CS390 Computational Game Theory and Mechanism Design July 4, 2013

Problem Set 1

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1. (a) Only one.

   We will prove that the only NE is when each player take Rock, Paper and Scissors with equal probility. For convenience we use a triple to represent the probality of Rock, Paper and Scissors.

   First we prove this is a NE. Notice for each player, given the other's strategy $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, Let this players strategy be (p_1, p_2, p_3), then $u = \sum(\frac{1}{3}p_i - \frac{1}{3}p_i) = 0$, so any mixed strategy is a best response. Particularly, $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is a best mixed response.

   On the other hand, if in another NE one player's mixed strategy (p_1, p_2, p_3), $p_i = \frac{1}{3}$ does not hold. Let's just suppose p_1 is the biggest one. Then the best mixed response for the other will be $(0, 1, 0)$, then (p_1, p_2, p_3) should be $(0, 0, 1)$, that is a controdiction.

   (b) We use a pair to represent the probility of B and S.

   $u(B, B) = (2, 1), u(S, S) = (1, 2)$, others euqal to zero.

   Let $\sigma_1 = (\frac{2}{3}, \frac{1}{3}), \sigma_2 = (\frac{1}{3}, \frac{2}{3})$, we will prove this is a mixed NE. On one hand, iven $\sigma_2 = (\frac{1}{3}, \frac{2}{3})$. Let σ_1 be (p, q). Then $u_1 = 2\frac{1}{3}p + \frac{2}{3}q = \frac{2}{3}(p + q) = \frac{2}{3}$, so the strategy of player 1 is a best mixed response. By symmetry, this is true for player 2 either.

   (c) There are two pure equilibrum in BoS. Both B and both S. Since in each case, for each player, change to another strategy won't get more score.

   (d) IN the constructed game, each player can choose a real number, and his utility is equal to the number he choose. Obviously, no matter what strategy one player choose, he can always change to a better one.