CS390 Computational Game Theory and Mechanism Design July 4, 2013

Problem Set 1

F1224004 *5112409048* 刘爽

1. (a) Only one.

   We will prove that the only NE is when each player take Rock, Paper and Scissors with equal probility. For convenience we use a triple to represent the probality of Rock, Paper and Scissors.

   First we prove this is a NE. Notice for each player, given the other's strategy $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, Let this players strategy be (p_1, p_2, p_3), then $u = \sum(\frac{1}{3}p_i - \frac{1}{3}p_i) = 0$, so any mixed strategy is a best response. Particularly, $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is a best mixed response.

   On the other hand, if in another NE one player's mixed strategy (p_1, p_2, p_3), $p_i = \frac{1}{3}$ does not hold. Let's just suppose p_1 is the biggest one. Then the best mixed response for the other will be $(0, 1, 0)$, then (p_1, p_2, p_3) should be $(0, 0, 1)$, that is a controdiction.

   (b) We use a pair to represent the probility of B and S.

   $u(B, B) = (2, 1), u(S, S) = (1, 2)$, others euqal to zero.

   Let $\sigma_1 = (\frac{2}{3}, \frac{1}{3}), \sigma_2 = (\frac{1}{3}, \frac{2}{3})$, we will prove this is a mixed NE. On one hand, iven $\sigma_2 = (\frac{1}{3}, \frac{2}{3})$. Let σ_1 be (p, q). Then $u_1 = 2\frac{1}{3}p + \frac{2}{3}q = \frac{2}{3}(p + q) = \frac{2}{3}$, so the strategy of player 1 is a best mixed response. By symmetry, this is true for player 2 either.

   (c) There are two pure equilibrum in BoS. Both B and both S. Since in each case, for each player, change to another strategy won't get more score.

   (d) In the constructed game, each player can choose a real number, and his utility is equal to the number he choose. Obviously, no matter what strategy one player choose, he can always change to a better one.

2. (a) Let's just suppose $n > 1$. First we formulate it as a normal form game.

$$
\begin{aligned}
&N = \{1, 2, 3, \cdots, n\} \\
&S = N_+^n \\
&u_i = \begin{cases} v_i - s_i & s_i = s_{max} \text{ and } i < j \text{ if } s_j = s_{max} \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

   Then, we will prove, if there is a NE, then player 1 obtains the object. If not, then Let k be the one obtains the object, $k \neq 1$. Then we have $s_k \leq v_k$, since any player won't choose to lose money. Thus $s_1 < s_k \leq v_k < v_1$. Then player 1 can change his bid to v_k to win this auction and win non-zero profit.

Afterwards, let's find all the NE. We have proved in a NE, player 1 submits the highest bid. If there are no other player submit a bid as high as player 1, player 1 can change his bid lower. In addition, if player 1 submit a bid lower than $v_2 - 1$, player2 can change his bid to $v_2 - 1$. So far, if a strategy profile is a NE, it should has three properties as following

- $v_1 \geq s_1 \geq v_2 - 1$
- $\forall j, s_j \leq s_1$
- $\exists j, s_j = s_i$

And we can easily verify that if the three properties are satisfied, it will be a NE.

(b) First we formalize weak dominance.
A strategy s_i weakly dominate s_i' if

$$\forall s_{-i}, u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$$

and a strategy is weakly dominant if it weakly dominate any other strategies. Then we will prove for any player i, bid for v_i is a weak dominance. In the first case, player i wins the auction. If he wants to get more profit, he will pay less than he should pay, and thus he cannot win the auction, that is a controdiction. In the second case, player i doesn't win the auction, then obviously he cannot get more profit.
Finally, let's consider a equilibriam in which the winner is not player 1.

$$v_1 = 3, v_2 = 2, v_1 = 1$$

$$s_1 = 1, s_2 = 10, s_3 = 2$$

It is indeed a NE and in this case player 2 wins the auction.

3. First we show the sequence as below.

$$\begin{aligned} S^1 &= \{1, 2, 3, \cdots, 100\}^{15} \\ S^2 &= \{1, 2, 3, \cdots, 99\}^{15} \\ S^3 &= \{1, 2, 3, \cdots, 98\}^{15} \\ &\vdots \\ S^{100} &= \{1\}^{15} \end{aligned}$$

Then we will explain why the sequence is like that.

Here we use a induction. We denote 1/3 of the class average as X. Suppose our last strategy profile is $\{1, 2, 3, \cdots, k\}^{15}$, then for each player, strategy k is strictly dominated by 1, since $X \leq k/3 < (1+k)/2$, which means strategy 1 is always better than strategy k.