Assignment-1
ECSE 506: Stochastic Control and Decision Theory

Sadia Khaf - KHAS23519206

[1]McGill University

1. Exercise 1

I solved by taking expectation and finding the minimum of it. The argument that minimizes the value of the expectation is the correct g for each x in $\mathcal{X}$.

1.1. MATLAB Code

```
%%%%%%%%%%%%%%% ELEC 506 Assignment-1 %%%%%%%%%%%%%%%%%%%%%%%%%%
% Submitted by: Sadia Khaf                                     %
% ETS Permanent Code: 23519206                                 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 clear all, close all, clc


%%%%%%%%%% Problem-1 %%%%%%%%%%%%%%%%%%%%
% Defining given information
X= [1,2];
U= [1,2,3];
W = [1,2,3];
 P_xw = [0.25 0.15 0.05;0.30 0.10 0.15];
 c_xu1 = [3 5 1;2 3 1];
 c_xu2 = [4 3 1;1 2 8];
 c_xu3 = [1 2 2;4 1 3];

% J(g) = E[c(X,g(X),W)]
% min_g E[c(X,g(X),W)]
% g^*(x) = arg min_u \in U E[c(x,u,W)|X=x]
% g^*(1) = arg min_u \in U E[c(1,u,W)|X=1]
% g^*(2) = arg min_u \in U E[c(2,u,W)|X=2]
 E_1uw = [c_xu1(1,:);c_xu2(1,:);c_xu3(1,:)]*P_xw(1,:)'/sum(P_xw(1,:));
 E_2uw = [c_xu1(2,:);c_xu2(2,:);c_xu3(2,:)]*P_xw(2,:)'/sum(P_xw(2,:));
 [val gstar] = min([E_1uw,E_2uw],[],1)
% gstar comes out to be 3,1 for the values given in the question
```

2. Exercise 2

I solved this problem using the similar conditional expectation formulation as that for problem 1. The information regarding Y seems to be redundant however.

2.1. MATLAB Code

```
%%%%%%%%%%%%%%%%%%% ELEC 506 Assignment-1 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Submitted by: Sadia Khaf                                              %
% ETS Permanent Code: 23519206                                          %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all, close all, clc

%%%%%%%%%%%%%% Problem-2 %%%%%%%%%%%%%%%%%%%%%%%%%%
%Defining given info
X= [1,2];
Y= [1,2];
U= [1,2,3];
W = [1,2,3];
QY1 = [0.15 0.10 0.00;0.15 0.05 0.10];
QY2 = [0.10 0.05 0.05;0.15 0.05 0.05];
c_xu1 = [3 5 1;2 3 1];
c_xu2 = [4 3 1;1 2 8];
c_xu3 = [1 2 2;4 1 3];
E_11uw = [c_xu1(1,:);c_xu2(1,:);c_xu3(1,:)]*QY1(1,:)'/sum(QY1(1,:));
E_12uw = [c_xu1(1,:);c_xu2(1,:);c_xu3(1,:)]*QY2(1,:)'/sum(QY2(1,:));
E_21uw = [c_xu1(2,:);c_xu2(2,:);c_xu3(2,:)]*QY1(2,:)'/sum(QY1(2,:));
E_22uw = [c_xu1(2,:);c_xu2(2,:);c_xu3(2,:)]*QY2(2,:)'/sum(QY2(2,:));
[val2 gstar2] = min([E_11uw,E_12uw,E_21uw,E_22uw],[],1)
```

2.2. Comments

The probability P seems to have been divided into two possible values for Y but the sum remains equal to the original probability matrix P in problem 1, i.e., $P = Q_{Y=1} + Q_{Y=2}$. The information regarding y is irrelevant information since W is conditionally independent of Y given X (Blackwell's principle of irrelevant information).

$$\mathbb{E}[c(x, g^*(x), W)|X = x] \leq \mathbb{E}[c(x, g(x, y), W)|X = x] \tag{1}$$

3. Exercise 3

The reward for the operator is given as

$$r(u, W) = \begin{cases} pu, & \text{if } W > u \\ pu - c(u - W), & \text{if } W < u. \end{cases} \tag{2}$$

Similar to the newspaper example covered in class, we formulate the reward as:

$$J(u) = \int_{W=0}^{u} pu - c(u - W)f(W)dW + \int_{W=u}^{\infty} puf(W)dW \tag{3}$$

We can find maxima by verifying first and second derivative test, and equating first derivative to zero, i.e.,

$$\frac{d}{du}J(u) = 0 \tag{4}$$

$$puf(u) + \int_{W=0}^{u} (p-c)f(W)dW - puf(u) + \int_{W=u}^{\infty} pf(W)dW = 0$$

$$(p-c)F(u) + p(1-F(u)) = 0$$

$$u = F^{-1}(\frac{p}{c}) \tag{5}$$