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Self-Focusing of High-Power Laser Beam through Plasma

Mahdi Habibi and Majid Davoodianidalik

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Abstract

One of the most significant factors in understanding the behaviour of laser beams during the propagation process through plasma is how the spot size of the beam changes with both laser and plasma parameters. Self-focusing and defocusing of the laser beam, therefore, play an important role in this study. This chapter is aimed at presenting a brief investigation into these common phenomena in laser-plasma interaction. In addition to a short overview on the types of self-focusing of laser beams through plasma, a detailed study on the relativistic self-focusing of high-intense laser beam in quantum plasma as a particular nonlinear medium is performed. In this case, the essential equations to model this phenomenon are derived; furthermore, a range of values of laser-plasma parameters which would satisfy these equations is considered.

Keywords: self-focusing of laser beam, laser-plasma interaction

1. Introduction

A self-focusing phenomenon is considered as one of the major self-action effects in laser-plasma interaction [1, 2]. During this process, the laser beams are able to modify their front medium by means of a nonlinear response of plasma so as to make it more suitable for propagation. In this situation, the refractive index of plasma could be expressed as $n = n_0 + n_2 I(r)$ in which n_0 presents a linear term, n_2 is considered as an optical constant characterizing the strength of optical nonlinearity, and $I(r)$ determines the beam intensity distribution along the radial coordinate r. In other words, when an intense laser beam with an intensity gradient along its cross-section propagates through plasma, the self-focusing and the

self-defocusing of laser beam occur frequently. In this stage, if the electric field is strong enough, the laser beam will create a dielectric waveguide in the path ahead. This typical waveguide results in reducing or entirely eliminating the divergence of the beam. From an optical perspective, the refractive index of the medium in such situations acts as a convex lens; consequently, the central part of the laser beam would move slower than the edge parts. Therefore, while the beam is propagated through the nonlinear medium, its wave front becomes increasingly distorted, as depicted in **Figure 1**.

Overall, the generation of self-focusing phenomenon could be connected with various physical causes. The basic physical mechanism which is responsible for self-focusing of laser beam is the nonlinearity of the medium which originates in its interaction with the laser field. Therefore, the self-focusing of laser beam through plasma is categorized into three options according to nonlinear mechanisms that they are listed here:

1. Thermal self-focusing (TS)

    This effect is due to collisional heating of plasma exposed to electromagnetic radiation. In fact, the rise in temperature induces the hydrodynamic expansion, which leads to an increase in refractive index and further heating [3].

2. Ponderomotive self-focusing (PS)

    A nonlinear radial ponderomotive force of the focused laser beam pushes electrons out of the propagation axis. It expels the plasma from the beam centre, high-intensity region, and increases the plasma dielectric function, leading to self-focusing of the laser in plasmas.

3. Relativistic self-focusing (RS)

    The increase of electrons' mass traveling by velocity approaching the speed of light modifies the plasma refractive index. This phenomenon has been observed in several experiments and has been proved to be an efficient way to guide a laser pulse over distances much longer than the Rayleigh length.

R.W. Boyd et al. [4] reviewed the self-focusing methods, which are recommended by the authors for more details on the topic.
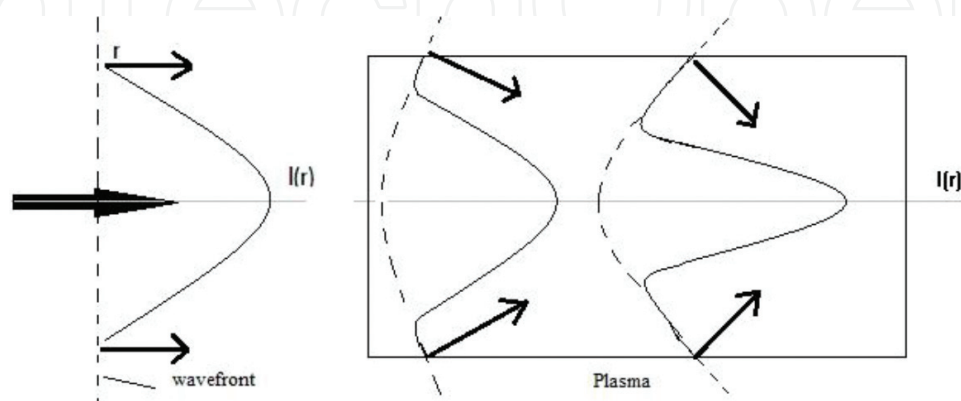
Figure 1. A schematic showing distortion of the wave-front and self-focusing of a laser beam in plasma.

According to the self-focusing of laser beam through plasma, the irradiance of a focused pulse laser with a range of $10^{15} - 10^{16} \mathrm{W/cm^2}$, for instance, can reach approximately $10^{18} \mathrm{W/cm^2}$. Under this high intensity, the relativistic motion of the electrons may be expected. This mechanism is of great importance because it is able to produce ultra-high laser irradiances ($10^{19} - 10^{20} \mathrm{W/cm^2}$) over distances much greater than the Rayleigh length [5]. Sun et al. [6], Barnes et al. [7], Kurki-Suonio et al. [8], and Borisov et al. [9] investigated both relativistic mass variation and charge displacement due to the ponderomotive force by solving the propagation within the envelope approximation and fluid equations. With advancement in laser technology using the chirped-pulse amplification technique [10, 11], the propagation of laser beam in inhomogeneous plasmas has been also studied [12, 13]. Furthermore, guiding nonlinear relativistic laser pulses in preformed plasma with density transition has recently become a subject of great interests [14–17].

The simplest wave equation describing self-focusing is a prototype of an important class of nonlinear partial differential equations in physics, such as the Landau Ginsberg equation for the macroscopic wave function of type II superconductors or the Schrodinger's equation for a particle with self-interactions [18]. In this situation, there are several approximate analytical approaches to analyse the effect of self-focusing, namely paraxial ray approximation, moment theory approach, variation approach, and source-dependent expansion method. However, each of these theories has limitations in describing completely the experimental and computer simulation results.

Our discussion begins with a review of the equations which are normally utilized to describe self-focusing in Wentzel-Kramers-Brillouin (WKB) and paraxial ray approximation (Section 2), followed by an outline of the ramp-density profile and its impact on self-focusing of high-intense laser beams. The next sections deal with consideration of relativistic self-focusing in classical and quantum plasma for Gaussian and Cosh-Gaussian laser beams.

2. Self-focusing equation for high-intense laser-plasma interaction

2.1. High-order paraxial theory

It is reasonable to assume that the paraxial wave equation presents an accurate description for laser beams propagating near the axis throughout the propagation. Akhmanove et al. [19] illustrated that in a limit when the eikonal term is expanded only up to the second power in r, the shape of the radial intensity profile remains unchanged. However, in the experimental situation with high-intense laser beams, one needs to go beyond the paraxial approximation for which the predictions of such an approximation are often not sufficiently accurate [20]. Thus, it would be interesting, on high-intense laser-plasma interaction, to investigate propagation of laser beams using the extended paraxial approximation. In this case, Liu [21] and Tripathi [22] reported a useful theoretical framework that accounts for the combined several effects of interaction of an intense short pulse laser with plasma, the laser frequency blue shift, self-defocusing, ring formation and self-phase modulation. The expansion of the eikonal term

to the fourth power in r makes significant difference in studying laser beam propagating through plasma and even other nonlinear materials.

For more details, the interaction of an intense laser beam with particular plasma is considered. From Maxwell's equations, it is noted that the propagation of such a beam can be investigated by solving the scalar wave equation in the cylindrical coordinate system and along the axis z:

$$\frac{\partial^2}{\partial z^2} E(r, z, t) + \nabla_\perp^2 E(r, z, t) - \frac{\varepsilon}{c^2} \frac{\partial^2}{\partial t^2} E(r, z, t) = 0 \tag{1}$$

where E shows amplitude of the electric field, c is the speed of light in vacuum and ε is the dielectric constant of plasma. In this stage, we consider the solution of the Eq. (1):

$$E(r, z, t) = A(r, z, t) \exp \left[i \left(\omega t - \int_0^z k(z) dz \right) \right] \tag{2}$$

$E(r, z, t)$ can be substituted with Eq. (2) in Eq. (1). This substitution leads to neglecting $\partial^2 A / \partial z^2$ and $\partial^2 A / \partial t^2$ on the assumption that A is a slowly varying function of z and t is compared with ω:

$$-2ik \frac{\partial A}{\partial z} - iA \frac{\partial k}{\partial z} - k^2 A + \frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \frac{\omega^2}{c^2} \varepsilon A = 0. \tag{3}$$

The complex amplitude of the electric vector $A(r, z, t)$ is expressed as,

$$A(r, z, t) = A_0(r, z, t) \exp \left[-ik(z) S(r, z) \right] \tag{4}$$

From Eq. (4), it is noticed that the envelope $A(r, z, t)$ has been separated into real amplitude and complex phase terms in which the eikonal function $S(r, z)$ is.

$$S(r, z) = S_0(z) + (r^2 / r_0^2) S_2(z) + (r^4 / r_0^4) S_4(z) \tag{5}$$

where $S_0(z)$ is the axial phase shift, $S_2(z) = (df(z)/dz)/2f(z)$ is indicative of the spherical curvature of the wave front, and $S_4(z)$ corresponds to its departure from the spherical nature. Moreover, the beam irradiance $A_\circ(r, z, t)$ of Cosh-Gaussian (ChG) laser beam can be written as:

$$
\begin{aligned}
A_0^2(r, z, t) = EE^* = {} & \frac{E_0^2}{4f^2(z)} \exp\left(\frac{b^2}{2}\right) \left(1 + \frac{r^2}{r_0^2 f^2(z)} a_2(z) + \frac{r^4}{r_0^4 f^4(z)} a_4(z) \right) \\
& \times \left(\exp\left[-\left(\frac{r}{r_0 f(z)} + \frac{b}{2}\right)^2 \right] + \exp\left[-\left(\frac{r}{r_0 f(z)} - \frac{b}{2}\right)^2 \right] \right)^2 g(t)
\end{aligned}
\tag{6}
$$

In the near-axis approximation (i.e. $a_2, a_4 \to 0$), Eq. (6) converts to a general solution of ChG laser beam. In Eq. (6), the initial laser intensity at the central position $r = z = 0$ is presented by E_0, and the beam-width parameter characterized by $f(z)$ depends on z. Temporal shape of the pulse can be considered as a step function, $g(t) = 1$ at $t \rangle 0$ and $g(t) = 0$ otherwise. In addition,

b is called the decentred parameter as well as r_0 introduced the initial spot size of the laser beam. Two functions $a_2(z)$ and $a_4(z)$, the coefficients of r^2 and r^4, respectively, are considered as an indicative of the departure of the beam from the Gaussian nature.

Furthermore, in the higher-order paraxial theory, the dielectric constant is expanded to the next higher power in r^2 to obtain $\varepsilon(r, z) = \varepsilon_0(z) - (r^2/r_0^2)\varepsilon_2(z) - (r^4/r_0^4)\varepsilon_4(z)$. The parameters $\varepsilon_0(z)$, $\varepsilon_2(z)$, and $\varepsilon_4(z)$ corresponding to the nonlinearity play an important role in investigating the self-focusing of laser beam through a nonlinear medium. By substituting both Eqs. (5) and (6) into Eq. (3) and equating the coefficients of r^0, r^2, and r^4 on both sides of the resulting equation, the differential equations governing the parameters a_2, a_4, $f(z)$, and $S_4(z)$ can be given by:

$$\frac{da_2}{d\xi} = -16S_4'f^2, \quad a_4 = (3a_2^2 - 4a_2)/4 \tag{7}$$

$$\frac{d^2f}{d\xi^2} = \frac{\{(3a_2^2 - 8a_2 + 4) - b^2(4 + b^2/3 - 2a_2)\}}{\varepsilon_0 f^3} - \frac{\varepsilon_2}{\varepsilon_0}\rho^2 f - \frac{1}{2\varepsilon_0}\frac{d\varepsilon_0}{d\xi}\frac{df}{d\xi} \tag{8}$$

$$\frac{dS_4'}{d\xi} = \frac{40a_2^2 - 52a_2^3 + 32a_2}{8\varepsilon_0 f^6} + \frac{(a_2^2 + (4 - 8a_2)/3 + 2b^6/15)b^2}{\varepsilon_0 f^6} - \frac{\varepsilon_4}{2\varepsilon_0}\rho^2 - \frac{S_4'}{2\varepsilon_0}\frac{d\varepsilon_0}{d\xi} - \frac{4S_4'}{f}\frac{df}{d\xi} \tag{9}$$

where $S_4' = S_4\omega/c$. Additionally, $\xi = (c/\omega r_0^2)z$ and $\rho = r_0\omega/c$ are considered as a dimensionless propagation distance and an original beam-width parameter, respectively. The first term on the right-hand side of Eq. (8) represents the diffraction effect, while the second term plays a vital role in self-focusing of laser beam. Both of them are nonlinear and responsible for the defocusing and focusing of the ChG laser beam through plasma. The behaviour of laser beam propagating through plasma can be investigated by solving these equations, Eqs. (7)–(9), with the initial conditions $f = 1$, $df/d\xi = 0$, $S_4' = 0$ and $a_2 = 0$ at $\xi = 0$. In non-stationary situations, the time derivative is removed from the wave equation using the independent variable transformation ($Z = z, T = t - z/v_g$) where v_g is group velocity. We have presented non-stationary self-focusing of high-intense Gaussian laser beams for different portions of a pulse in classical and quantum plasmas in the weakly relativistic as well as ponderomotive regimes [23, 24]. We have recorded a very significant focusing near the peak of the pulse and the rear portion of the pulse.

2.2. Importance of nonlinearity in self-focusing equation

As mentioned in the previous section, the refractive index of plasma, the second term in the right-hand side of the self-focusing equation, Eq.(8), depends on nonlinearity mechanisms. Therefore, this term plays an important role in investigation propagation laser beam through a nonlinear medium with a wide variety of nonlinearities. For example, in collisional nonlinearity condition:

$$\varepsilon(r, z) = 1 - \frac{\omega_p^2}{\omega^2} + \frac{\omega_p^2}{\omega^2}\frac{\alpha EE^*/2T_e}{1 + \alpha EE^*/2T_e}, \tag{10}$$

where $\alpha = e^2/3\delta m\omega^2 k_B$

or in the ponderomotive regime and in low-intensity laser, it should be expressed like,

$$\varepsilon(r,z) = 1 - \frac{\omega_p^2}{\omega^2} + \frac{\omega_p^2}{\omega^2}\left[1 - \exp\left(-\frac{\beta EE^*}{T_e}\right)\right], \tag{11}$$

where $\beta = \frac{e^2}{8m\omega^2 k_B}$

In addition, in the relativistic regime and high-intense laser-plasma interaction:

$$\varepsilon(r,z) = 1 - \frac{1}{\gamma}\frac{\omega_p^2}{\omega^2}, \gamma \simeq \left(1 + a_\circ^2/2\right)^{1/2} \text{Circular Polarization}, a_\circ = \frac{eE_\circ}{\omega mc} \tag{12}$$

where γ is the Lorentz factor which arises from the quiver motion of the electron in the laser field. The expressions earlier are just three forms of nonlinearities in plasma. In this case, we have investigated relativistic self-focusing of high-intense laser beam in cold and warm quantum plasma [25–28]. From a quantum-mechanical viewpoint, the de Broglie wavelength of the charge particle is comparable to the inter-particle distance. In this situation, the dielectric constant in relativistic cold quantum plasma (CQP) is expressed by:

$$\varepsilon(r,z) = 1 - \frac{1}{\gamma}\frac{\omega_p^2}{\omega^2}\left(1 - \hbar^2 k^4/4\gamma m_0^2\omega^2\right)^{-1} \tag{13}$$

and in the relativistic warm quantum plasma:

$$\varepsilon(r,z) = 1 - \frac{1}{\gamma}\frac{\omega_p^2}{\omega^2}\left(1 - 2k^2 k_B T_{Fe}/m_0^2\omega^2 - \hbar^2 k^4/4\gamma m_0^2\omega^2\right)^{-1} \tag{14}$$

2.3. Ramp density profile

Another important parameter in solving the self-focusing equation is plasma density profile. From mathematical and practical perspectives, it can be considered as a uniform or non-uniform function of propagation distance. In inhomogeneous plasmas [12], the propagation of a Gaussian high-intense laser beam in under-dense plasma with an upward increasing density ramp has been investigated. In this chapter, the effect of electron density profile on spot size oscillations of laser beam has been also shown. It leads to further fluctuations in the figure for the spot size of laser beam compared. Therefore, it was confirmed that an improved electron density gradient profile is an important factor in having a good stationary and non-stationary self-focusing in laser-plasma interaction. A mathematical function of non-uniform charge density profile for modelling inhomogeneous plasma can be considered as:

$$n_e(\xi) = n_0 F(\xi/d) \tag{15}$$

where $\xi = z/R_d$ is a dimensionless propagation distance, $R_d = \omega r_0^2/c$ is the Rayleigh length, r_0 is the focused laser beam radius, n_0 is the density of the plasma at $\xi = 0$, and $F(\xi) = 1 + (n_1/n_0)\tan(\xi/d)$ is the linear density profile function. The slope of ramp density profile can be determined by changing d and n_1 parameters. This model of density is just one kind of

plasma density transitions (the so-called ramp density profile), which is usable to investigate several laser-plasma mechanisms such as self-focusing of laser beam, electron acceleration and high harmonic generation. This mathematical model of density transition has been also supported by a wide range of numerical and experimental work [29–35]. For example, Chunyang et al. [36] observed the high harmonics in the reflection spectra from short-intense laser-pulse interaction with over-dense plasmas the particle-in-cell (PIC) simulations.

3. Relativistic self-focusing of ChG laser beam in quantum plasma

Theoretical investigations of quantum effects on propagation of Gaussian laser beams are carried out within the framework of quantum approach in dense plasmas [37–40]. Habibi et al. have also shown the effective role of Fermi temperature in improving relativistic self-focusing of short wavelength laser beam (X-ray) through warm quantum plasmas [26]. From a theoretical view-point, the relativistic effect would be effective as a result of increasing fermions' number density in dense degenerate plasmas. However, several recent technologies have made it possible to produce plasmas with densities close to solid state. Furthermore, considerable interest has recently been raised in production and propagation of a decentred Gaussian beam on account of their higher efficient power with a flat-top beam shape compared with that of a Gaussian laser beam and their attractive applications in complex optical systems. Generally, focusing of the ChG beam can be analysed like Gaussian beam in plasmas without considering quantum effects. In particular, the present section is devoted to study nonlinear propagation of a ChG laser beam in quantum plasma, including higher-order paraxial theory.

The figure for a ChG laser beam makes a substantial contribution with an even stronger self-focusing effect compared with that of a Gaussian laser beam in cold quantum plasma (CQP). In this chapter, the plasma dielectric function, Eq. (13), which is in the relativistic regime, is considered for unmagnetized and collision-less CQP. Then, it is expanded to the next higher power in r^4 to obtain $\varepsilon(r,z) = \varepsilon_0(z) - (r^2/r_0^2)\varepsilon_2(z) - (r^4/r_0^4)\varepsilon_4(z)$. The parameters $\varepsilon_0(z)$, $\varepsilon_2(z)$, and $\varepsilon_4(z)$ corresponding to the relativistic nonlinearity are:

$$\varepsilon_0(z) = 1 - \frac{\omega_p^2}{\omega^2}\left(1 + \frac{\Gamma}{f^2(z)}\right)^{-\frac{1}{2}}\left[1 - \hbar^2 k^4\left(1 + \frac{\Gamma}{f^2(z)}\right)^{-1/2}\bigg/4m_0^2\omega^2\right]^{-1} \tag{16}$$

$$\varepsilon_2(z) = -\frac{\Gamma(-2 + b^2 + a_2(z))}{2f^4(z)}\frac{\omega_p^2}{\omega^2}\left(1 + \frac{\Gamma}{f^2(z)}\right)^{-\frac{3}{2}}\left[1 - \hbar^2 k^4\left(1 + \frac{\Gamma}{f^2(z)}\right)^{-1/2}\bigg/4m_0^2\omega^2\right]^{-2} \tag{17}$$

$$\varepsilon_4(z) = -\frac{\Gamma(\beta_1 f^2(z) + \Gamma\beta_2/4)}{6f^8(z)}\frac{\omega_p^2}{\omega^2}\left(1 + \frac{\Gamma}{f^2(z)}\right)^{-\frac{5}{2}}$$
$$\times \left[-2\beta_1 f^2(z) + 3\Gamma\left(a_2^2(z) - 2a_4(z)\right) + 1 + 4b^4 + \hbar^2 k^4\left(1 + \frac{\Gamma}{f^2(z)}\right)^{-1/2}\bigg/4m_0^2\omega^2\right] \tag{18}$$

where $\Gamma = \alpha E_0{}^2$, $\beta_1 = 6 - 6b^2 + b^4 + 3a_4 - 6a_2 + 3a_2b^2$, and $\beta_2 = -3a_2^2 + (6b^2 - 12)a_2 - 12b^2 + b^4 + 12a_4 + 12$. By substituting these expressions in Eq. (8), the relativistic self-focusing of ChG laser beam through cold quantum plasma could be investigated. Eqs. (7)–(9) are numerically solved simultaneously using the fourth-order Runge-Kutta method with the initial conditions $f(0) = 1, f'(0) = 0, S_4{}' = 0, a_2 = 0$ at $\xi = 0$ as well as following the set of parameters: $\Gamma = 0.08, 0.12, 0.14$, $r_\circ = 20\,\mu m$, and $\omega_p = 0.59\omega$, $\omega = 1.77 \times 10^{16} s^{-1}$ which correspond to the gold metallic plasma at room temperature. It is noted that the case of a Gaussian beam $b = 0.0$ is similar to a dark ring, maximum irradiance on the axis. Therefore, no parts of the beam penetrate beyond the determined depth of penetration. While in the case of a ChG beam, where $b \neq 0.0$ like a bright ring, the irradiance is maximum on a ring and hence the portion of the beam around the bright ring is able to penetrate farther than that of on the axis. As a result, the decentred parameter plays an effective role in improving relativistic self-focusing of high-power Cosh-Gaussian laser beams in quantum plasma.

Therefore, a comparative analysis can be done among various decentred parameters so as to determine its role in relativistic self-focusing of laser beam. At the first step, the variation of f on ξ in both Gaussian and Cosh-Gaussian of intensity distribution through the Q-plasma for three decentred parameters is shown in **Figure 2**. As seen from this figure, the focusing term becomes dominant with an increasing value of the decentred parameter b. The self-focusing effect is stronger for a higher decentred parameter at $b = 0.8$ than Gaussian laser beam $b = 0$. Consequently, increase of decentred parameters in the ChG laser beams will result in better reduction in the focusing length and more enhancements in localization of non-Gaussian laser
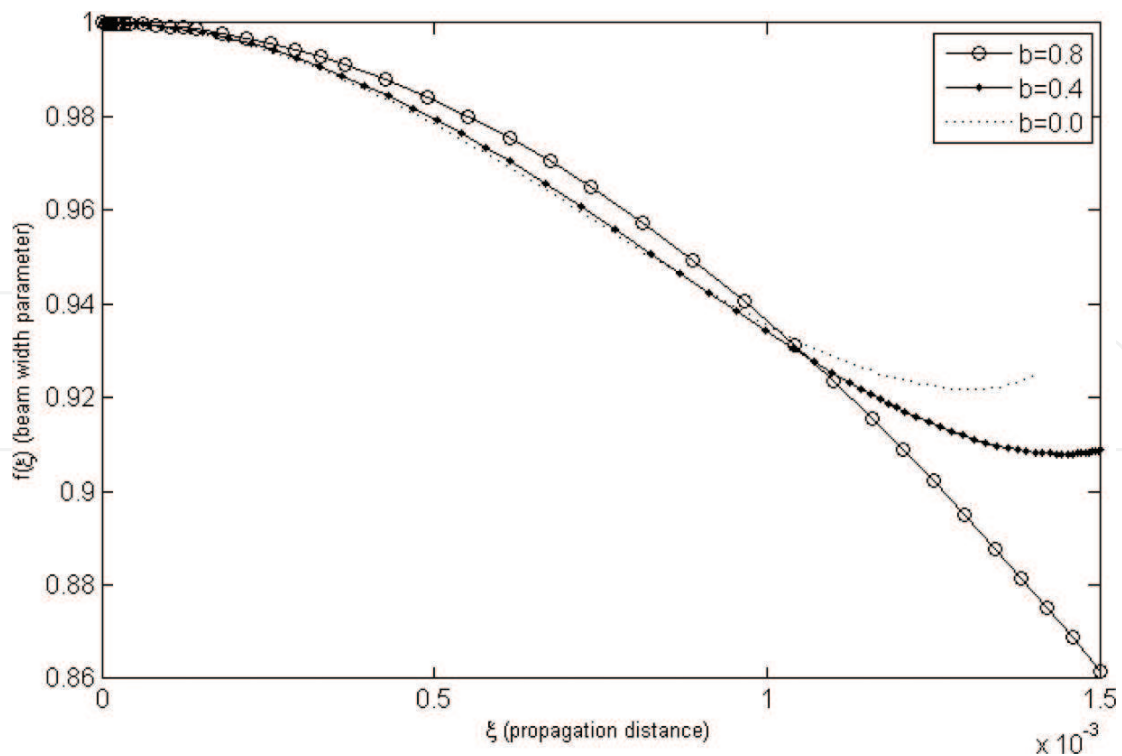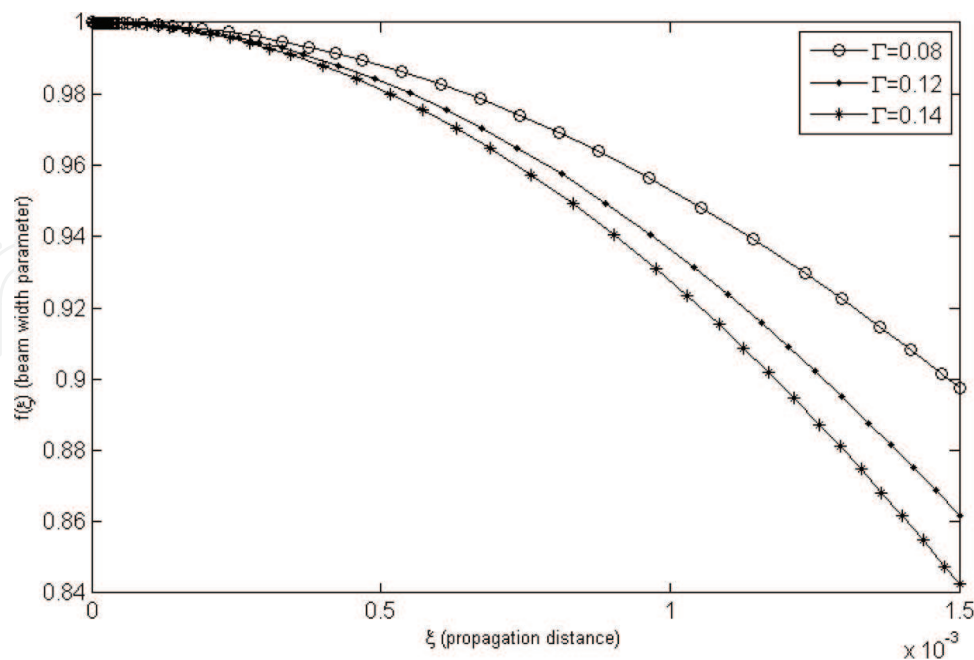

Figure 2. Variation of beam-width parameter f with the normalized propagation distance (ξ) for different values of decentred parameters $b = 0.0, 0.4, 0.8$ and $\Gamma = 0.12, r_0 = 20\,\mu m, \omega_p = 0.59\omega$.

beam as it is moving through plasma. For more investigation and better comparison, the variation of beam-width parameter with propagation distance for various powers of pump laser beam with $b = 0.8$ and through plasma density $\omega_p = 0.75\omega$ is shown in **Figure 3**. It is obvious that the laser beam focused substantially when the initial power of the laser beam grew from 0.08 to 0.14. As a result, the intensity of laser also plays an important role in enhancing the focusing of ChG laser beam through the cold dense plasmas along with the decentred parameter.

Therefore, if laser intensity increases, a beam with more relativistic electrons will travel with the laser beam and generate a higher current and consequently a very high quasi-stationery magnetic field. Consequently, while the pinching effect of the magnetic field is becoming stronger, focusing effect will become much more important. **Figure 4** illustrates the effects of changing plasma density on the relativistic self-focusing process for a given initial intensity of the laser beam. As seen from the results in the **Figure 4**, the focusing of laser beam increases while respective focusing length decreases with increasing the slope of ramp density profile.

It is clear that the inclusion of the quadratic r^4 term in the eikonal function modifies the radial profile. According to equation of RSF in CQP and using cost-effective decentred parameters, decreasing beam-width parameter is observed so that it could produce ultra-high laser irradiance over distances much greater than the Rayleigh length.

On the other hand, we know that an upward ramp density profile as transition density gives rise more reduction in amplitude of the laser spot size close to the propagation axis [12, 25]. Therefore, we show an analysis of joint relativistic and quantum effects on ChG laser beams in one-species axial inhomogeneous cold quantum plasma (ICQP), using the high-order paraxial
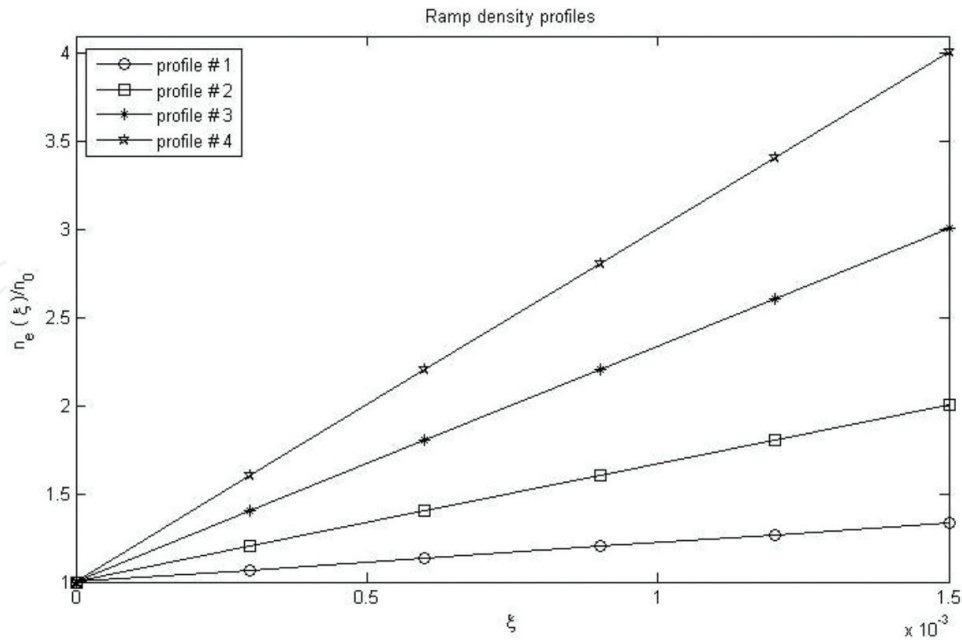

Figure 3. Variation of beam-width parameter f with propagation distance (ξ) for different powers of pump laser beam $\Gamma = 0.08, 0.12, 0.14$ at plasma density $\omega_p = 0.75\omega$, $b = 0.8$, and $r_0 = 20\,\mu m$.
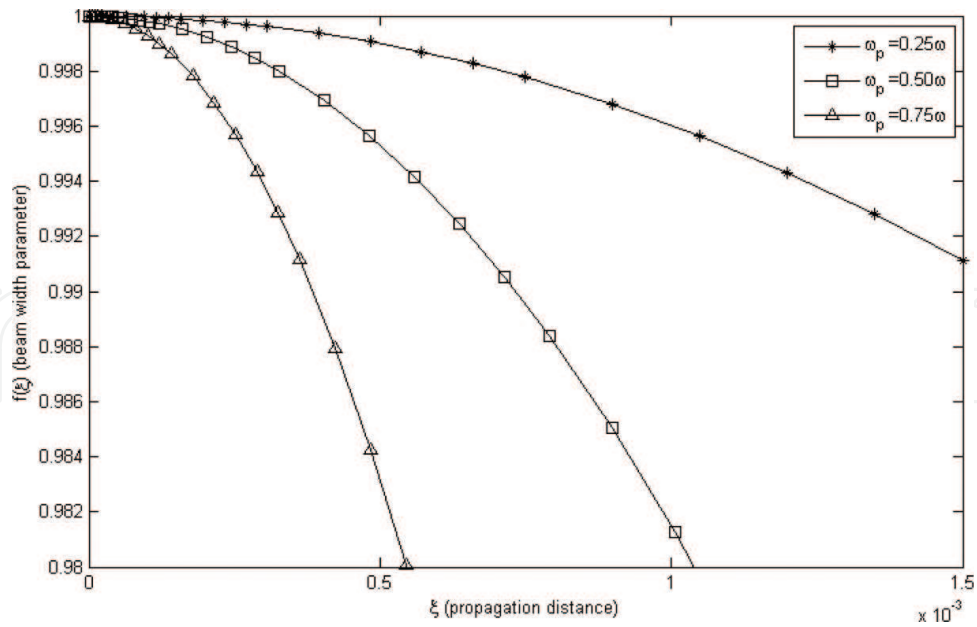
Figure 4. The effect of the change of plasma density on the relativistic self-focusing process for different densities $\omega_p = 0.25\omega, 0.50\omega, 0.75\omega$, $\Gamma = 0.14$, $b = 0.8$, and $r_0 = 20\,\mu m$.

approach. For this purpose, Eqs. (7)–(9) should be solved simultaneously again with considering ramp density profile $\omega_p(\xi) = 0.5F(\xi)\omega$ and $\Gamma = 0.10, 0.12, 0.14$. **Figure 5** provides information about four normalized upward density profiles with various slopes.

Figure 5. The normalized linear density profiles with different slopes: $n_1/n_0 \approx 1 + 6.746 \times 10^2 \tan(\xi/3)$ (profile#1), $n_1/n_0 \approx 1 + 33.531 \times 10^2 \tan(\xi/5)$ (profile#2), $n_1/n_0 \approx 1 + 40.178 \times 10^2 \tan(\xi/3)$ (profile#3), $n_1/n_0 \approx 1 + 20.079 \times 10^2 \tan(\xi)$ (profile #4).
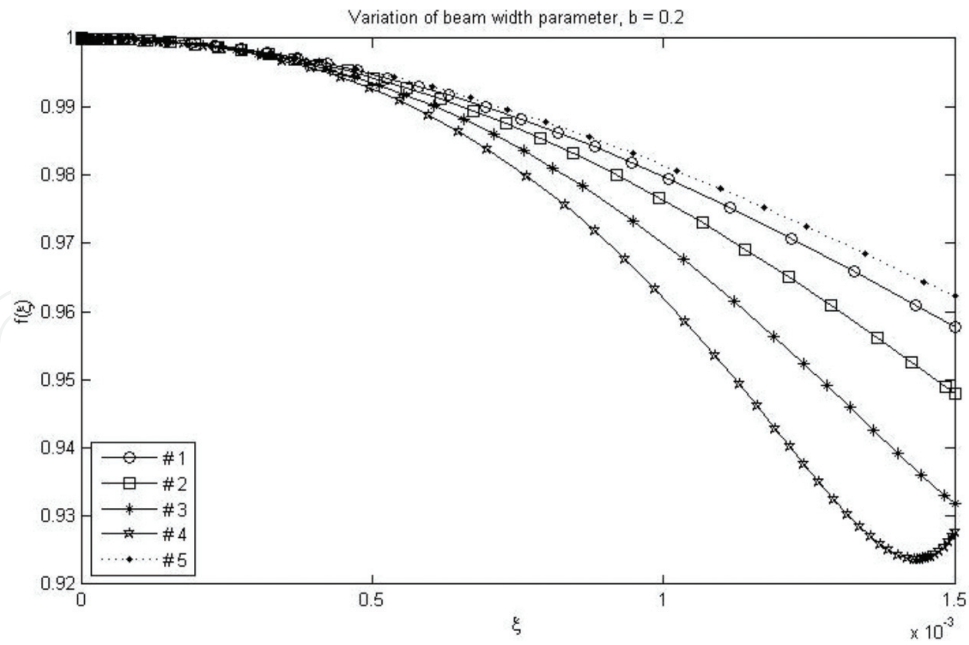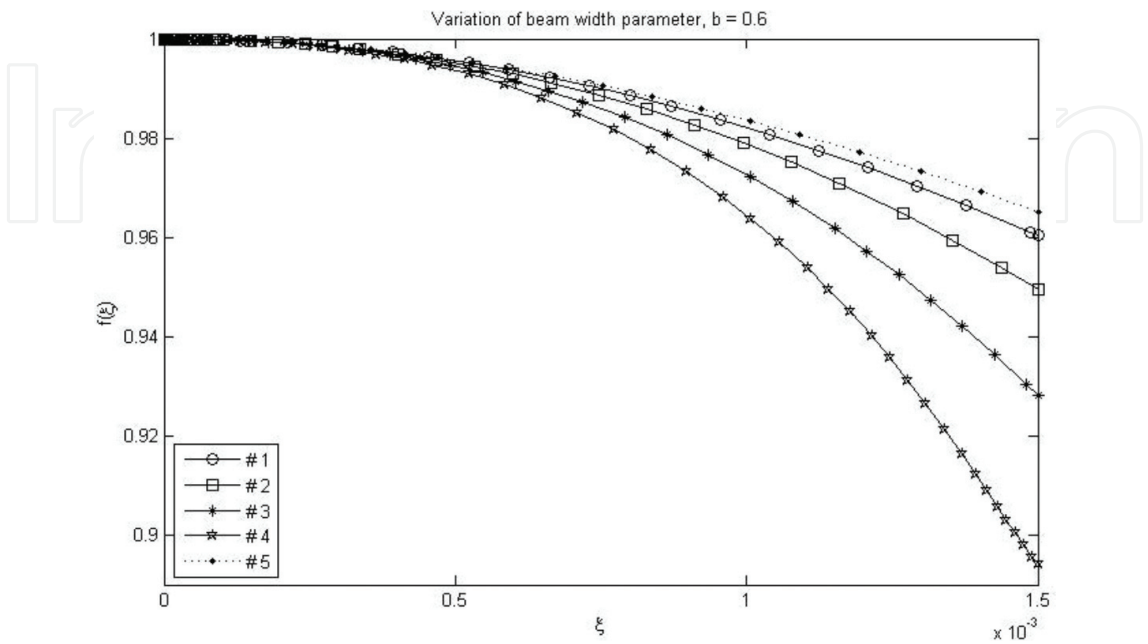
Figure 6. Variation of beam-width parameter through ICQP in the presence of ramp and uniform density profiles for $b = 0.2$.
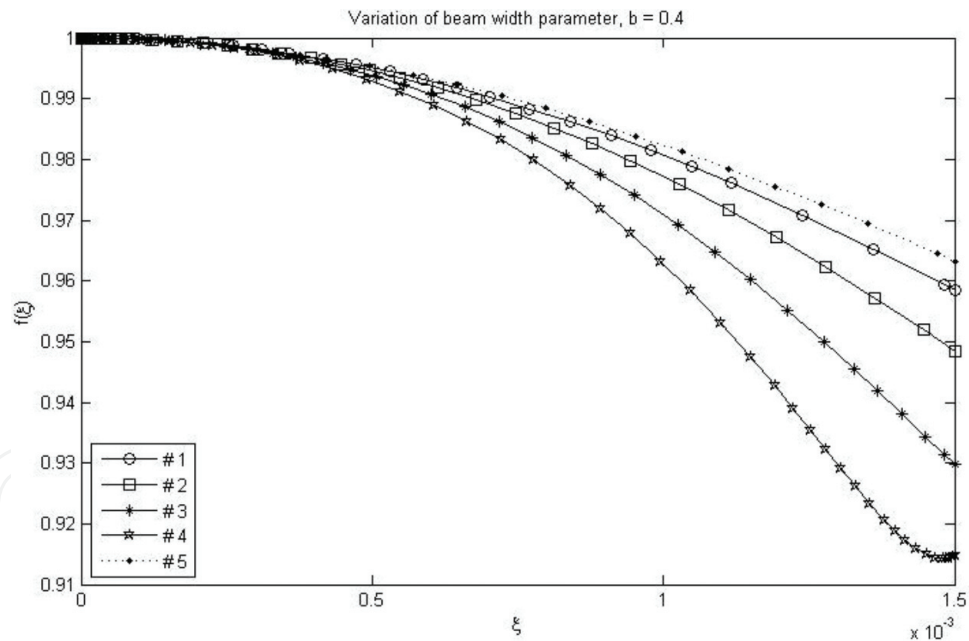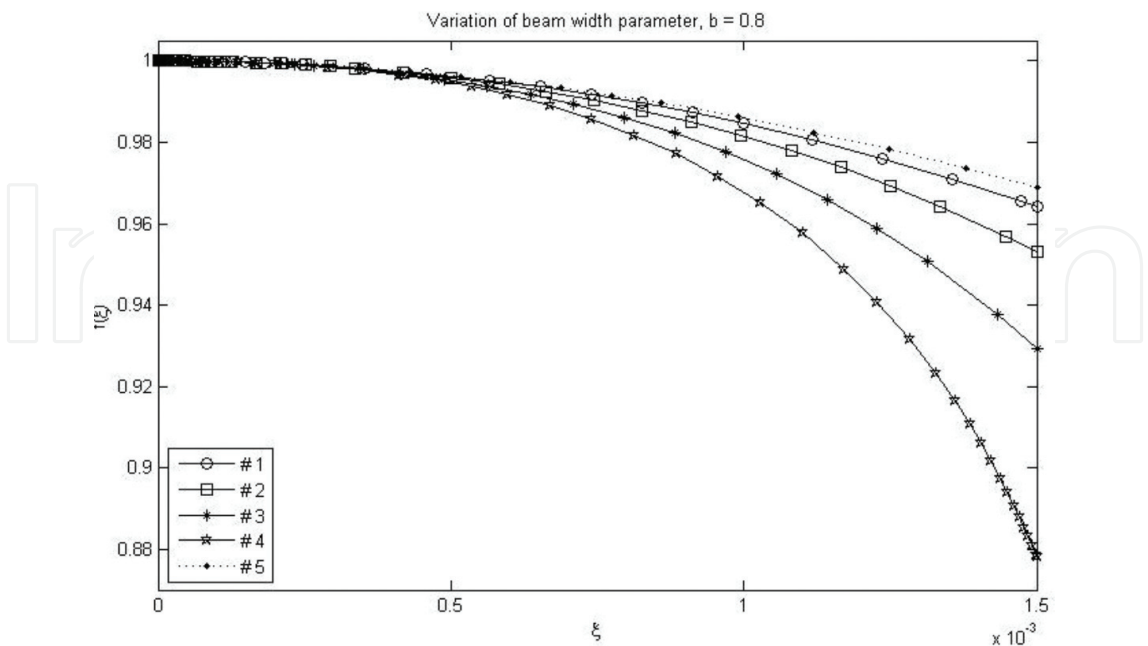

Figure 7. Variation of beam-width parameter through ICQP in the presence of ramp and uniform density profiles for $b = 0.4$.

The linear density profile function $F(\xi) = 1 + (n_1/n_0) \tan (\xi/d)$ for axial inhomogeneity has been chosen. Based on this mathematical function, the slope of ramp density profile is adjustable using appropriate d and n_1 so that the range of plasma density has been adjusted from $n_c/4$ to $n_c/3$, $n_c/2$, $3n_c/4$, and n_c. The profile number of each ramp density function has been

located in the top left-hand corner as a legend in **Figure 5**. **Figures 6–9** illustrate the results of a numerical solution carried out on the self-focusing equation to assess what behaves like a high-intense ChG laser beam through inhomogeneous cold quantum plasma (ICQP).
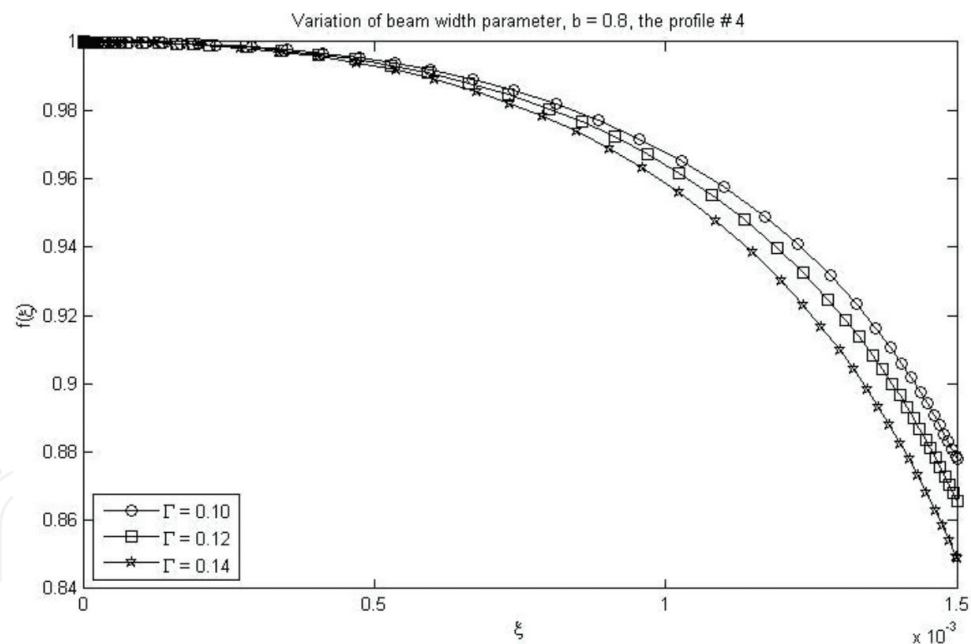

Figure 8. Variation of beam-width parameter through ICQP in the presence of ramp and uniform density profiles for $b = 0.6$.

Figure 9. Variation of beam-width parameter through ICQP in the presence of ramp and uniform density profiles for $b = 0.8$.
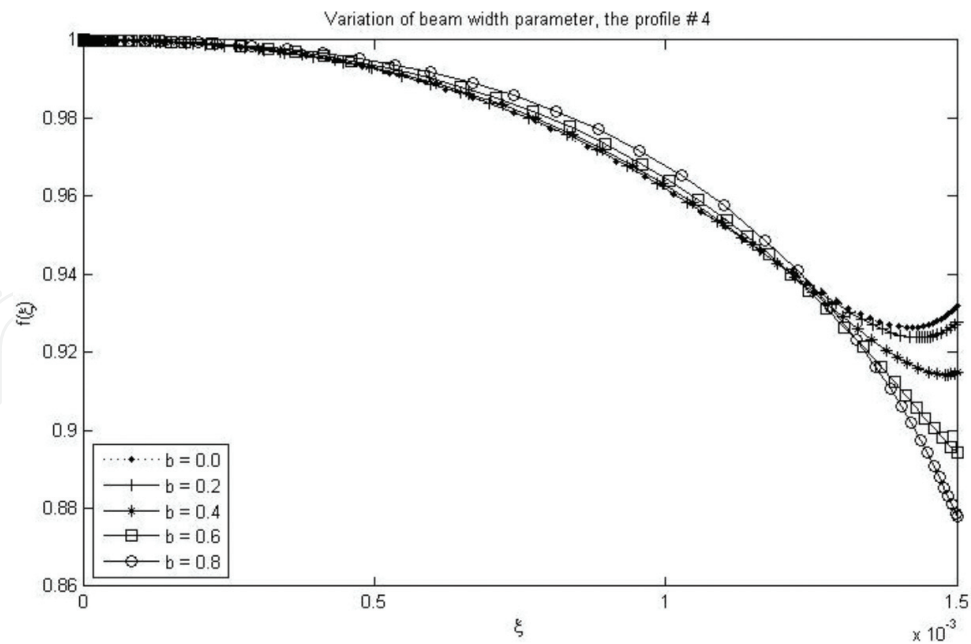
Figure 10. Variation of beam-width parameter through ICQP in the presence of ramp density profile #4 and different decentred parameters: $b = 0.0$ for Gaussian beam and $b = 0.2, 0.4, 0.6, 0.8$ for ChG beam.

Figure 11. Variation of beam-width parameter through ICQP in the presence of ramp density profile #4, $b = 0.8$, and different intensities: $\Gamma = 0.10, 0.12, 0.14$.

In all figures, it is clear that the self-focusing becomes stronger when the slope of ramp density profile increases. Furthermore, it is noticeable that from these figures, for $\xi = 1.5 \times 10^{-3}$, the beam-width parameter decreased significantly when the higher-slope ramp density profile was used. Generally, a comparison among **Figures 6–9** reveals that there was a substantial change from homogeneous to inhomogeneous quantum plasma by using various decentred parameters.

In addition, the influence of decentred parameter on the propagation of ChG laser beam in **Figure 10** is illustrated for the improved density profile (profile #4). Clearly, the final values of spot size for such a laser with b from zero (Gaussian profile) to 0.8 in the $\xi = 1.5 \times 10^{-3}$ dropped significantly.

For more details and better comparison among trends at $b = 0.8$, **Figure 11** shows different intensities. It is obvious that the laser beam becomes self-focused when the initial power of the laser beam increases from 0.10 to 0.14. A significant enhancement in laser self-focusing in underdense plasma with a localized plasma density ramp is observed. It is clear from this figure and the earlier ones that in addition to the type of density profile and decentred parameter, the intensity of laser also plays an effective role in enhancing the focusing of ChG laser beam through the inhomogeneous cold dense plasmas.

4. Conclusion

Generally, we aimed to review our results about self-focusing of high-intense laser beams through various plasmas such as cold and warm quantum plasmas. We, in addition, have introduced a ramp density profile for plasma as an effective factor in improving self-focusing and even ion acceleration. The propagation of cosh-Gaussian intensity profile for high-power and short-wavelength laser was also investigated by means of higher-order paraxial approximation. From these investigations, the effects of ramp density profile and off-axial contribution on enhancing the further focusing of laser beam can be drawn. The results show that inclusion of the quadratic r^4 term in the eikonal function modifies radial profile of the pulse. It can have a direct impact on producing ultra-high laser irradiance over distances much greater than the Rayleigh length which can be used for various laser-plasma applications.

Author details

Mahdi Habibi[1]* and Majid Davoodianidalik[2,3]

*Address all correspondence to: habibi.physics@gmail.com

1 Department of Physics, Payame Noor University (PNU), Tehran, Iran

2 Department of Energy Engineering and Physics, Amirkabir University of Technology, Tehran, Iran

3 Department of Physics, Iran University of Science and Technology, Tehran, Iran

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