Homework 9

Computer Science Theory for the Information Age

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1. Consider a random walk on the positive half line, that is the integers $1, 2, 3, \ldots$. At the origin, always move right one step. At all other integers move right with probability $2/3$ and left with probability $1/3$. What is the escape probability?

   Solution:

   Let us construct a electrical network. The current flow into the 1^{th} node, the resistance between the i^{th} node and the $(i+1)^{th}$ node is $\frac{1}{2^i}$.

   Then the escape probability

   $$P_{escape} = \frac{c_{eff}}{c_a} = \frac{1}{r_{eff}} = \frac{1}{2}$$

2. Using the Metropolis-Hasting Algorithm create a Markov chain whose stationary probability is that given in the following table.

x_1x_2	00	01	02	10	11	12	20	21	22
Prob	1/16	1/8	1/16	1/8	1/4	1/8	1/16	1/8	1/16

   Solution:

Since we have for $i \neq j$

$$p_{ij} = \frac{1}{r} \min(1, \frac{p_j}{p_i})$$

and

$$p_{ii} = 1 - \sum_{j \neq i} p_{ij}$$

We can construct the graph easily,