Homework 10

Computer Science Theory for the Information Age

致远 12 级 ACM 班　　刘爽　　5112409048

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1. Label the points in the plane that are within the circle of radius one as +1 and label the points in the annulus of inner radius two and outer radius three as -1. Find a function ϕ mapping the points to a higher dimensional space where the two sets are linearly separable.

   Solution:

   Define a function

   $$f : R^2 \to R^3$$
   $$f(x, y) = (x, y, \sqrt{x^2 + y^2})$$

   Then a plane $z = 1$ can separate these two kinds of points.

2. Prove that the VC-dimension of circles is three.

   Proof:

   Let's consider three points forming a non-degenerate triangle. Obviously, any set of them can be shatter by a circle.

   On the other hand, for any four points. In one case, one point is inside the convex hull of the other three points, and we cannot label the inside point +1 and the others -1. Otherwise, we call the four points a, b, c, d in the clockwise order. In this case, we cannot label a, b, c, d using one circle with +1, -1, +1, -1, and using another with -1, +1, -1, +1. If we can do this, we use two circles to divide the plane into four regions, which is not possible.

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