CS390 Computational Game Theory and Mechanism Design July 16, Year

Problem Set 4

F1224004 *5112409048* 刘爽

1. Suppose the player in the center prefer the opposite sides, the players around prefer the same sides. Then the player in the center choose strategy $(\frac{1+\epsilon}{2}H, (\frac{1-\epsilon}{2}T)$ and the players around choose stratege $(\frac{1}{2}H, \frac{1}{2}T)$, the player in the center has reached his best response, which with utility 0. Now consider any other player, his best response is $(1H, 0T)$, the expected utility of which is ϵ, and his current expected utility is 0. Then we can conclude this is an ϵ-NE.

2. First we consider the five players around. For each one of them, let's call him A, suppose the center player's strategy is (pH, qT). If $p = q$, then any strategy for A is a best response. If $p > q$, then A's best response is $(1H, 0T)$. If $p < q$, A's best response is $(0H, 1T)$.

   Next consider the center player. His utility can be calculated by $p(\sum H_i - \sum T_i) + q(\sum T_i - \sum H_i)$. If $q = p$, it is a NE if and only if $\sum H_i = \sum T_i$. If $p > q$, his opponents will all choose $(1H, 0T)$, and the center players best response is $(0H, 1T)$, that is a contradiction. For $p < q$ is the same.

   Finally, we can conclude that all the NE are look like this: the center player's strategy is $(\frac{1}{2}H, \frac{1}{2}T)$, other players can choose arbitrary strategy as long as $\sum H_i = \sum T_i$.

3. For each player p, suppose in the combined CE, p is recommended to choose strategy A. Let $u(A)$ be his expected utility if he chooses A. For any other strategy B for p, let $u(B)$ be his expected utility if he chooses B. We also let $u_i(A)$ and $u_i(B)$ be the utilities in the i_{th} original CE. Now we have

$$\begin{aligned} u(A) - u(B) &= \sum_{i=1}^{k} \lambda_i u_i(A) - \sum_{i=1}^{k} \lambda_i u_i(B) \\ &= \sum_{i=1}^{k} \lambda_i (u_i(A) - u_i(B)) \end{aligned}$$

   Since $u_i(A) \geq u_i(B)$ is always true. Then for each player we have $u(A) \geq u(B), \forall B$.