Graph

Graphs

- A data structure that consists of a set of nodes (*vertices*) and a set of edges that relate the nodes to each other
- The set of edges describes relationships among the vertices .
- A graph G is defined as follows:

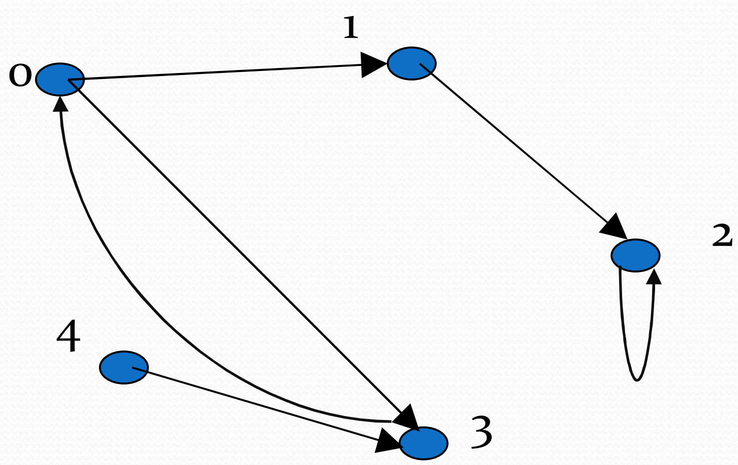
$$G=(V,E)$$

$V(G)$: a finite, nonempty set of vertices

$E(G)$: a set of edges (pairs of vertices)

Examples of Graphs

- V={0,1,2,3,4}
- E={(0,1), (1,2), (0,3), (3,0), (2,2), (4,3)}



When (x,y) is an edge,
we say that x is *adjacent to* y, and y is *adjacent from* x.

0 is adjacent to 1.
1 is not adjacent to 0.
2 is adjacent from 1.

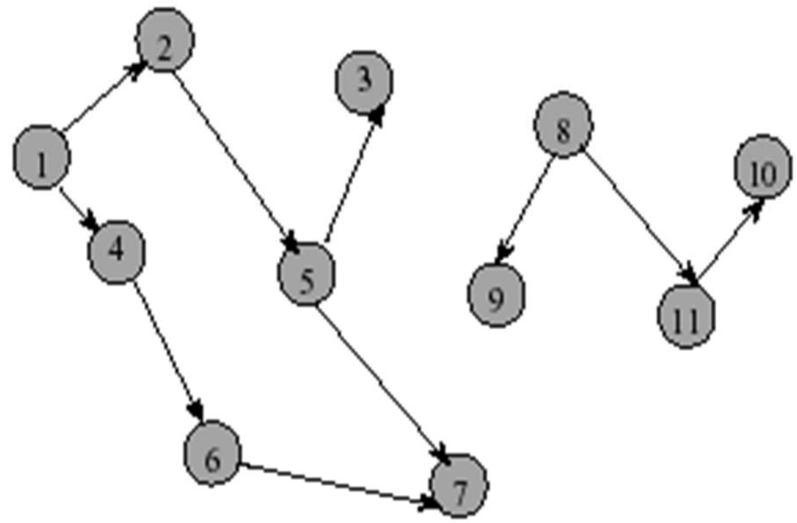
Directed vs. Undirected Graphs

- Undirected edge has no orientation (no arrow head)
- Directed edge has an orientation (has an arrow head)
- Undirected graph – all edges are undirected
- Directed graph – all edges are directed

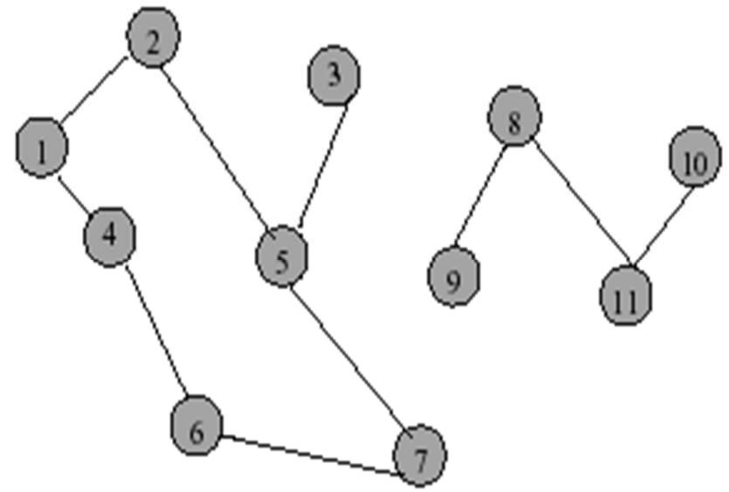
$$u \; \text{———} \; v \qquad\qquad u \longrightarrow v$$

undirected edge

directed edge



Directed graph



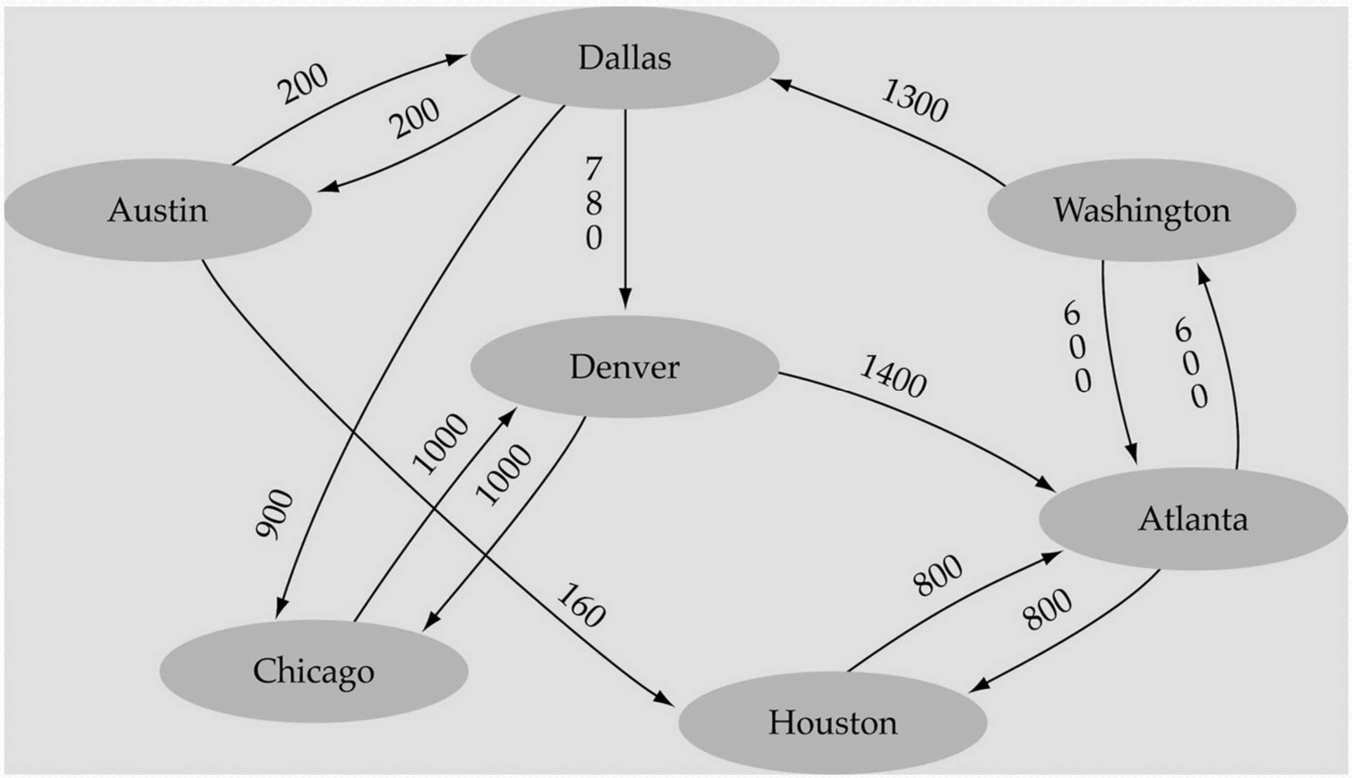
Undirected graph

Directed Graph

- Directed edge (i, j), i is incident to vertex j and j incident from vertex i
- Vertex i is adjacent to vertex j, and vertex j is adjacent from vertex i

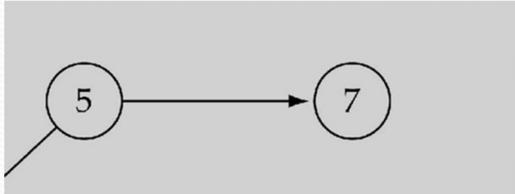
Weighted graph:

-a graph in which each edge carries a value



Graph terminology

- Adjacent nodes: two nodes are adjacent if they are connected by an edge

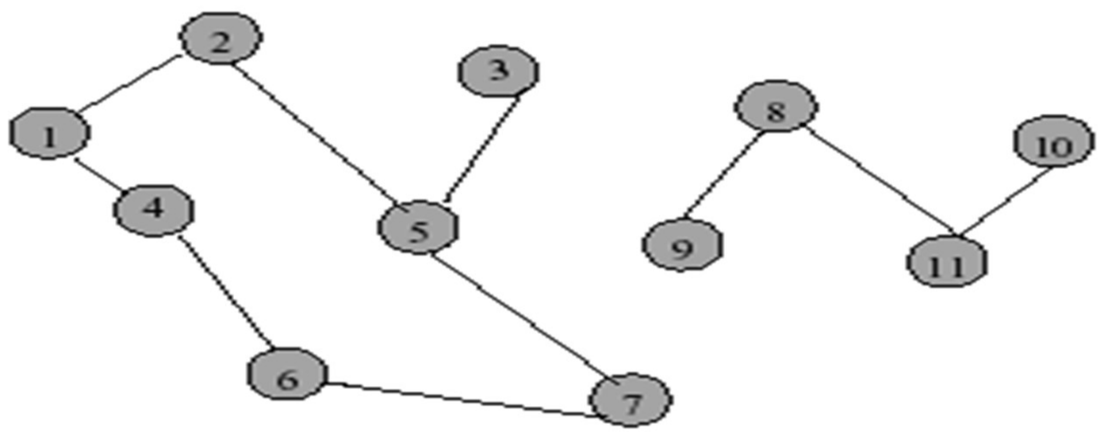


5 is adjacent to 7
7 is adjacent from 5

- Path: a sequence of vertices that connect two nodes in a graph
- Complete graph: a graph in which every vertex is directly connected to every other vertex

Continued…

- A cycle is a simple path with the same start and end vertex.
- The **degree** of vertex *i* is the no. of edges incident on vertex *i*.



e.g., degree(2) = 2, degree(5) = 3, degree(3) = 1

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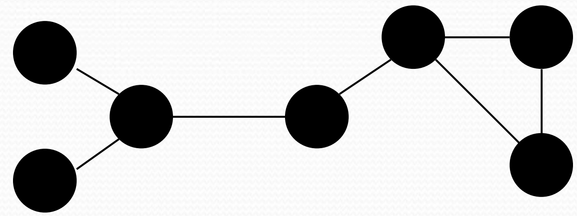
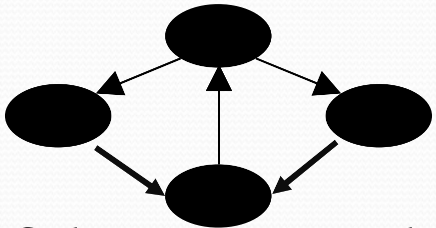
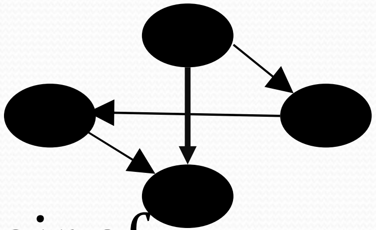
Undirected graphs are *connected* if there is a path between any two vertices



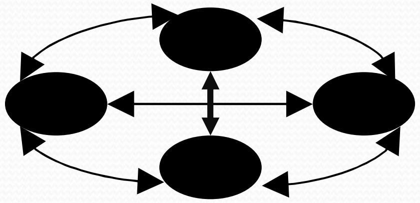
Directed graphs are *strongly connected* if there is a path from any one vertex to any other



Directed graphs are *weakly connected* if there is a path between any two vertices, *ignoring direction*



A *complete* graph has an edge between every pair of vertices



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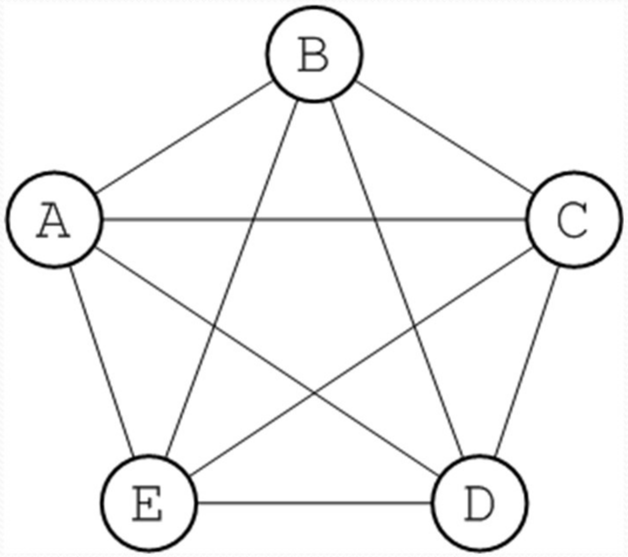
- *Loops*: edges that connect a vertex to itself
- *Paths*: sequences of vertices p0, p1, … pm such that each adjacent pair of vertices are connected by an edge
- *Multiple Edges*: two nodes may be connected by >1 edge
- *Simple Graphs*: have no loops and no multiple edges

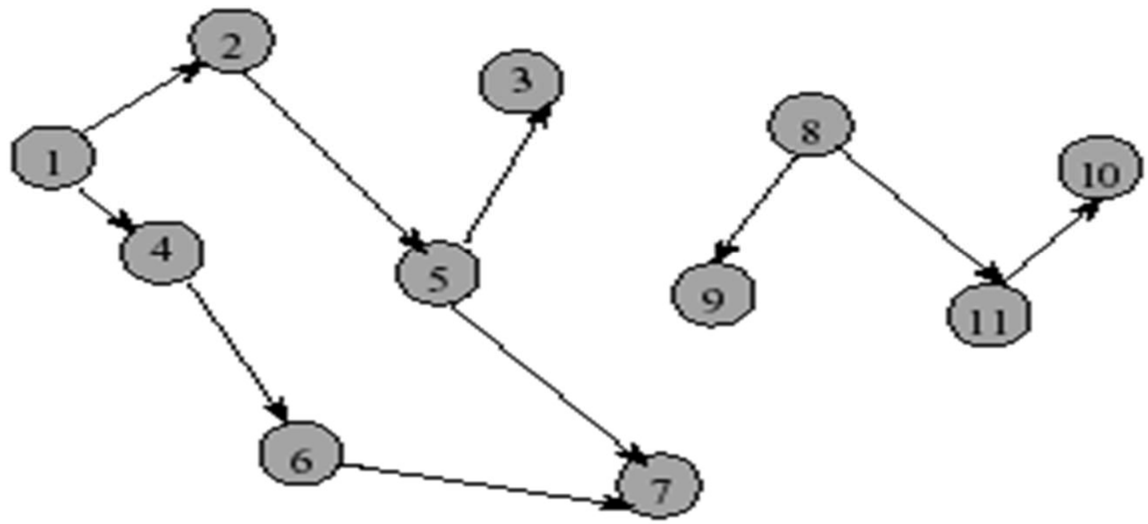
Graph Properties

Number of Edges

The no. of possible pairs in an n vertex directed graph is $n*(n-1)$. This type of graph is called complete graph.

Since edge *(u,v)* is **the same** as edge *(v,u)*, the number of edges in an undirected graph is $n*(n-1)/2$.





- In-degree of vertex i is the number of edges incident to i (i.e., the number of incoming edges).

  e.g., indegree(2) = 1, indegree(8) = 0

- Out-degree of vertex i is the number of edges incident from i (i.e., the number of outgoing edges).

  e.g., outdegree(2) = 1, outdegree(8) = 2

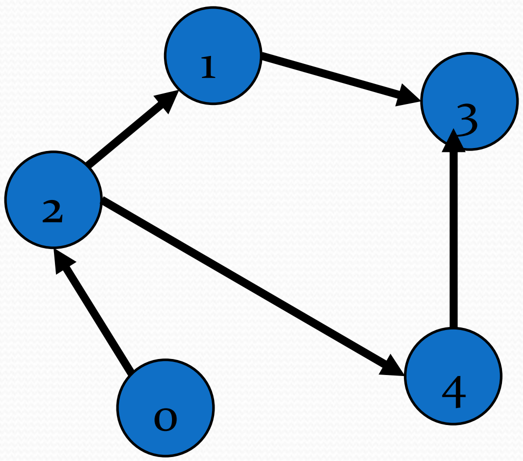
Graph Representation

- For graphs to be computationally useful, they have to be conveniently represented in programs
- There are two computer representations of graphs:
  - Adjacency matrix representation
  - Adjacency lists representation

Adjacency Matrix

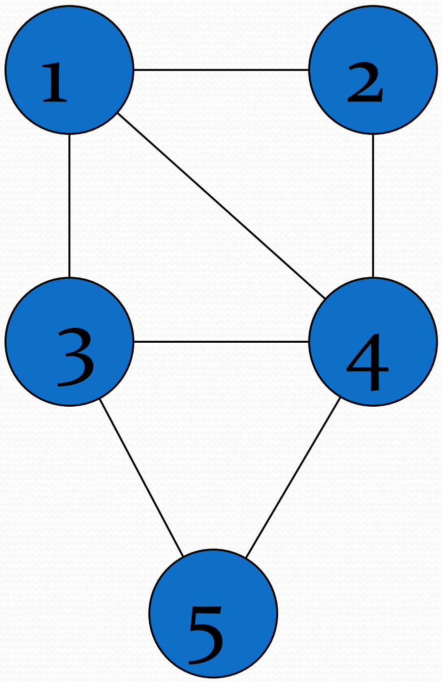
- A square grid of boolean values
- If the graph contains N vertices, then the grid contains N rows and N columns
- For two vertices numbered I and J, the element at row I and column J is true if there is an edge from I to J, otherwise false

Adjacency Matrix



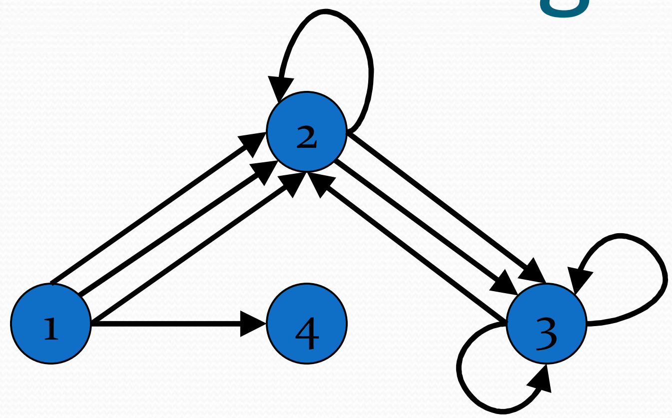
	0	1	2	3	4
0	false	false	true	false	false
1	false	false	false	true	false
2	false	true	false	false	true
3	false	false	false	false	false
4	false	false	false	true	false

Adjacency Matrix



	1	2	3	4	5
1	0	1	1	1	0
2	1	0	0	1	0
3	1	0	0	1	1
4	1	1	1	0	1
5	0	0	1	1	0

Adjacency Matrix -Directed Multigraphs



A:

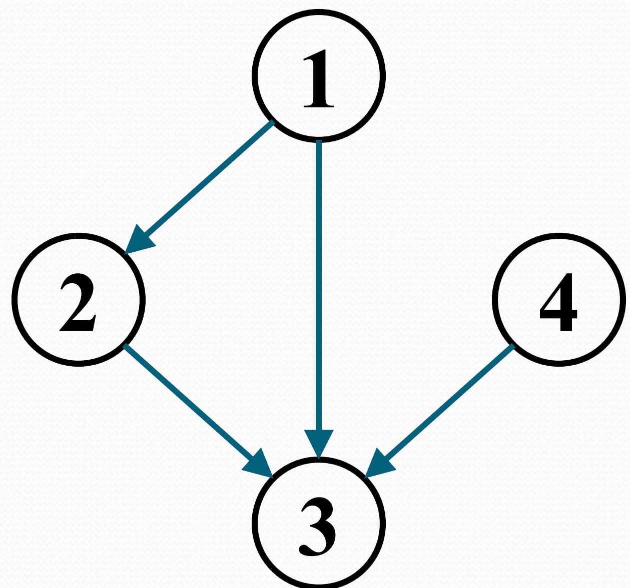
$$\begin{pmatrix} 0 & 3 & 0 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Adjacency Lists Representation

- A graph of n nodes is represented by a one-dimensional array L of linked lists, where
  - L[i] is the linked list containing all the nodes adjacent from node i.
  - The nodes in the list L[i] are in no particular order

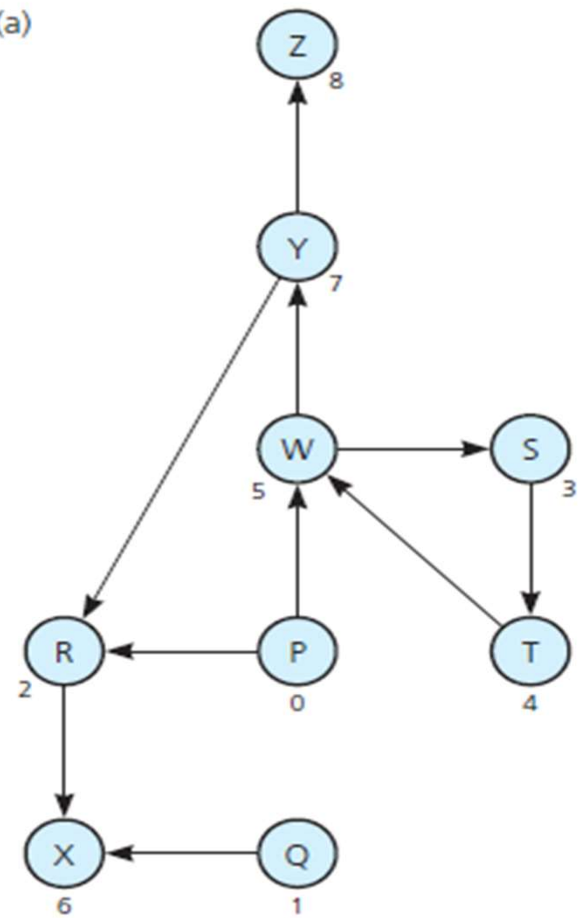
Graphs: Adjacency List

- Adjacency list: for each vertex $v \in V$, store a list of vertices adjacent to v
- Example:
  - Adj[1] = {2,3}
  - Adj[2] = {3}
  - Adj[3] = {}
  - Adj[4] = {3}
- Variation: can also keep a list of edges coming *into* vertex



Implementing Graphs

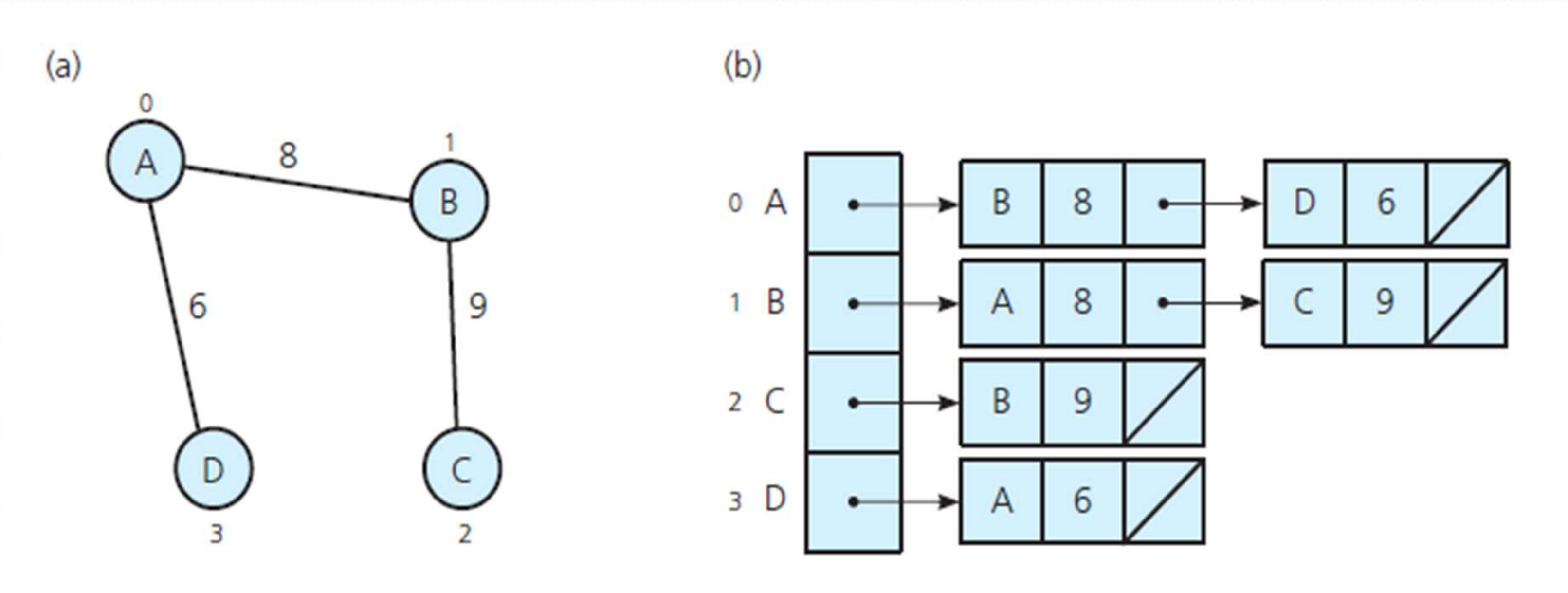
(a)



(b)

		0	1	2	3	4	5	6	7	8
		P	Q	R	S	T	W	X	Y	Z
0	P	0	0	1	0	0	1	0	0	0
1	Q	0	0	0	0	0	0	1	0	0
2	R	0	0	0	0	0	0	1	0	0
3	S	0	0	0	0	1	0	0	0	0
4	T	0	0	0	0	0	1	0	0	0
5	W	0	0	0	1	0	0	0	1	0
6	X	0	0	0	0	0	0	0	0	0
7	Y	0	0	1	0	0	0	0	0	1
8	Z	0	0	0	0	0	0	0	0	0

Implementing Graphs



- (a) A weighted undirected graph and
  (b) its adjacency list