Basis

A set S is a basis for V if

1. S spans V

2. S is LI.

If S is a basis for V then every vector in V can be written in one and only one way as a linear combo of vectors in S and every set containing more than n vectors is LD.

Basis Test

1. If S is a LI set of vectors in V, then S is a basis for V

2. If S spans V, then S is a basis for V

Change of Basis

P[x]_B' = [x]_B

[x]_B' = P^{-1} [x]_B

[B B'] -> [I P^{-1}]

[B' B] -> [I P]

Cross Product

if u = u1i + u2j + u3k

AND

v = v1i + v2j + v3k

THEN

u x v = (u2v3 - u3v2)i - (u1v3 - u3v1)j + (u1v2 - u2v1)k

Definition of a Vector Space

u + v is within V

u+v = v+u

u+(v+w) = (u+v)+w

u+0 = u

u-u = 0

cu is within V

c(u+v) = cu+cv

(c+d)u = cu+du

c(du) = (cd)u

1*u = u

Diagonalizable Matrices

A is diagonalizable when A is similar to a diagonal matrix. That is, A is diagonalizable when there exists an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix

Dot Products Etc.

length/norm ||v|| = sqrt(v_1^2 +...+ v_n^2

||cv|| = |c| ||v||

v / ||v|| is the unit vector

distance d(u,v) = ||u-v||

Dot product u•v = (u_1*v_1 +...+ u_n*v_n)

n cos(theta) = u•v / (||u|| ||v||)

u&v are orthagonal when dot(u,v) = 0

Eigenshit

The scalar lambda(Y) is called an **Eigenvalue** of A when there is a nonzero vector x such that Ax = Yx.

Vector x is an **Eigenvector** of A corresponding to Y.

The set of all eigenvectors with the zero vector is a subspace of R^n called the **Eigenspace** of Y.

1. Find Eigenvalues: det(YI - A) = 0

2. Find Eigenvectors: (YI - A)x = 0

If A is a triangular matrix then its eigenvalues are on its main diagonal

Gram-Schmidt Orthonormalization

1. B = {v1, v2, ..., vn}

2. B' = {w1, w2, ..., wn}:

w1 = v1

w2 = v2 - projw1v2

w3 = v3 - projw1v3 - projw2v3

wn = vn - ...

3. B" = {u1, u2, ..., un}:

ui = wi/||wi||

B" is an orthonormal basis for V

span(B) = span(B")

Important Vector Spaces

R^n

C(-inf, +inf)

C[a, b]

P

P_n

M_m,n

Inner Products

||u|| = sqrt<u,u>

d(u,v) = ||u-v||

cos(theta) = <u,v> / (||u|| ||v||)

u&v are orthagonal when <u,v> = 0

proj_v u = <u,v>/<v,v> * v

Kernal

For T:V->W The set of all vectors v in V that satisfies T(v)=0 is the kernal of T. ker(T) is a subspace of v.

For T:R^n ->R^m by T(x)=Ax ker(T) = solution space of Ax=0 & Cspace(A) = range(T)

Linear Combo

v is a linear combo of u_1 ... u_n

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Linear Independence

a set of vectors S is LI if c1v1 +...+ ckvk = 0 has only the trivial solution.

If there are other solutions S is LD. A set S is LI iff one of its vectors can

be written as a combo of other S vectors.

Linear Transformation

V & W are Vspaces. T:V->W is a linear transformation of V into W if:

1. T(u+v) = T(u) = T(v)

2. T(cu) = cT(u)

Non-Homogeny

If xp is a solution to Ax = b then every solution to the system can be written as x = xp

Nullity

Nullspace(A) = {x ε R^n : Ax = 0

Nullity(A) = dim(Nullspace(A)) = n - rank(A)

Orthogonal Sets

Set S in V is orthogonal when every pair of vectors in S is orthogonal. If each vector is a unit vector, then S is orthonormal

One-to-One and Onto

T is one-to-one **iff** ker(T) = {0}

T is onto **iff** rank(T) = dim(W)

If dim(T) = dim(W) then T is one-to-one **iff** it is onto

Rank and Nullity of T

nullity(T) = dim(kernal)

rank(T) = dim(range)

range(T) + nullity(T) = n (in m_x n)

dim(domain) = dim(range) + dim(kernal)

Rank of a Matrix

Rank(A) = dim(Rspace) = dim(Cspace)

Similar Matrices

For square matrices A and A' of order n, A' is similar to A when there exits an invertible matrix P such that A' = P^{-1} AP

Spanning Sets

S = {v1...vk} is a subset of vector space V. S spans V if every vector in v can be written as a linear combo of vectors in S.

Test for Subspace

1. u+v are in W

2. cu is in w