Homework 8

Computer Science Theory for the Information Age

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June 25, 2013

1. Read in a photo and convert to a matrix. Perform a singular value decomposition of the matrix. Reconstruct the photo using only 10%, 25%, 50% of the singular values.

   (a) Print the reconstructed photo. How good is the quality of the reconstructed photos?

From top to bottom, left to right, are the original picture, 50%, 25%, 10% pictures.

As we can see, the 50% compresion performs quite well, 25% performs a little bit noisy, while 10% has a lot of noisy points.

(b) What percent of the Forbenius norm is captured in each case.

Since I use SVD for the R, G, B matrices seperately, I captured Forbenius norm seperately too.
There are 99.9617% for R, 99.9617% for G, 99.9617% for B, captured in the 50% quality picture.
There are 99.7215% for R, 99.7574% for G, 99.7197% for B, captured in the 25% quality picture.
There are 99.0076% for R, 99.1092% for G, 98.9815% for B, captured in the 10% quality picture.

2. Consider the pairwise distance matrix for twenty US cities given below. Use the algorithm of Exercise 4.30 to place the cities on a map of the US. Suppose you had airline distances for 50 cities around the world. Could ou use these distances to construct a world map?

Solution:

Let X be a 20×2 matrix, Since we have

$$(XX^T)_{ij} = -\frac{1}{2}[d_{ij}^2 - \frac{1}{n}\sum_{j=1}^{n} d_{ij}^2 - \frac{1}{n}\sum_{i=1}^{n} d_{ij}^2 + \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^2]$$

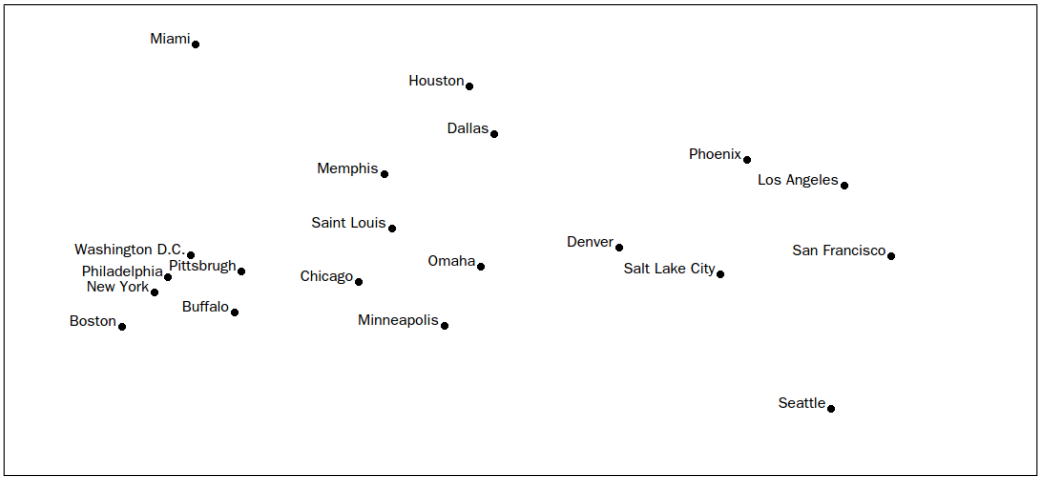
We can easily get XX^T, using singular value decomposition, we can get $XX^T = UDV^T$. Notice that the columns of U are the right singular vectors of $(XX^T)^T = XX^T$, which means $U^T = V^T$, thus $U = V$. Then we get

$$XX^T = VDV^T$$

$$X = \sqrt{D}V$$

where $\sqrt{D}$ means take the square root of each element in the diagonal of D. The rows of X are the coodinates of the cities.

See the map below.



However we cannot use the airline distance around the world to build the map, since the earch is a sphere, not a plane.