UNIVERSITY

Name : Sadman

Institute : 011221592

Subjects : Vector

Roll No : ______ Section : C

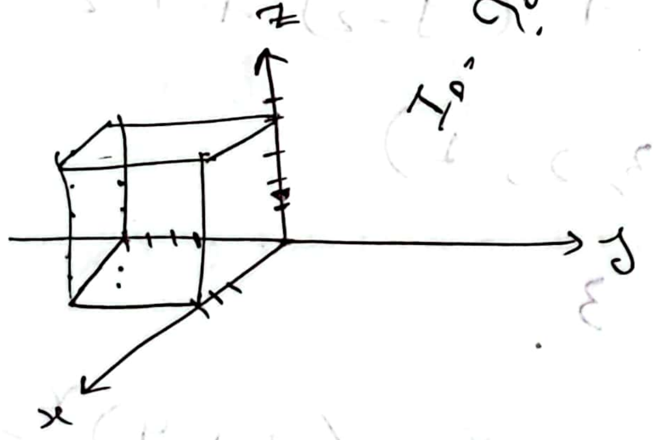
CW 28.5.23

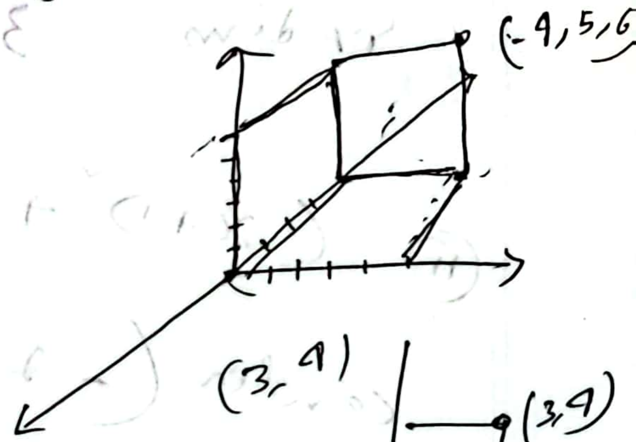
11

Vector

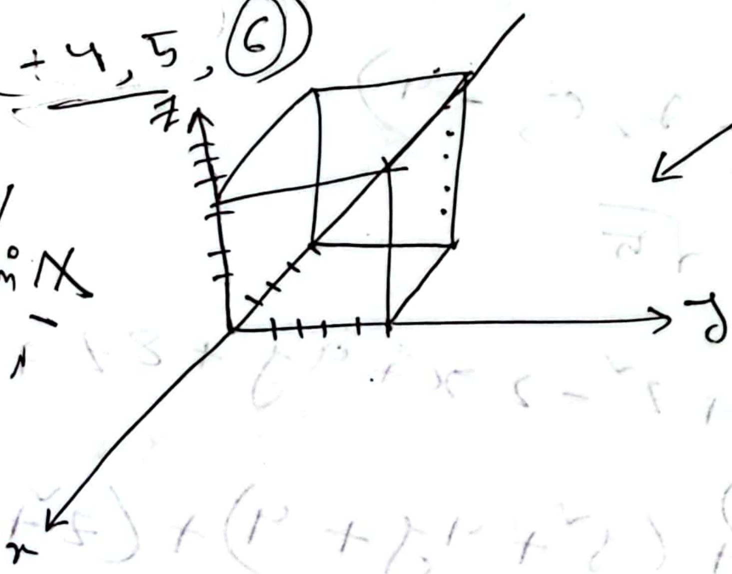
$I_D = ?d \; |V_D| / |V_N|$

*





* (+4, 5, 6)

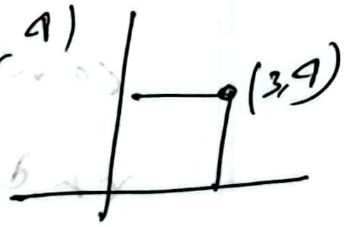


$30 \rightarrow 25$

$20 \rightarrow \frac{25}{30}$

$\frac{25 \times 20}{30} = 17$

(3, 4)



* distance between two points. (2D)

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$3D \rightarrow D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

(i) $(x-3)^2 + (y-2)^2 + (z-1)^2 = 9$

center $(3, 2, 1)$

radius $= 3$

(ii) $(x+1)^2 + y^2 + (z+4)^2 = 5$

center $(-1, 0, -4)$

radius $= \sqrt{5}$

(iii) $x^2 + y^2 + z^2 - 2x + 4y + 8z + 17 = 0$

$\Rightarrow (x^2 - 2x + 1) + (y^2 + 4y + 4) + (z^2 + 8z + 16) = -17 + 1 + 4 + 16$

$\Rightarrow (x-1)^2 + (y+2)^2 + (z+4)^2 = 4$

$\Rightarrow (1, -2, -4) = 2$

2.

$$x^2 - 6x + y^2 - 4y + z^2 = 12$$

$$\Rightarrow (x^2 - 6x + 9) + (y^2 - 4y + 4) + z^2 = 12 + 9 + 4$$

$$\Rightarrow (x-3)^2 + (y-2)^2 + z^2 = 25$$

$$(3, 2, 0), \; 5^2 \Rightarrow 5$$

3.

$$(x^2 + 2x + 1) + (y^2 - 4y + 4) +$$
$$(z^2 - 6z + 9) = 13 + 1 + \cancel{9} +$$

$$\Rightarrow (x+1)^2 + (y-2)^2 + (z-3)^2 = 27$$

$$\Rightarrow (-1, 2, 3) = \sqrt{27}$$

shortest distance between

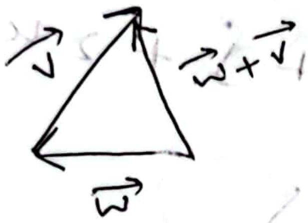
2, -2, 0

$$\therefore D = \sqrt{(2-4)^2 + (-2-0)^2 + (0-5)^2}$$

$$= \sqrt{38}$$

C.W
31.5.23

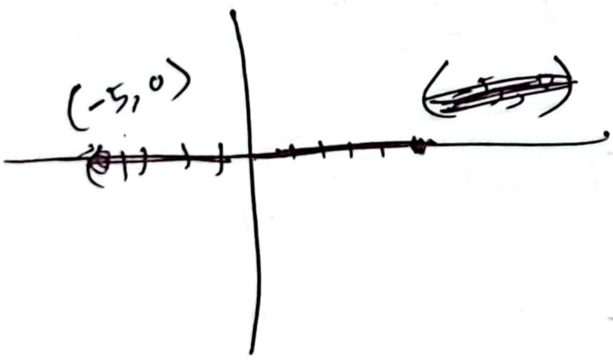
Vector



6

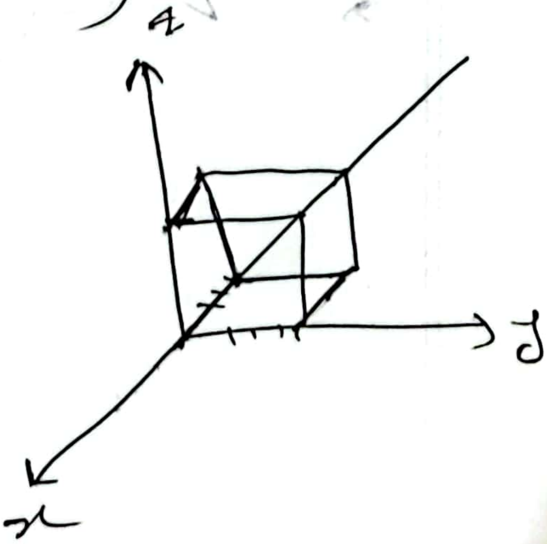
(a) $(-3-2,\ 3-3)$

$(-5,\ 0)$



(b) $(0-3,\ 4-0,\ 4-4)$

$(-3,\ 4,\ 0)$



7/

(b) $v = -3i + j + 2k$

$(5, 0, -1) \longrightarrow (x, y, z) = (8, -1, 3)$

$5 - x = -3$

$\Rightarrow x = 8$

$0 - y = 1$

$\Rightarrow y = -1$

$-1 + z = 2$

$\Rightarrow z = 3$

P.T.O.

15

(a) $u + v = 2i - 2j + 2k$

$\therefore \|u + v\| = \sqrt{2^2 + 2^2 + 2^2}$

$= \sqrt{4 + 4 + 4}$

$= \sqrt{12}$

(b) $u = \sqrt{1^2 + 3^2 + 2^2}$

$= \sqrt{1 + 9 + 4} = \sqrt{14}$

$v = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\therefore u + v = \sqrt{14} + \sqrt{2}$

P.T.

(c) $-2u = 2i - 6j + 4k$

$$\|-2u\| = \sqrt{2^2 + 6^2 + 4^2}$$

$$= \sqrt{4 + 36 + 16}$$

$$= \sqrt{56}$$

$$v = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$2\|v\| = 2\sqrt{2}$$

$$\therefore \|-2u\| + 2\|v\| = \sqrt{56} + 2\sqrt{2}$$

PTO

31/

$2u - v - w = 7x - x$

$\Rightarrow 6x = 2u - v - w$

$\Rightarrow x = \frac{1}{3}u - \frac{v}{6} - \frac{w}{6}$

$= \frac{1}{3}(1, 3) - \left(\frac{2}{6}, \frac{1}{6}\right)$

$= \left(\frac{1}{3}, \frac{3}{3}\right) - \left(\frac{2}{6}, \frac{1}{6}\right) - \left(\frac{4}{6}, -\frac{1}{6}\right)$

$=$

Sunday - Wednesday   Room: 310

9:30 - 10:45

Saturday - Tues (12:45 - 1:30)

4.6.23

Dot Product:

$\vec{U} = (U_1, U_2, U_3)$

$\vec{V} = (V_1, V_2, V_3)$

$\Rightarrow \vec{U} \cdot \vec{V} = (U_1 V_1) + (U_2 V_2) + (U_3 V_3)$

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

$Proj_{\vec{u}} \vec{v} = ?$

C.T (Next class) $\rightarrow$ (Sunday)

11.1

23.

$$x^2+y^2+z^2+10x+4y+2z-19=0$$

$$\Rightarrow (x^2+10x+25)+(y^2+4y+4)+(z^2+2z+1) = 19+25+4+1 = 49$$

$$\Rightarrow (x+5)^2+(y+2)^2+(z+1)^2 = 49$$

$\therefore$ center : $(-5,-2,-1)$

radius : 7 Ans.

11.2

(a) (i) $\quad (1,-2) \xrightarrow{\vec{v}=3i-2j} (x,y)$

$$x-1=3$$

$$\Rightarrow x=4$$

$$y-(-2)=-2$$

$$\Rightarrow y+2=-2$$

$$\Rightarrow y=-4$$

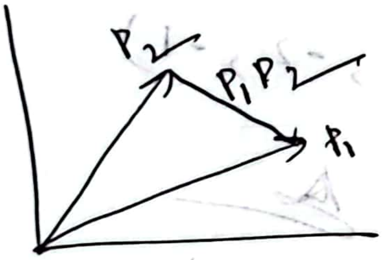
(b)

$$(x, y, z) \xrightarrow{v = (-3, 1, 2)} (5, 0, -1)$$

$\Rightarrow 5 - x = -3$ | $0 - y = 1$ | $-1 - z = 2$

$\Rightarrow -x = -3 - 5$ | $\Rightarrow y = -1$ | $\Rightarrow -z = 3$

$\Rightarrow x = 8$ | | $\Rightarrow z = -3$



15.

(c) $\|-2u\| = |-2|\ \|v\|$

$= 2 \cdot \sqrt{14}$

$2\|v\| = 2\sqrt{2}$

$\therefore \|-2u\| + 2\|v\| = 10.312$

11.3

15.

(a) $v = i + j - k$

$\|v\| = \sqrt{3}$

$$\frac{v}{\|v\|} = \frac{1}{\sqrt{3}} i + \frac{1}{\sqrt{3}} j - \frac{1}{\sqrt{3}} k$$

$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.73°$

$\beta = 54.73°$

$\gamma = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) = 125.26°$

16.

(b) $v = 3i - 4k$, $\|v\| = 5$

$$\frac{v}{\|v\|} = \frac{3}{5} i - \frac{4}{5} k$$

~~cos~~ $\alpha = \cos^{-1}\left(\frac{3}{5}\right) = 53.13°$

$\beta = 90°$

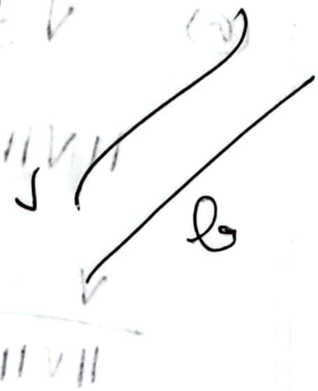
$\gamma = \cos^{-1}\left(-\frac{4}{5}\right) = \cancel{36.87}$ 143.13°

Parallel_V

25

(a) $v = 2i - j + 3k$ $\quad$ $proj_b v =$

$b = i + 2j + 2k$



$\|b\| = 3, \quad \|b\|^2 = 9$

$$Proj_b v = \frac{v \cdot b}{\|b\|^2} \cdot b$$

$$= \frac{2-2+6}{9}(i+2j+2k)$$

$$= \frac{\cancel{6}^{2}}{\cancel{9}_{3}}(i+2j+2k)$$

$$= \frac{2}{3}i + \frac{4}{3}j + \frac{4}{3}k$$

the vector component of v orthogonal to b :⇒

$$v - Proj_b v = (2i - j + 3k) - \left(\frac{2}{3}i + \frac{4}{3}j + \frac{4}{3}k\right)$$

$$= \frac{4}{3}i - \frac{7}{3}j + \frac{5}{3}k$$

37.

$W = Fs \cos 60^\circ$

$= 50 \times 15 \cos 60^\circ$

$= 375$ ft. lb.

Ans

38.

$\vec{F} = -3j$

$P = (1, 3)$, $Q = (4, 7)$

$\vec{PQ} = (3, 4)$

$\therefore W = \vec{F} \cdot \vec{PQ} = (-3j) \cdot (3i + 4j)$

$\cancel{= (-3j) \cdot (3i + 4j)}$

$\cancel{= |-12| = 12\,j}$

$= |-12| = 12$ ft. lb.

Ans

11.4

3

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ -4 & 1 & 2 \end{vmatrix}$$

$$= i(4+3) - j(2-12) + k(1+8)$$

$$= 7i + 10j + 9k$$

$\cancel{\therefore \|u \times v\| = 18.06}$

$$u \cdot (u \times v) = (i + 2j - 3k)(7i + 10j + 9k)$$

$$= \cancel{7 + 28 - 27}\ 7 + 20 - 27$$

$$= -20 + 20$$

$$= 0$$

Ans

24

$u = 2i - 3j + k$

$v = 4i + j - 3k$

$w = j + 5k$

$\therefore u \cdot (v \times w) = \begin{vmatrix} 2 & -3 & 1 \\ 4 & 1 & -3 \\ 0 & 1 & 5 \end{vmatrix}$

$= 80$

25

$u \cdot (v \times w) = \begin{vmatrix} 2 & -6 & -2 \\ 0 & 4 & -2 \\ 2 & 2 & -4 \end{vmatrix}$

$= |-16| = 16$ units

17

$$U \times V = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 0 & 3 & 1 \end{vmatrix}$$

$$= i(-1-6) - j(1-0) + k(3-0)$$

$$= -7i - j + 3k$$

$\therefore \|U \times V\| = \sqrt{59} = 7.68$ units

Ans

19

$P(1, 5, -2)$, $Q(0, 0, 0)$, $R(3, 5, 1)$

$PQ = -1, -5, 2$

$PR = 2, 0, 3$

P.T.O

$$PQ \times PR = \begin{vmatrix} i & j & k \\ -1 & -5 & 2 \\ 2 & 0 & 3 \end{vmatrix}$$

$\Rightarrow S$

$= i(-15-0) - j(-3-4) + k(0+10)$

$= -15i + 7j + 10k$

$\therefore \|S\| = \sqrt{374} = 19.34$

$\therefore$ Area of triangle $= \dfrac{19.34}{2}$

$= 9.67$ units

27

(r)

$$v \cdot (v \times w) = \begin{vmatrix} 1 & -2 & 1 \\ 3 & 0 & -2 \\ 5 & -4 & 0 \end{vmatrix}$$

$= 0$

Ans

CW 7.6.23

12.5

Vectors

Parametric equation of line:

$$x = x_0 + at$$

$$y = y_0 + bt$$

$$z = z_0 + ct$$

11.5

5, 6, 7, 8, 9, 10

9

(a) $(x, y) = (-3, 4) + t(2, 5)$

(b) $(x, y, z) = (2, -3, 0) + t(-1, 5, 1)$

10

(a) $(x, y) = (0, -2) + t(2, 1)$

(b) $(x, y, z) = (1, -7, 4) + t(2, 3, -5)$

C.W ~~13.6.23~~ ~~11.5~~

~~29~~

$x = 2 + t_1$, $y = 2 + 3t_1$, $z = 3 + t_1$

$x = 2 + t_2$, $y = 3 + 4t_2$, $z = 4 + 2t_2$

$2 + t_1 = 2 + t_2$

$\Rightarrow$ ~~$2 + t_1 \ldots$~~

$\Rightarrow t_1 - t_2 = -2 + 2$

$= 0$

$\Rightarrow$

$2 + 3t_1 = 3 + 4t_2$

$\Rightarrow 3t_1 - 4t_2 = 1$

$\Rightarrow t_1, t_2 = \left(\cancel{\ldots, \tfrac{7}{2}}\right) = (-1, -1)$

$\Rightarrow z = \cancel{3 + 5 = 8}$,

$\Rightarrow 3 - 1 = 2$

$z = \cancel{4 + 2 \times \tfrac{7}{2}}$

$= 4 + 2 \times (-1)$

$= 2$

~~33~~, ~~35~~, ~~37~~, ~~19~~, ~~21~~, ~~23~~, ~~27~~, ~~29~~, ~~30~~

C.W 14.6.23

11.6

Vector

15, 16, 17, 18, 25, ~~27, 29, 31, 33,~~
~~35, 37~~, 19, 20

41, 42, 43, 44, 45, 46, 47, 48

C.W 18.6.23

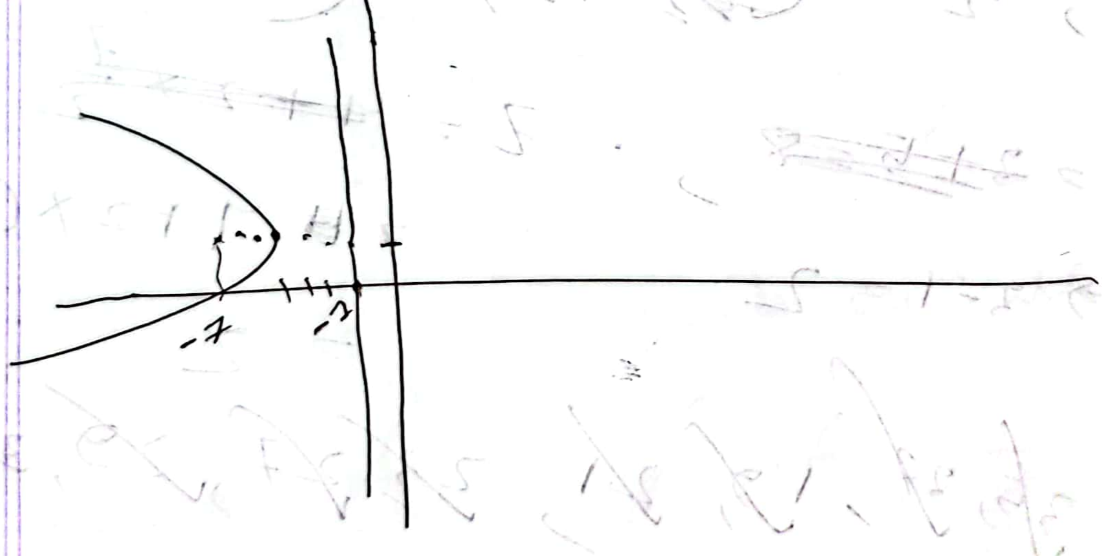
Vertex $(-4, 2)$

5

(a) $(y-2)^2 = -4 \cdot 3\,(x+4)$

$y = 2$

$x = -4$



6

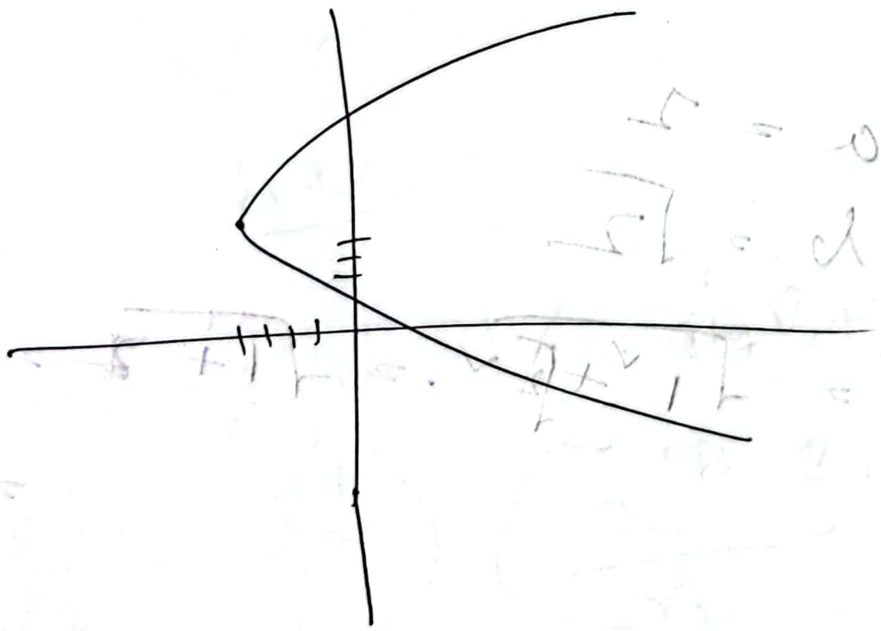
$$(y^2 - 6y + 9) - 2x + 1 = \cancel{8}\, 9$$

$$\Rightarrow (y-3)^2 = 2x + 8$$

$$\Rightarrow (y-3)^2 = 2(x+4)$$

$$\Rightarrow (y-3)^2 = 4 \cdot \frac{1}{2}(x+4)$$

$(-4, 3)$



CW 21.6.23

Hyperbola

Vector

~~Ellipse~~

13

(b) $16(x+1)^2 - 8(y-3)^2 = 16$

$\Rightarrow \dfrac{(x+1)^2}{1} - \dfrac{(y-3)^2}{2} = 1$

$\Rightarrow \dfrac{(x+1)^2}{1} - \dfrac{(y-3)^2}{2} = 1$

$a = 1$

$b = \sqrt{2}$

$c = \sqrt{1^2 + \sqrt{2}^2} = \sqrt{1+2} =$

$= \sqrt{3}$

CW 25.6.23

Vector

Rotation of Axes

$0 < \theta < 90^\circ$

$xy = 1$

$\cot 2\theta = \frac{0-0}{1}$

23, 24, 25, 26

$\cot 2\theta = 0$

$2\theta = 90^\circ$

$\theta = 45^\circ$

$\Rightarrow xy = 1$

$\Rightarrow (x'\cos\theta - y'\sin\theta)(x'\sin\theta + y'\cos\theta) = 1$

$\Rightarrow \left(\frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}\right)\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right) = 1$

$\Rightarrow \frac{x'^2}{2} - \frac{y'^2}{2} = 1$

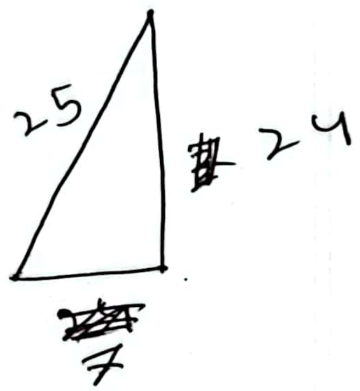
9/

$$9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$$

$$\cot 2\theta = \frac{16-9}{-24}$$

$$= +\frac{7}{24}$$

VL



$$\cancel{\cos 2\theta = \frac{24}{25}}$$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{}}$$

$$\cos 2\theta = \frac{7}{25}$$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \frac{4}{5}$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \frac{3}{5}$$

CW 26.7.23

12.1

Vector

~~20~~

$\cancel{f(x,y) = (x-y)(x+y)^{-1}}$

$\cancel{= (x+y) \cdot -1 (x+y)^{-2} +}$

$\cancel{(x+y)^{-1} \cdot 1}$

$$f(x,y) = \frac{x-y}{x+y}$$

$f(x,y) = (-1,-2)$

$f(x) = (x-y)(x+y)^{-1}$

$= (x-y) \cdot -1 (x+y)^{-2} \cdot 1 +$

$(x+y)^{-1} \cdot 1 = -\frac{4}{9} i$

$f(y) = (x-y) \cdot -1 (x+y)^{-2} \cdot 1 +$

$(x+y)^{-1} \cdot 1$

$= -\frac{4}{9} j$

$\vec{u} = \cos\theta \, \hat{i} + \sin\theta \, \hat{j}$ $\quad \theta = 90^\circ$

$\Rightarrow \vec{u} = 0 \, \hat{i} + 1 \, \hat{j}$

$D_u f = 0 + \left(-\frac{4}{9}\right)$

$= -\frac{4}{9}$ $\cancel{\theta}$

$$\boxed{D_u f = (f_x \hat{i} + f_y \hat{j}) \cdot \vec{u}}$$

$P(-1, +1)$

$\cancel{53}$

$f_x = \cancel{4x^3 \cdot 2y + y^2 \cdot 12x^2}$

$= \cancel{9} \, 4x^3 y^2$

$= 12x^2 y^2 = 12$

$f_y = 8x^3 y = -8$

$\nabla f(-1,1) = 12\hat{i} - 8\hat{j}$

$\Rightarrow \vec{v} = \dfrac{12}{4\sqrt{13}}\hat{i} - \dfrac{8}{4\sqrt{13}}\hat{j}$

$= \dfrac{3}{\sqrt{13}}\hat{i} - \dfrac{2}{\sqrt{13}}\hat{j}$

$\phi(x,y) = -\dfrac{1}{(x^2+y^2)^{\frac{1}{2}}}$

$f_x = -(x^2+y^2)^{-\frac{1}{2}}$

$= \frac{1}{2}(x^2+y^2)^{-\frac{3}{2}} \cdot (2x \cancel{+ y^2})$

$f_y = \frac{1}{2}(x^2+y^2)^{-\frac{3}{2}} \cdot (\cancel{x^2} + 2y)$

$$\operatorname{Curl} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & 2y^3 z & 3z \end{vmatrix}$$

$$\cancel{= i\left(\frac{\partial}{\partial y}(3z) - \frac{\partial}{\partial z}(2y^3 z)\right) - j}$$

$$\left(\frac{\partial}{\partial x}(3z) - \frac{\partial}{\partial z}(x^2 y)\right) +$$

$$k\left(\frac{\partial}{\partial x}(2y^3 z) - \frac{\partial}{\partial y}(x^2 y)\right)$$

$$\cancel{= i\left(3z - 2y^3\right) - j\left(3z - x^2 y\right) +}$$

$$\cancel{k\left(2y^3 z - x^2\right)}$$

$$= i(0 - 2y^3) - j(0 - 0) + k(0 - x^2)$$

$$= -2y^3 i - x^2 k$$

$\cancel{15}$ SSS

$\cancel{20}$

$$f(x,y,z) = e^{xy}\hat{i} + \cos xy\,\hat{j} + \sin^2 z\,\hat{k}$$

$\operatorname{div} F =$

$$f_x = e^{xy} = e^{xy} \cdot y$$

$$f_y = -\sin xy \cdot x$$

$$f_z = 2 \sin z \cdot \cos z$$

C.W 28.2.23 15.2

line Integral

Vector

$* \; M = \int_C f(x(t), y(t), z(t)) \, \|r'(t)\| \, dt$

Exm 1

(a) $\int_C (1 + xy^2) \, ds$

$x = t$

$y = 2t$

$C: r(t) = t\hat{i} + 2t\hat{j}$

$0 \leq t \leq 1$

$\Rightarrow \int_0^1 (1 + (2t)^2) \, ds$

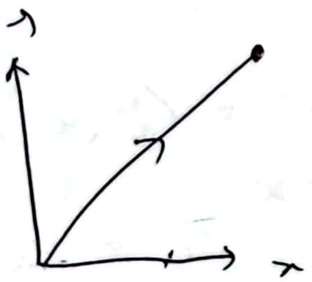
$ds = \|r'(t)\| \, dt$

$r'(t) = \hat{i} + 2\hat{j}$

$\|r'(t)\| = \sqrt{1^2 + 2^2}$

$= \sqrt{5}$

$\Rightarrow \int_0^1 (1 + t(2t)^2) \sqrt{5} \, dt$

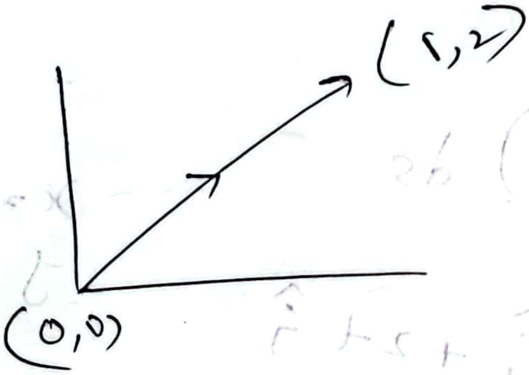


$r = r_0 + (r_1 - r_0) t$

13.6, 15.1, 15.2

Exm: 5

(a) $(0,0)$ to $(1,2)$



for x,

$r = r_0 + (r_1 - r_0)t$

$= 0 + (1-0)t$

$= t$

for, y

$r = r_0 + (r_1 - r_0)t$

$= 0 + (2-0)t$

$= 2t$

$\therefore (x,y) = (t, 2t)$

13.6

Exm: 1

$f(x, y) = xy$ $P(1, 2)$

$f_x = y = 2$

$f_y = x = 1$

$\therefore d_u f = (2, 1) \cdot \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$= \frac{1 + 2\sqrt{3}}{2} = 2.23$ Ans

Exm: 2

$v = \cos\frac{\pi}{3} i + \sin\frac{\pi}{3} j$ $P = (-2, 0)$

$= \frac{1}{2} i + \frac{\sqrt{3}}{2} j$

$= \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$f(x, y) = e^{xy}$

$f_x = e^{xy} \cdot y = 0$

$f_y = e^{xy} \cdot x = -2$

$\therefore d_u f = (0, -2) \cdot \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$= -\sqrt{3}$

≈ -1.732

Ans

Exm : 3

$a = 2i + j - 2k$

$u = \frac{2}{3}i + \frac{1}{3}j - \frac{2}{3}k = \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$

$P = (1, -2, 0)$

$f(x, y, z) = x^2 y - y z^3 + z$

$f_x = 2xy = -4$

$f_y = x^2 - z^3 = 1$

$f_z = -3yz^2 + 1 = 1$

$\therefore d_u f = (-4, 1, 1) \cdot \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$

$= -3$ Ans

Exm: 4

$f(x,y) = x^2 e^y$, $P(-2,0)$

$f_x = 2xe^y = -4$

$f_y = x^2 e^y = 4$

$\nabla f = (-4, 4)$

$\|\nabla f\| = 4\sqrt{2}$

$\therefore u = -\frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j$ Ans

Exercise

$(1-23)$

4

$f(x,y) = \frac{cx+dy}{x-y}$ $P(3,4)$

$f_x = (cx+dy)(x-y)^{-1}$

$= (cx+dy) \cdot -(x-y)^{-2} \cdot 1 + (x-y)^{-1} \cdot c$

$= -(3c+4d) - c$

$= -4c - 4d$

P.T.O

$f_y = \left((cx+dy)\cdot(x-y)^{-1}\right)$ $(3,4)$

$= \left[(cx+dy)\cdot -(x-y)^{-2}\cdot(-1) + (x-y)^{-1}\cdot d\right]$

$= (cx+dy) - d = (3c+4d) - d$

$= 3c+3d$

$d_u f = (-4c-4d,\ 3c+3d)\cdot\left(\frac{4}{5}, \frac{3}{5}\right)$

$= (-4c-4d)\times\frac{4}{5} + (3c+3d)\times\frac{3}{5}$

$= -\frac{16}{5}c - \frac{16}{5}d + \frac{9}{5}c + \frac{9}{5}d$

$= -\frac{7}{5}c - \frac{7}{5}d$

18

$a = 20i - 4j + 5k$ $\qquad P(-2, 2, -1)$

$$u = \frac{20}{21}\hat{i} - \frac{4}{21}\hat{j} + \frac{5}{21}\hat{k}$$

$$f_{xyz} = e^{x+y+3z}$$

$$f_x = e^{x+y+3z} \cdot 1 = 0.05$$

$$f_y = e^{x+y+3z} \cdot 1 = 0.05$$

$$f_z = e^{x+y+3z} \cdot 3 = 0.15$$

$$\therefore d_u f = (0.05, 0.05, 0.15) \cdot \left(\frac{20}{21}, -\frac{4}{21}, \frac{5}{21}\right)$$

$$= \frac{31}{420} \approx 0.07$$

Ans

~~27~~

$u = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$P = (30, 60)$

$f(x, y) = \tan(2x + y)$

$f_x = \sec^2(2x + y) \cdot 2 = 8$

$f_y = \sec^2(2x + y) = 4$

$\therefore d_u f = (8, 4) \cdot \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$= 2\sqrt{2}$ ~~Ans~~

(33 - 40, 41 - 46)

15.1

Exm: 4

$F(x,y,z) = x^2 y\, i + 2y^3 z\, j + 3z\, k$

$\operatorname{div} F = \frac{\partial}{\partial x}(x^2 y) + \frac{\partial}{\partial y}(2y^3 z) + \frac{\partial}{\partial z}(3z)$

$= 2xy + 6y^2 z + 3$

$\operatorname{curl} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f & g & h \end{vmatrix}$

$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & 2y^3 z & 3z \end{vmatrix}$

$= i\left(\frac{\partial}{\partial y} 3z - \frac{\partial}{\partial z} 2y^3 z\right) - j\left(\frac{\partial}{\partial x} 3z - \frac{\partial}{\partial z} x^2 y\right) + k\left(\frac{\partial}{\partial x} 2y^3 z - \frac{\partial}{\partial y} x^2 y\right)$

$= i(0 - 2y^3) - j(0 - 0) + k(0 - x^2)$

$= -2y^3 i - x^2 k$

15.2 (17-22), (22-27), (28-30)

22

$$\int \frac{e^{-z}}{x^2+y^2}\, ds$$

$$= \sqrt{5}\int \frac{e^{t}}{4}\, dt$$

28

$$\int_C (y-x)\, dx + xy\, dy$$

$$r = r_0 + (r_1 - r_0)$$

$$= (3,4) + \{(2,1) - (3,4)\}\, t$$

$$= (3,4) + (-1,-3)\, t$$

$$= (\underbrace{3-t}_{x},\ \underbrace{4-3t}_{y})$$

$$\int_0^1 (4-3t-3+t)(-1)\,dt + \int_0^1 (3-t)(4-3t)(-3)\,dt$$

~~34(b)~~

~~15.3~~

~~Exm: 1~~

$$\sinh = \frac{e^x - e^{-x}}{2}$$

$$\cosh = \frac{e^x + e^{-x}}{2}$$

19)

$$\int_C \frac{1}{1+x}\, ds \qquad 0 \leq t \leq 3$$

C: $r(t) = t\,i + \frac{2}{3} t^{\frac{3}{2}}\, j$

$r'(t) = i + t^{\frac{1}{2}}\, j$

$$ds = \sqrt{(1)^2 + (t^{\frac{1}{2}})^2}\, dt$$

$$= \sqrt{1+t}\, dt$$

$$\therefore \int_0^3 \frac{1}{1+t} \sqrt{1+t}\, dt$$

$= 2$

20.

$$\int_c \frac{x}{1+y^2}\, ds \qquad 0 \le t \le 1$$

$x = 1+2t, \quad y = t$

$$ds = \sqrt{(2)^2 + (1)^2}\, dt$$

$$= \sqrt{5}\, dt$$

$$\therefore \int_0^1 \frac{1+2t}{1+t^2} \sqrt{5}\, dt$$

$$= 3.31$$

P.T.O

21

$$\int_C 3x^2yz\,ds \qquad 0<t<1$$

$x=t,\quad y=t^2,\quad z=\frac{2}{3}t^3$

$\therefore ds=\sqrt{1^2+(2t)^2+(2t^2)^2}\,dt$

$$\therefore \int_C 3x^2yz\,ds = 0.730$$

22

$$\int_C \frac{e^{-z}}{x^2+y^2}\,ds$$

$$\therefore \int_0^{2\pi} \frac{e^{-t}}{(2\cos t)^2+(2\sin t)^2}\sqrt{(-2\sin t)^2+(2\cos t)^2+1}\,dt$$

$= \cancel{}\ 0.558$

27

$$\int_C (x+2y)\,dx + \int_C (x-y)\,dy$$

$$= \int_0^{\frac{\pi}{4}} (2\cos t + 8\sin t)(-2\sin t) + (2\cos t - 4\sin t)(4\cos t)\,dt$$

$$= \int_0^{\frac{\pi}{4}} -4\cos t\sin t - 16\sin^2 t + 8\cos^2 t - 16\sin t\cos t\,dt$$

$$= \int_0^{\frac{\pi}{4}} (-20\cos t\sin t - 16\sin^2 t + 8\cos^2 t)\,dt$$

$= 8 - 14 = -6$ Ans

25.

$$\int_c -y\,dx + x\,dy$$

$y^2 = 3x$, $(3,3)$ to $(0,0)$

for, x

$$r = r_0 + (r_1 - r_0)t$$

$$= 3 + (0-3)t$$

$$= 3 - 3t, \text{ for } x$$

for, y

$$r = 3 - 3t, \text{ for } y$$

$$\frac{dx}{dt} = -3$$

$$dx = -3\,dt$$

$$dy = -3\,dt$$

$x = 3,\quad 3 = 3 - 3t,\quad t = 0$

$x = 0,\quad 0 = 3 - 3t,\quad -3t = -3 \Rightarrow t = 1$

$$\therefore \int_0^1 \left\{-(3-3t)(-3)\right\} + (3-3t)(-3)\,dt$$

$$= 0$$

15.2

$x^2+y^2=r^2$

$x^2+y^2=1$ $\quad 1=1$

29

$(\sin^2 t+\cos^2 t)=1$

$\int yz\,dx - x\cancel{z}\,dy + xy\,dz$

$x=e^t \qquad y=e^{3t} \qquad z=e^{-t}$

$\frac{dx}{dt}=e^t \qquad \frac{dy}{dt}=3e^{3t} \qquad \frac{dz}{dt}=-e^{-t}$

$\int_0^1 (e^{3t}\cdot e^{-t}\cdot e^t - e^t\cdot e^{-t}\cdot 3e^{3t} + e^t\cdot e^{3t})\,dt$

$\cancel{= \int_0^1 e^{3t} - 3e^{3t} + e^{4t}\,dt}$

$\cancel{= \int_0^1 e^{4t} - 2e^{3t}\,dt}$

$\cancel{= 0.176}$ $\cancel{= (-6.36 - 7.08 + 13.39)}$

$\cancel{= 0.57}$

$$= \cancel{\int_0^1 \left(-2e^{3t} + 3e^{3t} + e^{4t}\right) dt}$$

$$= \cancel{\int_0^1 \left(2e^{3t} + e^{4t}\right) dt}$$

$$=$$

$$= \int_0^1 \left(-e^{3t} \cdot e^{-t} \cdot e^{t}\right) + \left(e^{t} \cdot e^{-t}\, 3e^{3t}\right) + \left(-e^{t} \cdot e^{3t}\, e^{-t}\right) dt$$

$$= \int_0^1 \left(-e^{3t} + 3e^{3t} - e^{3t}\right) dt$$

$$= \int_0^1 \left(-2e^{3t} + 3e^{3t}\right) dt$$

$$=$$

27

$0 < t < 90^\circ$

$x = \cos t, \quad y = \sin t$

$\frac{dx}{dt} = -\sin t, \quad \frac{dy}{dt} = \cos t$

$$\therefore \int_0^{90} \left(-\sin t \cdot -\cos t \right) dt$$

$= -102.29$

P.T.O.

$r_0 (\ )^{x_1}$ $\quad r_1 (\ )$

34

$C = (1-t) r_0 + t r_1$

$C_1 = (1-t)(2,0) + t(0,0)$

$= (2-2t, 0) + (0,0)$

$= (\underbrace{2-2t}_{x}, \underbrace{0}_{y}) = (\underbrace{-2}_{\cancel{dx/dt}}, \underbrace{0}_{dy/dt})$

$\int_0^1 y\,dx - x\,dy$

$= \int_0^1 \cancel{\ldots}\ 0$

$= \cancel{\ldots}, \quad C_1$

C_2

$C_2 = (1-t)\,r_0 + t\,r_1$

$= (1-t)(1,1) + t\,(2,0)$

$= (1-t,\ 1-t) + (2t,\ 0)$

$= (1+t,\ 1-t) = (1,\ -1)$

$\therefore \int_0^1 y\,dx - x\,dy$

$= \int_0^1 \{(1-t) + (1+t)\}\,dt$

$= 2$

$C_3 = 0$

$\therefore C = C_1 + C_2 + C_3 = 2$

Ans

25

$$\int_C y\,dx + x\,dy$$

t	x	y	t
$\sqrt{3}$	1	-1	-1
$\sqrt{3}$	1	1	1

$y^2 = 3x$, $y = t$

$\Rightarrow x = \dfrac{t^2}{3}$

$\Rightarrow \dfrac{dx}{dt} = \dfrac{2}{3}t$, $\dfrac{dy}{dt} = 1$

$$\therefore \int_{-1}^{1} \left(t \cdot \frac{2}{3}t + \frac{t^2}{3}\right) dt$$

$=$ 9

13.6

53

$f_x = 12x^2 y^2 = 12$

$f_y = 8x^3 y = -8$

$\therefore \nabla f = 12i - 8j$

$\vec{u} = \left(\frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}} \right)$

$\therefore D_u f = (12, -8) \cdot \left(\frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}} \right)$

$= 4\sqrt{13}$

15.3

* $F(x, y, z) = f(x, y, z)\,\hat{i} + g(x, y, z)\,\hat{j} + h(x, y, z)\,\hat{k}$

$\phi(x, y, z) \longrightarrow$ potential Function

$\rightarrow$ gradient field of $\phi(x, y, z)$

$$\frac{\partial \phi}{\partial x} = f(x, y, z)$$

$$\frac{\partial \phi}{\partial y} = g(x, y, z)$$

$$\frac{\partial \phi}{\partial z} = h(x, y, z)$$

$$W = \int_{x_0}^{x_1} F(x)\, dx = \phi(x_1, y_1) - \phi(x_0, y_0)$$

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

P.T.O

F will be conservative vector field if

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x} \quad \text{where}$$

$$F = f(x, y)\,\hat{i} + g(x, y)\,\hat{j}$$

<u>Exmiy</u>

$$\phi \xrightarrow{\text{Derivative}} F$$

$$F \xrightarrow{\text{Integrate}} \phi$$

$$\sqrt{x^2+y^2+z^2} = r$$

$$F(x,y,z) = \frac{xi + yj + zk}{r}$$

$$\text{Curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{r} & \frac{y}{r} & \frac{z}{r} \end{vmatrix}$$

$= \cancel{i(\partial - \partial)}$

$\Rightarrow \cancel{i(}$

$$\frac{\partial}{\partial x} = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$\Rightarrow i\left(\frac{\partial}{\partial y}\frac{z}{\sqrt{x^2+y^2+z^2}} - \frac{\partial}{\partial z}\frac{y}{\sqrt{x^2+y^2+z^2}}\right)$$

$\Rightarrow z(x^2+y^2+z^2)^{-\frac{1}{2}} - y(x^2+y^2+z^2)^{-\frac{1}{2}}$

$\Rightarrow -\cancel{z}\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-\frac{3}{2}} \cdot \cancel{2yz} + \frac{\cancel{3}}{\cancel{2}}(x^2+y^2+z^2)^{-\frac{3}{2}} \cdot \cancel{2}y$

$= -(x^2+y^2+z^2)^{-\frac{3}{2}} + (x^2+y^2+z^2)^{-\frac{3}{2}}$

$= 0$

$$\hat{i}\left(\frac{\partial}{\partial y}\frac{z}{\sqrt{x^2+y^2+z^2}} - \frac{\partial}{\partial z}\frac{y}{\sqrt{x^2+y^2+z^2}}\right)$$

$$\frac{\partial}{\partial y} z\cdot\left(x^2+y^2+z^2\right)^{-1/2} - \frac{\partial}{\partial z} y\cdot\left(x^2+y^2+z^2\right)^{-1/2}$$

$$= -\frac{1}{2}z\left(x^2+y^2+z^2\right)^{-3/2}2y - \left(-\frac{1}{2}y\left(x^2+y^2+z^2\right)^{-3/2}\cdot 2z\right)$$

$$= -\frac{1}{\cancel{2}}\cancel{z}\left(x^2+y^2+z^2\right)^{-3/2}\cancel{2y} + \frac{1}{\cancel{2}}y\left(x^2+y^2+z^2\right)^{-3/2}\cdot\cancel{2z}$$

$$= 0$$ Ans.

Exm: 6

$$F(x,y) = e^{y}\,\hat{i} + xe^{y}\,\hat{j} \longrightarrow \nabla\phi$$

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x} \qquad \frac{\partial \phi}{\partial y} \qquad \int_a^b f(x)\,dx$$

$$\Rightarrow e^{y} = e^{y} \qquad = F(b) - F(a)$$

$$\frac{\partial \phi}{\partial x} = f$$

$$\phi = \int f\,dx$$

$$= \int e^{y}\,dx$$

$$\phi = xe^{y} + k(y) \qquad \therefore \phi = xe^{y} + C$$

$$\frac{\partial \phi}{\partial y} = xe^{y} + k'(y)$$

$$\Rightarrow xe^{y} = xe^{y} + k'(y)$$

$$\Rightarrow k'(y) = 0$$

$$\Rightarrow k(y) = C$$

15.3

(1–6), (9–14)

5

$F(x, y) = (\cos y + y\cos x)\, i + (\sin x - x\sin y)\, j$

$\dfrac{\partial g}{\partial x} = \cos x - 1 \cdot \sin y$

$\dfrac{\partial f}{\partial y} = -\sin y + \cos x$

$\dfrac{\partial f}{\partial y} = \dfrac{\partial g}{\partial x}$ (conservative)

$\dfrac{\partial \phi}{\partial y} = g$

$\dfrac{\partial \phi}{\partial x} = f$

$\phi = \int f$

$= \int (\cos y + y\cos x)\, dx$

17.2

$= x\cos y + y\sin x + k(y)$

$$\frac{\partial \phi}{\partial y} = g$$

$\Rightarrow \sin x - x\sin y = \cancel{\quad} - x\sin y + \sin x + k'(y)$

$\Rightarrow k'(y) = 0$

$\Rightarrow k(y) = C$

$\therefore \phi = x\cos y + y\sin x + C$

(1-12), (13-16)

14.7

double integration

Exm: 2

(a) $$\int_1^3 \int_2^4 (40 - 2xy)\, dy\, dx$$

$$= \int_1^3 \left[40y - xy^2\right]_2^4 dx$$

$$= \int_1^3 \left\{(160 - 16x) - (80 - 4x)\right\} dx$$

$$= \int_1^3 (80 - 12x)\, dx$$

$$= \left[80x - 12 \cdot \frac{x^2}{2}\right]_1^3$$

$$= 112$$

$$\underline{9} \quad \int_0^1 \int_0^1 \frac{x}{(xy+1)^2}\, dy\, dx$$

$$\cancel{= \int_0^1 \left[\frac{x(xy+1)^{-1}}{-1} \right]_0^1 dx}$$

$1 + xy = u$

$x\,dy = du$

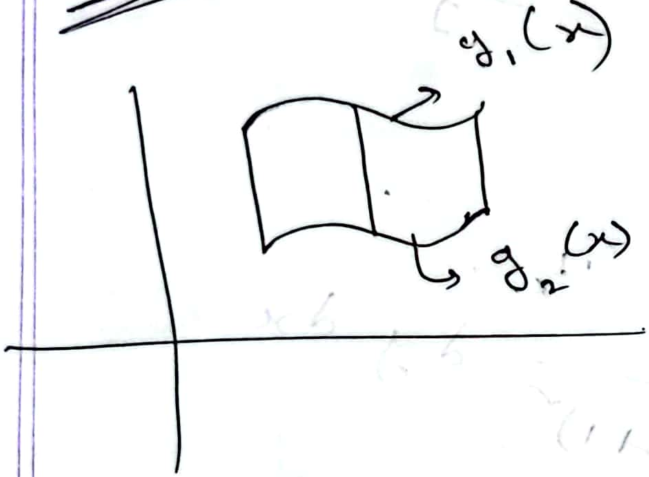
y	0	1
u	1	$1+x$

$$= \int_0^1 \int_1^{1+x} \left[\frac{du}{(u)^2}\right] dx$$

$$= \int_0^1 \left(\frac{u^{-1}}{-1}\right)_1^{1+x} dx$$

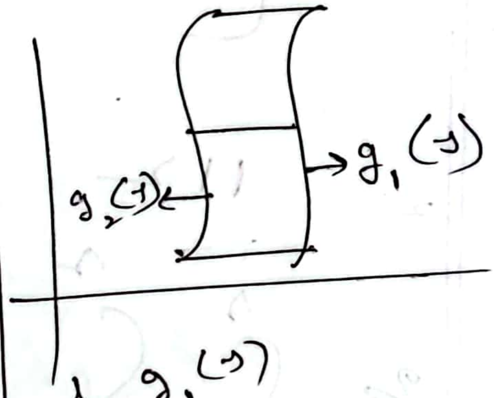
CW

Type: 1



$$\int_a^b \int_{g_2(x)}^{g_1(x)} f(x, y)\, dy\, dx$$

Type: 2



$$\int_a^b \int_{g_2(y)}^{g_1(y)} f(x, y)\, dx\, dy$$

Ex: 3

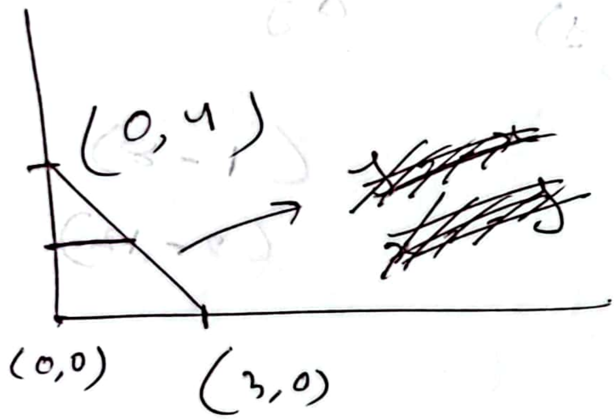
$y = \frac{1}{2}x, \quad y = \sqrt{x}, \quad x = 2, \quad x = 4$

$$\int_2^4 \int_{\frac{1}{2}x}^{\sqrt{x}} xy\, dy\, dx$$

Exm: 5 , 6 , 8

(1-8), (15-18)

14.2



$y = -x$

$\frac{y}{4} = -\frac{x}{3}$

$\Rightarrow x = -\frac{3y}{4}$

$$= \int_0^4 \int_0^{-3y/4} f(x,y)\,dx\,dy$$

CW
20.8.23

14.5

Triple Integration

$$\iint \int_{g_1(x,y)}^{g_2(x,y)} dz \quad dx\, dy \quad \text{— (I)}$$
$$\qquad\qquad\qquad dy\, dx \quad \text{— (II)}$$

(1-8)
(9-12)

Exm: 2, 3

11.8 (Box)

ρ
ϕ
θ

GW 23.8.23 146

Vector

Exm 1

Exm 2

* Cylindrical Co-ordinate

$r \geq 0$

$z = z$

$x = r\cos\theta$, $y = r\sin\theta$

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\int_{z_1}^{z_2} dz\, dr\, d\theta,$$

* Spherical Co-ordinate:

$\rho \geq 0$

ϕ

θ

$x = \rho\cos\phi \sin\theta$

$y = \rho\sin\phi \sin\theta$

$z = \rho\cos\phi$

$$\iiint d\rho\, d\phi\, d\theta$$

15-4

$$\oint_C f(x,y)\,dx + g(x,y)\,dy = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA$$

Exm: 1

15.3, 14.1, 14.2 C.T (w.d)

GW 27.8.23 15-

Vector

* $\iint F \cdot n \, ds = \iint F \cdot n \left\| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right\| dA$

$$= \iint F \cdot \frac{\left(\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v}\right)}{\left\| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right\|} \left\| \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right\| dA$$

$$= \iint F \cdot \left(\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right) dA$$

* Find the flux over the field $F(x, y, z) = z\hat{k}$ outward oriented sphere $x^2 + y^2 + z^2 = 1$

$$= \iint \cos\phi \, \hat{k} \left(-\sin^2\phi \cos\theta \, i + \sin^2\phi \sin\theta \, j + \sin\phi \cos\phi \, \hat{k} \right) dA$$

$$= \int_0^{2\pi} \int_0^{\pi} \sin\phi \cos^2\phi \, d\phi \, d\theta = \frac{4\pi}{3}$$

divergence Theorem

$$\boxed{\iint_S F\,n\,ds = \iiint_G \operatorname{div} F \, dv}$$

$$\Phi = \iiint 1 \, dv$$

$$\rho = 0 \le \rho \le r$$

$$\theta = 0 \le \theta \le 2\pi$$

$$\phi = 0 \le \phi \le \pi$$

C.W 30.8.23

14.2

Vector

6.

$$\int_{-x^2}^{x^2} (x^2 - y)\, dy$$

$$= \left[x^2 y - \frac{y^2}{2}\right]_{-x^2}^{x^2}$$

$$= x^4 - \frac{x^4}{2} + x^4 + \frac{x^4}{2}$$

$$= \int_{-1}^{1} \frac{2x^4}{}\, dx = \left[\frac{2x^5}{5}\right]_{-1}^{1}$$

$$= \frac{2}{5} + \frac{2}{5}$$

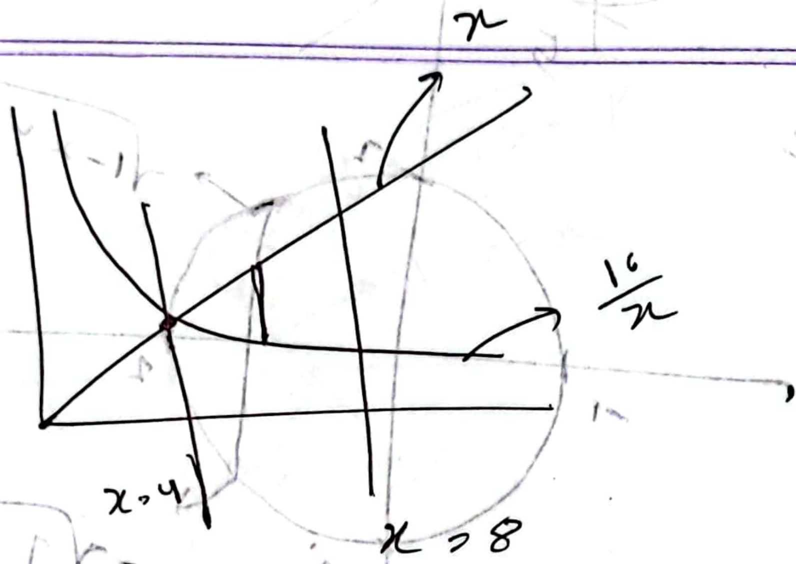
$$= \frac{4}{5}$$

8/ $\int_0^{\ln 2} e^{\frac{x}{\ln 2}}\, dx$

$= \left[\frac{e^{\frac{x}{\ln 2}}}{\frac{1}{\ln 2}} \right]$

$= \int_1^2 \cancel{\frac{e^1}{\frac{1}{\ln 2}} - \ln 2 = 4}$

1.5



$$\frac{16}{x} = x$$

$$\Rightarrow x^2 = 16$$

$$\Rightarrow x = 4$$

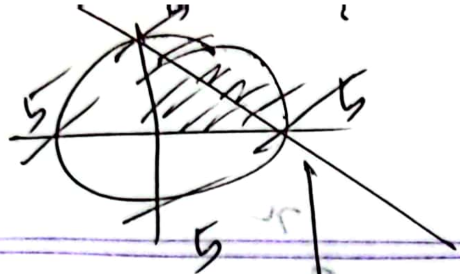
$$\int_{16/x}^{x} x^2 \, dy$$

$$= \left[x^2 y \right]_{16/x}^{x}$$

$$= \int_4^8 \left(x^3 - 16x \right) dx$$

$$= 576$$

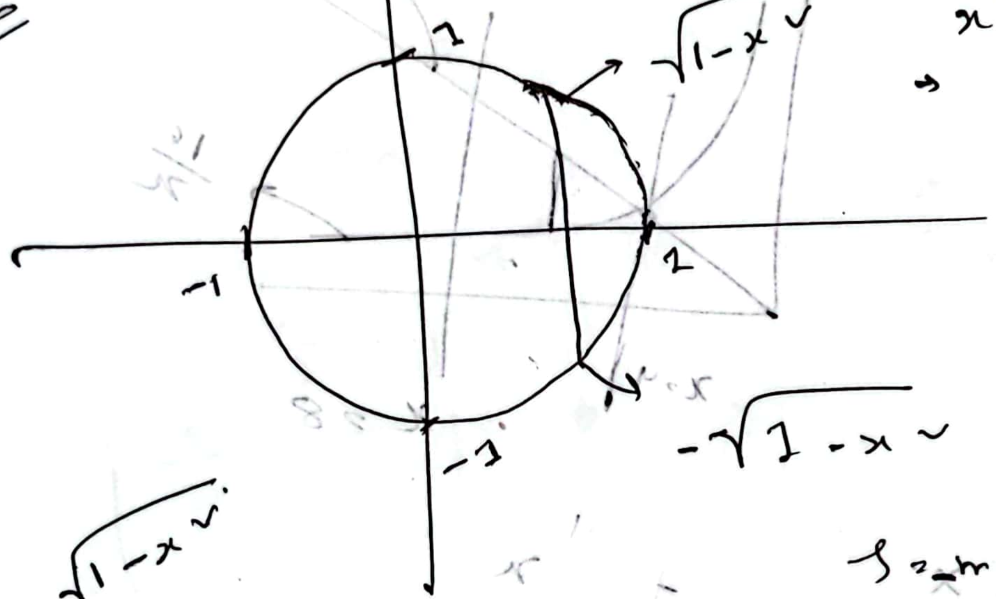
$y \geq 5 - x$



$y = \sqrt{1-x^2}$

$= -\sqrt{1-x^2}$

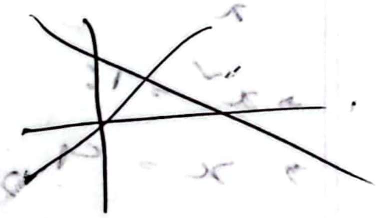
17



$x^2 + y^2 = 1$

$dy\,dx$

$y = mx + c$



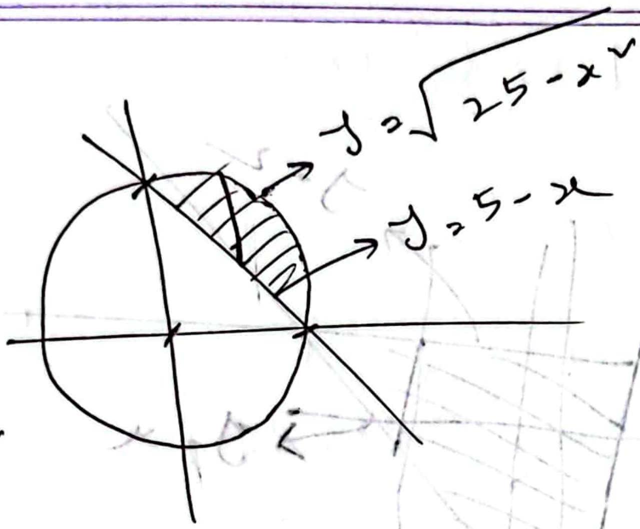
$$\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (3x - 2y)\,dy$$

$$\left[3xy - y^2\right]_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}$$

$$= \int_{-1}^{1} \left\{3x\sqrt{1-x^2} - \cancel{\left(\sqrt{1-x^2}\right)^2}\right\} + 3x\sqrt{1-x^2} \cancel{+ \left(\sqrt{1-x^2}\right)^2}$$

$$= \int_{-1}^{1} 6x\sqrt{1-x^2}\,dx = 0$$

18



$dy\,dx$

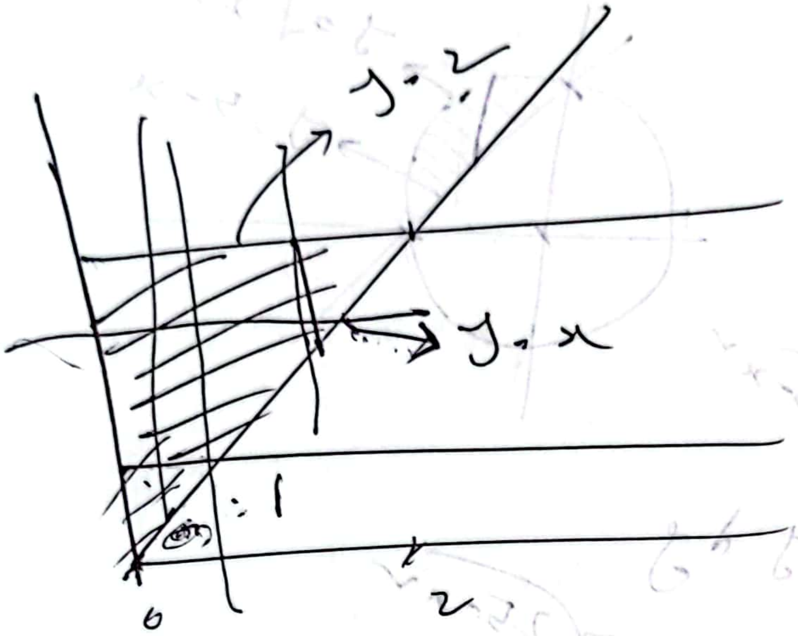
$$\int_{5-x}^{\sqrt{25-x^2}} y\,dy$$

$$= \left[\frac{y^2}{2}\right]_{5-x}^{\sqrt{25-x^2}}$$

$$= \int_0^5 \frac{25-x^2}{2} - \frac{(5-x)^2}{2}\,dx$$

$$= \frac{125}{6}$$

16



$xy^2 \, dy$

$= \int_x^2 \left[\frac{xy^3}{3}\right]_x^2$

$= \int_0^2 \frac{8x}{3} - \frac{x^4}{3} \, dx$

$=$

$$\int_0^y x y^2 \, dx$$

$dx \, dy$

$x = 0$

$x = y$

$$= \left[\frac{x^2 y^2}{2} \right]_0^y$$

$$= \int_1^2 \frac{y^4}{2} - 0$$

$$= \frac{31}{10} \quad \text{Ans}$$

Vector Note   Maliha Madam

13.7 {1, 2, (3-12), 29, 31}

11.8

* Cylinder to rectangular :-

$(r, \theta, z) \to (x, y, z)$

* Rectangular to Cylindrical

$(x, y, z) \to (r, \theta, z)$

$x = r\cos\theta$, $y = r\sin\theta$, $z = z$

$r = \sqrt{x^2+y^2}$, $\tan\theta = \frac{y}{x}$, $z = z$

---

* spherical to cylinder

$(\rho, \theta, \phi) \to (r, \theta, z)$

* cylinder to spherical

$(r, \theta, z) \to (\rho, \theta, \phi)$

$r = \rho\sin\phi$, $\theta = \theta$, $z = \rho\cos\phi$

$\rho = \sqrt{r^2+z^2}$, $\theta = \theta$, $\tan\phi = \frac{r}{z}$

* spherical to rectangular

$(\rho, \theta, \phi) \rightarrow (x, y, z)$

* Rectangular to spherical

$(x, y, z) \rightarrow (\rho, \theta, \phi)$

$x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$,

$z = \rho \cos\phi$, $\rho = \sqrt{x^2 + y^2 + z^2}$, $\tan\theta = \frac{y}{x}$

$\cos\phi = \sqrt{x^2 + y^2 + z^2}$

$$\boxed{\begin{array}{l} r \geq 0, \quad \rho \geq 0 \\ 0 \leq \theta \leq 2\pi \\ 0 \leq \phi \leq \pi \end{array}}$$

* rectangular $\rightarrow (x, y, z)$
* Cylinder $\rightarrow (r, \theta, z)$
* Spherical $\rightarrow (\rho, \theta, \phi)$

* $(4\sqrt{3}, 4, -4) \rightarrow (r, \theta, z)$

$r = \sqrt{(4\sqrt{3})^2 + 4^2} = 8$

$\theta = \tan^{-1}\left(\frac{4}{4\sqrt{3}}\right)$

$= 30^\circ$

$z = -4$

$(r, \theta, z) = (8, 30^\circ, -4)$

* $(8, \frac{3\pi}{4}, -2) \rightarrow (x, y, z)$

r, θ, z

$x = r\cos\theta = -4\sqrt{2}$

$y = r\sin\theta = 4\sqrt{2}$

$z = -2$

$\therefore (x, y, z) = (-4\sqrt{2}, 4\sqrt{2}, -2)$

$\left(2, \frac{3\pi}{2}, \frac{\pi}{2}\right) \rightarrow (x, y, z)$

ρ, θ, ϕ

$x = \rho \sin\phi \cos\theta$

$= 0$

$y = \rho \sin\phi \sin\theta = -2$

$z = \rho \cos\phi = 0$

$(x, y, z) = (0, -2, 0)$

P.T.O

15.3

$$\int_C F \cdot dr = \int_C f(x,y)\,dx + g(x,y)\,dy$$

$$F(x,y) = \nabla \phi(x,y)$$

$$\int_C F(x,y)\,dr = \int_C \nabla\phi \cdot dr$$

$$= \phi(x_1,y_1) - \phi(x_0,y_0)$$

$$\frac{\partial \phi}{\partial x} = f$$

$$\Rightarrow \phi = \int f$$

$$= x^2y^3 + k(y)$$

$$\Rightarrow \frac{\partial \phi}{\partial y} = g$$

$$\Rightarrow 3x^2y^2 + k'(y) = 1 + 3x^2y^2$$

$$\Rightarrow k'(y) = 1$$

$$\Rightarrow k(y) = y + C$$

$\therefore \phi = x^2y^3 + y + C$

$\cancel{\int_{(1,4)}^{(3,1)} 2xy^3 \, dx}$

$\Rightarrow \cancel{\left[x^2y^3\right]_{(1,4)}^{(3,1)}}$

$= \cancel{-55}$

$\cancel{\int_{(1,4)}^{(3,1)} (1+3x^2y^2) \, dy}$

$= \cancel{\left[y + x^2y^3\right]_{(1,4)}^{(3,1)}}$

$\Rightarrow \int_{(1,4)}^{(3,1)} x^2y^3 + y$

$= -58$ Ans.

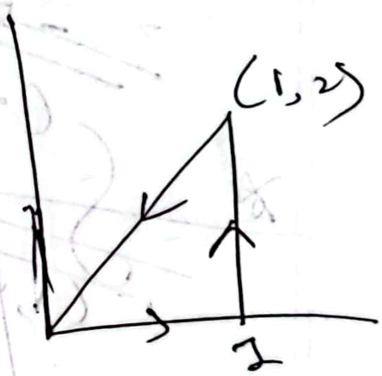
15.4

Green's theorem:

$$\int_C f(x,y)\,dx + g(x,y)\,dy$$

$$= \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA$$

* $\displaystyle\int_C \underbrace{x^2 y}_{f}\,dx + \underbrace{x}_{g}\,dy$



~~(x_1, y_1)~~

$(x_1, y_1) = (0,0)$
$(x_2, y_2) = (1,2)$

$$y = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) x$$

$$= \left(\frac{2-0}{1-0}\right) x = 2x$$

$$\Rightarrow \int_0^1 \int_0^{2x} (1 - x^2)\, dy\, dx$$

$$= \left[y - x^2 y\right]_0^{2x}$$

$$= \int_0^1 2x - 2x^3\, dx$$

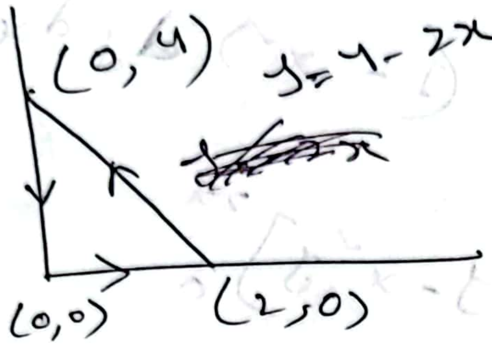
$$= \left[x^2 - \frac{x^4}{2}\right]_0^1$$

$$= \frac{1}{2} \text{ Ans}$$

~~$y = \frac{4-0}{0-2}x \Rightarrow y = -2x$~~

$(x_1, y_1) = (2,0)$
$(x_2, y_2) = (0,4)$

∗



$$\iint \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA$$

$$\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$$

$$\Rightarrow \frac{y-0}{0-4} = \frac{x-2}{2-0}$$

$$\Rightarrow -\frac{y}{4} = \frac{x-2}{2}$$

$$y = \frac{-4x+8}{2}$$

$$y = -2x+4$$

$$\Rightarrow 4-2x$$

$$\oint_C \ln(1+y)\,dx - \frac{xy}{1+y}\,dy$$

$$= \int_0^2 \int_0^{4-2x} \left(\frac{-y}{1+y} - \frac{1}{1+y}\right) dy\,dx$$

$$\Rightarrow -\int_0^2 \int_0^{4-2x} \frac{y+1}{y+1}\,dy\,dx$$

$$= -\int_0^2 \left[y\right]_0^{4-2x} dx$$

$$= -\int_0^2 4-2x\,dx = -4$$

Ex: 5, 9

$$\cancel{\int_0^{2\pi} \frac{x^3}{3}\, dy}$$

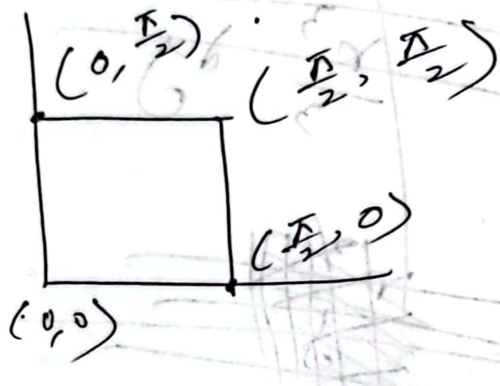
$$A = \oint_c x\,dy = -\oint_c y\,dx$$

$$= \frac{1}{2}\oint_c -y\,dx + x\,dy$$

$$\boxed{dy\,dx = r\,dr\,d\theta}$$

$$\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}$$

c



$$\iint x\cos y\, dx - y \sin x\, dy$$

$$\Rightarrow \int_0^{\pi/2}\int_0^{\pi/2} (-y\cos x + x \sin y)\, dy\, dx$$

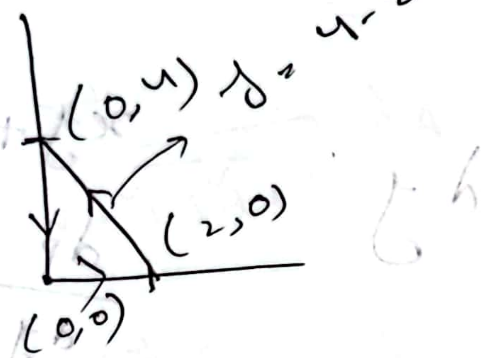
$$\Rightarrow \left[\left(-\frac{y^2 \cos x}{2} - x\cos y\right)\right]_0^{\pi/2}$$

$$\Rightarrow \int_0^{90} \left(-\frac{8100 \cos x}{2} + x\right) dx$$

$$= \left[-4050 \sin x + \frac{x^2}{2}\right]_0^{90}$$

$$= \left[-4050 + 4050\right] = 0 \quad \underline{\text{Ans}}$$

*



$$\int -\frac{y}{1+y} - \frac{1}{1+y}\, dy\, dx$$

$$= -\int_0^2\int_0^4 1\, dy\, dx$$

$$= -\int_0^2 [y]_0^{4-2x}\, dx$$

$$= -\int_0^2 4-2x\, dx$$

$$\Rightarrow - \cancel{[\;]}_0 - [4x - x^2]_0^2$$

$$= -[8-4] = -4$$

$$= -8$$

14.1

a)

$$\int_0^1 \frac{x}{(xy+1)^2}\,dy \qquad xy+1 = p$$

$$\frac{dp}{dy} = x$$

$$= \int_0^1 \frac{1}{p^2}\,dp \qquad dp = x\,dy$$

$$= \left[-\frac{1}{p}\right]_0^1 \qquad y = (1, 0)$$

$$= -\frac{1}{xy+1}$$

$$= \int_0^1 \left(-\frac{1}{x+1} + 1\right) dx$$

$$= \left[-\ln(x+1) + x\right]_0^1$$

$$= 0.307$$

14

$$\iint \frac{xy}{\sqrt{x^2+y^2+1}}\, dA \qquad x^2+y^2+1=p$$

$$\Rightarrow 2y\, dy = dp$$

$$= \int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}}\, dy$$

$$= \int_0^1 \frac{1\,x}{2\sqrt{p}}\, dp$$

$$= \left[x\sqrt{p}\right]_0^1$$

$$= \left[x\sqrt{x^2+y^2+1}\right]_0^1$$

$$= \int_0^1 \left(x\sqrt{x^2+2} - x\sqrt{x^2+1}\right) dx$$

$$= 0.1798$$ Ans

14.2

$$\int_0^{\cos y} x \sin y \, dx$$

$$= \frac{x^2}{2} \sin y$$

$$= \cancel{\int_0^{60} \frac{\cos^2 y}{2} \sin y}$$

$$= \cancel{\frac{1-\sin^2 y}{2}} \sin y$$

$$= \int_0^{60} \left(\frac{\sin y - \sin^3 y}{2} \right) dy$$

$$= \left[-\frac{\cos y}{2} + \frac{\cos^3 (y)}{6} \right]_0^{60}$$

$$= -\frac{1}{4} + \frac{1}{48} + \frac{1}{2} - \frac{1}{6} =$$

$$= \int_0^{60} \frac{\cos^2 y}{2} \sin y \, dy$$

~~$\sin y \, dy = p$~~

~~$\frac{dp}{dy} = \cos y$~~

~~$= dp = \cos y \, dy$~~

$\cos y = p$

$-\sin y \, dy = dp$

$$= -\int_0^{60} \frac{p^2}{2} dp$$

$$= -\left[\frac{p^3}{6}\right]_0^1$$

$$= -\left(\frac{\cos^3 y}{6}\right)_0^{60}$$

$$= -\frac{1}{48} + \frac{1}{6} = \frac{7}{48} \text{ Ans}$$

Sum-20

1

$(0,0) \rightarrow (2,1)$

$$y \quad \frac{y-0}{2-0} \quad x$$

$$\Rightarrow y = \frac{1}{2}x$$

$$\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$$

$$\Rightarrow \frac{y-0}{0-1} = \frac{x-0}{0-2}$$

$$\Rightarrow -y = -\frac{x}{2}$$

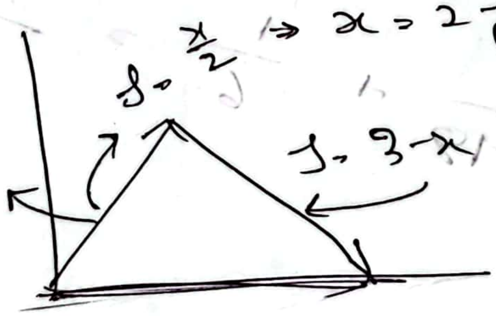
$$\boxed{\Rightarrow y = \frac{x}{2}}$$

$$\frac{y-1}{1-0} = \frac{x-2}{2-3}$$

$$\Rightarrow y-1 = -x+2$$

$$\Rightarrow y = -x+3 \Rightarrow y = 3-x$$

$y = \frac{x}{2} \Rightarrow x = 2y$



$y = 3-x \Rightarrow x = 3-y$

$$\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \qquad f =$$

$$\int_{3-y}^{2y} \qquad\qquad \frac{\partial f}{\partial y} = 0$$

$$\frac{\partial g}{\partial x} = 0$$

$$\frac{\partial \phi}{\partial x} = f$$

$$\phi = \int f$$

$$= \frac{x^2}{2} + x + k(y)$$

$$\frac{\partial \phi}{\partial y} = g$$

$$k'(y) = 2y$$

$$k(y) = y^2$$

$$\therefore \phi = \frac{x^2}{2} + x + y^2$$

$$\therefore \int_{3-y}^{2y} \frac{x^2}{2} + x + y^2$$

$$\Rightarrow \int_{3-y}^{2y} \frac{4y^2}{2} + 2y + y^2 - \frac{(3-y)^2}{2} - (3-y) - y^2$$

$$= \int_{0}^{2} 2y^2 + 2y + \cancel{y^2} - \frac{9-6y+y^2}{2} - 3 + y - \cancel{y^2}$$

$$= \frac{7}{2}$$

14.5

Tripple Integration

* $\int_{-1}^{2}\int_{0}^{3}\int_{0}^{2} 12xy^2z^3 \, dz \, dy \, dx$

$\Rightarrow \int_0^2 12xy^2z^3 \, dz$

$\Rightarrow \dfrac{12xy^2z^4}{4}$

$\Rightarrow \left[3xy^2z^4\right]_0^2 \Rightarrow 48xy^2$

$\Rightarrow \int_0^3 48xy^2 \, dy$

$\Rightarrow \left[16xy^3\right]_0^3 = 432x$

$\Rightarrow \int_{-1}^{2} 432x \, dx = \left[216x^2\right]_{-1}^{2}$

$= 864 - 216$

$= 648$ Ans

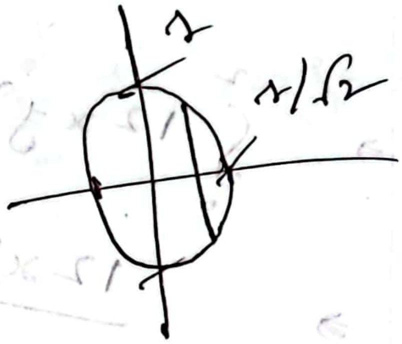
$2x^2 = 1$

$x = \frac{1}{\sqrt{2}}$

*

$z = 5x^2 + 5y^2 , \quad z = 6 - 7x^2 - y^2$

$5x^2 + 5y^2 = 6 - 7x^2 - y^2$



$\Rightarrow 12x^2 + 6y^2 = 6$

$\Rightarrow 2x^2 + y^2 = 1$

$$\int_{5x^2+5y^2}^{6-7x^2-y^2} dz$$

$$\left[z\right]_{5x^2+5y^2}^{6-7x^2-y^2}$$

$= 6 - 7x^2 - y^2 - 5x^2 - 5y^2$

$$= \int_{-\sqrt{1-2x^2}}^{\sqrt{1-2x^2}} 6 - 12x^2 - 6y^2 \, dy$$

$$= \left[6y - 12x^2 y - 2y^3 \right]_{-\sqrt{1-2x^2}}^{\sqrt{1-2x^2}}$$

$$= 6\sqrt{1-2x^2} - 12x^2\sqrt{1-2x^2} - 2(1-2x^2)^{\frac{3}{2}} +$$

$$6\sqrt{1-2x^2} - 12x^2\sqrt{1-2x^2} - 2(1-2x^2)^{\frac{3}{2}}$$

$$= \int_{-1/\sqrt{2}}^{1/\sqrt{2}} 12\sqrt{1-2x^2} - 24x^2\sqrt{1-2x^2} - 4(1-2x^2)^{\frac{3}{2}} \, dx$$

$= 6.66$ ~~ho~~

14.6

Convention

* $dz\,dy\,dx = r\,dz\,dr\,d\theta$
(cylindrical convension)

* $\cos^2\theta = 1 + \cos 2\theta$
* $\sin^2\theta = 1 - \sin 2\theta$

spherical conversition

* $\iiint dz\,dy\,dx = \iiint \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

* for cone, $\left\{ z = \sqrt{x^2+y^2} \text{ similar} \right\}$

$0 \le \phi \le \frac{\pi}{4}$

* hemisphere : $\Rightarrow$

$0 \le \phi \frac{\pi}{2}$.

15.5

Surface Integral, (6)

xy * $\iint_R f(x, y, g(x,y)) \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$

xz * $\iint_R f(x, g(x,z), z) \sqrt{\left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2 + 1}\, dA$

yz $x=$ * $\iint_R f(g(y,z), y, z) \sqrt{\left(\frac{\partial x}{\partial y}\right)^2 + \left(\frac{\partial x}{\partial z}\right)^2 + 1}\, dA$

* $\iint_R xz\, ds$ $\quad$ xy projection

$x + y + z = 1$

$z = 1 - x - y$

$x(1-x-y) = x - x^2 - xy$

0.04163

$$ds = \sqrt{(-1)^2 + (-1)^2 + 1}\ dA$$

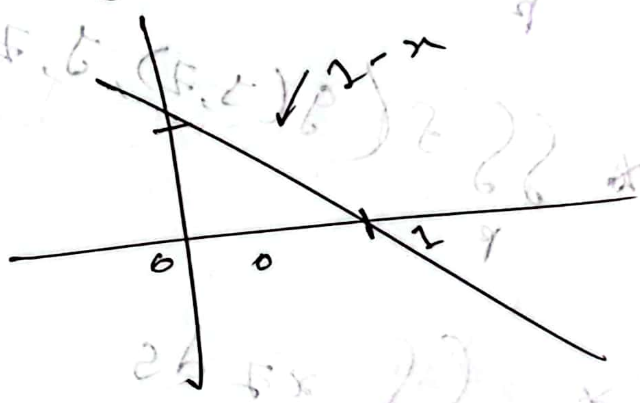
$$= \sqrt{3}\ dA$$

Now removing z term to sketch the graph for $dy\,dx$.

$1 - x - y = 0$

$\Rightarrow x + y = 1$

$\Rightarrow y = 1 - x$



$$\int_0^{1-x} x - x^2 - xy\ dy$$

$$\Rightarrow \left[xy - x^2 y - \frac{xy^2}{2} \right]_0^{1-x}$$

$$\Rightarrow \int_0^1 \left(x(1-x) - x^2(1-x) - \frac{1}{2}x(1-x)^2 \right) dx$$

$=$

$$= \int_0^1 x - x^2 - x^2 + x^3 - \frac{1}{2}x(1-2x+x^2)\,dx$$

$$= \int_0^1 x - x^2 - \cancel{x^2} + x^3 - \frac{1}{2}x + \cancel{x^2} - \frac{x^3}{2}\,dx$$

$$= \int_0^1 \frac{x}{2} + x^3 - x^2 - \frac{x^3}{2}$$

$$= \int_0^1 \frac{x}{2} - x^2 + \frac{x^3}{2}\,dx$$

$$= \left[\frac{x^2}{4} - \frac{x^3}{3} + \frac{x^4}{8}\right]_0^1 \times \sqrt{3}$$

$$= \frac{\sqrt{3}}{24}$$ A

15.6

Flux

$$\phi = \iint_{\sigma} F \cdot n \, ds$$

* $\iint_{\sigma} F \cdot n \, ds = \iint_{R} F \cdot \left(-\frac{\partial z}{\partial x} i - \frac{\partial z}{\partial y} j + k \right) dA$

[oriented up]

* $\iint_{R} F \cdot \left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} - k \right) dA$

(oriented down]

15.7

Divergence theorem

$$\iint_{\sigma} F \cdot n \, dS = \iiint \underline{div} \, f \, dv$$

[closed region]

outward orientation.

33. c) $(0,0)$; $(1,0)$

$r = \cancel{(\;)} \; (1-t) \, r_0 + t \, (r_1)$

$= (1-t)(0,0) + t(1,0)$

$= \frac{t}{x}$

$x = t$

$\frac{dx}{dt} = 1$

$dx = dt$

$\int_C y \, dx$

$= 0$

P.T.O

$0 < 1$

$\underline{\underline{C_2}}$

$(1,0), (0,1)$

$r = (1-t)(1,0) + t(0,1)$

$= (1-t, 0) + (0, t)$

$= \underset{x}{\underline{1-t}}, \underset{y}{\underline{t}}$

$\frac{dx}{dt} = -1$

$dx = -dt$

$dy = dt$

$$\int_0^1 y\,dx - x\,dy$$

$-t\,dt - (1-t)\,dt$

$$\int_0^1 \{-t-(1-t)\}\,dt = -1$$

C_3

$(0,1) \rightarrow (0,0)$ $\qquad dx = 0$

$(1-t)(0,1) + t(0,0)$

$r = (1-t)(0,1)$ $\qquad \frac{dy}{dt} = -1$

$= \underset{x}{0}, \frac{1-t}{y}$ $\qquad dy = -dt$

$\int_C -x\,dy$

$= 0 = 0$

$\therefore 0 - 1 + 0 = -1$

2

$$\int_0^{r^2} r\sin\theta\, dz$$

$$\left[r\sin\theta\, z\right]_0^{r^2}$$

$$= \int_0^{\cos\theta} r^3 \sin\theta\, dr$$

$$= \frac{r^4}{4}\sin\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^4\theta}{4}\sin\theta\, d\theta$$

$\cos\theta = p$

$\frac{dp}{d\theta} = \sin$

$dp = \sin\theta$

$$= \int \frac{(-p)^4}{4}\, dp$$

=

$$= \int_0^{\frac{\pi}{2}} \frac{p^4}{4}$$

$$\left[\frac{p^5}{20}\right]_0^{\frac{\pi}{2}} = \frac{1}{20}\ \text{A}$$

$=$