Homework

Computer Science Theory for the Information Age

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1. What is the expected number of squares (4-cycles) in $G(n, \frac{d}{n})$ What is the expected number of 4-cliques in $G(n, \frac{d}{n})$?

   Solution:

   Let X be the number of squares and Y be the number of 4-cliques.

$$\begin{aligned} E(X) &= \frac{1}{2}\frac{n(n-1)(n-2)(n-3)}{4}(\frac{d}{n})^4 \\ &= \frac{d^4(n-1)(n-2)(n-3)}{8n^3} \end{aligned}$$

$$\begin{aligned} E(Y) &= \frac{n(n-1)(n-2)(n-3)}{4!}(\frac{d}{n})^6 \\ &= \frac{d^6(n-1)(n-2)(n-3)}{24n^5} \end{aligned}$$

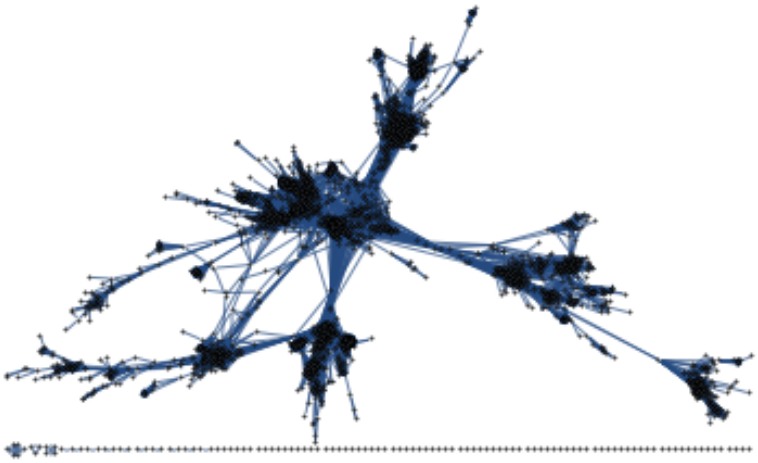
2. Search for WWW for an undirected graph or a data base that can be counted to a graph. find the connected components and count the number of each size.

Solution:

I find a dataset in **http://snap.stanford.edu/data/egonets-Facebook.html**, which provides 'circles'(or 'friends list')from Facebook. This data was collected from survey participants using this Facebook app. For the convenience of computation, I simply remove the node with index zero, and finally get a graph which has 4038 nodes and 88233 edges. Here is a piece of Mathematica code to deal with it.

```
rawData = Flatten[Import[#] & /@ FileNames["~/
MCS-homework/Course-2013-6/Homework4/*.dat"], 1];
Data = DeleteDuplicates[Map[If[#[[1]] < #[[2]],
       #, {#[[2]], #[[1]]}] &, rawData]];
g = Graph[Data]
Tally[Map[Length, ConnectedComponents[g]]]
```

And here is what the network looks like



And the commponents' size distribution, in the form '{size, count}'

```
{{3926, 1}, {6, 1}, {4, 1}, {3, 2}, {2, 8}, {1, 80}}
```