COMP206P Statistics Component: questions to prepare for the in-course assessment.

- These questions are provided to help you prepare for the test which will take place at **3pm on Monday 13th March 2017**. The time allowed for the test will be 50 minutes. The wording of the test questions will be very similar to that overleaf, but **the precise numbers and some other details used in the test questions will be different, and the order of part questions may be changed**.
- The test will take place in two different lecture rooms. The allocation of students to venues will be available on Moodle by Friday 10th March. **You must take the test in the venue allocated to you**; swaps between students will not be allowed.
- The test will be "closed-book". You will NOT be allowed to consult your solutions to the questions overleaf, nor any books or notes.
- Your solutions should be all your own work. Any detected plagiarism will normally result in disciplinary action against all students involved.
- The use of a UCL-approved calculator will be allowed in the test. No other calculators will be allowed.
- The following formulae will be provided:
  * An Exponentially distributed random variable X with mean $E(X) = 1/\lambda$ has probability density function
    $$f(x) = \lambda e^{-\lambda x} \text{ for } x > 0,$$
    and cumulative distribution function
    $$F(x) = 1 - e^{-\lambda x} \text{ for } x > 0.$$
  * A Poisson distributed random variable Y with mean μ has
    $$P(Y = k) = \frac{e^{-\mu}\mu^k}{k!} \text{ for } k = 0, 1, 2, \ldots$$
- The questions are of different lengths and hence do **not** carry equal weight.

PLEASE TURN OVER

1. An online pharmacy sells a certain type of flu medication. Customers can order either 1 or 2 boxes of the flu medication; because of legal restrictions, orders of more than 2 boxes are not allowed. Assume that the pharmacy receives orders for the flu medication according to a Poisson Process of rate 4 per hour, and that each order is for 1 box with probability 0.2 or for 2 boxes with probability 0.8, independently of all other orders.

   (a) For a randomly sampled order of flu medication received by the pharmacy, denote by X the number of boxes required. Find $E(X)$ and $\mathrm{Var}(X)$.

   (b) Let Y be the number of orders of flu medication received up to and including the first one that only requires 1 box. Name the distribution of Y and find $P(Y \geq 20)$.

   (c) Let N be the number of orders of flu medication received during a time interval of 3 hours. Name the distribution of N and find $P(N = 10)$.

   (d) Let T be the time (in minutes) between one order of flu medication and the next. Name the distribution of T and state its mean. Find $P(5 < T < 10)$.

   Suppose that, on a particular day, the pharmacy receives 100 orders of flu medication.

   (e) Let R_1 be the number of orders (out of these 100) that only require 1 box, and denote by R_2 the number of orders that require 2 boxes. Name the distribution of R_1 and find $P(R_1 = 20)$. Are the random variables R_1 and R_2 independent? Justify your answer.

   (f) Let S be the total number of boxes of flu medication required for these 100 orders. Find the mean and the variance of S. What distribution does S have, approximately (state its name)?

2. A heavy container with chemical waste has been illegally dumped into the sea, and is now lying in a fixed but unknown position on the bottom of the sea. The container is believed to be situated in British territorial waters with probability 0.8. The Royal Navy are planning a search through the British territorial waters only, to try to find the container. If the container is situated in British territorial waters, then the probability that it will be found by such a search is 0.7 (while the probability that it will be overlooked is 0.3).

   (a) Calculate the probability that the container will be discovered by this search through the British territorial waters.

   (b) Suppose that this search through the British territorial waters turns out to be unsuccessful, i.e. the container is not found. Given this information, what is now the probability that the container is situated in British territorial waters?

3. An experiment consists of randomly forming a sequence with the 10 letters

   A, C, I, I, S, S, S, T, T, T

   where all possible orders of these 10 letters are equally likely. Find the probability of each of the following events (*you may leave your answers in the form of fractions and/or in factorial or combinatorial notation*):

   (a) the first three letters include no 'S';

   (b) the first three letters include exactly one 'S';

   (c) the first three letters include no 'S' *or* the last three letters include no 'S', *but* <u>*not*</u> *both*;

   (d) the word 'STATISTICS' is obtained;

   (e) the sequence contains the word 'ASCII'.