Project: Quantum Walks and Monte Carlo - Summary

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Project Overview

This project explores the implementation of a Quantum Galton Box, a quantum analogue of the classical bean machine used to generate statistical distributions. The primary goal is to analyze the performance of this quantum walk under both ideal, noiseless conditions and on a realistic noisy simulator. Crucially, the project quantifies the performance degradation caused by hardware noise and measures the effectiveness of a foundational error mitigation strategy—circuit optimization via transpilation—in improving simulation fidelity. The theoretical basis for such statistical simulations is grounded in concepts like the Universal Statistical Simulator framework.

Methodology & Simulation Setup

The project is structured around four distinct quantum walk simulations designed to compare ideal, realistic, and mitigated outcomes.

1. **Simulations Performed:**
   - **Ideal Binomial Walk:** A simulation of a perfect Quantum Galton Box where the quantum "coin" is fair (50/50 probability), achieved using a Hadamard (H) gate. This serves as the theoretical gold standard, producing a perfect binomial distribution.
   - **Ideal Exponential Walk:** A demonstration of the quantum advantage in generating custom distributions. The Hadamard gate is replaced with a Y-axis rotation RY(θ) gate to create a biased coin, resulting in a skewed binomial distribution that approximates an exponential decay curve.
   - **Noisy Baseline:** The ideal binomial walk circuit is executed on a realistic noisy simulator. This establishes a baseline to measure the impact of hardware errors on the accuracy of the results.
   - **Optimized Noisy:** The same binomial walk circuit is first optimized using the Qiskit transpile function before being executed on the noisy simulator. This serves as the test case for error mitigation.

1. **Realistic Noise Model:** To simulate the behavior of a real-world quantum computer, a Noise Model was derived from FakeVigo, a software model containing the specific error characteristics of the retired 5-qubit IBM Quantum system "Vigo". This model incorporates gate errors, decoherence, and the device's physical coupling map.

2. **Error Mitigation Strategy:** The primary error mitigation technique investigated was circuit optimization. By using the Qiskit transpile function, the quantum circuit was recompiled into a more efficient version tailored to the backend's native gate set. This process reduces the circuit's overall depth and total gate count, minimizing its exposure to decoherence and the opportunities for gate errors to occur.

3. **Analysis and Verification Metrics:** To provide a robust comparison, the probability distribution from each simulation (P) was compared against its ideal theoretical target (Q).

   - **Primary Metric (Hellinger Distance):** The Hellinger Distance, defined as $\mathrm{H}(\mathrm{P},\mathrm{Q}) = \frac{1}{\sqrt{2}}\sqrt{\sum_i(\sqrt{p_i - q_i})^2}$, was used to measure the similarity between the two distributions. It is a true statistical metric bounded between 0 (identical) and 1 (no overlap).
   - **Stochastic Uncertainty Analysis:** To distinguish real hardware error from statistical randomness (shot noise), the uncertainty of the Hellinger Distance was calculated using the bootstrapping method. This provides statistically significant error bars for the final measurements.

Results & Analysis

The analysis quantitatively demonstrates both the challenge of hardware noise and the success of the mitigation strategy. The Hellinger Distance for each of the four simulations is compared in the chart below, with lower values indicating higher fidelity to the target distribution.

Key Observations:

- **Impact of Noise:** The **Noisy Baseline** simulation (Hellinger Distance: 0.0171 ± 0.0037) showed a nearly **3x increase in error** compared to the Ideal Binomial simulation (Hellinger Distance: 0.0058 ± 0.0030). This confirms that realistic hardware noise significantly degrades the accuracy of the quantum walk.
- **Effectiveness of Error Mitigation:** The **Optimized Noisy** simulation achieved a Hellinger Distance of 0.0087 ± 0.0032. This represents a **49% reduction in error** compared to the noisy baseline, bringing the result remarkably close to the ideal, shot-noise-limited performance. This proves that even a simple software-based mitigation technique like transpilation is highly effective.
- **Statistical Significance:** The calculated error bars, representing stochastic uncertainty, confirm that the difference in performance between the noisy baseline and the optimized run is statistically significant and not an artifact of random shot noise.
- **Custom Distribution Fidelity:** The **Ideal Exponential** walk (Hellinger Distance: 0.0079 ± 0.0017) also showed high fidelity, confirming the simulator's ability to accurately generate custom, non-symmetric distributions when noise is not a factor.

Conclusion & Future Work

This project successfully implemented a Quantum Galton Board and demonstrated a key challenge of the NISQ (Noisy Intermediate-Scale Quantum) era: the degradation of results due to hardware noise.

The key takeaway is that **software-based error mitigation is a powerful and essential tool.** By simply transpiling the circuit to a more efficient form, nearly half of the accuracy lost to noise was recovered. This highlights the critical role of the quantum software stack in extracting meaningful results from today's imperfect quantum hardware.

Future Work could expand on this project by:

- Implementing more advanced error mitigation techniques like Zero-Noise Extrapolation (ZNE) or Dynamical Decoupling.
- Running the optimized circuits on actual IBM Quantum hardware to compare real-world performance against the FakeVigo simulation.
- Exploring more complex, multi-dimensional quantum walks.

References

- Carney, M., & Varcoe, B. (2022). *Universal Statistical Simulator*. arXiv:2202.01735 [quant-ph].
- Kempe, J. (2003). Quantum random walks: An introductory overview. *Contemporary Physics, 44*(4), 307–327.