1. Let T be the linear transformation such that: 1 point

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)=\begin{bmatrix}1\\3\\-4\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)=\begin{bmatrix}2\\-1\\-3\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)=\begin{bmatrix}4\\5\\-11\end{bmatrix}$$

Find its rank.

2

2. Let M be a square matrix. 1 point

Check all that are true.

- [x] If $\det(M)=5$, then $\det(M^n)=5^n$.
- [x] If M is non-singular, then so is M^{-1}.
- [ ] If M has size n, then it has n distinct eigenvalues.
- [ ] The determinant is the area of a parallelogram spanned by M and the vectors $\begin{bmatrix}1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$. Therefore, it is always positive.

3. Let 1 point

$$M=\begin{bmatrix}1&0&1\\0&1&0\\1&1&1\end{bmatrix}$$

$$N=\begin{bmatrix}2&8&7\\4&3&9\\1&9&5\end{bmatrix}$$

The value for $\det(M\cdot N)$ is:

0

4. What is the span of the following vectors vectors? 1 point

$$\begin{bmatrix}2\\1\\1\end{bmatrix},\begin{bmatrix}1\\0\\2\end{bmatrix},\begin{bmatrix}1\\0\\1\end{bmatrix}$$

- (●) The entire 3 dimensional space.
- ( ) A plane in a 3 dimensional space

5. Select all the options that are a basis for the 3D space 1 point

- [x] $\begin{bmatrix}2\\3\\1\end{bmatrix},\begin{bmatrix}0\\-1\\2\end{bmatrix},\begin{bmatrix}-1\\1\\4\end{bmatrix}$
- [x] $\begin{bmatrix}1\\0\\1\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix},\begin{bmatrix}1\\0\\0\end{bmatrix}$
- [ ] $\begin{bmatrix}-1\\2\\1\end{bmatrix},\begin{bmatrix}1\\2\\2\end{bmatrix},\begin{bmatrix}1\\6\\5\end{bmatrix}$
- [x] $\begin{bmatrix}2\\3\\1\end{bmatrix},\begin{bmatrix}0\\2.5\\3.5\end{bmatrix},\begin{bmatrix}0\\5.5\\4.5\end{bmatrix}$
- [x] $\begin{bmatrix}-1\\2\\1\end{bmatrix},\begin{bmatrix}1\\2\\2\end{bmatrix},\begin{bmatrix}1\\0\\1\end{bmatrix}$

6. Select the characteristic polynomial for the given matrix. 1 point

$$M=\begin{bmatrix}2&1\\-3&6\end{bmatrix}$$

- (●) $\lambda^2-8\lambda+15$

○ $\lambda^2 + 8\lambda + 15$

○ $\lambda^2 - 8\lambda - 1$

○ $\lambda^3 - 8\lambda + 15$

7. Consider the following matrix: **1 point**

$$M = \begin{bmatrix} 3 & 2 \\ 5 & 8 \end{bmatrix}$$

The covariance matrix related to this matrix is:

Hint: Remember you need to centralize M for each column to first get the matrix denoted in lectures as X, then use the correct formula. You may want to watch again the lecture on PCA - Mathematical Formulation

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◉ $\begin{bmatrix} 2 & 6 \\ 6 & 18 \end{bmatrix}$

○ $\begin{bmatrix} 36 & 46 \\ 46 & 68 \end{bmatrix}$

○ $\begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix}$

○ $\begin{bmatrix} 0.5 & -1.5 \\ -1.5 & 4.5 \end{bmatrix}$

8. Consider the following matrix **1 point**

$$M = \begin{bmatrix} 3 & 0 \\ -2 & 1 \end{bmatrix}$$

Check all the options that represent the eigenvectors of this matrix.

Hint:

- *The characteristic polynomial for M is given by* $(3-\lambda)(1-\lambda)$.
- *Remember that for each eigenvalue, if there is a non-zero eigenvector related to it, then there are infinitely many more eigenvectors related to the same eigenvalue. In other words, if v is an eigenvector for an eigenvalue λ, then kv is also an eigenvector for the same eigenvalue λ, for any real valued number k.*
- *You may want to watch again the lecture on "*Eigenvalues and eigenvectors*".*

☐ $\begin{bmatrix} 0 \\ k \end{bmatrix}$, for any k real.

☑ $\begin{bmatrix} k \\ -k \end{bmatrix}$, for any k real.

☐ $\begin{bmatrix} k \\ k \end{bmatrix}$, for any k real.

☐ $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

9. Suppose you have the following dataset **1 point**

	Size (m^2)	No. Bedrooms	No. Bathrooms
House 1	70	2	2
House 2	110	4	2

Which matrix is the $X - \mu$ matrix, used in the covariance matrix computation? The matrix $X - \mu$ is defined in the lecture Covariance Matrix. Remember that the covariance matrix is defined by $\Sigma = \frac{1}{n-1}(X-\mu)^T(X-\mu)$.

○ $X - \mu = \begin{bmatrix} 70 & 2 & 2 \\ 110 & 4 & 2 \end{bmatrix}$

○ $X - \mu = \begin{bmatrix} -20 & 20 \\ -1 & 1 \\ 0 & 0 \end{bmatrix}$

○ $X - \mu = \begin{bmatrix} 70 & 110 \\ 2 & 4 \\ 2 & 2 \end{bmatrix}$

◉ $X - \mu = \begin{bmatrix} -20 & -1 & 0 \\ 20 & 1 & 0 \end{bmatrix}$

10. For the dataset from question 9, what are the eigenvalues of the covariance matrix? 1 point

- (selected) $\lambda_1 = 802,\ \lambda_2 = 0,\ \lambda_3 = 0$
- $\lambda_1 = 0,\ \lambda_2 = 0$
- $\lambda_1 = 17027,\ \lambda_2 = 0,\ \lambda_3 = 0$