ kg.

$$M_a = \begin{bmatrix} 1.6038 & 0 & 0 & 0 & 0 & 0 \\ 0 & \int_L \pi\rho r^2\, dx & 0 & 0 & 0 & \int_L x\pi\rho r^2\, dx \\ 0 & 0 & \int_L \pi\rho r^2\, dx & 0 & -\int_L x\pi\rho r^2\, dx & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\int_L x\pi\rho r^2\, dx & 0 & \int_L x^2\pi\rho r^2\, dx & 0 \\ 0 & \int_L x\pi\rho r^2\, dx & 0 & 0 & 0 & \int_L x^2\pi\rho r^2\, dx \end{bmatrix}$$

We have:

$$\int_L \pi \rho r^2\, dx = \pi\rho \int_L r^2(x)\, dx$$

To calculate this integral, we must separate it in three different parts, the cylinder and two half ellipsoids. For the cylinder, we have a constant radius:

$$r(x) = R$$

For the ellipsoids, we can describe the radius with the following function:

$$\frac{r^2}{R^2} + \frac{(x - x_0)^2}{l^2} = 1$$

$$r^2(x) = R^2 \left(1 - \frac{(x - x_0)^2}{l^2}\right)$$

where l is the length in the X axis direction of the half ellipsoid.

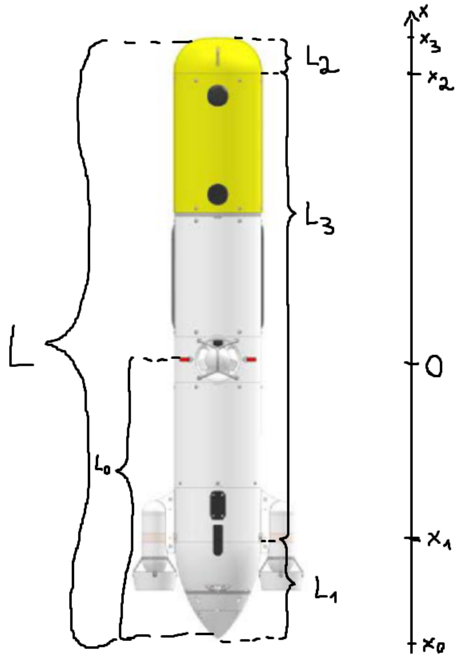


Figure 2: Main hull dimensions and limits of integration for the added mass matrix.

Let's consider the points marked on Figure 2 as the integration limits for each part of the main hull. Considering $x = 0$ in the center of gravity of the AUV, we have:

$$x_0 = -L_0;$$

$$x_1 = x_0 + L_1;$$

$$x_2 = x_1 + L_3;$$

$$x_3 = L - L_0;$$

$$\pi\rho \int_{x_0}^{x_3} r^2(x)\,dx = \pi\rho \left[\int_{x_0}^{x_1} R^2 \left(1 - \frac{(x - x_1)^2}{L_1^2} \right) dx + \int_{x_1}^{x_2} R^2\,dx + \int_{x_2}^{x_3} R^2 \left(1 - \frac{(x - x_2)^2}{L_2^2} \right) dx \right]$$

Using MATLAB to calculate the integral above and replacing L_0, L_1, L_2, L_3 and L with the real dimensions of the body, we have:

$$m_{22} = m_{33} = \int_L a_{22}\,dx = \int_L a_{33}\,dx = 61.5264\,\text{Kg}$$

Likewise, we can calculate $\int_L x a_{22}\,dx = \int_L x a_{33}\,dx = \int_L x \pi\rho r(x)^2\,dx$:

$$\int_L x a_{22}\,dx = \int_L x a_{33}\,dx = 4.8920 \text{ Kg}$$

Lastly, we calculate $\int_L x^2 a_{22}\,dx = \int_L x^2 a_{33}\,dx = \int_L x^2 \pi\rho r(x)^2\,dx$:

$$\int_L x^2 a_{22}\, dx = \int_L x^2 a_{33}\, dx = 11.7625 \text{ Kg}$$

With the values computed above, we can build the hull's added mass matrix at the gravity center of the AUV:

$$M_{a,CG}^{hull} = \begin{bmatrix} 1.60 & 0 & 0 & 0 & 0 & 0 \\ 0 & 61.53 & 0 & 0 & 0 & 4.89 \\ 0 & 0 & 61.53 & 0 & -4.89 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4.89 & 0 & 11.76 & 0 \\ 0 & 4.89 & 0 & 0 & 0 & 11.76 \end{bmatrix}$$

To obtain the added mass at the buoyancy center, we need to apply:

$$M_{a,CB}^{hull} := \mathbf{H}^\top \left(r_{CB}^{hull}\right) M_{a,CG}^{hull} \mathbf{H} \left(r_{CB}^{hull}\right)$$

where,

$$r_{CB}^{hull} = \begin{bmatrix} 0 \\ 0 \\ -0.02\ m \end{bmatrix}$$

Which gives us:

$$M_{a,CB}^{hull} = \begin{bmatrix} 1.6 & 0 & 0 & 0 & -0.03 & 0 \\ 0 & 61.53 & 0 & 1.23 & 0 & 4.89 \\ 0 & 0 & 61.53 & 0 & -4.89 & 0 \\ 0 & 1.23 & 0 & 0.02 & 0 & 0.1 \\ -0.03 & 0 & -4.89 & 0 & 11.76 & 0 \\ 0 & 4.89 & 0 & 0.1 & 0 & 11.76 \end{bmatrix}$$

Antenna

We can consider the antenna to be slender along the z direction, so that we have:

$$M_a = \begin{bmatrix} \int_L a_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & \int_L a_{22}\, dz & 0 & 0 & 0 & \int_L z a_{22}\, dz \\ 0 & 0 & m_{a33} & 0 & m_{35} & 0 \\ 0 & 0 & 0 & \int_L z^2 a_{22}\, dz & 0 & 0 \\ 0 & 0 & m_{53} & 0 & \int_L z^2 a_{11}\, dz & 0 \\ 0 & \int_L z a_{22}\, dz & 0 & 0 & 0 & \int_L a_{66}\, dx \end{bmatrix}$$

$m_{33} = 0$ because its associated projected area is already included in that of the main body and $m_{53} = m_{35} = 0$ due to the symmetry of the antenna. With MATLAB, we get:

$$M_a^{antenna} = \begin{bmatrix} 0.20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.18 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

To translate it to the center of buoyancy of the vehicle, we use:

$$M_{a,CB}^{antenna} := \mathbf{H}^\top \left(r_{CB}^{antenna}\right) M_a^{antenna} \mathbf{H} \left(r_{CB}^{antenna}\right)$$

where,

$$r_{CB}^{antenna} = \begin{bmatrix} -0.389\ m \\ 0 \\ -0.222\ m \end{bmatrix}$$

$$M_{a,CB}^{antenna} = \begin{bmatrix} 0.20 & 0 & 0 & 0 & -0.04 & 0 \\ 0 & 1.18 & 0 & 0.26 & 0 & -0.46 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.26 & 0 & 0.06 & 0 & -0.1 \\ -0.04 & 0 & 0 & 0 & 0.01 & 0 \\ 0 & -0.46 & 0 & -0.1 & 0 & 0.18 \end{bmatrix}$$

Thrusters

$m_{22} = 0$ because its associated projected area is already included in that of the main body.
With Lamb's K factors, we have $m_{11} = 0.1308$.

$$M_a^{thruster} = \begin{bmatrix} 0.13 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.41 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.01 \end{bmatrix}$$

To translate it to the center of buoyancy of the vehicle, we use:

$$M_{a,CB}^{thruster} := \mathbf{H}^\top \left(r_{CB}^{thruster}\right) M_a^{thruster} \mathbf{H} \left(r_{CB}^{thruster}\right)$$

where,

$$r_{CB}^{thruster1} = \begin{bmatrix} -0.5\ m \\ -0.1585\ m \\ 0.02\ m \end{bmatrix}$$

$$r_{CB}^{thruster2} = \begin{bmatrix} -0.5\ m \\ 0.1585\ m \\ 0.02\ m \end{bmatrix}$$

$$M_{a,CB}^{thruster1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.41 & -0.22 & 0.7 & 0 \\ 0 & 0 & -0.22 & 0.04 & -0.11 & 0 \\ 0 & 0 & 0.7 & -0.11 & 0.36 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.01 \end{bmatrix}$$

$$M_{a,CB}^{thruster2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.41 & 0.22 & 0.7 & 0 \\ 0 & 0 & 0.22 & 0.04 & 0.11 & 0 \\ 0 & 0 & 0.7 & 0.11 & 0.36 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.01 \end{bmatrix}$$

Total Added Mass

The total added mass at the center of buoyancy of the AUV, we have to sum the added mass matrices from each part of the body at the buoyancy center of the AUV.

$$M_{a,CB}^{total} = M_{a,CB}^{hull} + M_{a,CB}^{antenna} + M_{a,CB}^{thruster1} + M_{a,CB}^{thruster2}$$

$$M_{a,CB}^{total} = \begin{bmatrix} 0.20 & 0 & 0 & 0 & -0.04 & 0 \\ 0 & 62.71 & 0 & 1.49 & 0 & 4.43 \\ 0 & 0 & 64.34 & 0 & -3.49 & 0 \\ 0 & 1.49 & 0 & 0.16 & 0 & 0 \\ -0.04 & 0 & -3.49 & 0 & 12.49 & 0 \\ 0 & 4.43 & 0 & 0 & 0 & 11.95 \end{bmatrix}$$

Problem 4

Compare this matrix at CG and CB. Is it important to take into account the distance between the two points ?

$$M_{a,CG}^{total} := \mathbf{H}^{\top}(-r_b)\, M_{a,CB}^{total} \mathbf{H}(-r_b)$$

$$M_{a,CG}^{total} = \begin{bmatrix} 0.2 & 0 & 0 & 0 & -0.04 & 0 \\ 0 & 62.71 & 0 & 0.24 & 0 & 4.43 \\ 0 & 0 & 64.34 & 0 & -3.49 & 0 \\ 0 & 0.24 & 0 & 0.13 & 0 & -0.09 \\ -0.04 & 0 & -3.49 & 0 & 12.49 & 0 \\ 0 & 4.43 & 0 & -0.09 & 0 & 11.95 \end{bmatrix}$$

$$|M_{a,CB}^{total} - M_{a,CG}^{total}| = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.25 & 0 & 0.03 & 0 & 0.09 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.09 & 0 & 0 \end{bmatrix}$$

Comparing the matrix at the two points, we can conclude that apart from the coupling between linear acceleration in the y direction and moment about the x axis, the influence of the distance between the two points is negligible. Due to the asymmetry about the xy plane (mainly due to the antenna), it makes sense that a vertical shift between the two points causes some difference in the observed moment, even if not very significant.

Problem 5

Compare the values of the main solid with the others and conclude.

$$M_{a,CB}^{hull} = \begin{bmatrix} 1.6 & 0 & 0 & 0 & -0.03 & 0 \\ 0 & 61.53 & 0 & 1.23 & 0 & 4.89 \\ 0 & 0 & 61.53 & 0 & -4.89 & 0 \\ 0 & 1.23 & 0 & 0.02 & 0 & 0.1 \\ -0.03 & 0 & -4.89 & 0 & 11.76 & 0 \\ 0 & 4.89 & 0 & 0.1 & 0 & 11.76 \end{bmatrix}$$

$$M_{a,CB}^{antenna} = \begin{bmatrix} 0.20 & 0 & 0 & 0 & -0.04 & 0 \\ 0 & 1.18 & 0 & 0.26 & 0 & -0.46 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.26 & 0 & 0.06 & 0 & -0.1 \\ -0.04 & 0 & 0 & 0 & 0.01 & 0 \\ 0 & -0.46 & 0 & -0.1 & 0 & 0.18 \end{bmatrix}$$

$$M_{a,CB}^{thruster1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.41 & -0.22 & 0.7 & 0 \\ 0 & 0 & -0.22 & 0.04 & -0.11 & 0 \\ 0 & 0 & 0.7 & -0.11 & 0.36 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.01 \end{bmatrix}$$

$$M_{a,CB}^{thruster2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.41 & 0.22 & 0.7 & 0 \\ 0 & 0 & 0.22 & 0.04 & 0.11 & 0 \\ 0 & 0 & 0.7 & 0.11 & 0.36 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.01 \end{bmatrix}$$

We can see that the significant difference in size results in the main hull having substantially larger added mass coefficients compared to those of the other components. When comparing it to the total matrix, we can also conclude that the total added mass matrix is influenced mainly by the matrix of the main hull, even after adding the contributions of all the other parts.

The added mass matrices of the two lateral thrusters have the same absolute value, but some inverted signs, due to the symmetry of its placement. Because of that, some moments that would be generated due to one thruster, such a moment about x due to a linear acceleration in the z direction, are cancelled out by the other thruster and don't affect the total added mass matrix.

Problem 6

Compare the added and real mass matrix and conclude.

$$M_{a,CG}^{total} = \begin{bmatrix} 0.2 & 0 & 0 & 0 & -0.04 & 0 \\ 0 & 62.71 & 0 & 0.24 & 0 & 4.43 \\ 0 & 0 & 64.34 & 0 & -3.49 & 0 \\ 0 & 0.24 & 0 & 0.13 & 0 & -0.09 \\ -0.04 & 0 & -3.49 & 0 & 12.49 & 0 \\ 0 & 4.43 & 0 & -0.09 & 0 & 11.95 \end{bmatrix}$$

$$M_{RB}^{CG} = \begin{bmatrix} 52 & 0 & 0 & 0 & -0.1 & 0 \\ 0 & 52 & 0 & 0.1 & 0 & -1.3 \\ 0 & 0 & 52 & 0 & 1.3 & 0 \\ 0 & 0.1 & 0 & 0.5 & 0 & 0 \\ -0.1 & 0 & 1.3 & 0 & 9.4 & 0 \\ 0 & -1.3 & 0 & 0 & 0 & 9.5 \end{bmatrix}$$

While the added mass coefficients depend entirely on the geometry of the vehicle, together with the density of the surrounding fluid, the real mass matrix depends on the total mass of the vehicle, the distribution of mass in its body and the displacement between the origin of the body and its center of gravity.

The coupling terms are caused by asymmetries along the body, maily due to the smaller parts of the vehicle, such as thrusters and antenna. We can see that in both matrices the off-diagonal terms are mostly smaller than the diagonal ones, which can be explained by the small size of the other parts of the body.

Since the added mass matrix depends on the geometry of the body, we observe that the first element of the matrix (m_{a11}) is considerably smaller than the first element in the real mass matrix, which depends on the actual total mass of the body.

Problem 7

Estimate all drag matrices.

We know the drag matrices should have the following shape:

$$D_{NL} = \begin{bmatrix} X_{uu} & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{vv} & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{ww} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{pp} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{qq} & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{rr} \end{bmatrix}_{Rb}^{CB} = \begin{bmatrix} K_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & K_{66} \end{bmatrix}_{Rb}^{CB}$$

Main hull

First, let us calculate the drag matrix for the main hull.

For the first coefficient, we have $K_{11} = \frac{1}{2}\rho S_x C_{D11}$, with S_x and C_{D11} being respectively the projected surface and the 3D drag coefficient in the x direction. For the main hull, we will consider its approximated shape to be an ellipsoid.

With the ratio $L/D = 1.60/0.23 = 6.96$ and assuming a turbulent flow with Reynolds number $Re \geq 2 \times 10^6$, we have drag coefficient $C_{D11} = 0.1$, which gives us:

$$K_{11} = 2.0774$$

$K_{22} = \frac{1}{2}\rho C_{D22} D_y L$, where D_y and C_{D22} are respectively the characteristic width and 2D drag coefficient along the length in the y direction.

For calculating the 2D drag coefficient along the y direction, we can approximate the shape of the 2D slices of the body as a circle. Again assuming turbulent motion with a high Reynolds number, we have $C_{D22} = 0.3$. Therefore, we have:

$$K_{22} = 55.2$$

$K_{33} = \frac{1}{2}\rho C_{D33} D_z L$, where D_z and C_{D33} are respectively the characteristic width and 2D drag coefficient along the length in the z direction.

For calculating the 2D drag coefficient along the z direction, we can use the same approximation used for the y direction. Therefore, we have:

$$K_{33} = 55.2$$

Then, we have:

$$K_{44} = 0$$

$$K_{55} = \frac{1}{64}\rho L^4 C_{D33} D_z = 7.0656$$

$$K_{66} = \frac{1}{64}\rho L^4 C_{D22} D_y = 7.0656$$

$$D^{hull} = \begin{bmatrix} 2.0774 & 0 & 0 & 0 & 0 & 0 \\ 0 & 55.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 55.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.0656 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7.0656 \end{bmatrix}$$

Antenna

Since the antenna stretches along the z axis (with $L = 0.254$ m), we will have the following equations for the terms in its drag matrix:

$$K_{11} = \frac{1}{2}\rho C_{D11} D_x L$$

$$K_{22} = \frac{1}{2}\rho C_{D22} D_y L$$

$$K_{33} = 0$$

$$K_{44} = \frac{1}{64}\rho L^4 C_{D22} D_y$$

$$K_{55} = \frac{1}{64}\rho L^4 C_{D11} D_x$$

$$K_{66} = 0$$

Where C_{D11} and C_{D22} are respectively the 2D drag coefficients along the length in the x and y directions and $K_{33} = 0$ because its associated projected area is already included in that of the main body.

For K_{11}, we can approximate its shape as a rectangular rod. With $L/D = 2.7$, we have:

$$C_{D11} = 1.3$$

For the side facing the y direction, we again use the rectangular rod, but with the upstream velocity coming from the y direction, which gives us:

$$C_{D22} = 2.5$$

Applying these values to the equations above, we have the following drag matrix for the antenna:

$$D^{antenna} = \begin{bmatrix} 4.0284 & 0 & 0 & 0 & 0 & 0 \\ 0 & 20.9550 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0107 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0021 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Thruster

The side thrusters shape can be approximated as two cylinders whose diameters correspond to the average of the diameter of the upper part and the diameter of the bottom part. Its drag matrices on its centers of buoyancy will look the same, so we first will calculate one drag matrix for both thrusters and then apply the transformation to represent it w.r.t. the AUV's center of buoyancy.

For the first coefficient, facing the x direction, considering a horizontal cylinder with $L/D = 2.7$, we have:

$$C_{D11} = 0.9$$

and

$$S_x = \pi R^2 = \pi \left(\frac{\frac{0.115}{2} + \frac{0.059}{2}}{2} \right)^2 = \pi \times 0.0435^2 = 0.0059\ m^2$$

$$K_{11} = 2.655$$

For the y direction, we have $K_{22} = 0$ and $K_{66} = 0$, because its associated projected area is already included in that of the main body.

For the z direction, we can use the circular rod coefficient:

$$C_{D33} = 0.3$$

$$K_{33} = 3.0863$$

The projected area is:

$$D_z L = 0.0206\ m^2$$

$$K_{55} = 0.0013$$

That gives us the following drag matrix as a result:

$$D^{thruster} = \begin{bmatrix} 2.655 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.086 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0013 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 8

Complete the simulator with some simple experiments to validate it.

For the first experiment, we applied 30% of forward propulsion power to both side thrusters and turned off the central thruster.

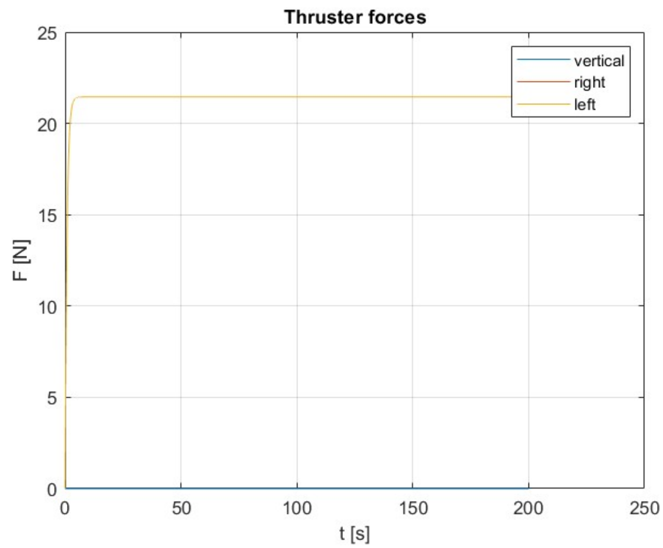
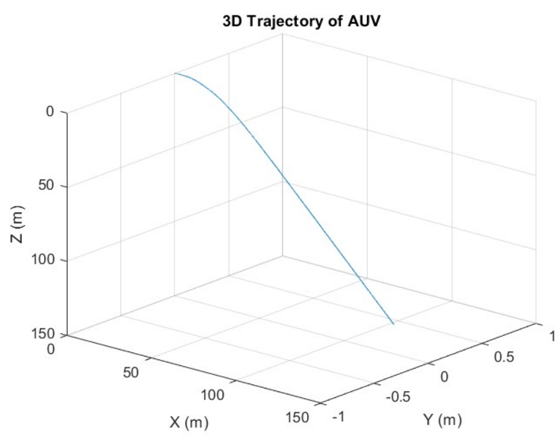
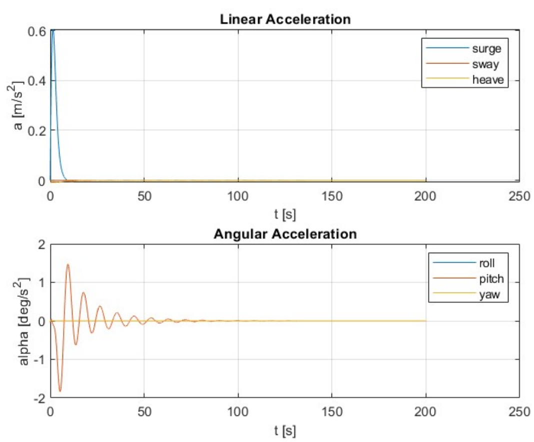


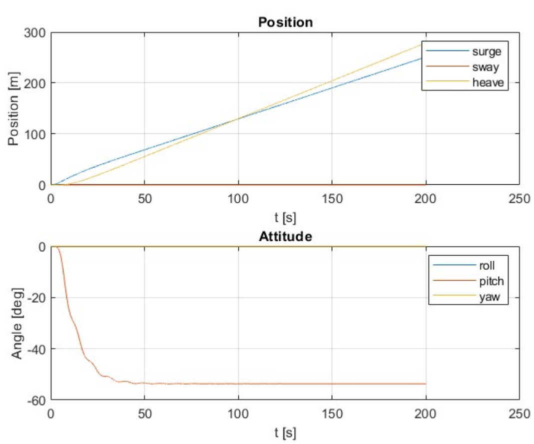
Figure 3: Forces from each thruster.



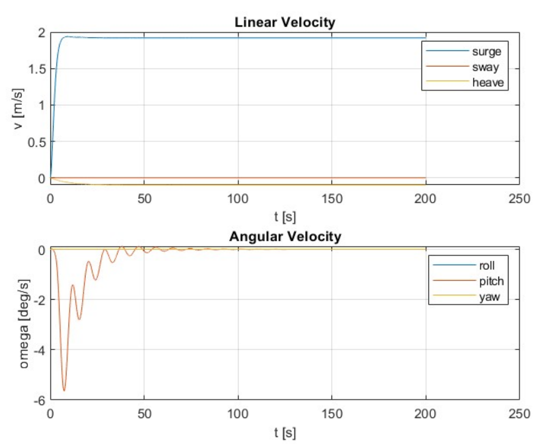
(a) Trajectory of the AUV.



(b) Linear and angular acceleration in the body frame.



(c) Position and attitude of the AUV.

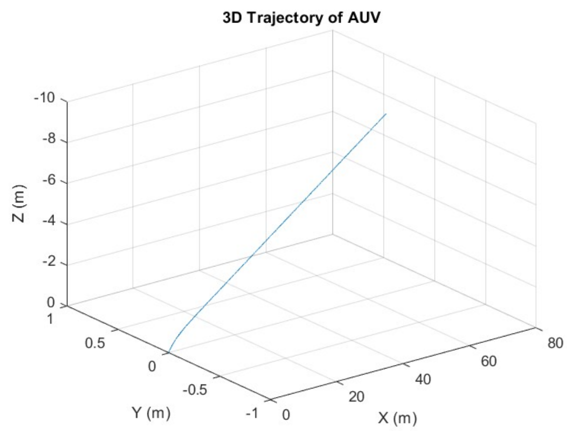


(d) Linear and angular velocity in the body frame.

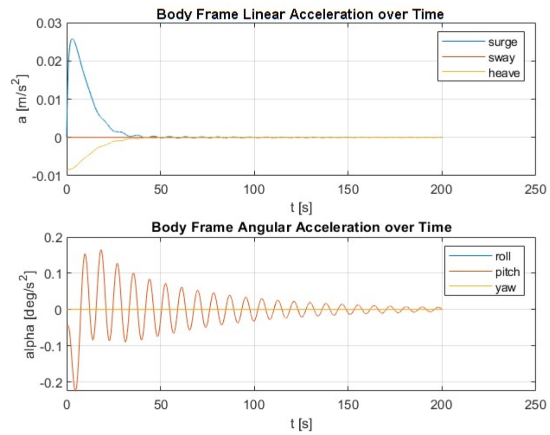
Figure 4: Simulation results with [0%, 30%, 30%] thruster configuration.

We can see that after the initial acceleration the robot is able to achieve a very smooth motion. As expected, it moves forward. However, it was expected to move upwards, not downwards. Instead, due to some torque it reaches a pitch angle of approximately 45° and keeps that orientation.

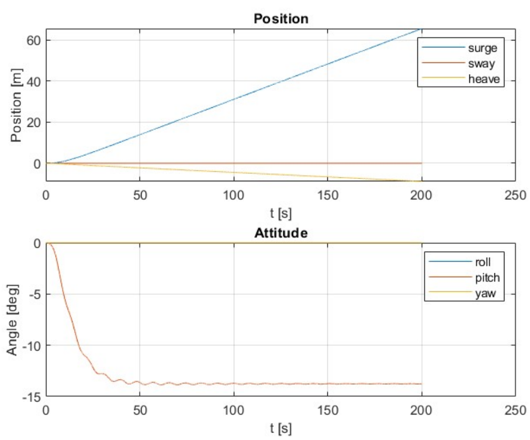
However, if we repeat the experiment with 1% power in both thrusters, we get results closer to the expected motion, as shown in Figure 5. We can see that here is a higher pitch oscillation, but the robot is able to move at a constant velocity, going up and forward.



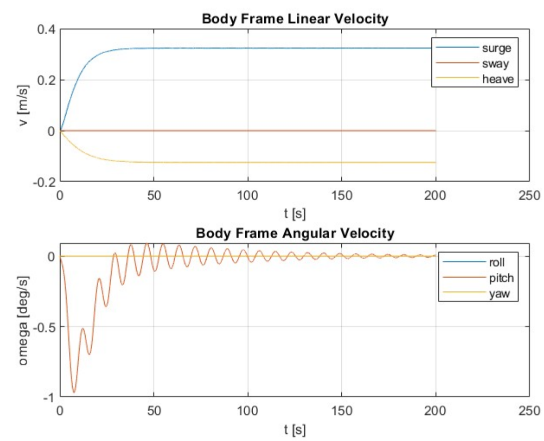
(a) Trajectory of the AUV.



(b) Linear and angular acceleration in the body frame.



(c) Position and attitude of the AUV.



(d) Linear and angular velocity in the body frame.

Figure 5: Simulation results with [0%, 1%, 1%] thruster configuration.

For the second experiment, we applied 30% of forward propulsion power to the central thruster and turned off the two side thrusters.

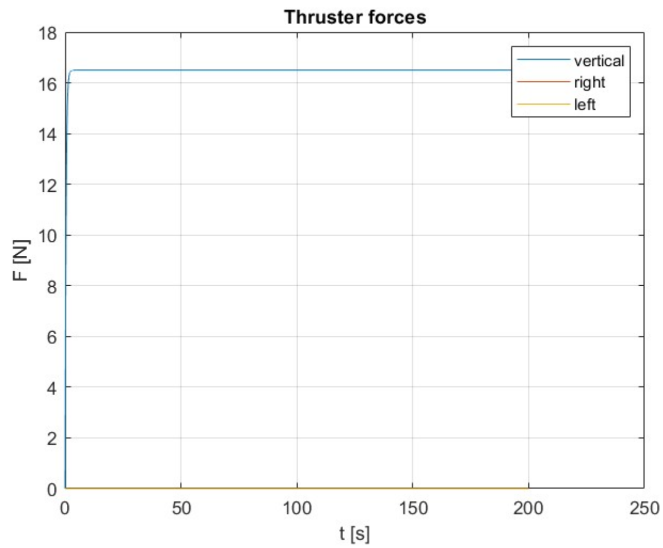
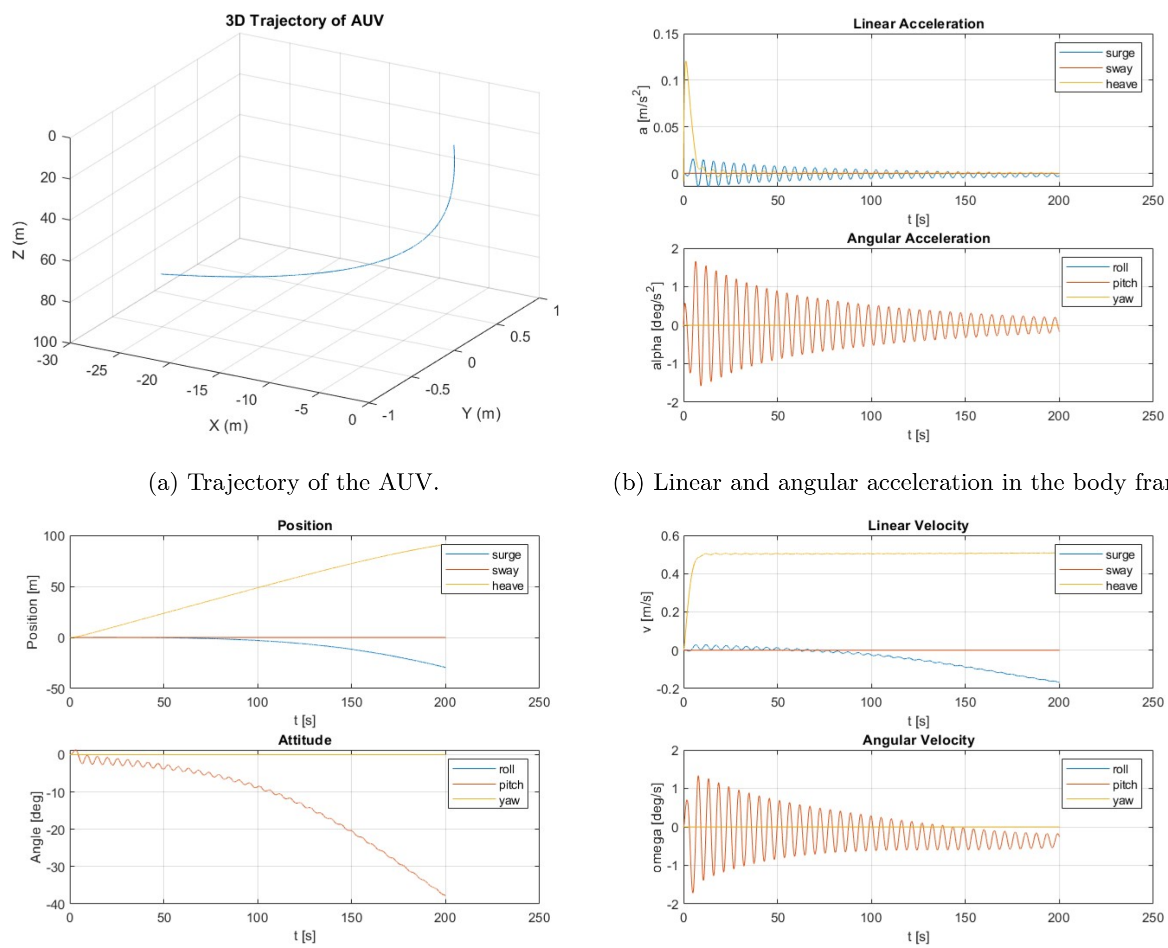


Figure 6: Forces from each thruster.



(a) Trajectory of the AUV.

(b) Linear and angular acceleration in the body frame.

(c) Position and attitude of the AUV.

(d) Linear and angular velocity in the body frame.

Figure 7: Simulation results with [30%, 0%, 0%] thruster configuration.

In that case, the AUV moves down but we also observed in the simulation a negative surge motion. We also observe some oscillation in the velocities and accelerations plots, which decreases over time.

For the third experiment, we applied 30% of forward propulsion power to the left thruster and 40% to the right thruster, while having the central thruster off.

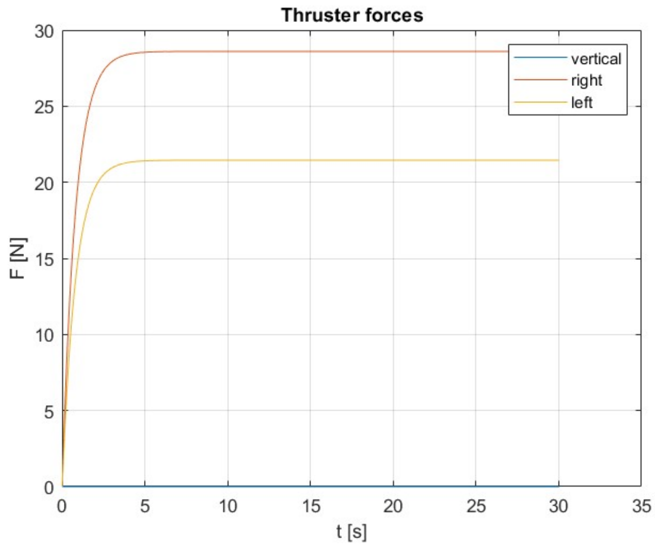
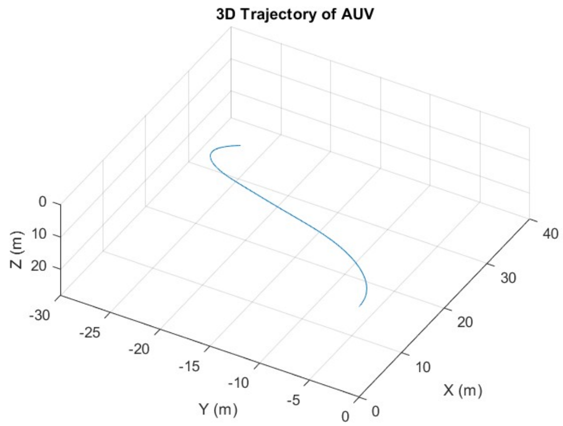
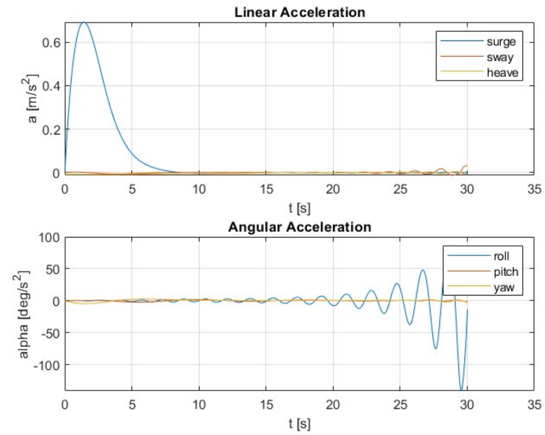


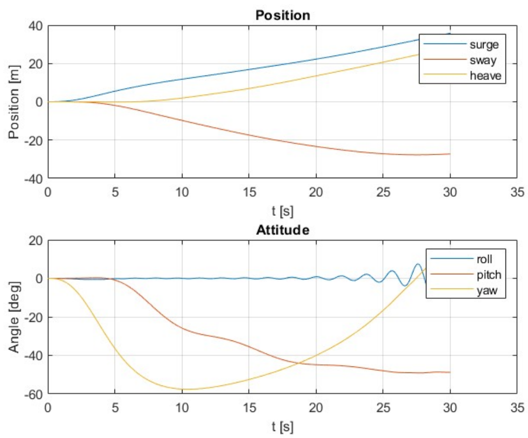
Figure 8: Forces from each thruster.



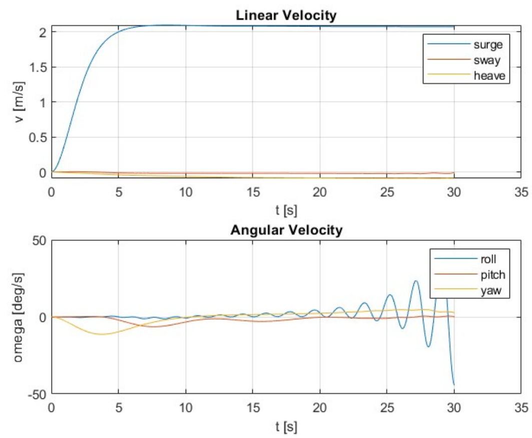
(a) Trajectory of the AUV.



(b) Linear and angular acceleration in the body frame.



(c) Position and attitude of the AUV.



(d) Linear and angular velocity in the body frame.

Figure 9: Simulation results with [0%, 40%, 30%] thruster configuration.

We observe that because of the higher power on the right side, the AUV turns left as expected. However, the system is unstable and there is a growing oscillation in the roll acceleration that eventually crashes the simulation.

Problem 9

Find some simulations to highlight the impact of the different coefficients in the global mass matrix. (Impose linear accelerations)

The global mass matrix in the gravity center of the AUV is given by the sum of the added mass matrix with the real mass matrix at the same point.

$$M_{global}^{CG} = M_{a,CG}^{total} + M_{RB}^{CG} = \begin{bmatrix} 52.2 & 0 & 0 & 0 & -0.14 & 0 \\ 0 & 114.71 & 0 & 0.34 & 0 & 3.13 \\ 0 & 0 & 116.34 & 0 & -2.19 & 0 \\ 0 & 0.34 & 0 & 0.63 & 0 & -0.09 \\ -0.14 & 0 & -2.19 & 0 & 21.89 & 0 \\ 0 & 3.13 & 0 & -0.09 & 0 & 21.45 \end{bmatrix}$$

To understand the impact of its coefficients, we can apply external forces in each direction separately, while turning off the effect of the other forces on the body, to see how the body reacts to a single force. To obtain the results shown below, we applied a single force of 10 N in each direction in three different experiments.

Force in the x direction

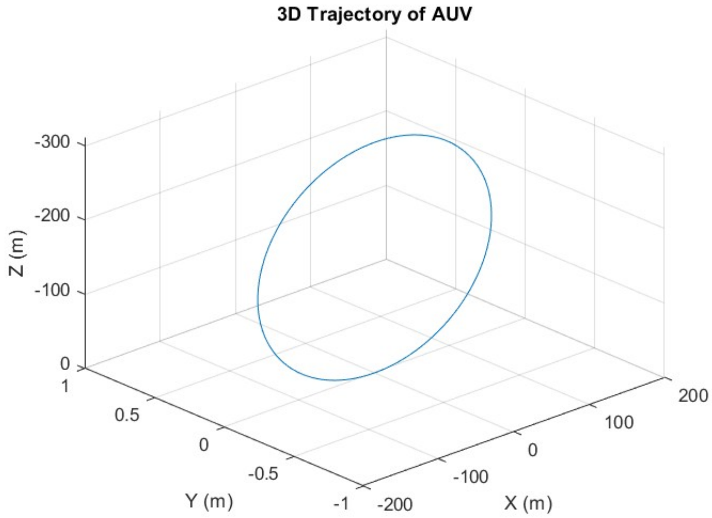


Figure 10: Trajectory of the AUV when applying an external force in the x direction

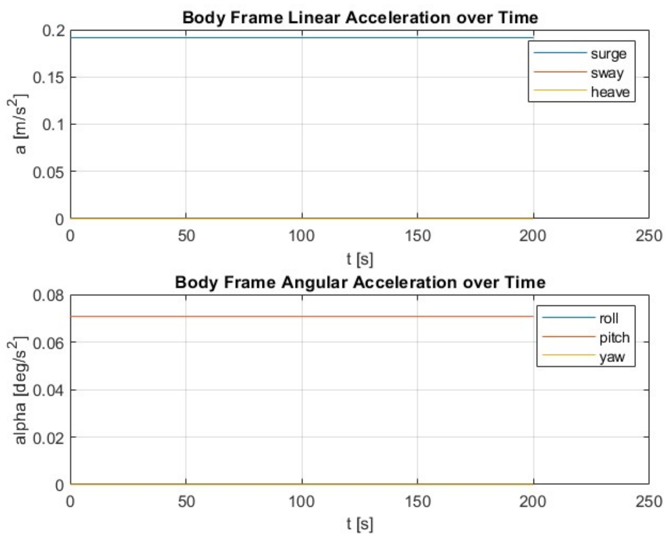


Figure 11: Accelerations resulting from the application of an external force in the x direction

In Figure 11 we can clearly see the effect of the coefficients from the mass matrix. Since only the coefficients m_{11} and m_{15} are non-zero in the first row, we observe only a surge and a pitch acceleration. Figure 10 shows that the resulting trajectory looks like a circle in the xz plane.

Force in the y direction

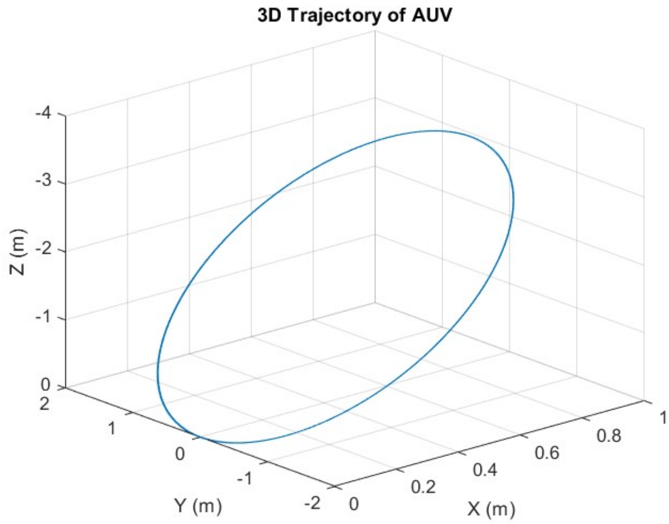


Figure 12: Accelerations resulting from the application of an external force in the y direction

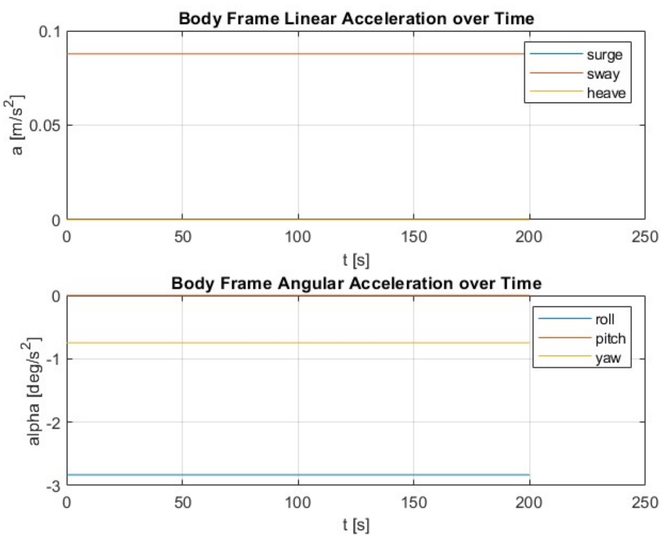


Figure 13: Accelerations resulting from the application of an external force in the y direction

In Figure 13 we can see the effect of the coefficients from the mass matrix. Since only the coefficients m_{22}, m_{24} and m_{26} are non-zero in the second row, we observe only a sway, a pitch and a yaw acceleration. Figure 12 shows that the resulting trajectory looks like a circle.

Force in the z direction

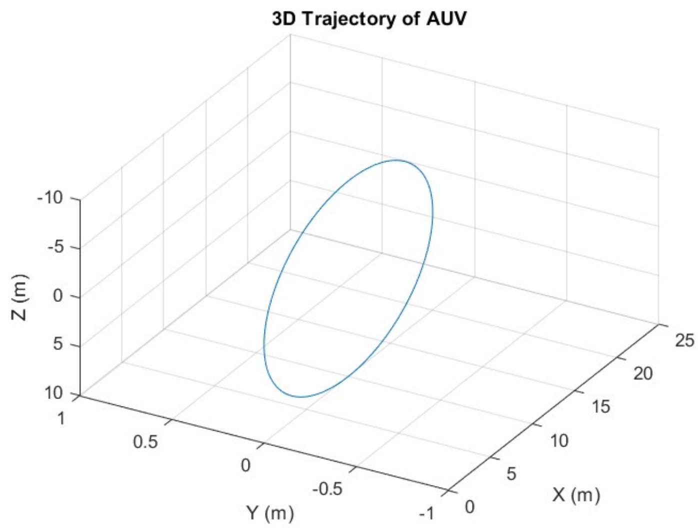


Figure 14: Accelerations resulting from the application of an external force in the z direction

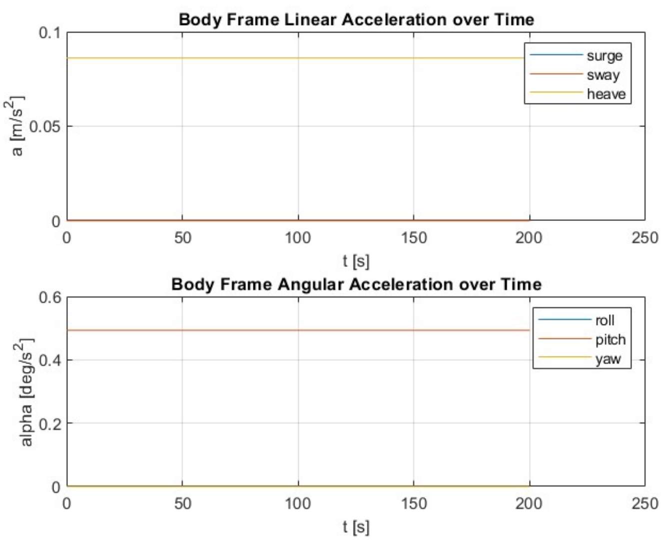


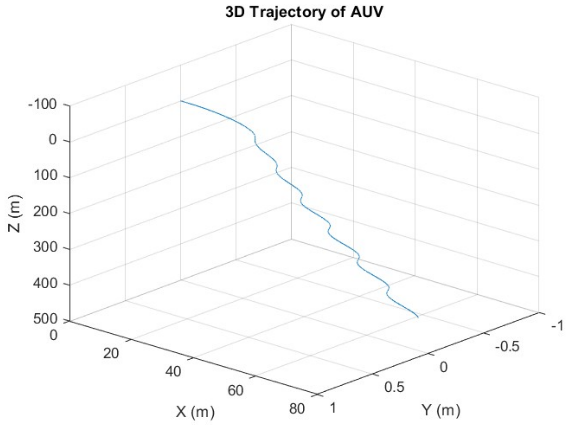
Figure 15: Accelerations resulting from the application of an external force in the z direction

In Figure 15 we can see the effect of the coefficients from the mass matrix. Since only the coefficients m_{33} and m_{35} are non-zero in the third row, we observe only a heave and a pitch acceleration. Figure 12 shows that the resulting trajectory looks like a circle in the xz plane0.

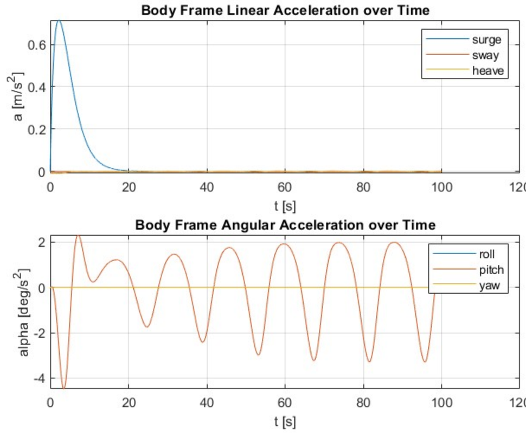
Problem 10

Find some simulations to highlight the impact of the drag forces of the different bodies (Impose constant linear speed)

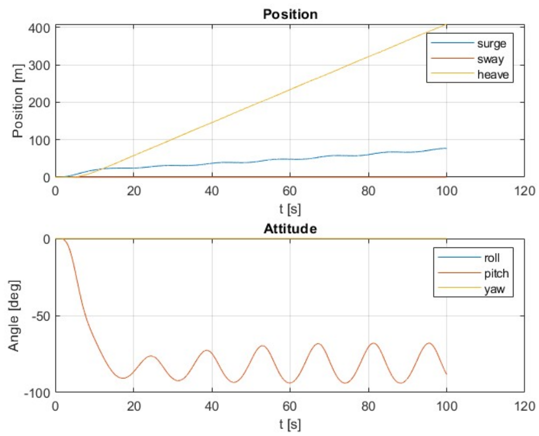
In the experiment illustrated below, we have removed the drag forces all of the parts of the body except for the main hull, while turning on only the side thrusters with 30% of its maximum power. We can see that the drag forces on the smaller parts actually help to stabilize the motion of the vehicle and without them it has a lot of angular oscillation, when compared to the experiment represented in Figure 4.



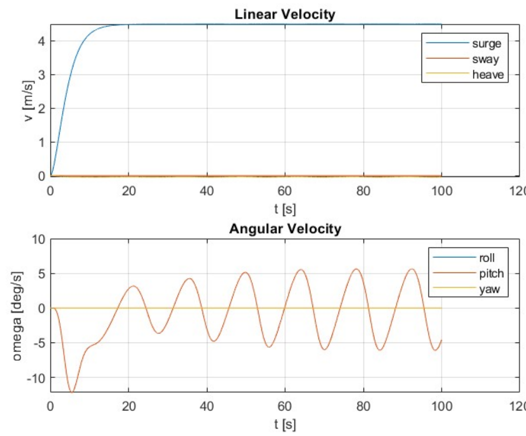
(a) Trajectory of the AUV.



(b) Linear and angular acceleration in the body frame.



(c) Position and attitude of the AUV.



(d) Linear and angular velocity in the body frame.

Figure 16: Simulation results with [0%, 30%, 30%] thruster configuration and after removing the effect of all drag forces except for the main hull.