CS390 Computational Game Theory and Mechanism Design July 15, 2013

Handout 6: Problem Set 5

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Due by Monday, July 22, 8am.

Problem 1 (10pt). Let the environment E be such that $\forall i \in N, \theta_i \in \Theta_i, o, o' \in O$, $u_i(\theta_i, o) \neq u_i(\theta_i, o')$. That is, the utility function does not have ties. The Condorcet correspondence, f^{CON}, is such that, $\forall \theta \in \Theta$,

$$f^{CON}(\theta) = \left\{ o \;\middle|\; \forall o' \in O, |\{i : u_i(\theta_i, o) > u_i(\theta_i, o')\}| \geq |\{i : u_i(\theta_i, o') > u_i(\theta_i, o)\}| \right\}.$$

That is, for $o \in f^{CON}(\theta)$ and any other o', if the players vote between o and o' according to their utilities, then o gets a (weak) majority.

Prove that f^{CON} satisfies monotonicity.

Problem 2 (10pt). Consider the environment E with n players and m outcomes, $O = \{o_1, \ldots, o_m\}$. Let $\Theta = \Theta_1 \times \cdots \times \Theta_n$, where each Θ_i is the set of all permutations on $\{1, \ldots, m\}$. That is, each θ_i assigns scores $1, 2, \ldots, m$ to the m outcomes. The utilities are $u_i(\theta_i, o) = \theta_i(o)$.

The Borda Court social-choice correspondence (i.e., rank-order voting), f^{BC}, is defined as follows: $\forall \theta \in \Theta$,

$$f^{BC}(\theta) = \left\{ o \;\middle|\; \sum_i \theta_i(o) = \max_{o' \in O} \sum_i \theta_i(o') \right\}.$$

Let $m = 3$ and $n = 2$. Is the corresponding f^{BC} monotone? Prove your conclusion.

Problem 3 (10pt). Consider n-player single-good auctions with $n \geq 3$. Let $\Theta_i = \mathbb{R}^+ \cup \{0\}$ and $O = N \times (\mathbb{R}^+ \cup \{0\})^n$. Is the following social choice correspondence monotone? Prove your conclusion.

$$\forall \theta \in \Theta, \; f(\theta) = \left\{ o = (w, p) \;\middle|\; w \in \operatorname*{argmax}_i \theta_i \right\}.$$