Project 1: Accelerating Elastic Ferris Wheel

Consider a 1 m long flexible heavy cord (e.g., bungee cord) represented with an array of 14 point-masses connected to one another by springs and dampers. The cord is connected to the end of a 5-meter long accelerating arm as shown in Figure 1.

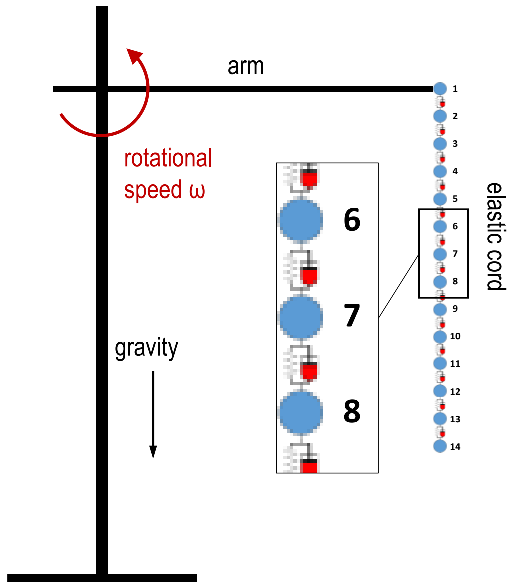

Figure 1: Mass-Spring-Damper (MSD) representation of a flexible cord.

The forces acting on a point mass i are shown in the free body diagram illustrated in Figure 2. In addition to gravity, there are spring and damper forces that act on each point-mass. Expressions for these forces are listed below:

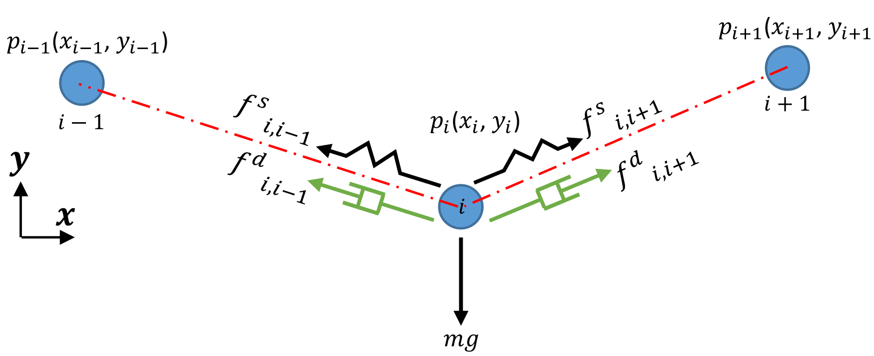

Figure 2: Free body diagram illustrating forces acting on point mass *i*.

$$\vec{f}^{\,s}_{i,i-1} = -k_s\left(\left\|\vec{p}_i - \vec{p}_{i-1}\right\| - l_r\right)\frac{\left(\vec{p}_i - \vec{p}_{i-1}\right)}{\left\|\vec{p}_i - \vec{p}_{i-1}\right\|} \tag{1}$$

$$\vec{f}^{\,s}_{i,i+1} = -k_s\left(\left\|\vec{p}_i - \vec{p}_{i+1}\right\| - l_r\right)\frac{\left(\vec{p}_i - \vec{p}_{i+1}\right)}{\left\|\vec{p}_i - \vec{p}_{i+1}\right\|} \tag{2}$$

$$\vec{f}^{\,d}_{i,i-1} = -k_d\left(\vec{u}_i - \vec{u}_{i-1}\right) \tag{3}$$

$$\vec{f}^{\,d}_{i,i+1} = -k_d\left(\vec{u}_i - \vec{u}_{i+1}\right) \tag{4}$$

where $\vec{f}^{\,s}_{i,i-1}$, $\vec{f}^{\,d}_{i,i-1}$, $\vec{f}^{\,s}_{i,i+1}$, and $\vec{f}^{\,d}_{i,i+1}$ are the spring and damper forces acting on point-mass i by its neighboring point-masses, and $\vec{p}_i$, $\vec{p}_{i-1}$, and $\vec{p}_{i+1}$ are the position vectors of point-masses i, $i-1$, and $i+1$, respectively. k_s and k_d are the spring and damping constants, respectively. The un-stretched length of the springs is shown with l_r. The velocity vectors for point-masses i, $i-1$, and $i+1$, are shown with $\vec{u}_i$, $\vec{u}_{i-1}$, and $\vec{u}_{i+1}$, respectively. The instantaneous distance between neighboring point-masses are

$$\left\|\vec{p}_i - \vec{p}_{i-1}\right\| = \sqrt{\left(x_i - x_{i-1}\right)^2 + \left(y_i - y_{i-1}\right)^2} \tag{5}$$

$$\left\|\vec{p}_i - \vec{p}_{i+1}\right\| = \sqrt{\left(x_i - x_{i+1}\right)^2 + \left(y_i - y_{i+1}\right)^2} \tag{6}$$

Therefore, the fractions on the right-hand side of Equations 1 and 2 are the unit vectors for the distance between the corresponding point-masses. The position and velocity of the point-mass can be obtained by solving Newton's 2nd law written for each point-mass:

$$\begin{aligned} m\vec{a}_i = &-k_s\left(\left\|\vec{p}_i - \vec{p}_{i-1}\right\| - l_r\right)\frac{\left(\vec{p}_i - \vec{p}_{i-1}\right)}{\left\|\vec{p}_i - \vec{p}_{i-1}\right\|} - k_s\left(\left\|\vec{p}_i - \vec{p}_{i+1}\right\| - l_r\right)\frac{\left(\vec{p}_i - \vec{p}_{i+1}\right)}{\left\|\vec{p}_i - \vec{p}_{i+1}\right\|} \\ &-k_d\left(\vec{u}_i - \vec{u}_{i-1}\right) - k_d\left(\vec{u}_i - \vec{u}_{i+1}\right) + m\vec{g} \end{aligned} \tag{7}$$

where $\vec{g}$ is the gravitational acceleration, $\vec{a}_i$ is the acceleration of the point-mass i, and m is the mass of each point-mass (see [1] for more information).

What you should submit:

1. Develop a ***Mathematica*** code that solves the above equations for each point-mass.
2. Assume an initial angular velocity of 0.4 rad/s for the arm and increase the speed at a constant rate of 0.02 rad/s zero. Simulate the rotation of the cord for 100 seconds. Assume zero initial stretching for the cord. Plot the profile and velocity of the cord (the $x-y$ coordinates and velocity of each point-mass) at four different locations of horizontal right, upward, horizontal left, and downward for four different angular velocities. Assume spring and damping constants of $\frac{k_s}{m} = 100\ N/mkg$ and $\frac{k_d}{m} = 10\ Ns/mkg$, respectively.
3. Write a short, but yet clean and professional report describing your work. Up to 25% of your grade will be based solely on the style and formatting of your report. Use proper heading for each section of your report. Be consistent in your font size. Use **Arial Narrow size 11** only. Make sure that figures have proper self-explanatory captions and are cited in the body of the report. Make sure that your figures have legends as well as x and y labels with proper and consistent fonts. Don't forget that any number presented in the report or on the figures has to have a proper unit. Equations and pages in your report should be numbered. Embed your figures in the text. Make sure they do not have unnecessary frames around them or are not plotted on a grey background (default setting of some software programs!).

Note: While you can work together on your projects, what you submit should be YOUR OWN original work.