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CMPE462 - Spring '20
Linear Algebra Assessment Solutions

1. (10 Points) **Solution 1:**
   Let say $V_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $V_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$
   Since 2 times v_1 is equal to the v_2 , column space of A is the span of the vector v_1.

   Let say $V_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $V_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $V_3 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$
   Since V_1 and V_2 linearly dependent but not V_3, column space of B is the span of the vector V_1 and V_3.

2. (10 points)

   - **Solution 2(a):**
     Take any vector that passes through (0,0,0), if you multiply any vector in that line by a scalar, then the result is also in that line. So it is a subspace of R^3.
   - **Solution 2(b):**
     Since when we multiply some vector in that area with a negative scalar, it won't be in the positive quarter-plane.So,It is not a subspace.

3. (10 points)

   - **Solution 3(a):**

     $V_1^T * V_2 = (-2.0) + (0.1) + (1.0) = 0$
     $V_2^T * V_3 = (0.2) + (1.0) + (0.4) = 0$
     $V_1^T * V_3 = (-2.2) + (0.0) + (1.4) = 0$
     Since standard Euclidean inner products is zero but V_1 and V_3's lenght is not 1, they form an orthogonal set but not an orthonormal set.
   - **Solution 3(b):**
     Since they are already a orthogonal set,we can divide vectors with their length to get an orthonormal set. So,
     $V_a = V_1/[(-2. - 2) + (0.0) + (1.1)]$
     $V_b = V_2/[(0.0) + (1.1) + (0.0)]$
     $V_c = V_3/[(2.2) + (0.0) + (4.4)]$
     V_a, V_b, V_c form an orthonormal set.

4. (10 points) **Solution 4:**
   Since column space of y^T has dimension one and $x.y^T$ will be in the column space of y^T by default, all columns of $x.y^T$ will be linearly dependent and its rank will be 1.

5. (10 points) **Solution 5:**
   $Y^T = [y_1, y_2, ..., y_n]$ where $y_i \epsilon R^p$ for all i

   We can rewrite Y as: $Y = \begin{pmatrix} y_1^T \\ y_2^T \\ ... \\ y_n^T \end{pmatrix}$ then XY becomes:

   $\begin{pmatrix} x_1, x_2, ..., x_n \end{pmatrix} . \begin{pmatrix} y_1^T \\ y_2^T \\ ... \\ y_n^T \end{pmatrix}$ which equals $\sum x_i . y_i^T$

6. (10 points) **Solution 6:**
$(A^TA)^T = A^T(A^T)^T = A^TA$ So, it is symmetric.
$X^T(A^TA)X = (X^TA^T)AX = (AX)^TAX \geqslant 0$ So, it is positive semi-definite.

We want $(AX)^TAX > 0$ to become positive definite. Then $(AX)^TAX$ should not be zero. It is zero when AX=0, So we need A to have independent columns for positive definite.

7. (15 points)

   - (5 points) **Solution 7(a)**
     When A^TA is low rank matrix, we can not take inverse of it and can not calculate the c.
   - (10 points) **Solution 7(b)**
     $(A^TA)v = \lambda v$
     Since A^TA is symmetric and positive semi-definite, $\lambda \geqslant 0$ If we add a positive term then $\lambda > 0$ and A^TA becomes positive definite and its columns will be linearly independent, so we can take its inverse and be able to calculate c.

8. (25 points) **Solution 8:**
Since the minimum natural number that minimize the equation is 1,the answer is 1.