6W 9.10.22

Calculus

Reflection

Reflection + translation

→ 1st reflection

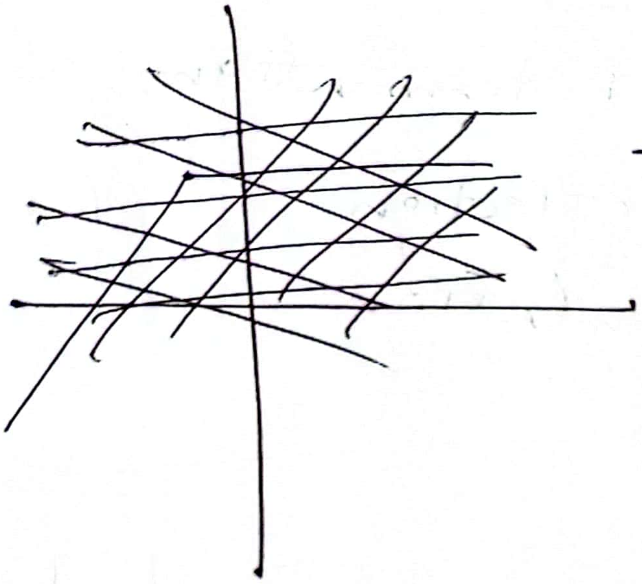
→ 2nd translation

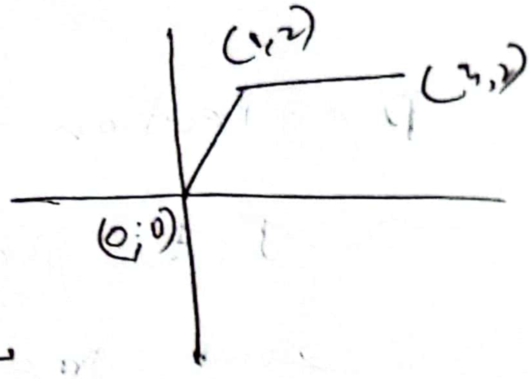
$y = x, x^2, x^3, \sqrt[3]{x}, |x|, \frac{1}{x}, \cos x, \sin x$

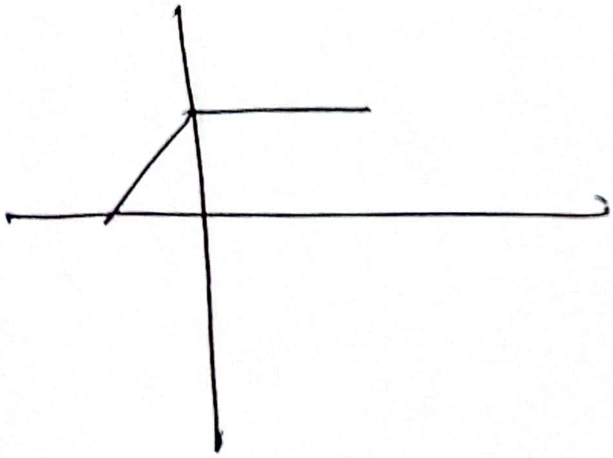
CT

$y = f(x-1)$

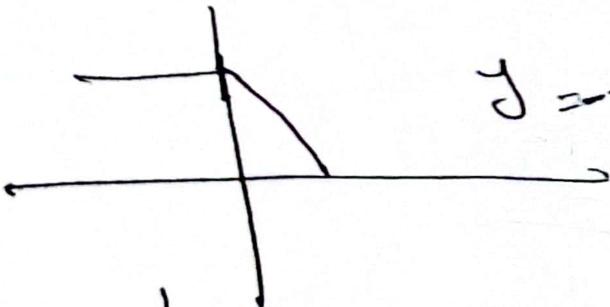
(v)



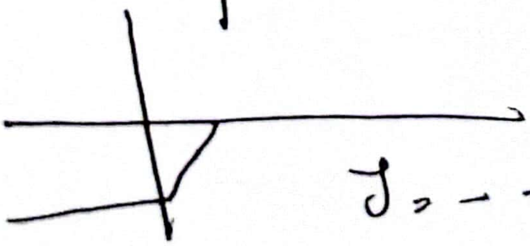


* 

$y = f(x)$

* 

$y = -f(x)$

* 

$y = -f(-x)$

(c)

$y = 1 - f(2-x)$

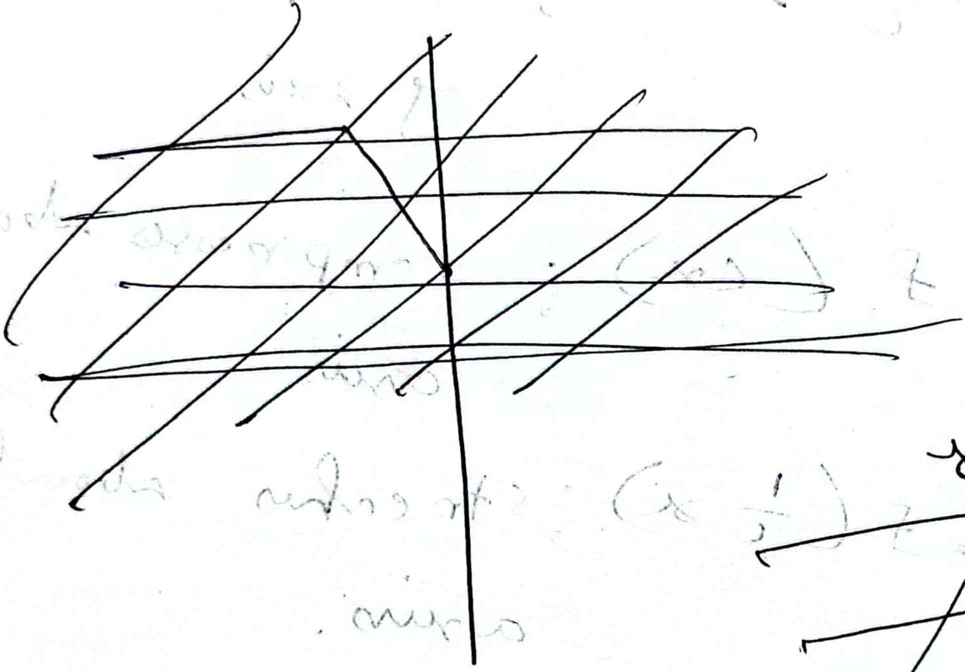
$= -f(2-x) + 1$

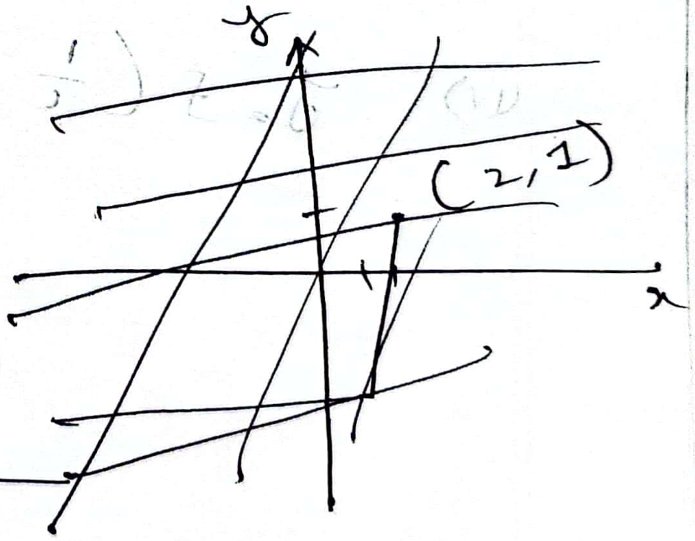
~~$-f(-(x-2))$~~

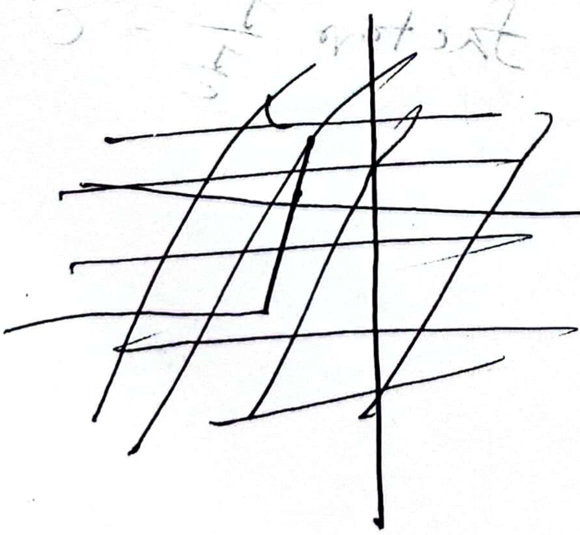
$-f(-x+2)$

$x = 2$

$y = 1$







P.T.O

(i) $y = cf(x)$, stretches about y axis

(ii) $y = \frac{1}{c} f(x)$; compresses about y axis

(iii) $y = f(cx)$; compresses about x axis

(iv) $y = f(\frac{1}{c} x)$; stretches about x axis.

factor $\frac{1}{\frac{1}{c}} = c$

* $y = 2f(2x)$

* $y = -f(2x)$

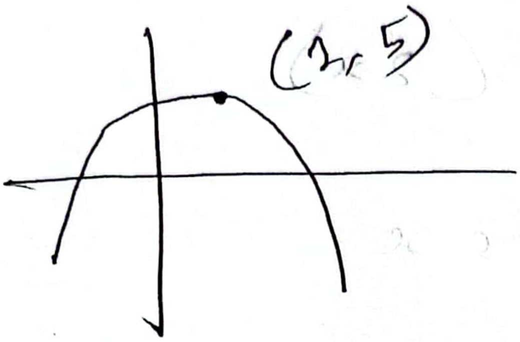
* $y = \frac{1}{2}\cos x$

* $y = \sin 2x$

* $y = \cos\left(\frac{x}{2}\right)$

2

(i)

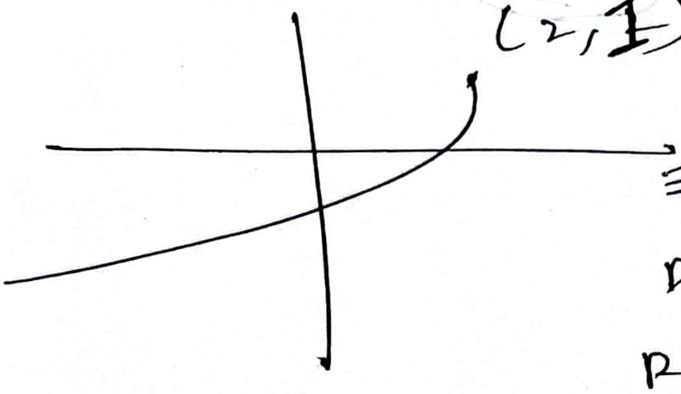


$y = 5 - (x-1)^2$

Dom: $(-\infty, +\infty)$

Range: $(-\infty, +5]$

(ii)

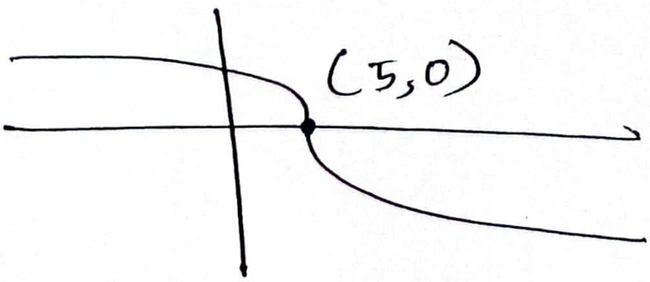


Dom $(-\infty, +2]$

Range $(-\infty, +1]$

$y = -\sqrt{-x+2} + 1$

(iii)



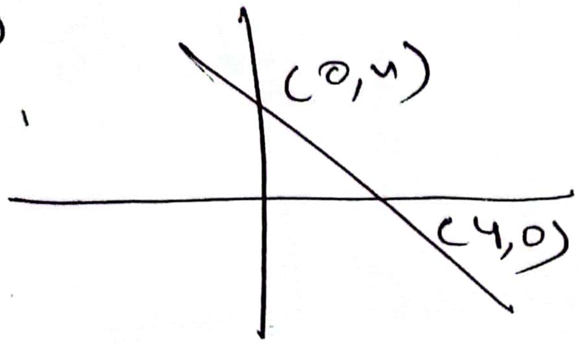
Dom $(-\infty, +\infty)$

Range: $(+\infty, -\infty)$

$y = -(x-5)^{\frac{1}{3}}$

P.T.O.

(iv)

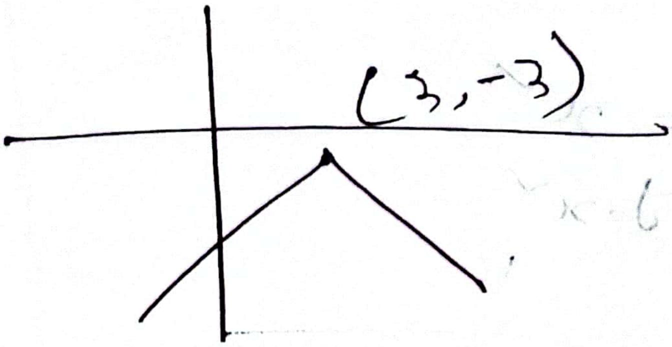


Dom: $(-\infty, +\infty)$

Range: $(+\infty, -\infty)$

$x + y = 4$

(v)



$y = -|x-3| - 3$

Dom $(-\infty, +\infty)$

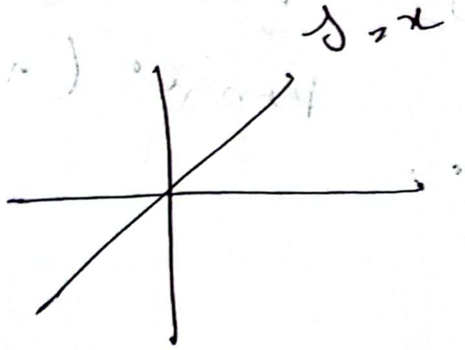
Range $(-\infty, -3]$

Calculus

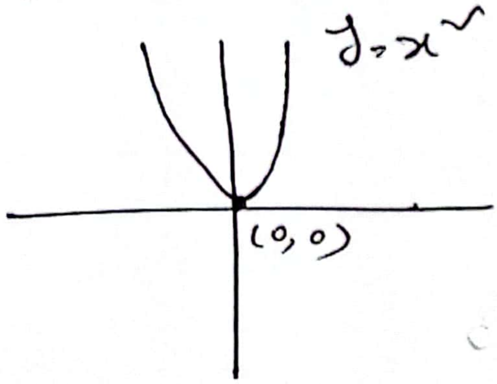
CT Practice

Graph

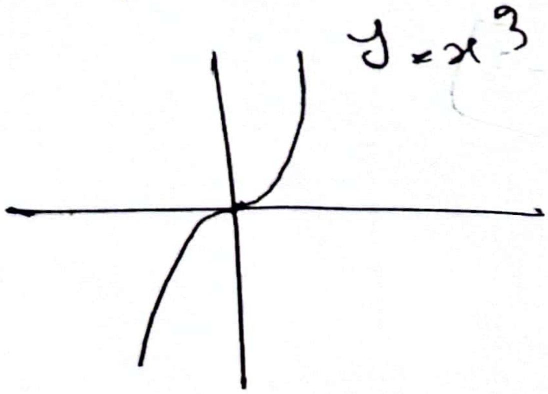
* $f(x) = x$



* $y = f(x) = x^2$



* $y = x^3$

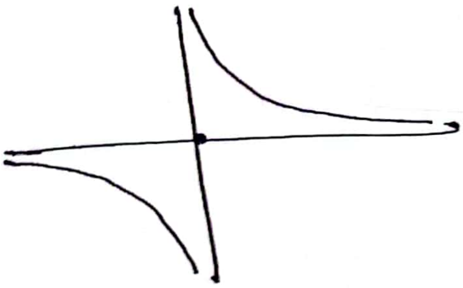


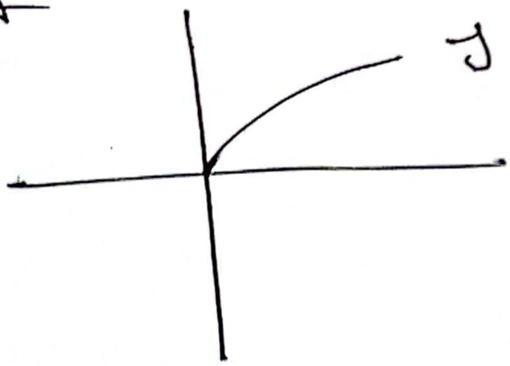
P.T.O

* $y = \frac{1}{x}$

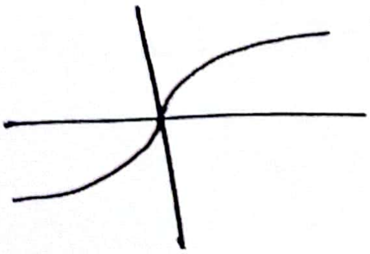
$x \neq 0$

$y \neq 0$



* 

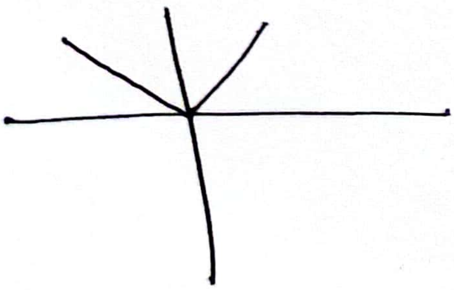
$y = \sqrt{x}$

* 

$y = \sqrt[3]{x}$

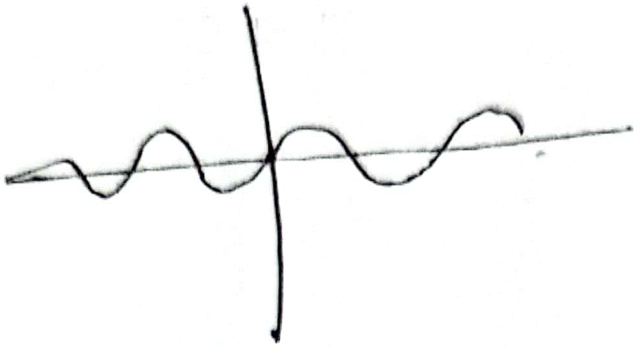
* $y = f(x) = |x|$

$x = 0$

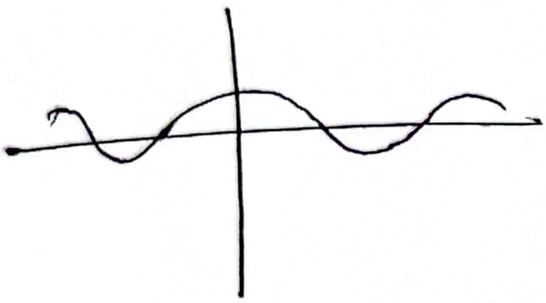


P.T.O

* $y = \sin x$



* $y = \cos x$



C.W 2.11.22

Calculus

Inverse function

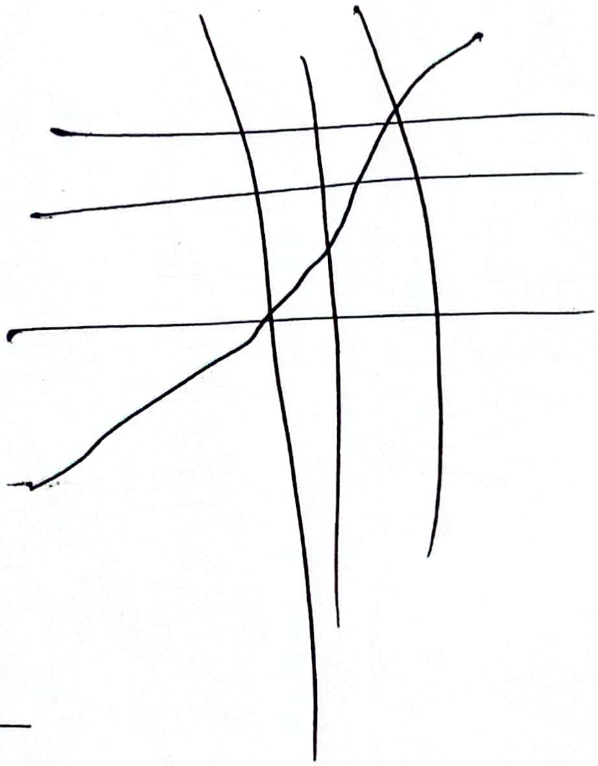
$y = f(x)$

$\Rightarrow x = f^{-1}(y) = g(y)$

$\downarrow$ dependent $\quad\quad$ $\downarrow$ Independent

- $y = x^3$

$\Rightarrow x = y^{-3}$



$f(a) = a^3$

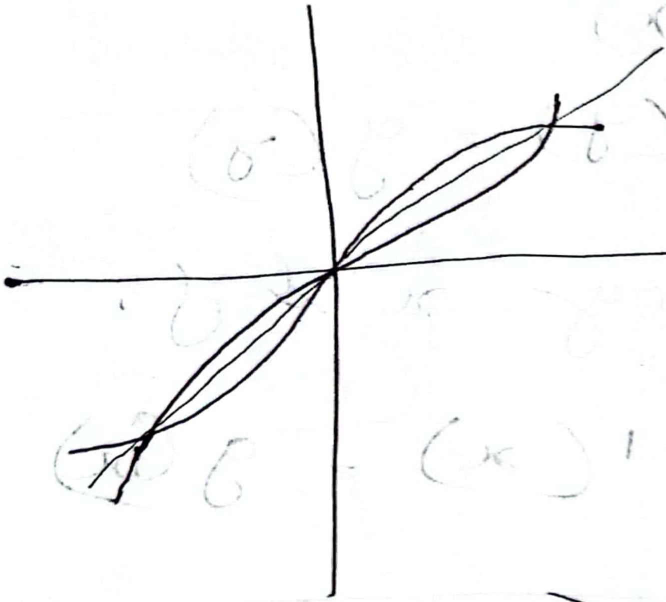
$f(b) = b^3$

$a^3 = b^3$

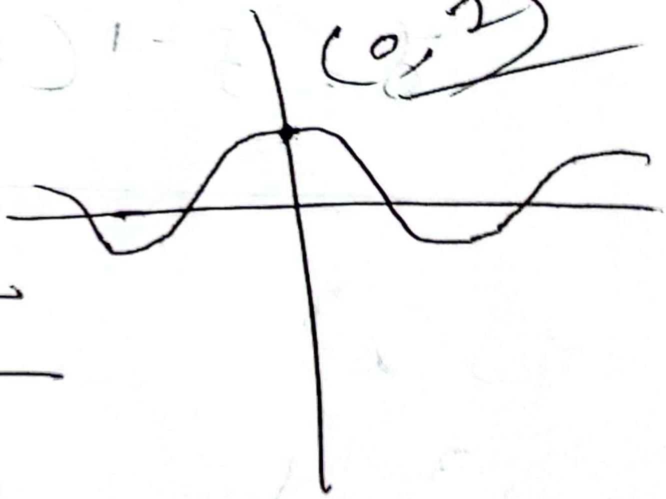
$\Rightarrow \sqrt[3]{a} = \sqrt[3]{b}$

$\Rightarrow a = b$

This is one to one function



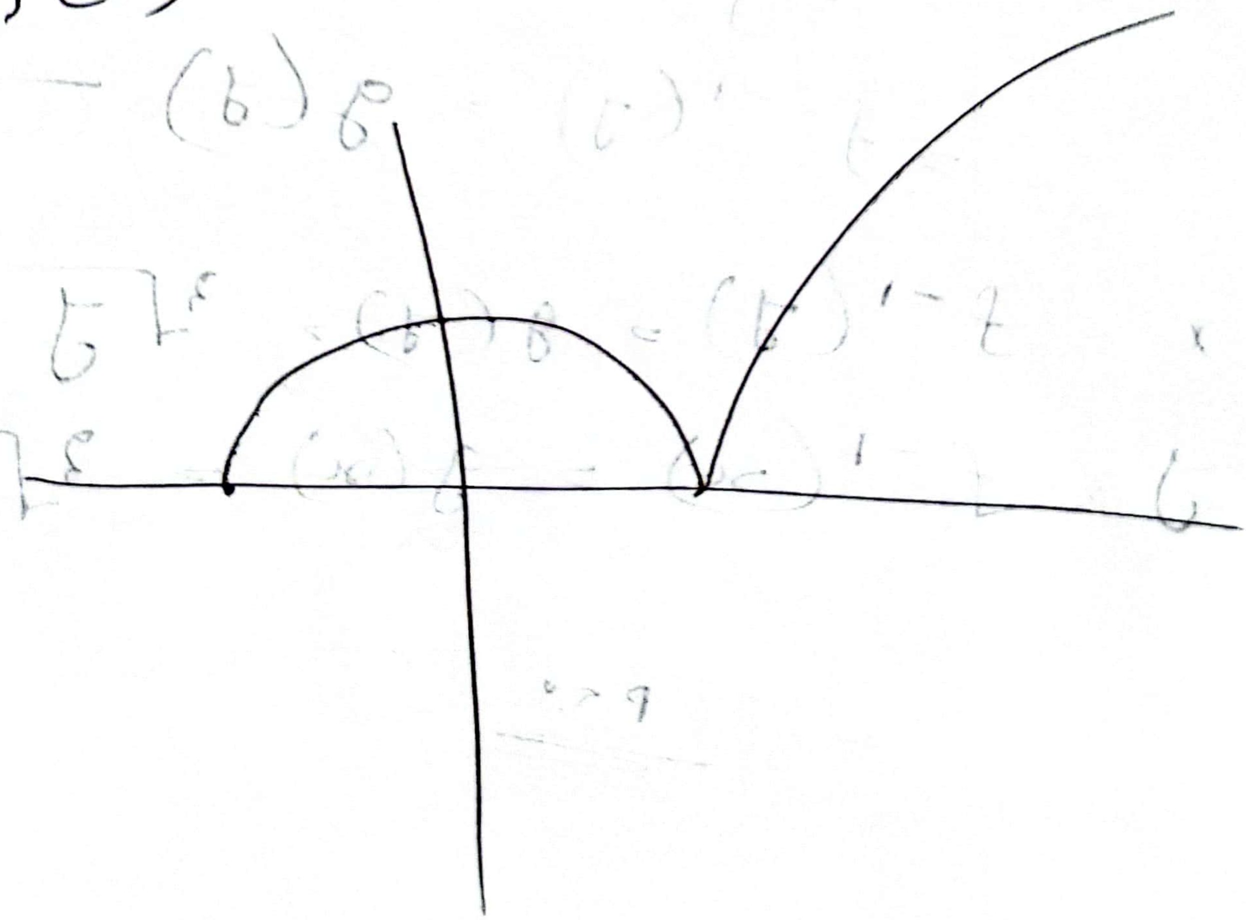
$y = \cos x$

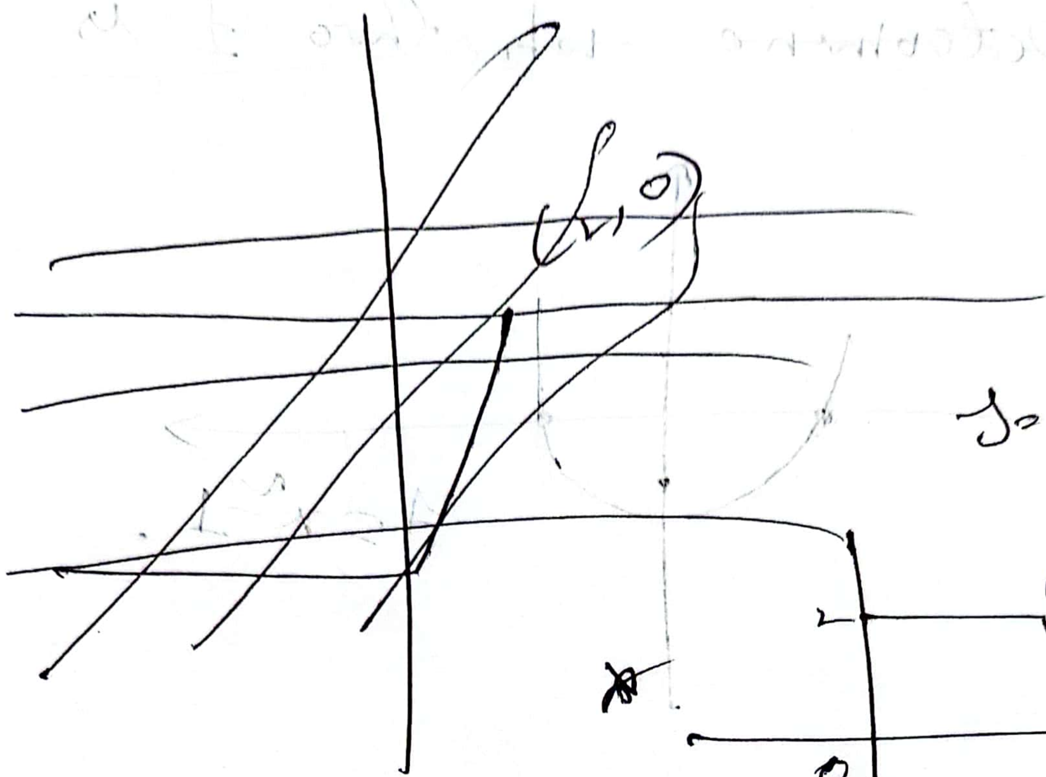


(i) $f(x) = \sqrt{3x-2}$

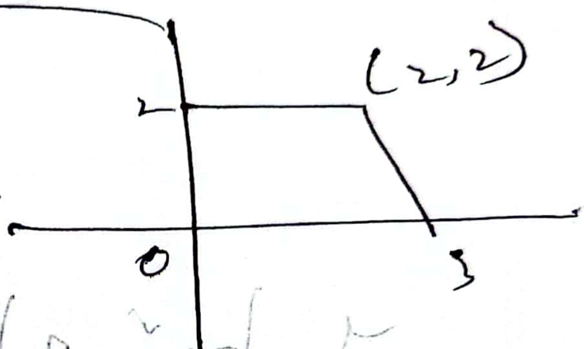
(ii) $f(x) = x^3 + 1$

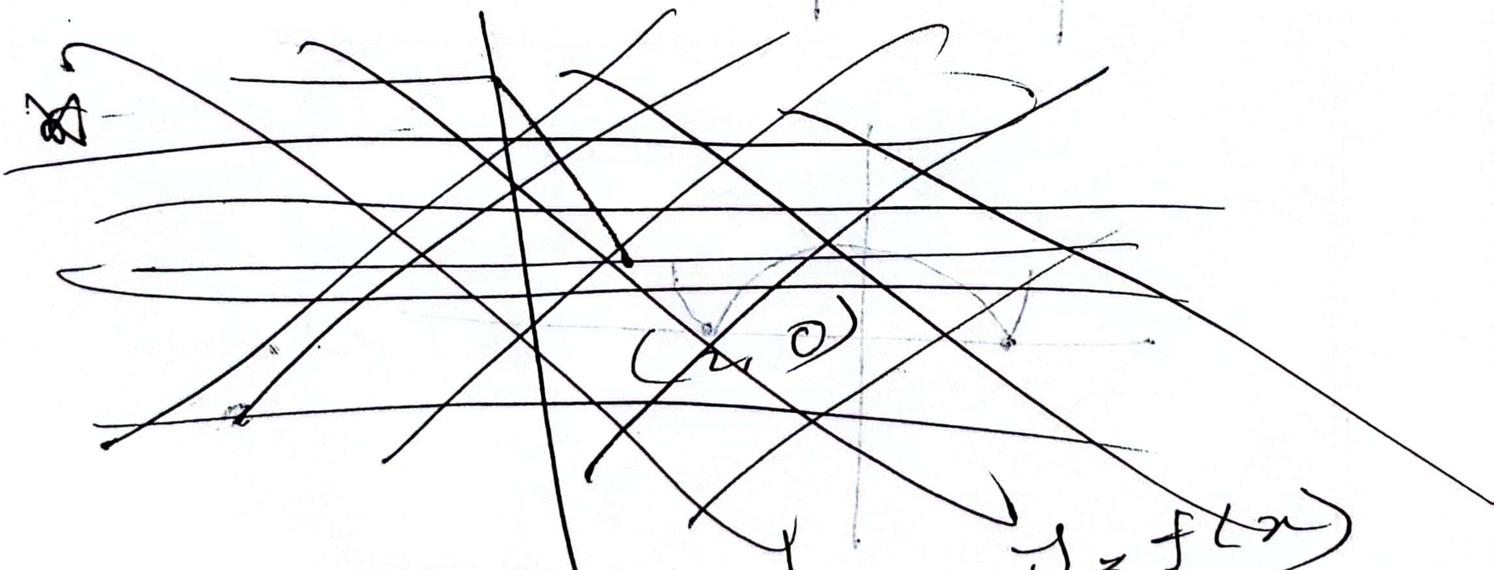
(iii) $f(x) = \sqrt[3]{x-1}$





$y = f(2-x)$

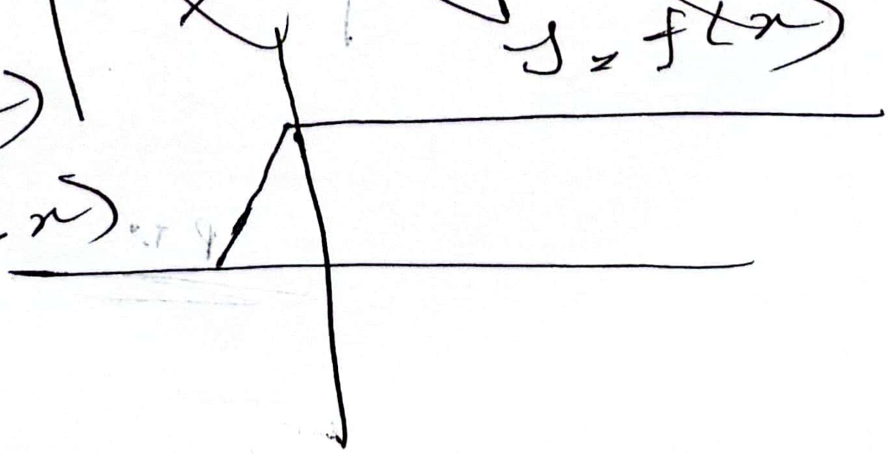




$y = f(x)$

$y = 2f(2-x)$

$y = -f(2-x)$

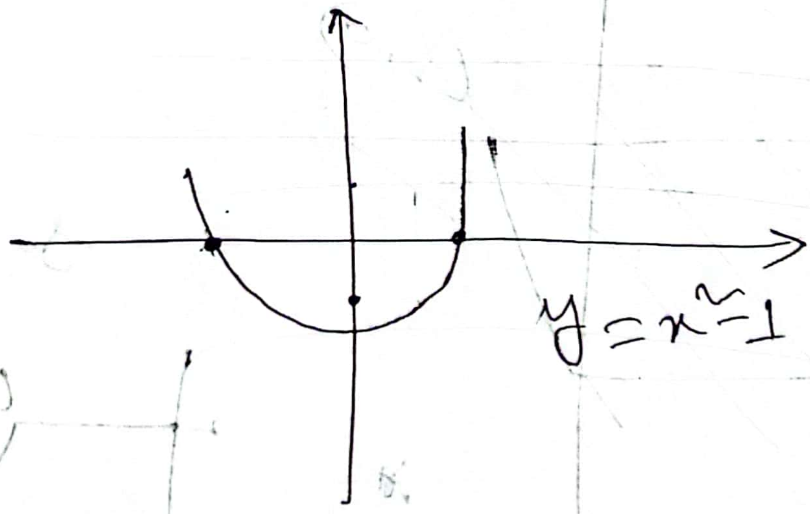


C.W
5.11.22

Calculus

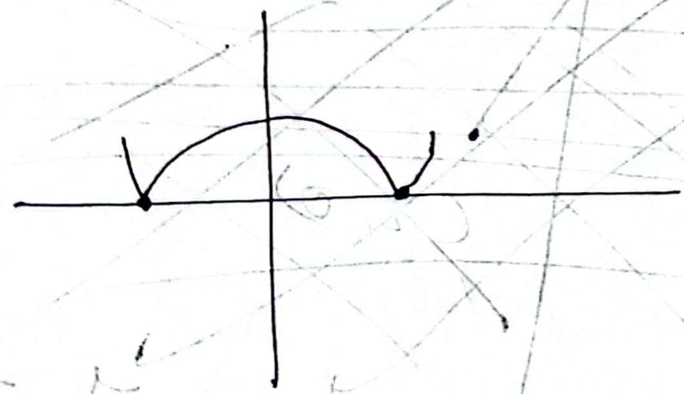
Chapter - 0.4

* ~~Determine whether~~ f is

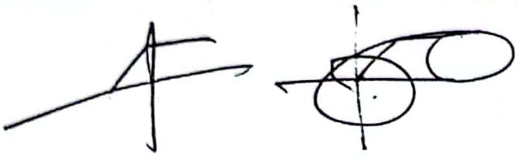


$y = x^2 - 1$

$y = |x^2 - 1|$



P.T.O



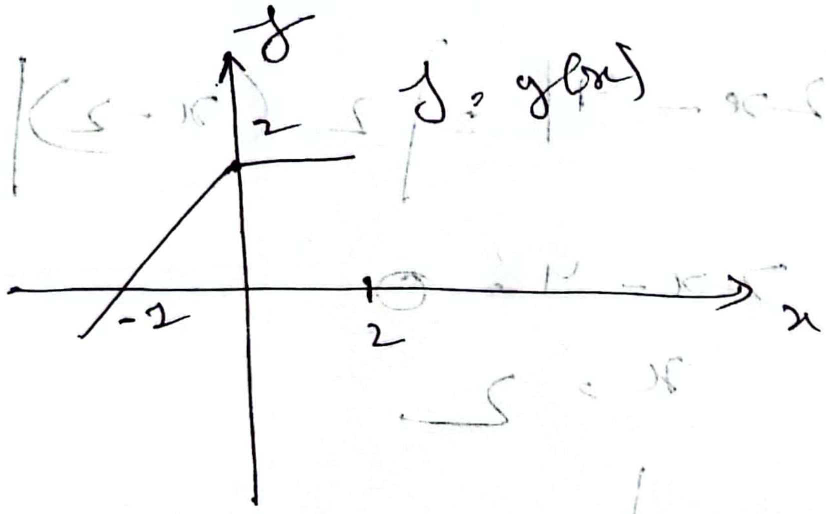
Chapter – 0.2

1 - 24

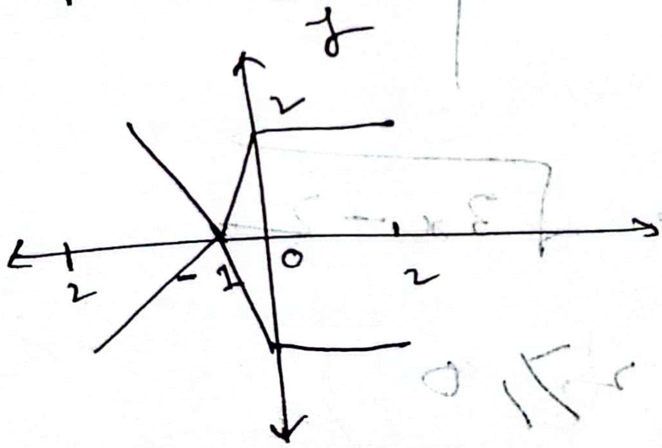
Exercise

(H.W)

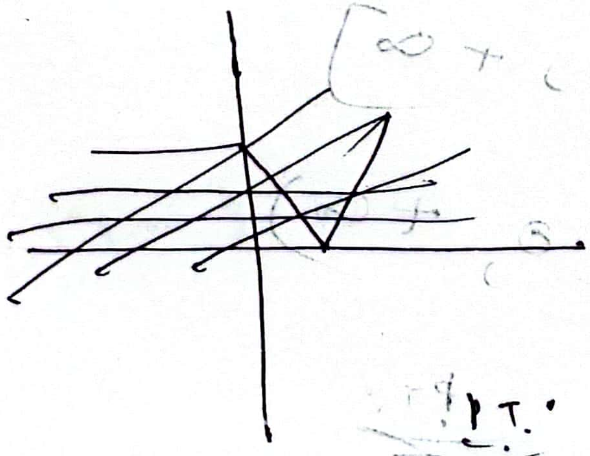
* $y = g(x)$



(i) $y = |g(x)|$



(ii) $y = |-g(x)|$

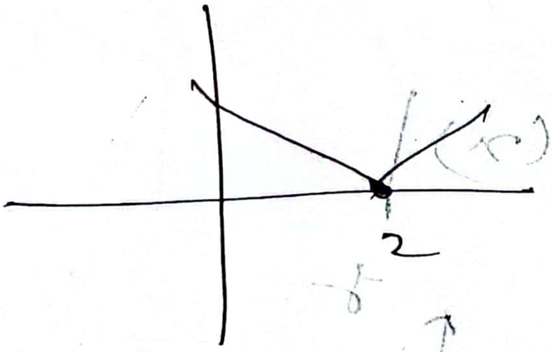


P.T.O.

* $y = |2x - 4| = |2(x-2)|$

$2x - 4 = 0$

$x = 2$



* $f(x) = \sqrt{3x-2}$

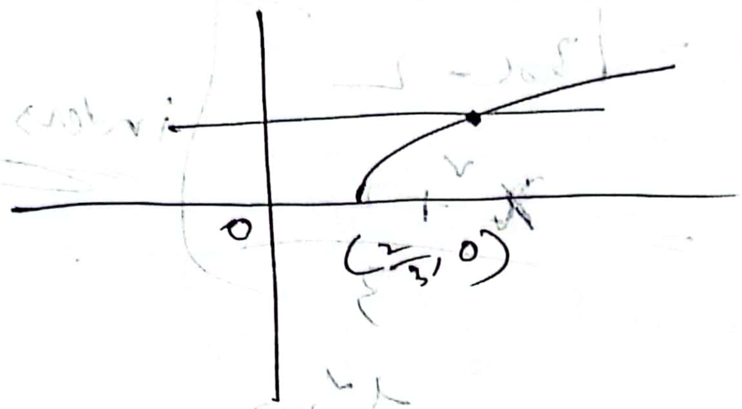
$3x - 2 \geq 0$

$\Rightarrow x \geq \frac{2}{3}$

Dom $\left[\frac{2}{3}, +\infty\right)$

Range: $[0, +\infty)$

PT.



Since the horizontal line test cuts the graph only at one point so f has an inverse function.

* $y = (x-1)^3$
$\Rightarrow y^{\frac{1}{3}} = x-1$

* $y = \sqrt[5]{x-1}$
$\Rightarrow y^5 = x-1$

* $y = \sqrt[3]{x-1}$
$\Rightarrow y^3 = x-1$

k.T.

$*$ $f(x) = \sqrt{3x-2}$

$g(x) = \dfrac{x^2+2}{3}$

inverse

$x = f^{-1}(y) = \dfrac{y^2+2}{3} = g(y)$

$y = \sqrt{3x-2}$

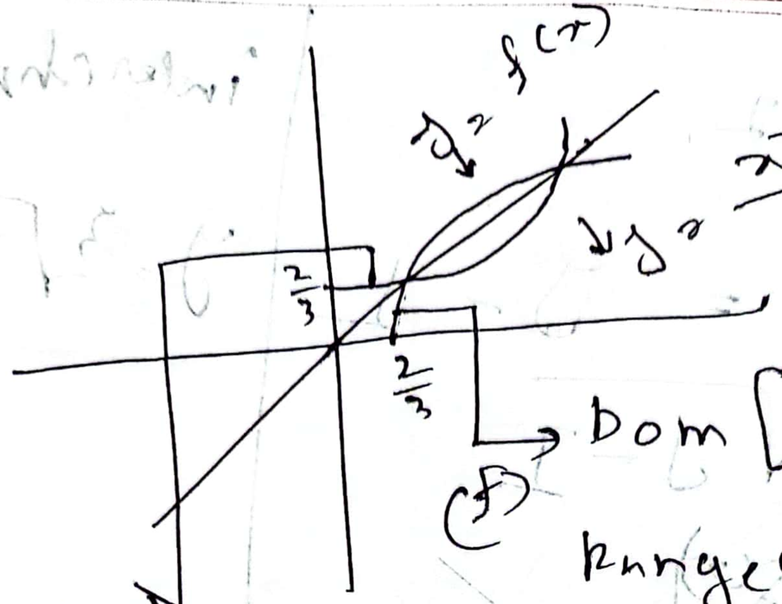
$\Rightarrow y^2 = 3x-2$

$\Rightarrow 3x = y^2+2$

$\Rightarrow x = \dfrac{y^2+2}{3} = g(y)$

P.T.O

$$2\sqrt{2x-3}$$



$y = \frac{x^2+2}{3}$; $y \geq \frac{2}{3}$

(f) Dom $[\frac{2}{3}, +\infty)$;

Range $[0, +\infty)$;

f^{-1} ~~Dom $[0, +\infty)$~~

(x) Dom $[0, +\infty)$;

Range $[\frac{2}{3}, +\infty)$,

P.T.O.

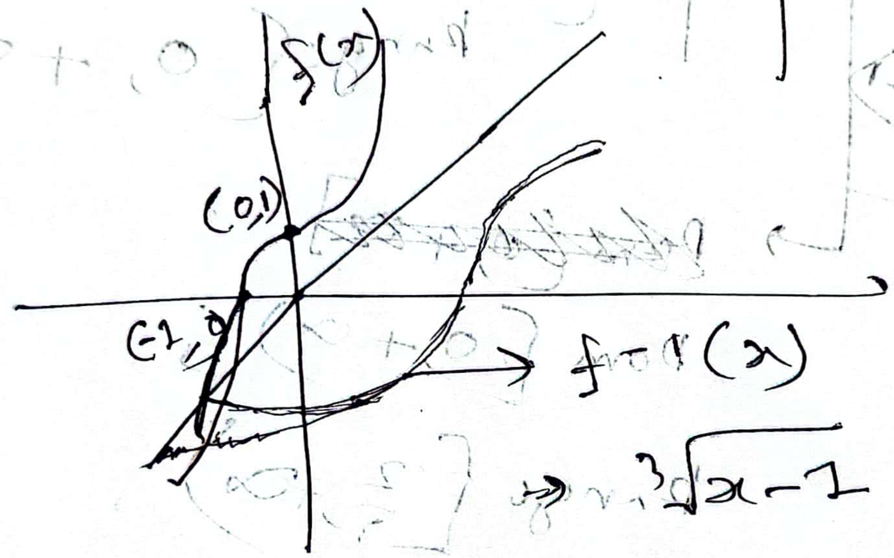
$y = x^3 + 1$

$\Rightarrow x^3 = y - 1$

$\Rightarrow x = \sqrt[3]{y-1}$

interchange

$y = \sqrt[3]{x-1}$

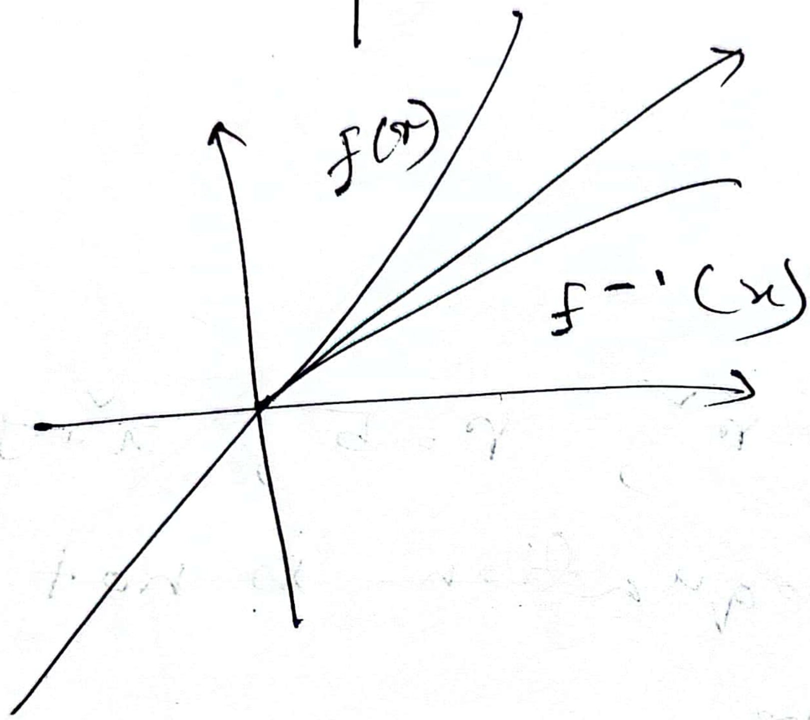


PT0

$y = x^2 \; \{x \geq 0\}$

$x = \sqrt{y}$

$y = \sqrt{x}$



P.T.O

SW
6.11.22

* Exercise [12 - 16]

1A LD

Calculus

Circle

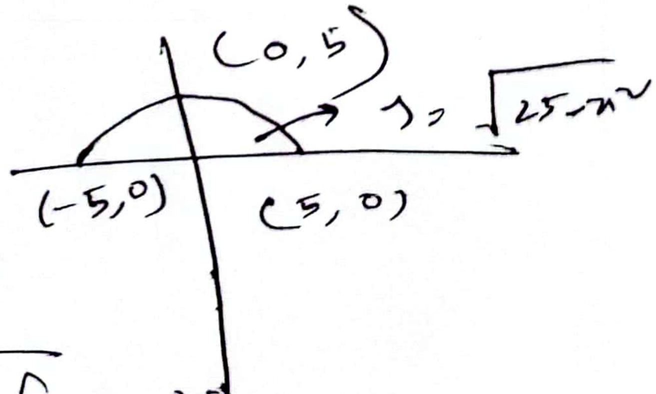
$x^2 + y^2 = r^2$, $r = 5$, $x^2 + y^2 = 25$

Circle equation is not function.

$y = f(x)$

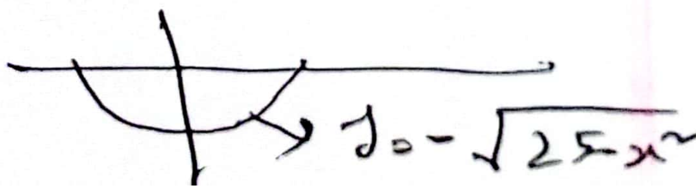
$\Rightarrow y^2 = 25 - x^2$

$\Rightarrow y = \pm\sqrt{25 - x^2}$



This is function.

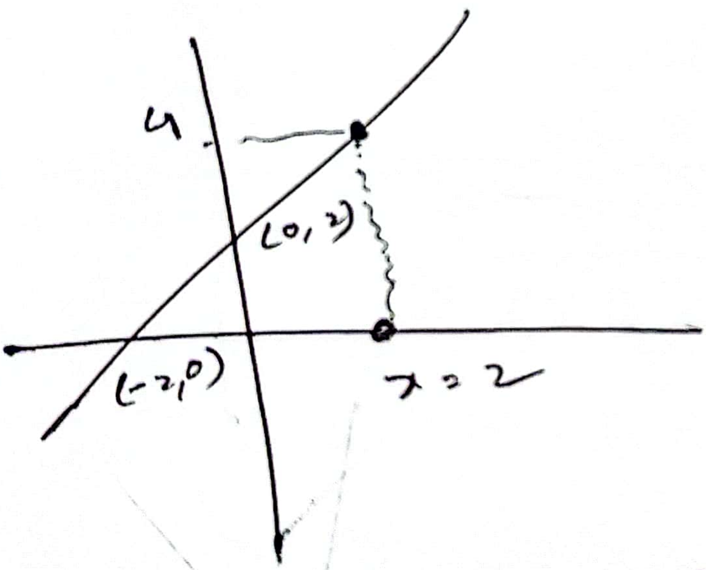
P.T.:



$$f(x) = \frac{x^2 - 4}{x - 2} \qquad x \neq 2$$

$$= \frac{(x+2)(x-2)}{(x-2)}$$

$$= x + 2$$



Dom : $x \in \mathbb{R} - \{2\}$

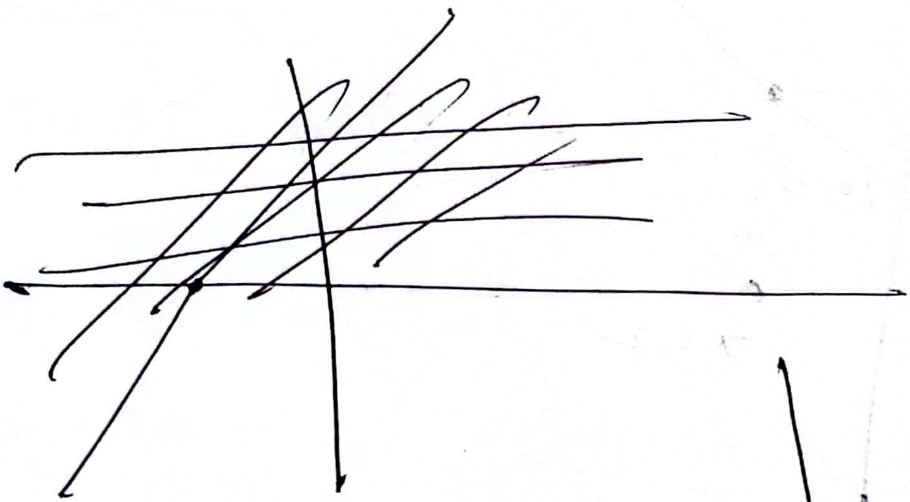
R : $y \in \mathbb{R} - \{4\}$

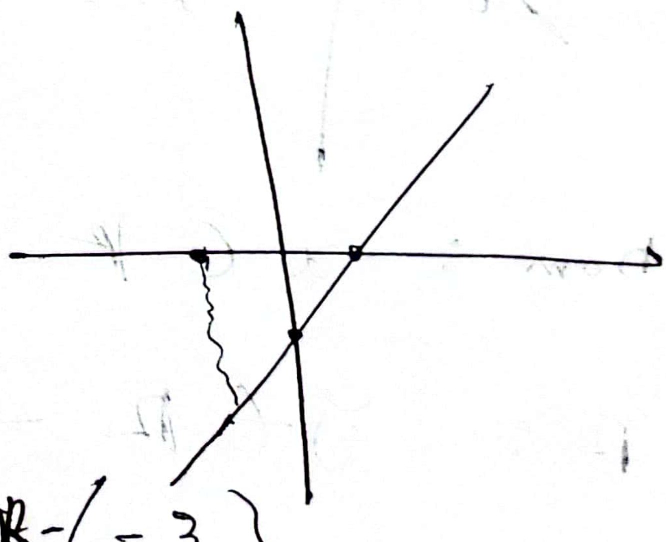
P.T.O

$$y = \frac{x^2-9}{x+3} \qquad x \neq -3$$

$$= \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}}$$

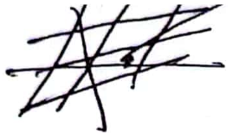
$$= x-3$$

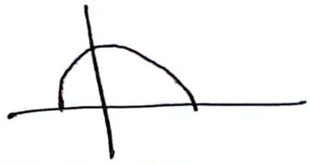




Dom: $\{x \in \mathbb{R} - \{-3\}\}$

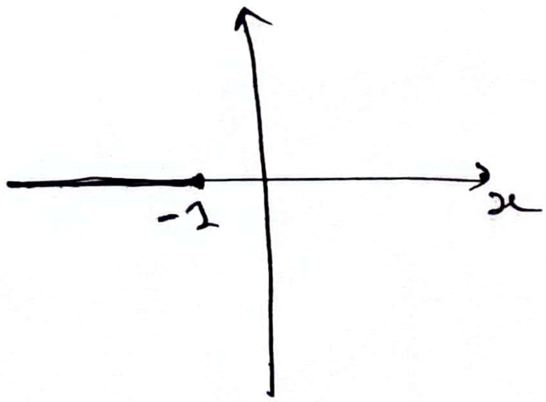
R: $y \in \mathbb{R} - \{-6\}$





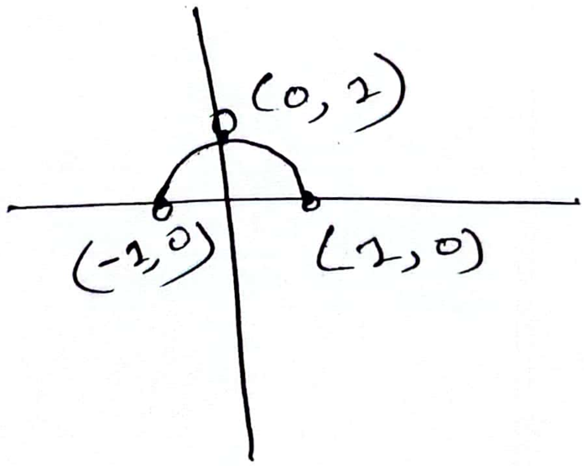
$$f(x) = \begin{cases} 0 & x \le -1 \\ \sqrt{1 - x^2} & -1 < x < 1 \\ x & x \ge 1 \end{cases}$$

$f(x) = 0, \; x \le -1$

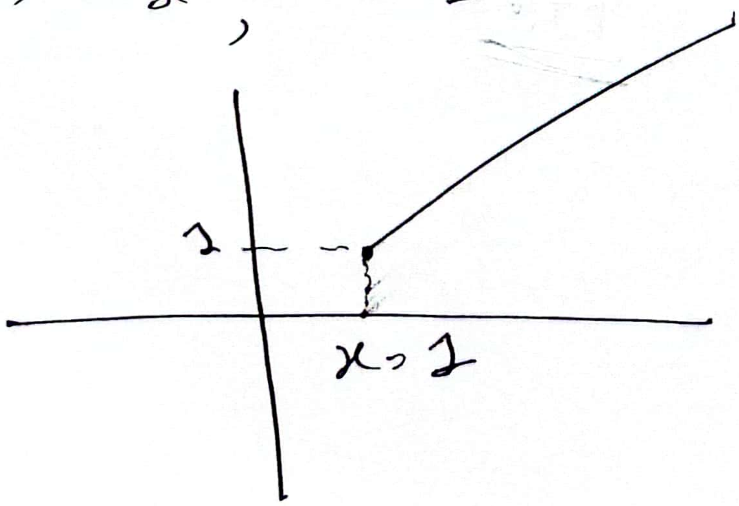


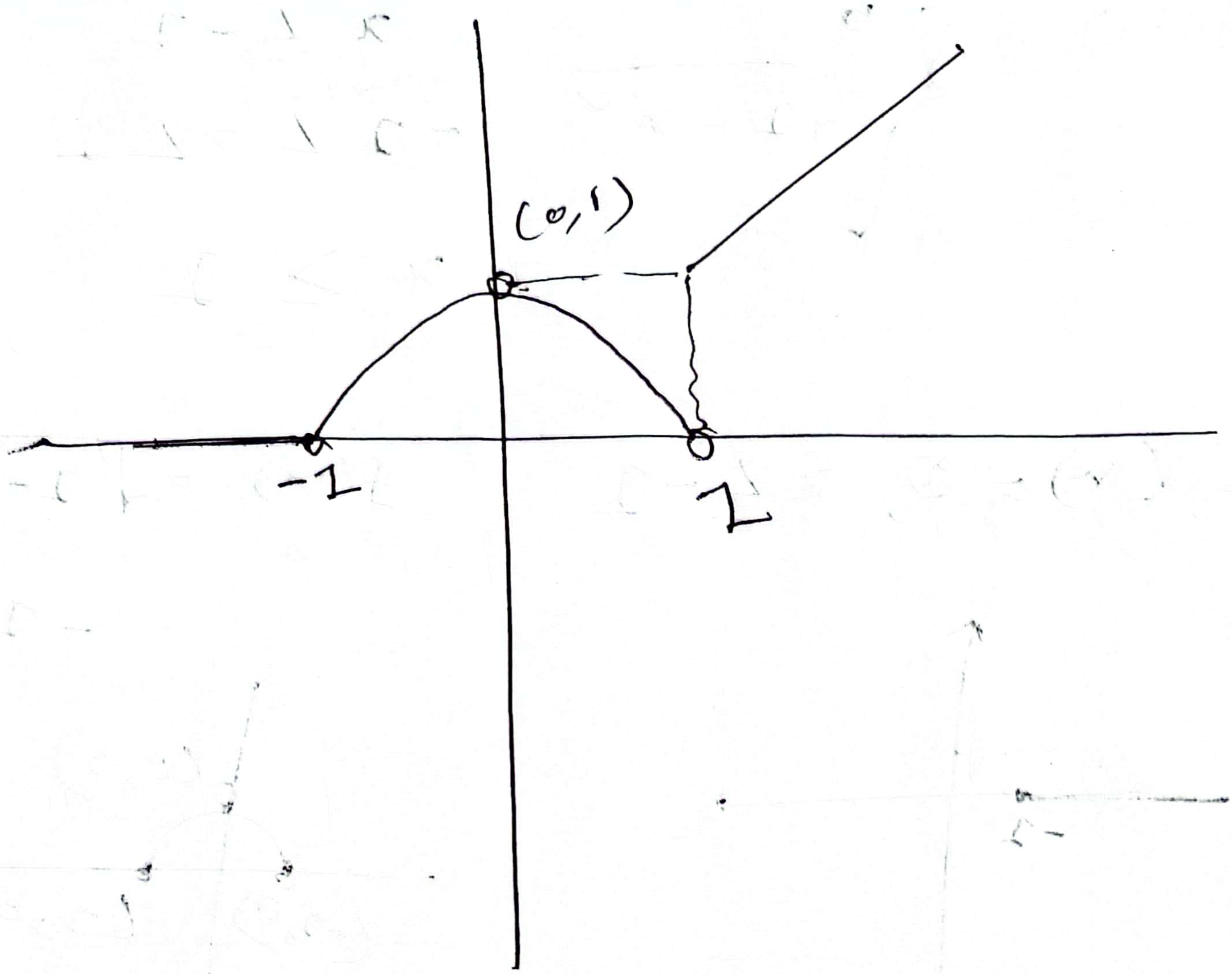
$f(x) = \sqrt{1 - x^2},$

$-1 < x < 1$



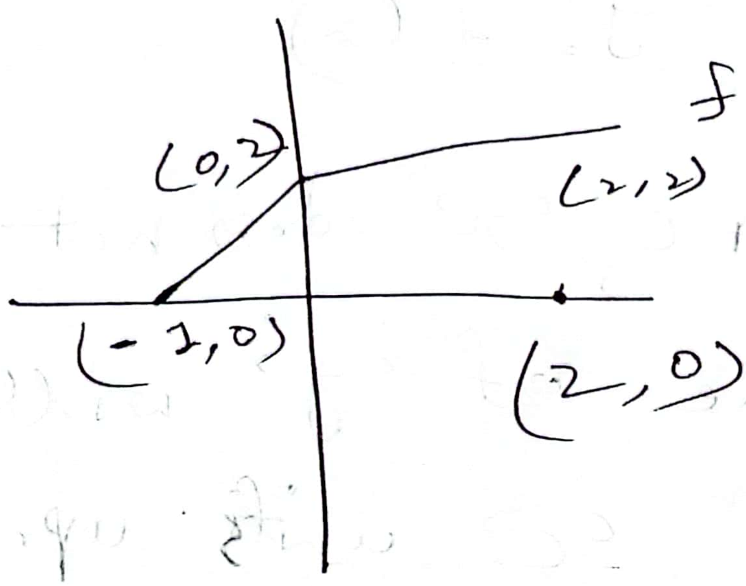
$f(x) = x, \quad x \ge 1$



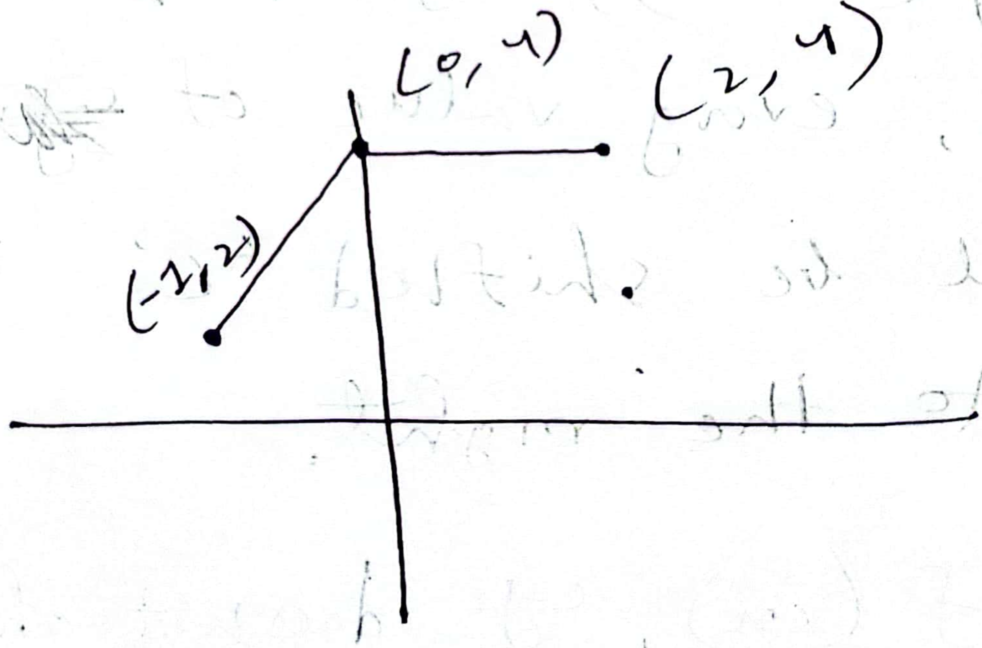


$P_{7,0}$

*



$y = f(x) + 2$



P.T.O

Given, $y = f(x)$

(i) $y = f(x) + c$, 'x' does not change, every value of 'y' will be shifted 'c' units up.

(ii) $y = f(x - c)$, 'y' does not change, every value of ~~y~~ 'x' will be shifted 'c' units to the right.

(iii) $y = f(cx)$, 'y' does not change, multiply each value of 'x' by $\frac{1}{c}$.

P.T.O.

(iv) $y = c\,f(x)$, 'x' will not change & multiply each value of 'y' by 'c'.

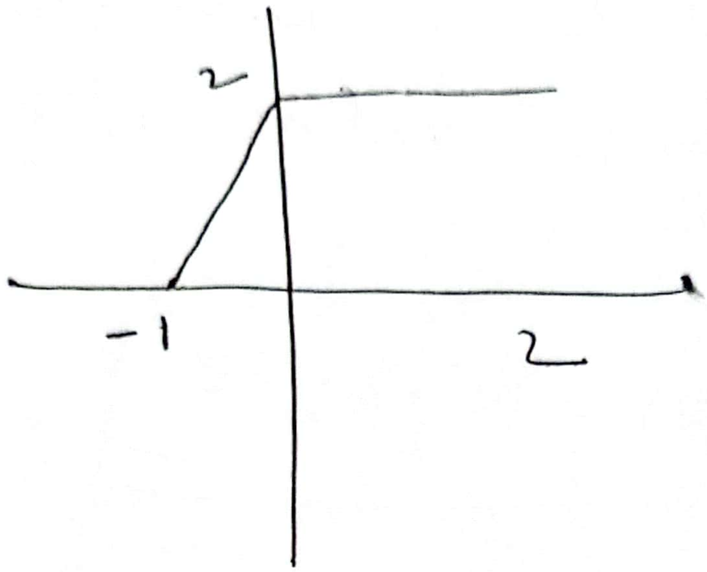
$$\boxed{f\left(\frac{x}{2}\right) = f\left(\frac{1}{2}\cdot x\right) = \frac{z}{\frac{1}{2}} = 2z}$$

(v) $y = f(-x)$ means reflection of $y = f(x)$ with respect to the 'y' axis.

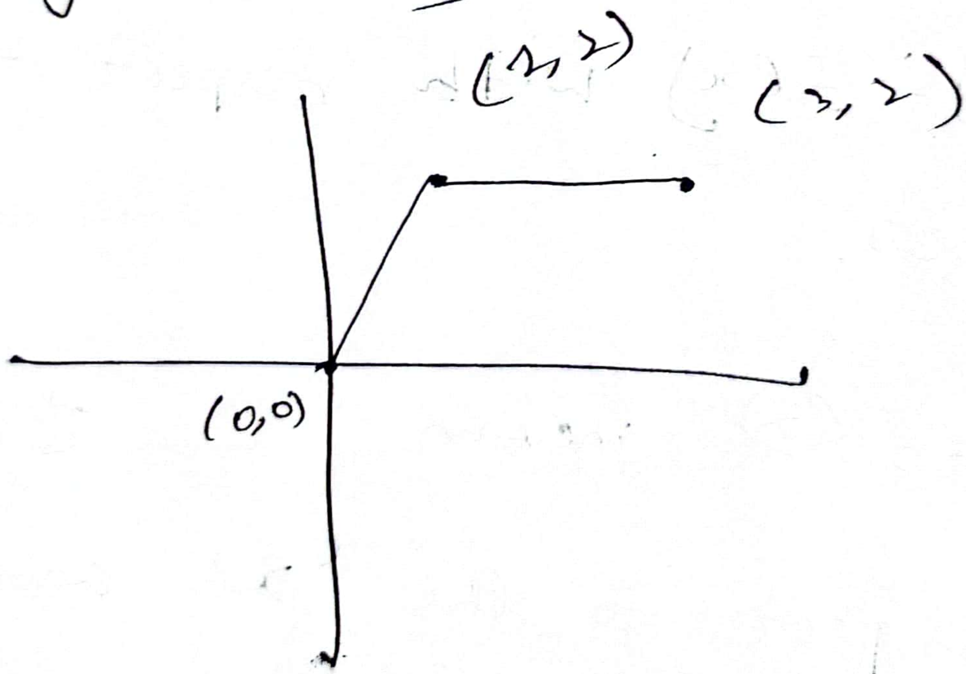
(vi) $y = -f(x)$ means $y = f(x)$ with respect to the 'x' axis.

P.T.O.

$f(x)$

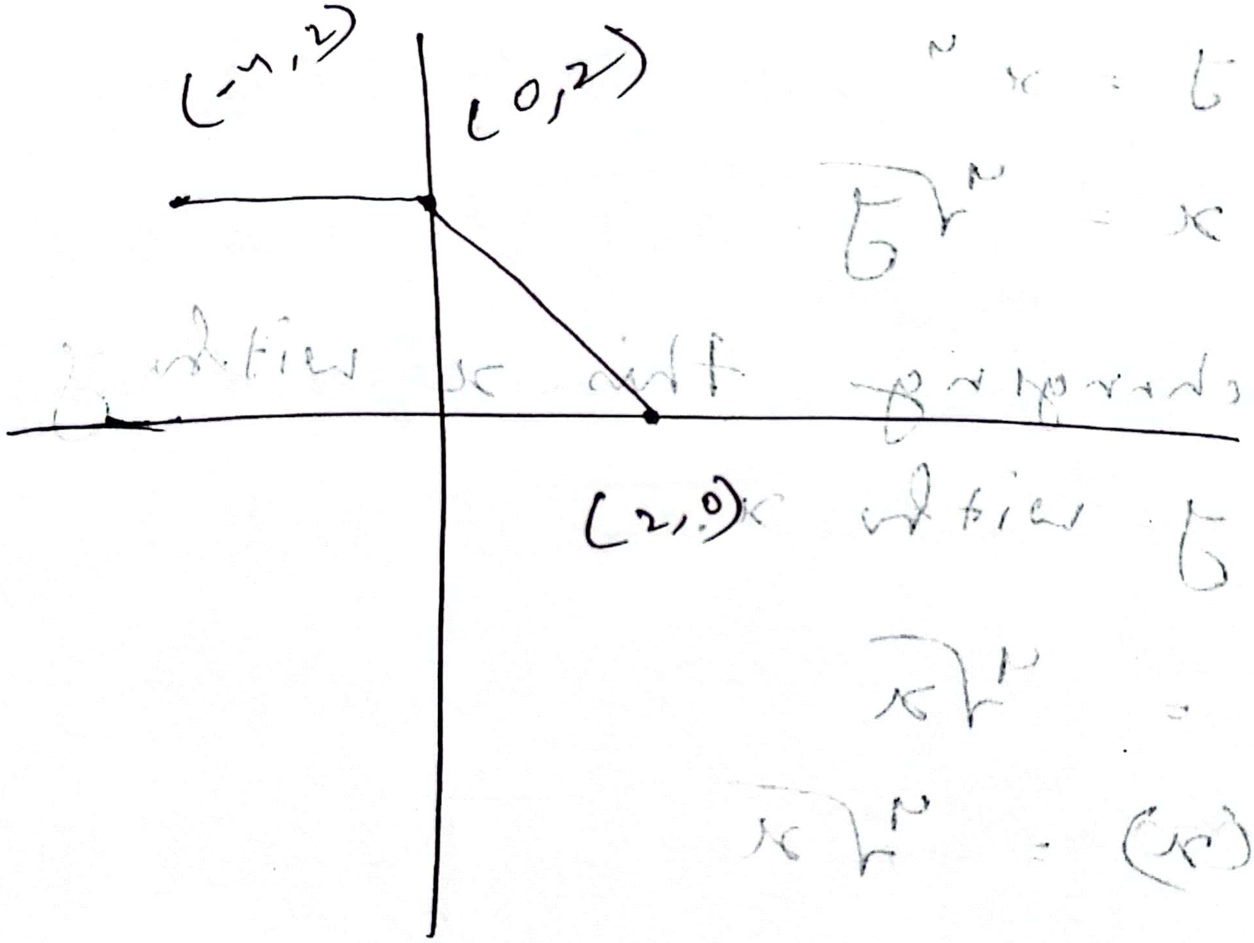


$* f(x-1)$



P.T.O

(d) $y = f\left(-\frac{1}{2}x\right)$



H.W 6.11.22

Assignment - 2

Calculus

5

(i) $f(x) = x^4$

$\Rightarrow y = x^4$

$\Rightarrow x = \sqrt[4]{y}$

interchanging this x with y and y with x...

$y = \sqrt[4]{x}$

$\Rightarrow g(x) = \sqrt[4]{x}$

(ii) $f(x) = x^5 = y$

$\Rightarrow x = \sqrt[5]{y}$

interchanging...

$y = \sqrt[5]{x}$

$\Rightarrow g(x) = \sqrt[5]{x}$

P.T.O

(iii) $f(x) = x^3 + 5$

$\Rightarrow y = x^3 + 5$

$\Rightarrow x^3 = y - 5$

$\Rightarrow x = \sqrt[3]{y-5}$

interchanging

$y = \sqrt[3]{x-5}$

$\Rightarrow g(x) = \sqrt[3]{x-5}$

(iv) $f(x) = \sqrt{2x+3}$

$y = \sqrt{2x+3}$

$\Rightarrow y^2 = 2x+3$

$\Rightarrow 2x = y^2 - 3$

$\Rightarrow x = \dfrac{y^2-3}{2}$

$\Rightarrow x = \frac{1}{2}(y^2-3)$

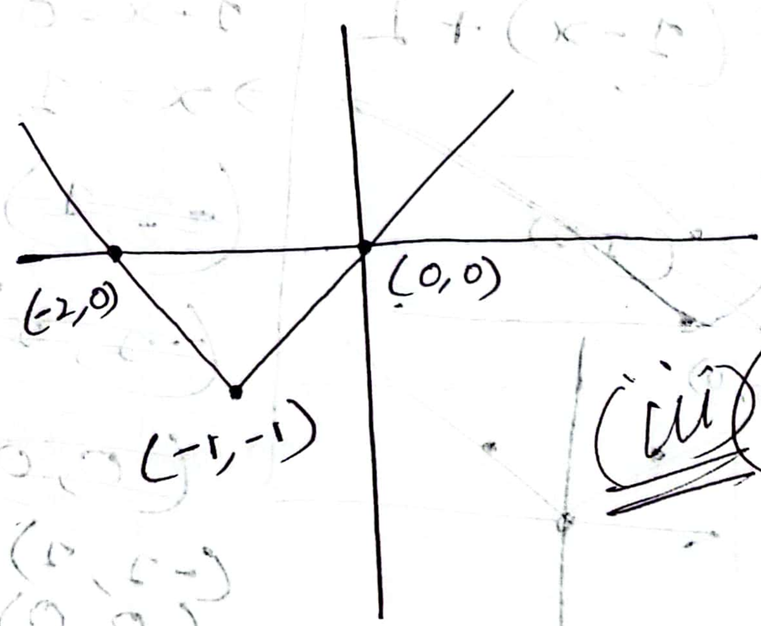
interchanging,

$y = \frac{1}{2}(x^2-3)$

$\Rightarrow g(x) = \frac{1}{2}(x^2-3)$

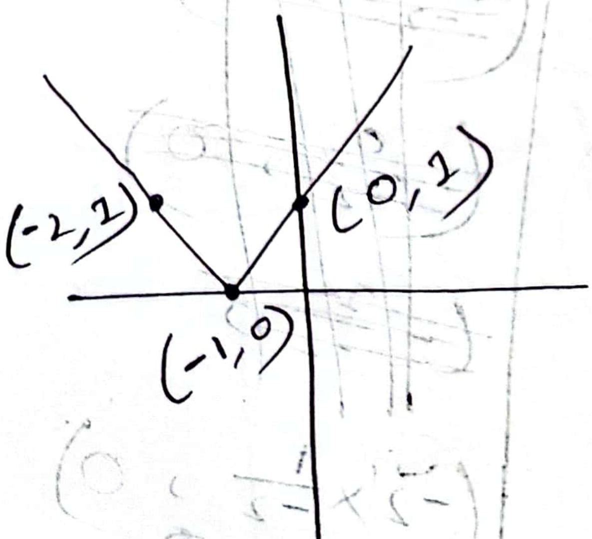
This function is inverse of $f(x)$ when $(x \geq 0)$.

I



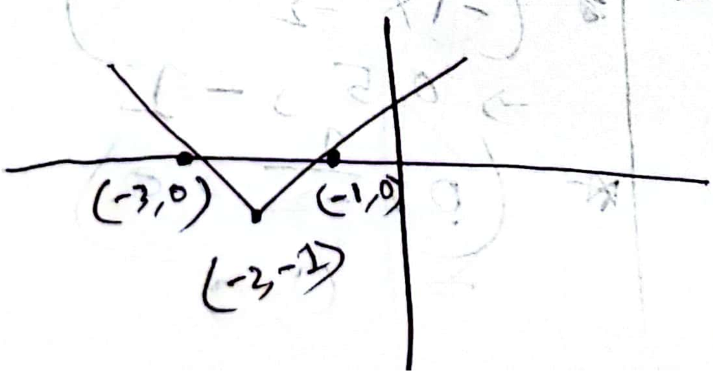
(iii) $(1 - f(x)) + 1$

(i) $y = f(x) + 1$



(-2, 1)
(0, 1)
(-1, 0)

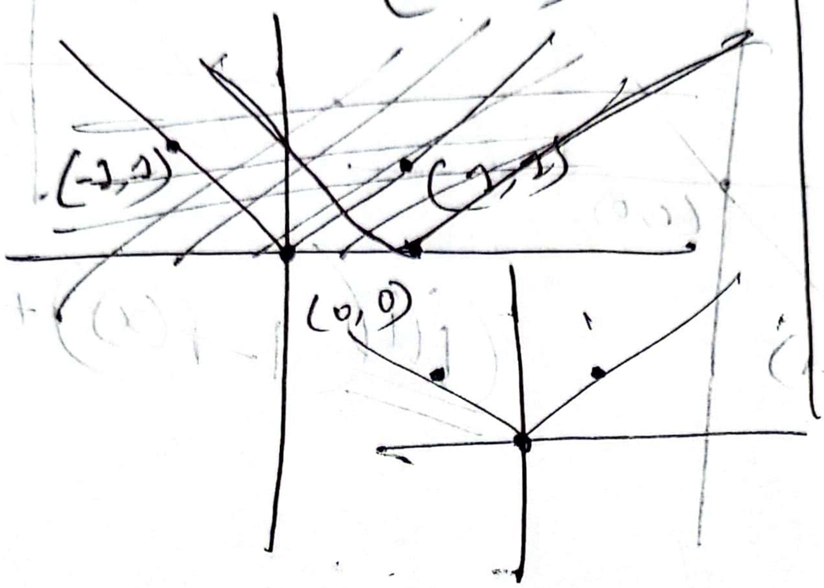
(ii) $y = f(x+1)$ | $x+1 = 0$
$\Rightarrow x = -1$



(-3, 0)
(-2, -1)
(-1, 0)

$(-2,0)$, $(-1,-1)$, $(0,0)$ $f(-x+)+1$

(iii) $y = f(1-x)+1$



$1-x=0$

$\Rightarrow x=1, y=1$

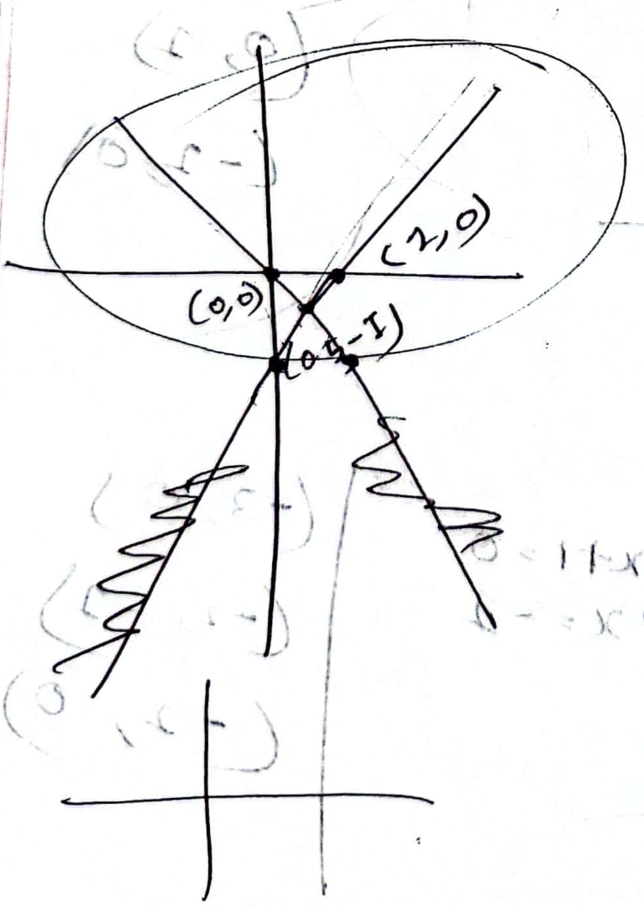
$\cancel{(-1,1)}$

$\cancel{(-1,1)}$

$\cancel{(0,0)}$

$(-1,1)$
$(0,0)$
$(1,1)$

(iv) $y = f(-2x)$



$\cancel{\left(\frac{-2}{-2}, 0\right)}$

$\cancel{\left(\frac{0}{-2}, 0\right)}$

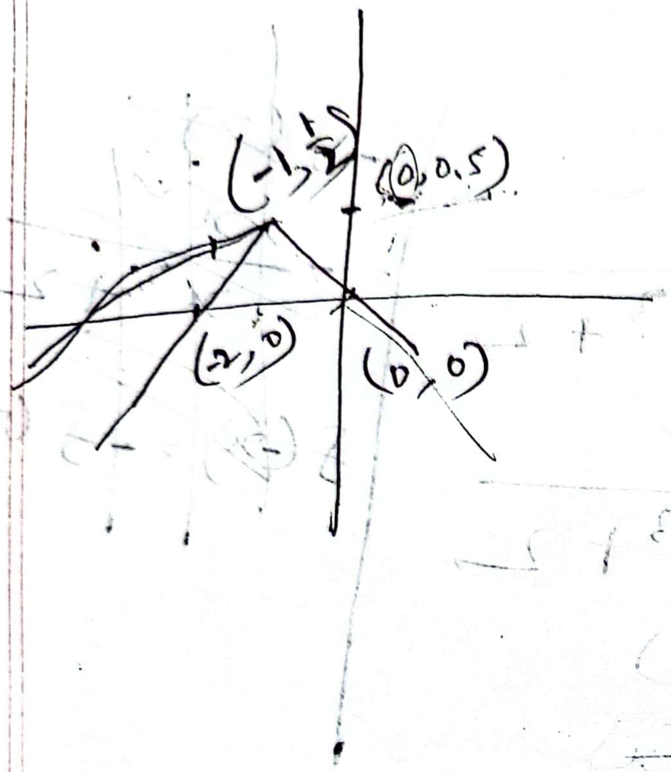
$\cancel{\left(\frac{1}{-2}\right)}$

$\left(-2 \times \frac{1}{-2}, 0\right)$
$1, 0$

$\left(-1 \times \frac{1}{-2}, -1\right)$
$\rightarrow 0.5, -1$

$\left(0 \times \frac{1}{-2}, 0\right)$
$\rightarrow 0, 0$

(v) $y = -\frac{1}{2} f(x)$



$(-2, (0 \times -\frac{1}{2})$

$\Rightarrow (-2, 0)$

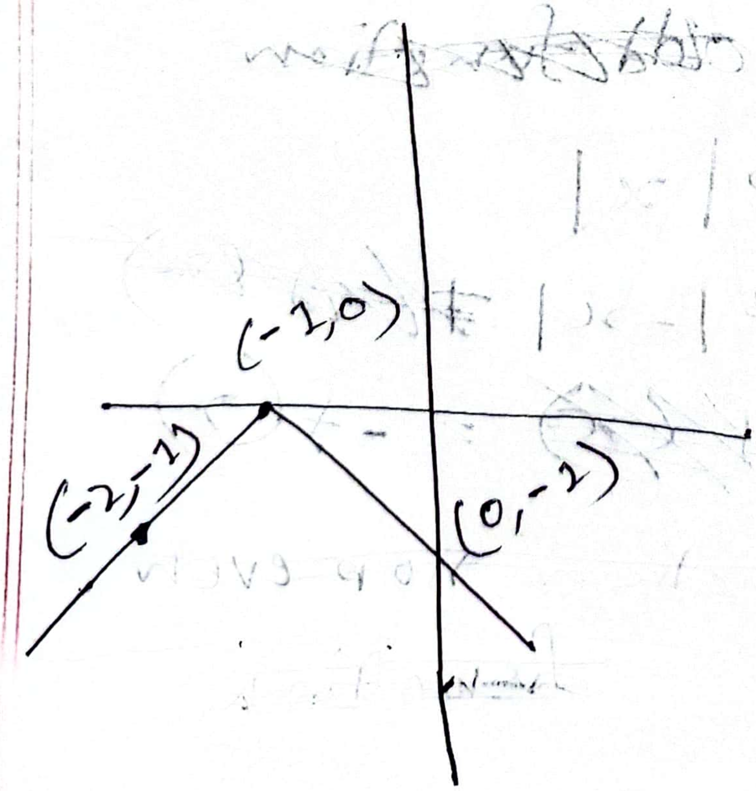
$(-1, \{-1 \times (-\frac{1}{2})\}$

$\Rightarrow (-1, \frac{1}{2})$

$(0, 0)$

(vi) $y = 2 - |f(x)|$

$y = -|f(x)| + 2$



3

(i) $f(x) = \frac{x^2}{x^3+2}$

$\Rightarrow f(-x) = \frac{(-x)^2}{-x^3+2}$

$= \frac{x^2}{-x^3+2}$

$\neq f(x)$

$\cancel{\neq -f(x)}$

$\cancel{-f(x)} = \cancel{-\frac{x^2}{x^3+2}}$

$\cancel{f(-x)} = -f(x)$

~~$f(x)$ is neither even nor odd function.~~ ~~odd function~~

(ii) $f(x) = x^3|x|$

$f(-x) = -x^3|-x| \cancel{\neq -f(x)}$

$= \cancel{f(x)} = -f(x)$

~~This is neither odd function nor even~~

odd ~~function~~

(iii) $f(x) = \dfrac{x^2}{x^6 - 3}$

$\Rightarrow f(-x) = \dfrac{(-x)^2}{(-x)^6 - 3}$

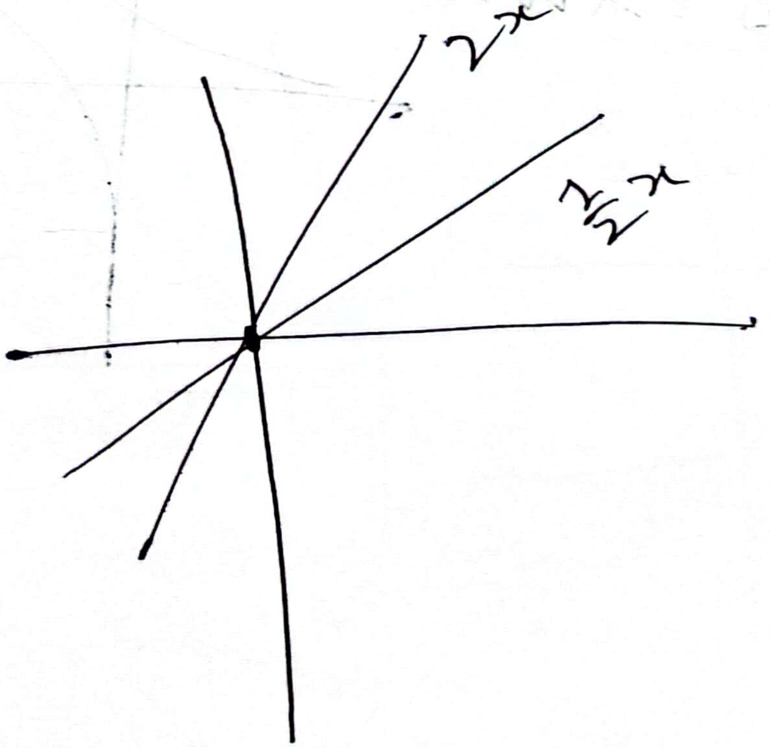
$= \dfrac{x^2}{x^6 - 3}$

$\therefore f(x) = f(-x)$

This is even function

$y = 2x$

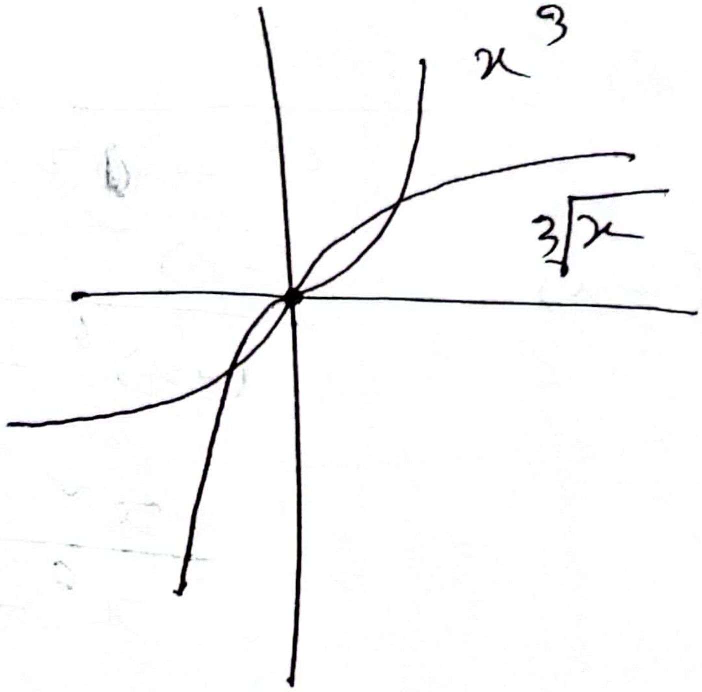
$y = \frac{1}{2}x$



P.T.O

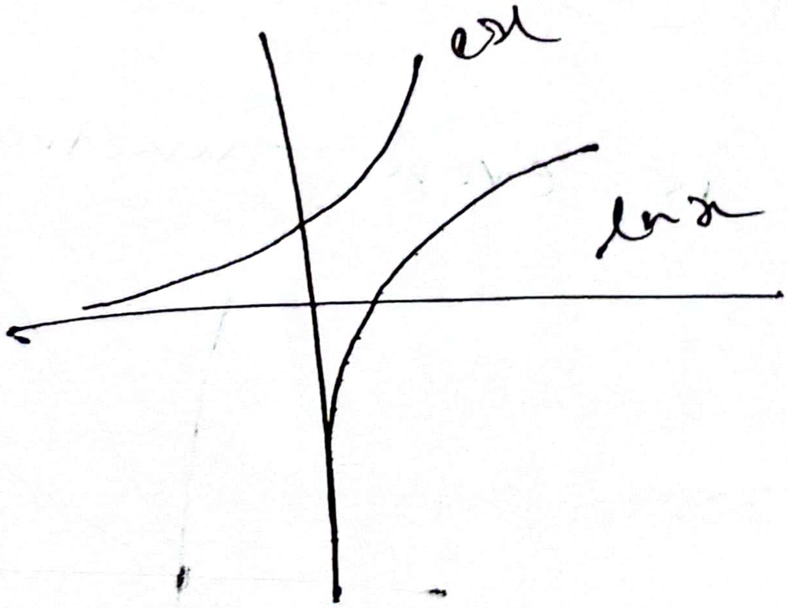
$y = x^3$

* $y = \sqrt[3]{x}$



* $y = e^x$

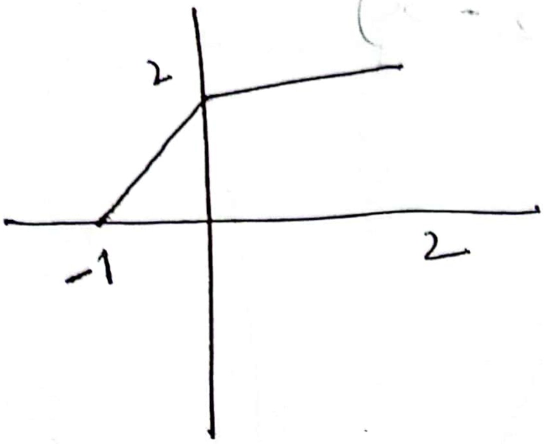
$y = \ln x$



C.W
7.11.22

Calculus

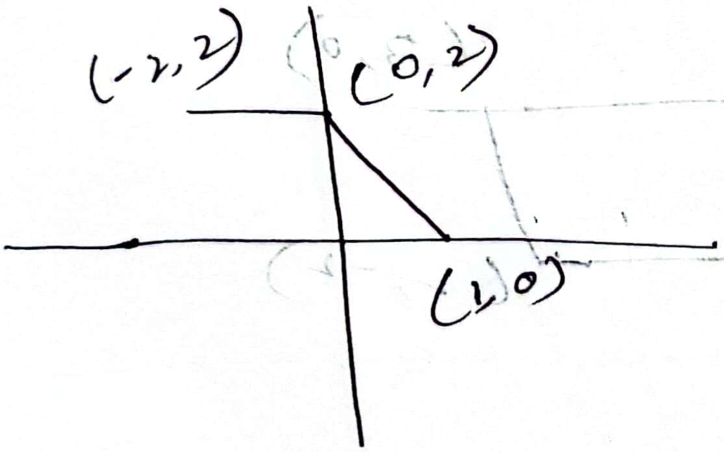
2



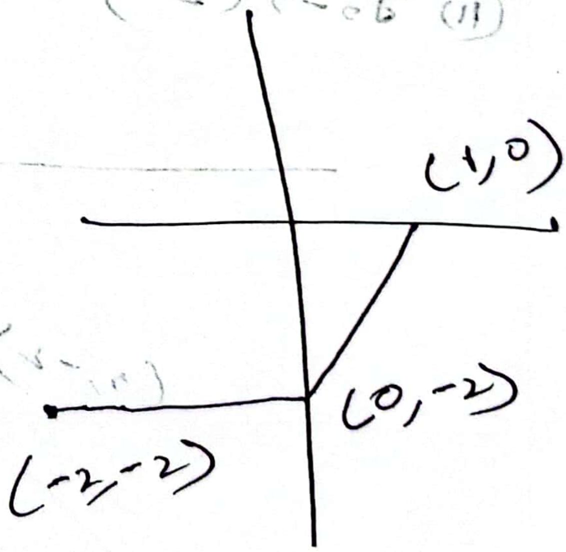
$f(x)$

(a) $y = -f(-x)$

(i) $y = f(-x)$

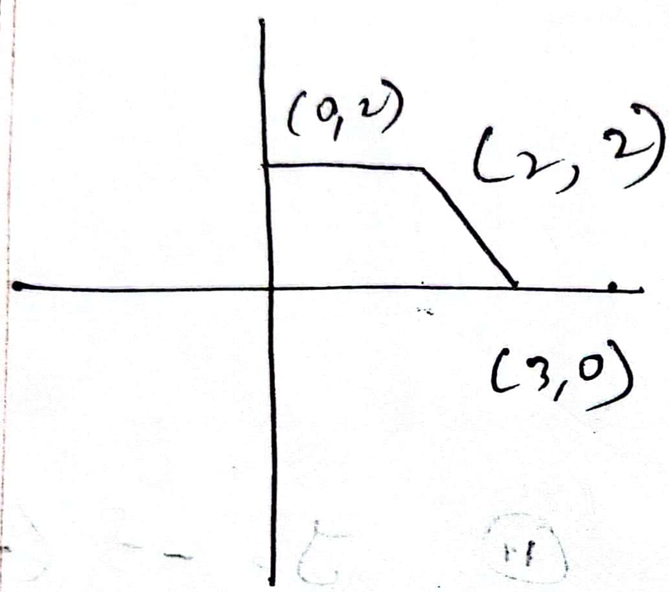


(ii) $y = -f(-x)$

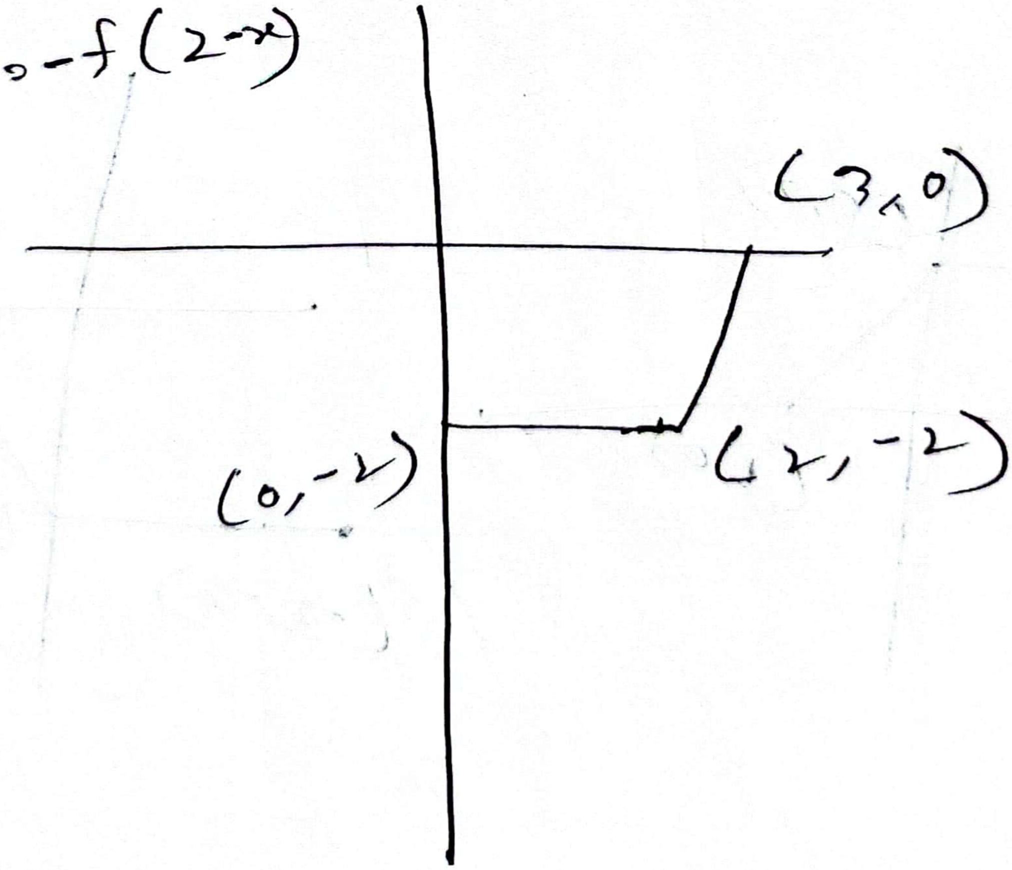


(c) $y = 1 - f(2-x)$

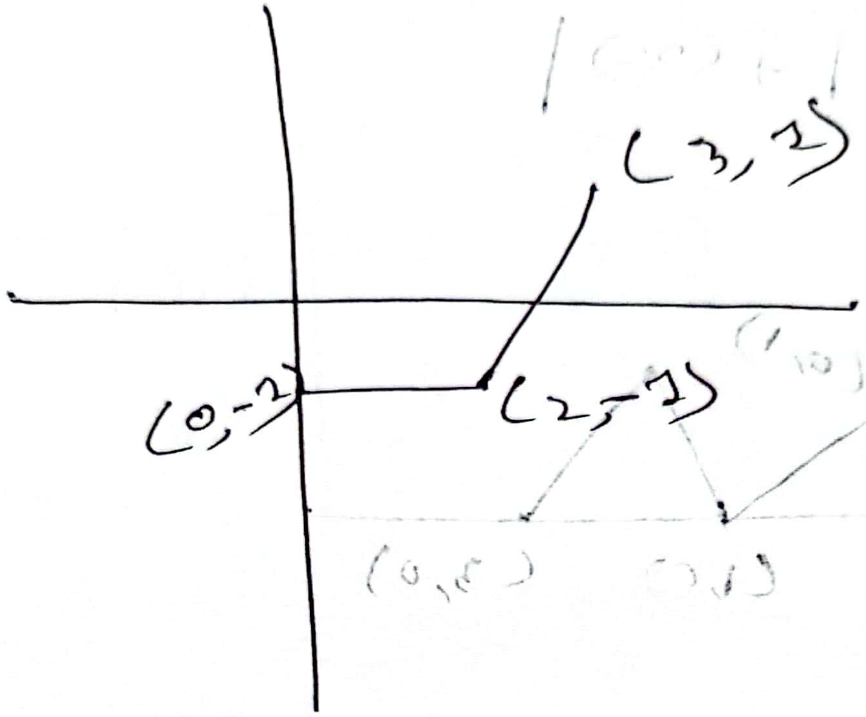
(i) $y = f(2-x)$



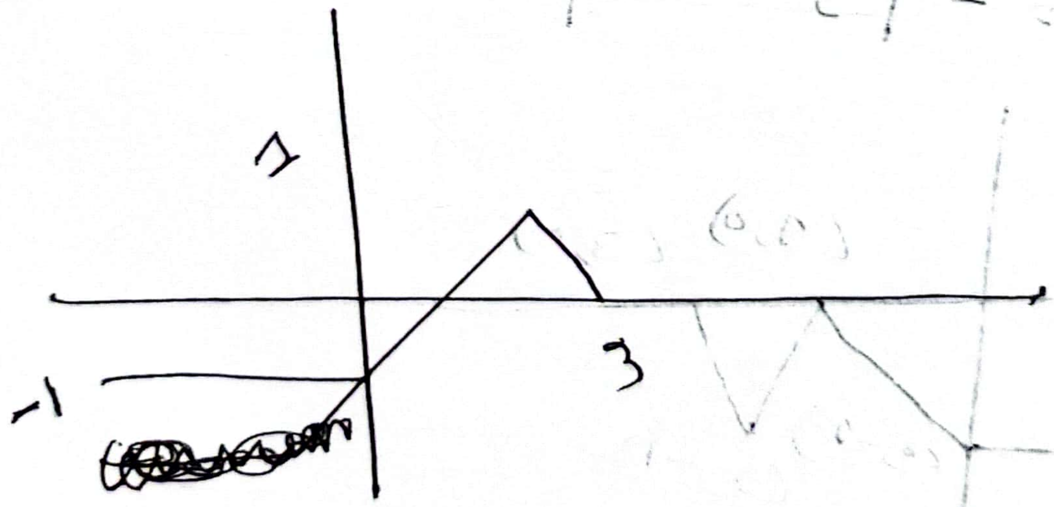
(ii) $y = -f(2-x)$



(1)(ii) $y = 1 - f(2-x) - 1$



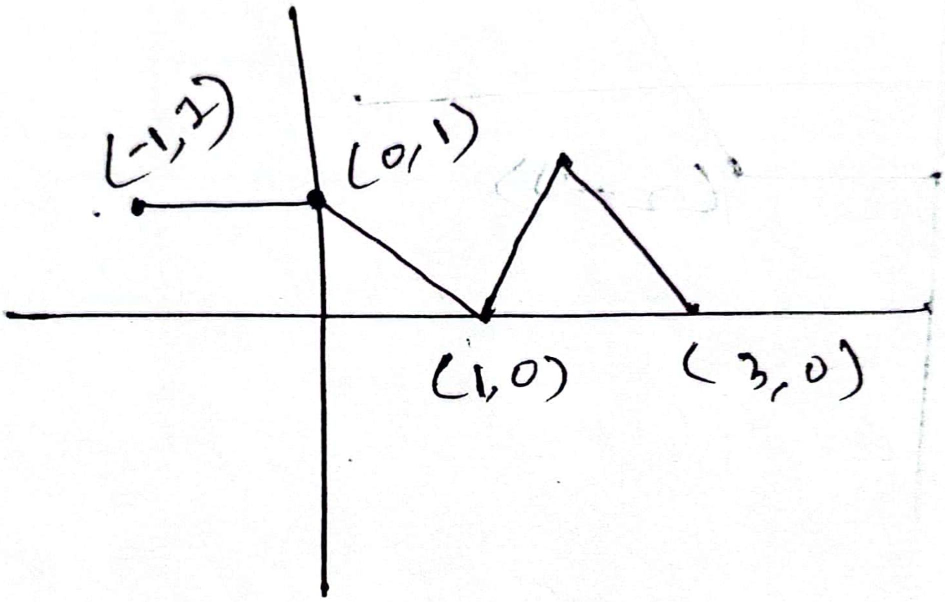
3)



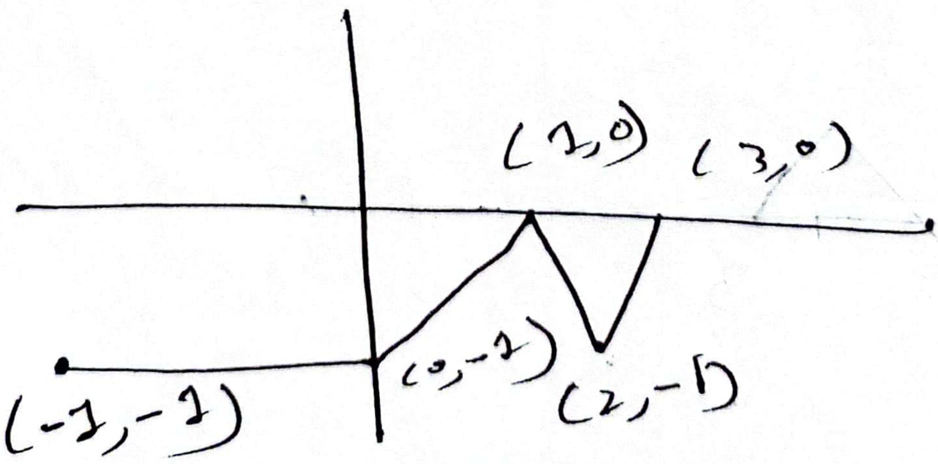
P.T.O.

(d) $y = 1 - |f(x)|$ -

(i) $y = |f(x)|$

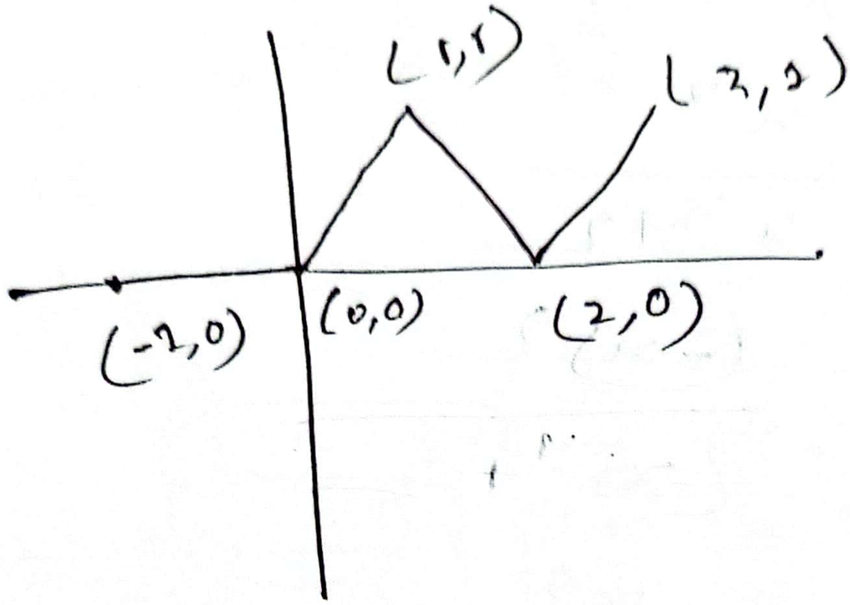


(ii) $y = -|f(x)|$

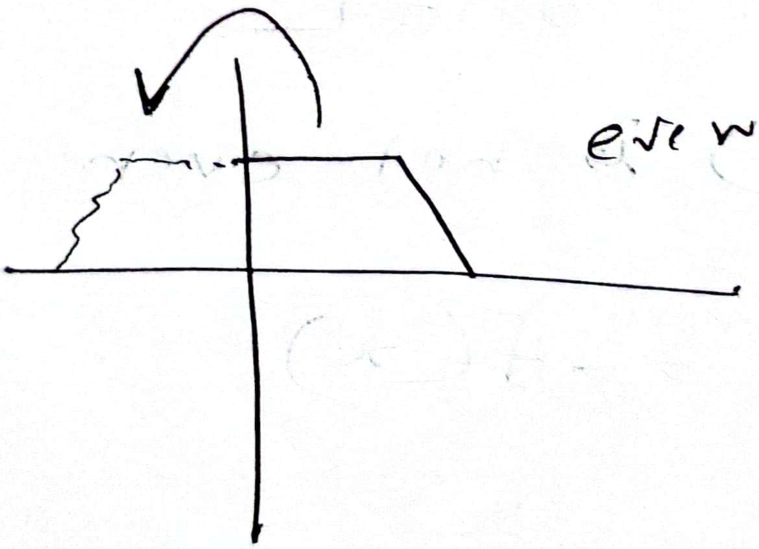


P.T.O

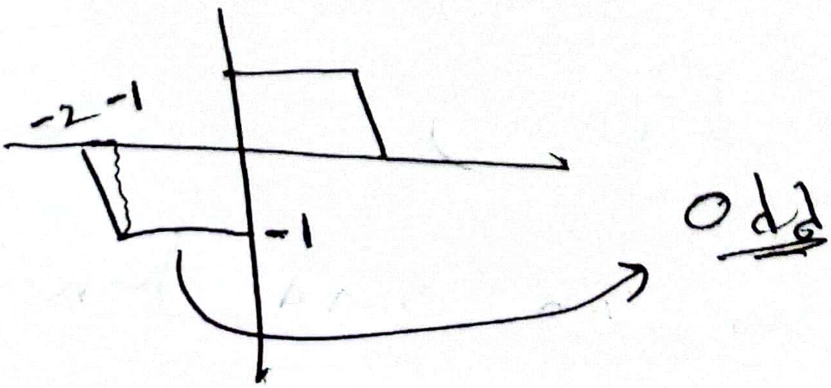
(iii) $y = 2 - |f(x)|$



(ii)



#



(1) $y = \dfrac{x^3}{x^4+2}$

$$f(x) = \frac{x^3}{x^4+2}$$

$$f(-x) = \frac{(-x)^3}{(-x)^4+2}$$

$$= \frac{-x^3}{x^4+2}$$

$\therefore y = f(x)$ is not even

$$f(-x) = -f(x)$$

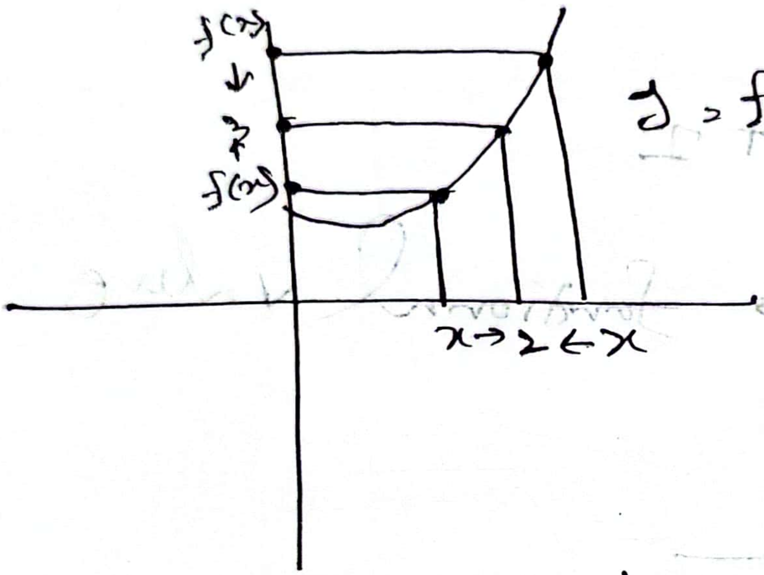
$$-f(x) = -\frac{x^3}{x^4+2}$$

$$= f(-x)$$

$\therefore f(x)$ is an odd function

Limits

$$\lim_{x \to 2} (x^2 - x + 1) = 3$$



$y = f(x) = x^2 - x + 1$

Left hand limit	Right hand limit
$= \lim_{x \to 2^-} f(x)$	$= \lim_{x \to 2^+} f(x)$
$= 3$	$= 3$

P.T.

Since $L.H.L = R.H.L$

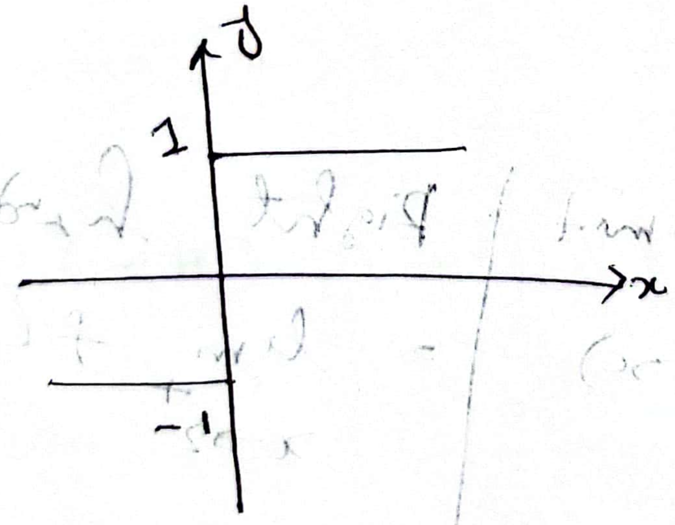
So $\lim_{x \to 2} f(x) = 3$

↑

limiting value

$f(2) = 2^2 - 2 + 1$

$= 3 \rightarrow$ functional value.



$L.H.L = \lim_{x \to 0^-} f(x) = -1$

$R.H.L = \lim_{x \to 0^+} f(x) = 1$

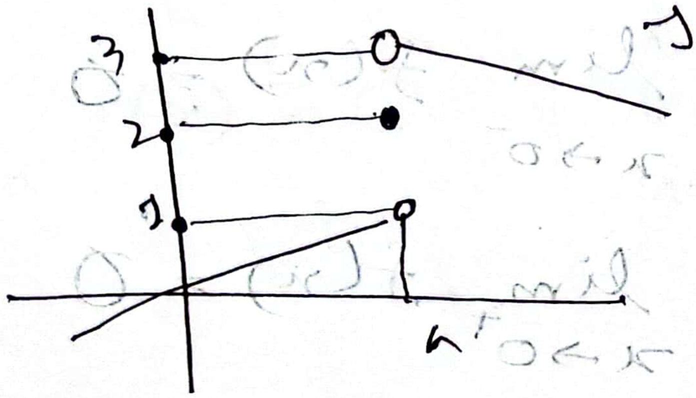
Since L.H.L ≠ R.H.L

~~So lim f(x)~~

So $\lim_{x \to 0} f(x)$ doesn't exist.

f(0) is undefined and doesn't exist.

8)



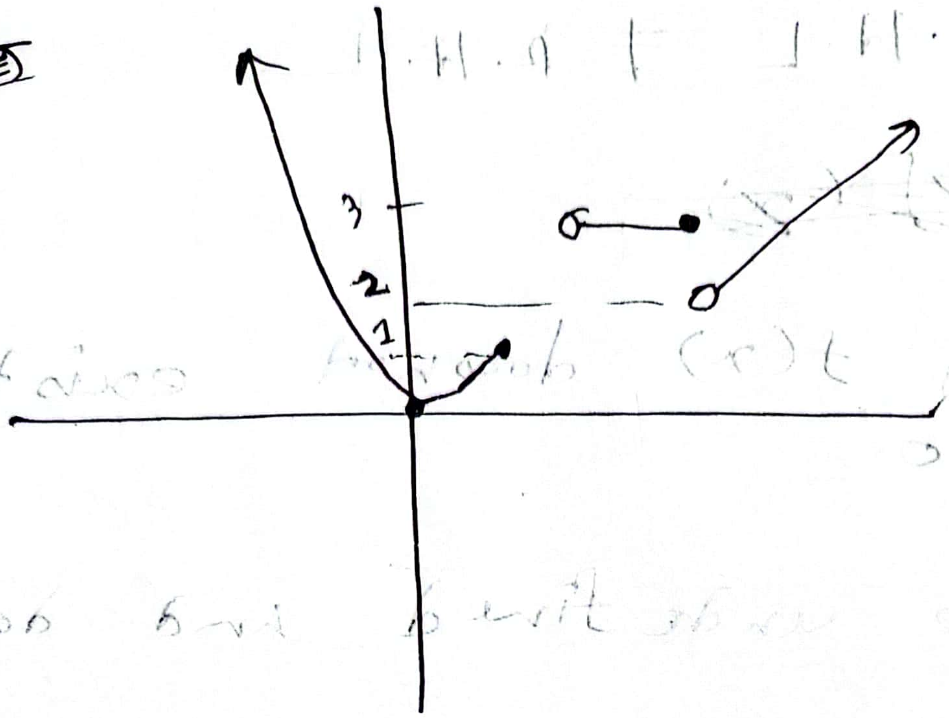
L.H.L = $\lim_{a^-} f(x) = 1$

R.H.L = $\lim_{a^+} f(x) = 3$

limit exist नहीं है

functional value ⇒ 2

(3)



(i) $\lim_{x \to 0} f(x) =$

L.H.L $= \lim_{x \to 0^-} f(x) = 0$

R.H.L $= \lim_{x \to 0^+} f(x) = 0$

(ii) $\lim_{x \to 1} f(x)$

L.H.L $= 1$

R.H.L $= 3$

(1) $\lim_{x \to 2} f(x) =$

$L.H.L = 3$

$R.H.L = 2$

(1)

$\lim_{x \to 0^-} f(x) = 1$, $\lim_{x \to 0^+} f(x) = 3$

(11) $\lim_{x \to -5^-} f(x) =$

Assignment-2

(1)

~~$\lim_{x \to -1^+} f(x) = 2$~~

~~$\lim_{x \to 3^-} f(x) = 1$~~

~~$\lim_{x \to 1^+} f(x) = 2$, $\lim_{x \to 1^-} f(x) = 1$~~

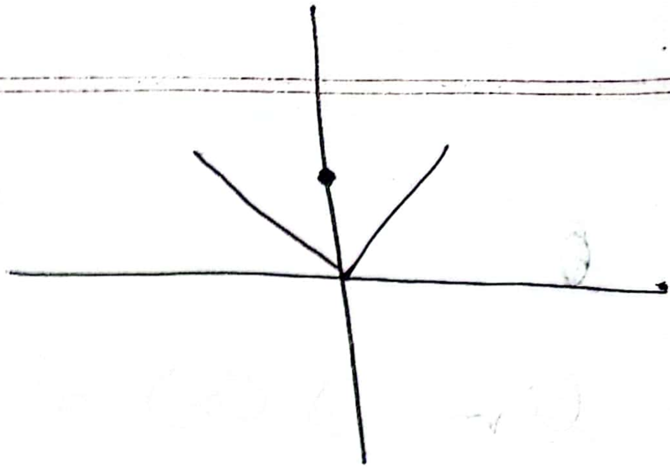
(I) $\lim_{x \to -1^+} f(x) = 1$

(II) $\lim_{x \to 3} f(x) = 0$

(III) $\lim_{x \to 1^-} f(x) = -1$, $\lim_{x \to 1^+} f(x) = 0$

$f(0) =$ (Undefined)

$f(1) = 2$



* $\lim_{x \to 0^-} f(x) = 0$

$\lim_{x \to 0^+} f(x) = 0$

$\cancel{\lim f(0)}$ $f(0) = 0$

$\therefore \lim_{x \to 0} f(x) = f(0)$

PT.

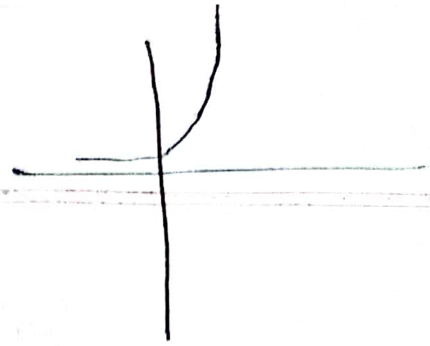
$$\lim_{x \to a^-} f(x) = 1$$

$$\lim_{x \to a^+} f(x) = 3$$

~~$f(a) = 2$~~ $f(a) = 2$

~~$\lim_{x \to a} f(x) \neq f(a)$~~

$\therefore$ continuous is not available



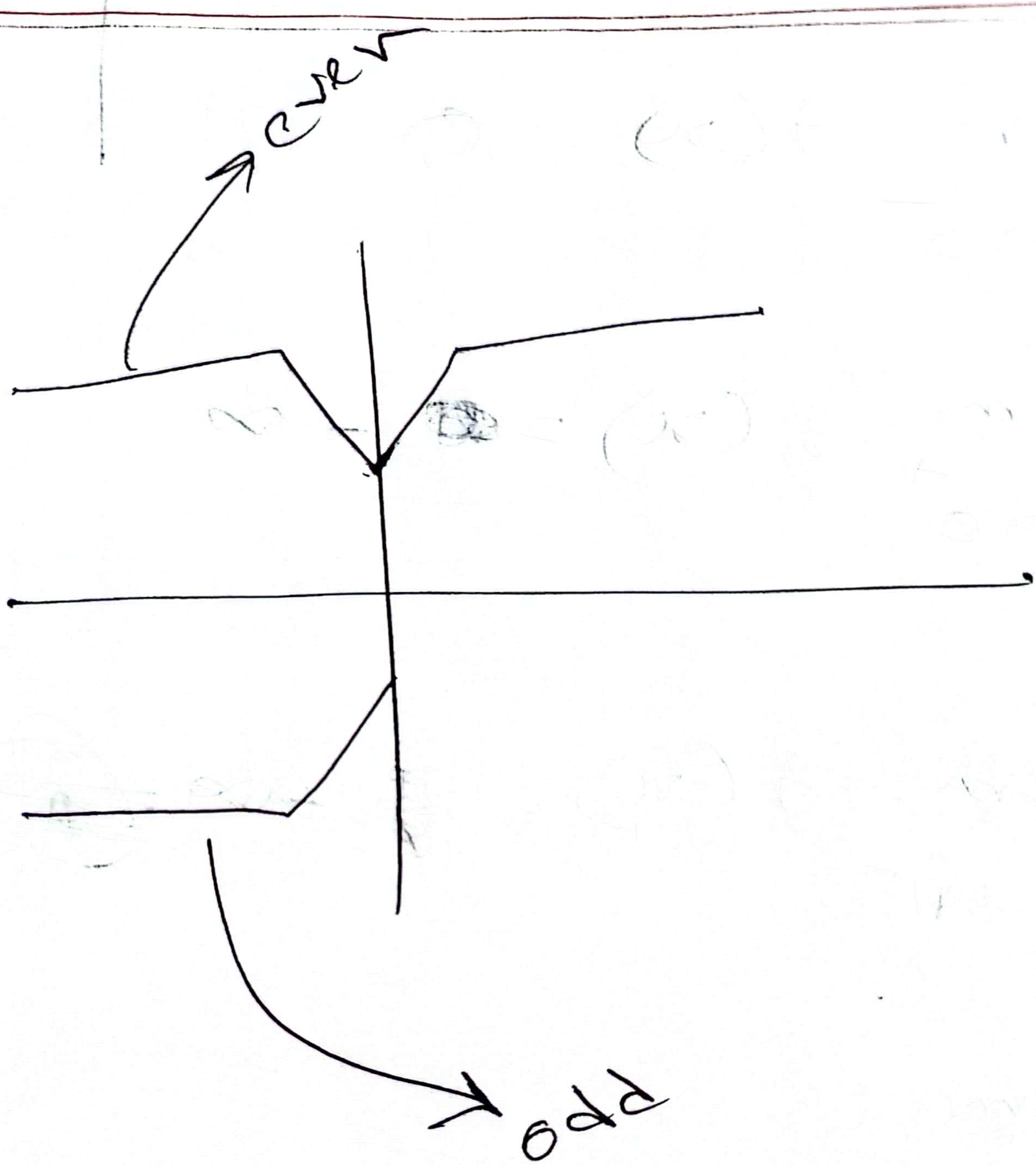
$$\lim_{x \to 0^-} f(x) = 0$$

$$\lim_{x \to 0^+} f(x) = -\infty$$

$$\lim_{x \to 4^-} f(x) = \cancel{\infty}\ \cancel{3}\ -0$$

lim

$$\lim_{x \to 4^+} f(x) = +\infty$$


odd

8

(ii) $\cancel{f(x) = \sqrt{4x+3}}$

$\cancel{f(x) = 1+e^{-x}}$

$\cancel{f^{-1}(x) = \log(x-1)}$

$f(x) = \sqrt{4x+3}$

$\Rightarrow y = \sqrt{4x+3}$

$\Rightarrow y^2 = 4x+3$

$\Rightarrow 4x = y^2 - 3$

$\Rightarrow x = \dfrac{y^2-3}{4}$ $[x \geq 0]$

interchanging, $y = \dfrac{x^2-3}{4}$

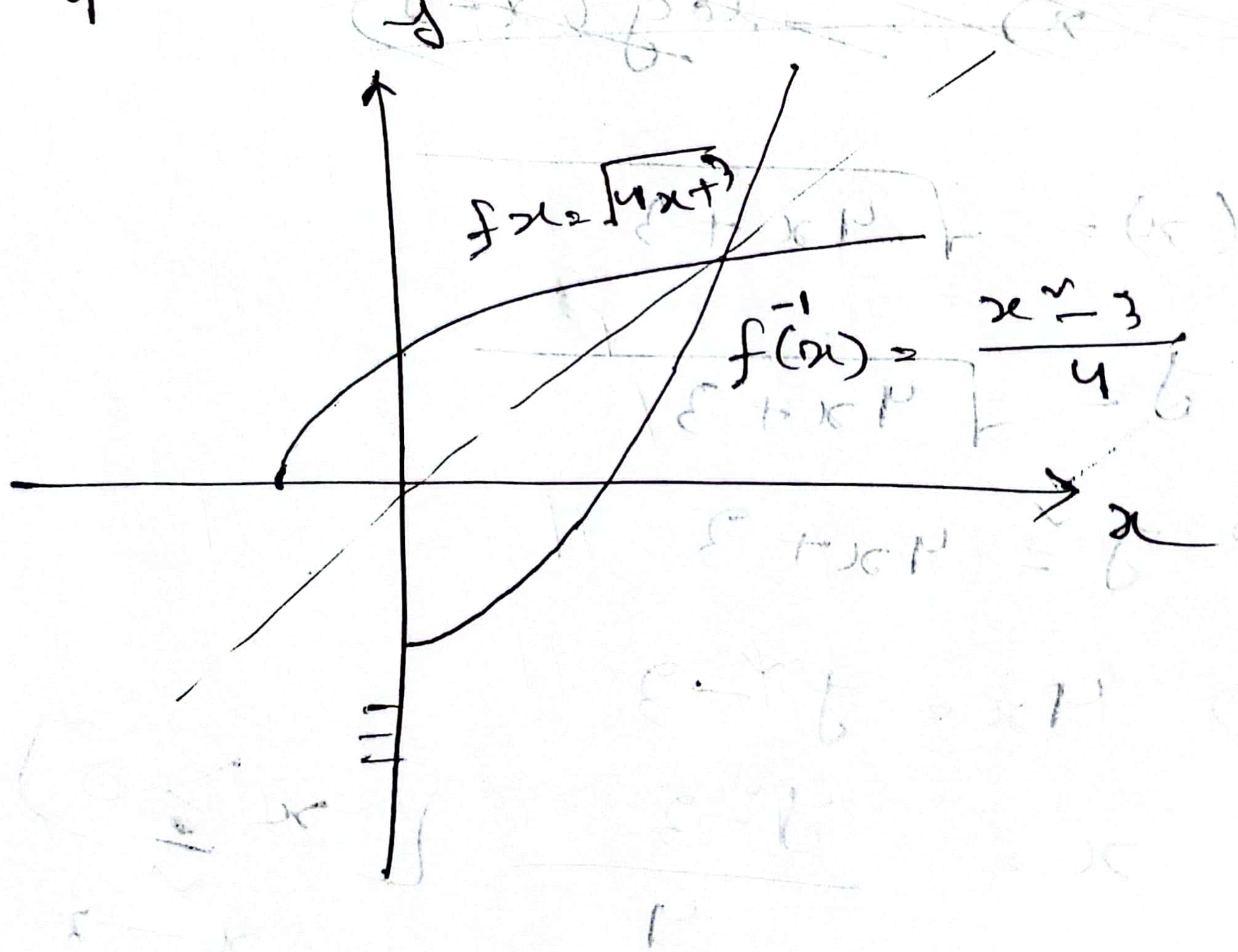
The domain of $f(x)$ is $4x+3 \geq 0$

$\Rightarrow x \geq -\dfrac{3}{4}$

P.T.O.

CamScanner

This is the range for f^{-1}, that is f^{-1} must be greater than or equal to $-\frac{3}{4}$.



PT.

$e^{\ln x}$

$$f(x) = 1 + e^{-x}$$

$$y = 1 + e^{-x}$$

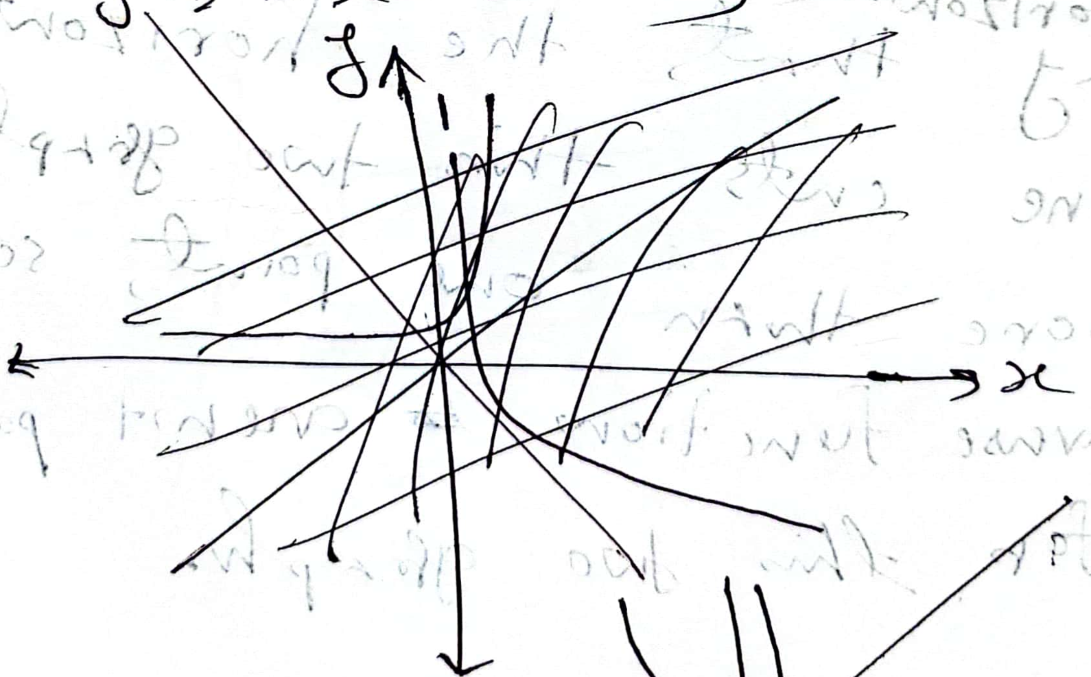
$$\Rightarrow x = 1 + e^{-y}$$

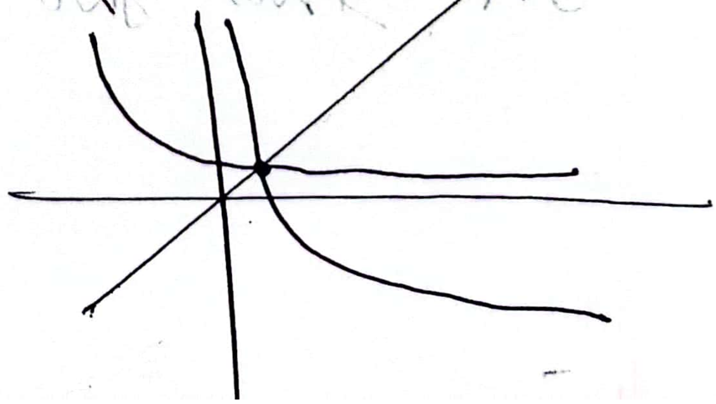
$$\Rightarrow 1 + e^{-y} = x$$

$$\Rightarrow e^{-y} = x - 1$$

$$\Rightarrow -y = \ln(x-1)$$

$$\Rightarrow y = -\ln(x-1)$$





4

In this (ii) & (iii) number problem for the following function inverse function are not possible. Because in this two graph from ~~vertical~~ Horizontal line test we can say that, the horizontal line cuts this two graph more than one point, so inverse function ~~is~~ are not possible for this two graph.

GW

Calculus

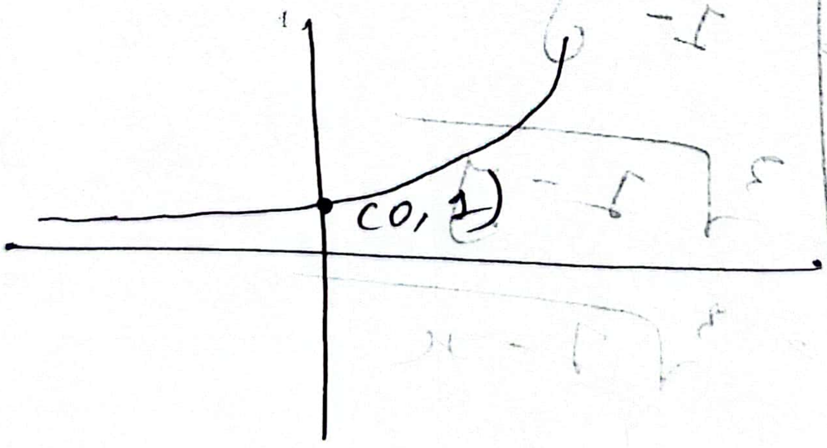
$$y = \sqrt[3]{x-3}$$

$$\Rightarrow y^3 = x - 3$$

$$\Rightarrow x = y^3 + 3$$

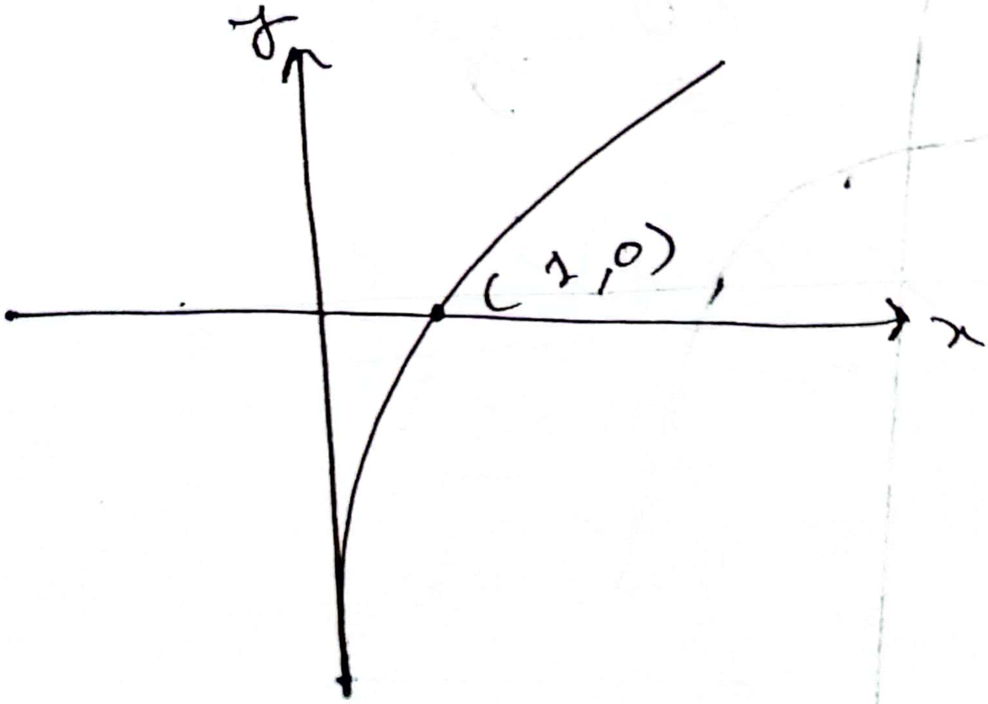
$$\Rightarrow y = x^3 + 3$$

* $y = e^x$

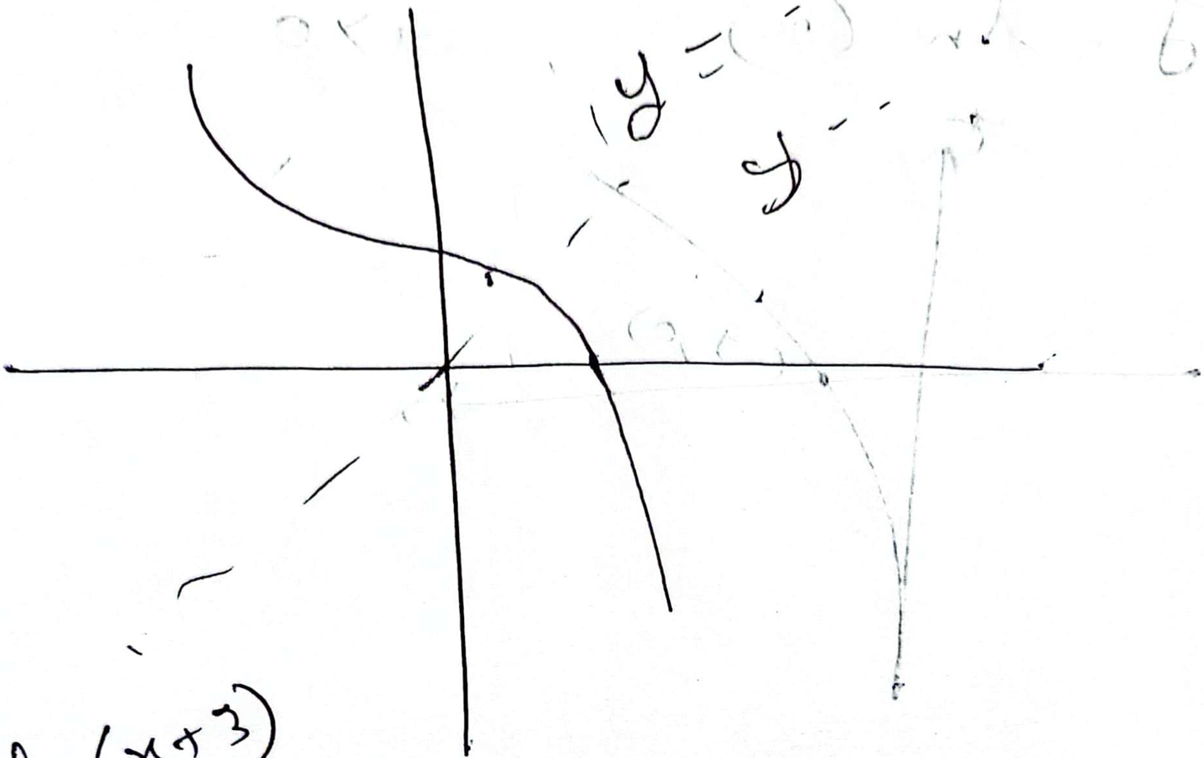


PTO

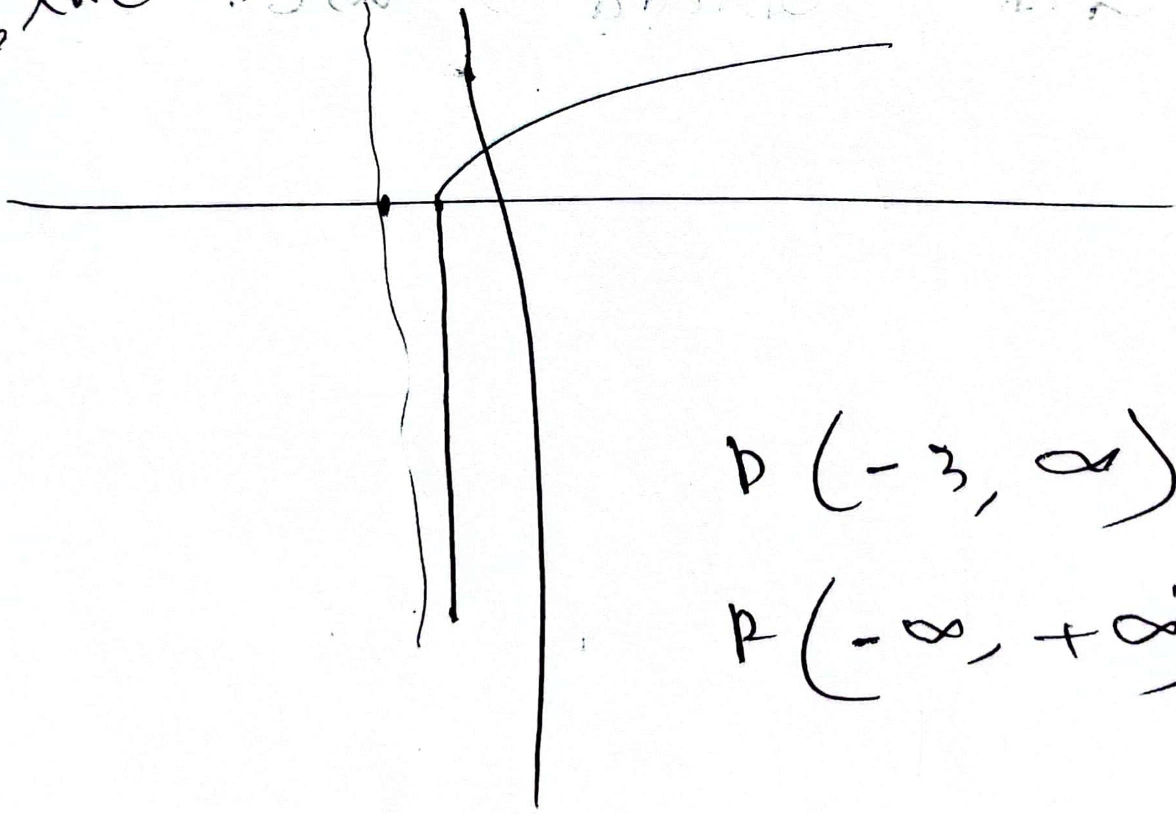
* $y = \ln(x)$, $x > 0$



এরা একে অপরের inverse.



$y = \ln(x+3)$



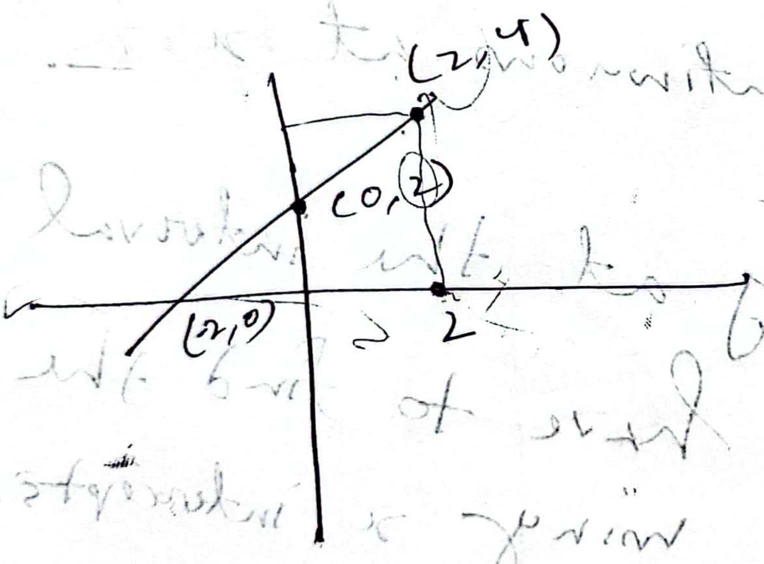
$D\ (-3, \infty)$

$R\ (-\infty, +\infty)$

* $y = \left\{ \frac{x^2 - 4}{x - 2} \right.$ find $\lim_{x \to 2} f(x)$

$$= \frac{(x+2)(x-2)}{(x-2)} \quad [x \neq 2]$$

$$= x + 2$$



$$\lim_{x \to 2^+} f(x) = 2 + 2 = 4$$

$$\lim_{x \to 2^-} f(x) = 2 + 2 = 4$$

P.T.O

~~Since~~

$f(x) = 4$, when, $x = 2$

$f(2) = 4$

since $\lim_{x \to 2} f(x) = 4 = f(2)$

so, f is continuous at $x = 2$.

* Continuity at the interval

(i) First, we have to find the interval by using x intercepts.

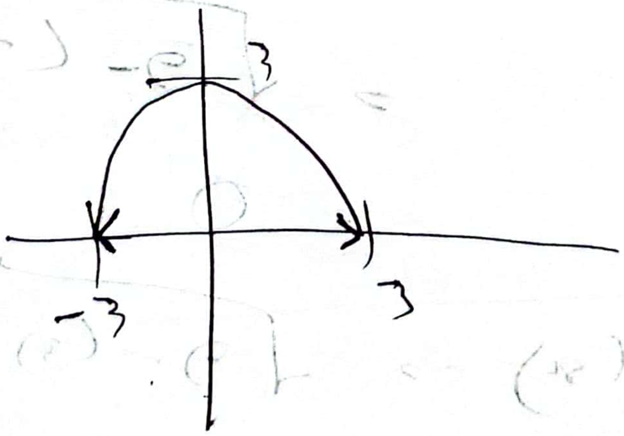
(ii) 2nd, we have to check the continuity about the interval $(a, b) \rightarrow f(a) = f(b)$

PTO

(iii) f is continuous from right at a.

(iv) f " " " left at b.

* $y = \sqrt{9 - x^2}$



So the continuity about the function about interval $[-3, 3]$

P.T.O.

If, $y = 0$

$\sqrt{9-x^2} = 0$

$\Rightarrow 9 - x^2 = 0$

$\Rightarrow x = \pm 3$

$\sqrt{9-(-3)^2} = 0$

$f(3) = \sqrt{9-(3)^2}$

$= 0$

$\lim_{x \to -3^+} f(x) = \sqrt{9-x^2}$

$= \sqrt{9-(-3)^2}$

$= 0$

$\lim_{x \to 3^-} f(x) = \sqrt{9-(3)^2}$

$= 0$

#

$$f(x) = y = \frac{x^2 - 9}{x^2 - 5x + 6}$$

f is undefined or discontinuity

if, $x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow (x-3)(x-2) = 0$

$x = 2$ and $x = 3$

$$y = f(x) = \frac{x^2 - 9}{x^2 - 5x + 6}$$

$$= \frac{(x+3)(x-2)}{(x-2)(x-3)}$$

1/2(cos x⁻¹)

cos⁻¹ 1/2

0.2 → Ex → (1-24) (Page-15)

strecteches, compress, even, odd

0.4 → Inverse (Page : 38)

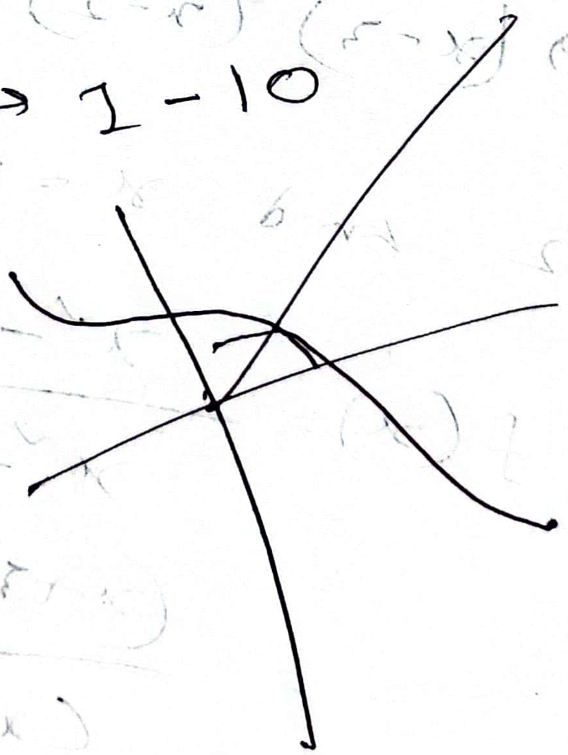
cos⁻¹ 2π

1.1 → limit (Page : 67)

cos⁻¹(2π)

Example → 1-8

Exercise → 1-10



The exponential and logarithmic function :-

1. Exponential :

$y = b^x$ / $y = e^x$ / $y = 2^x$

$b > 0$

$x = 0$

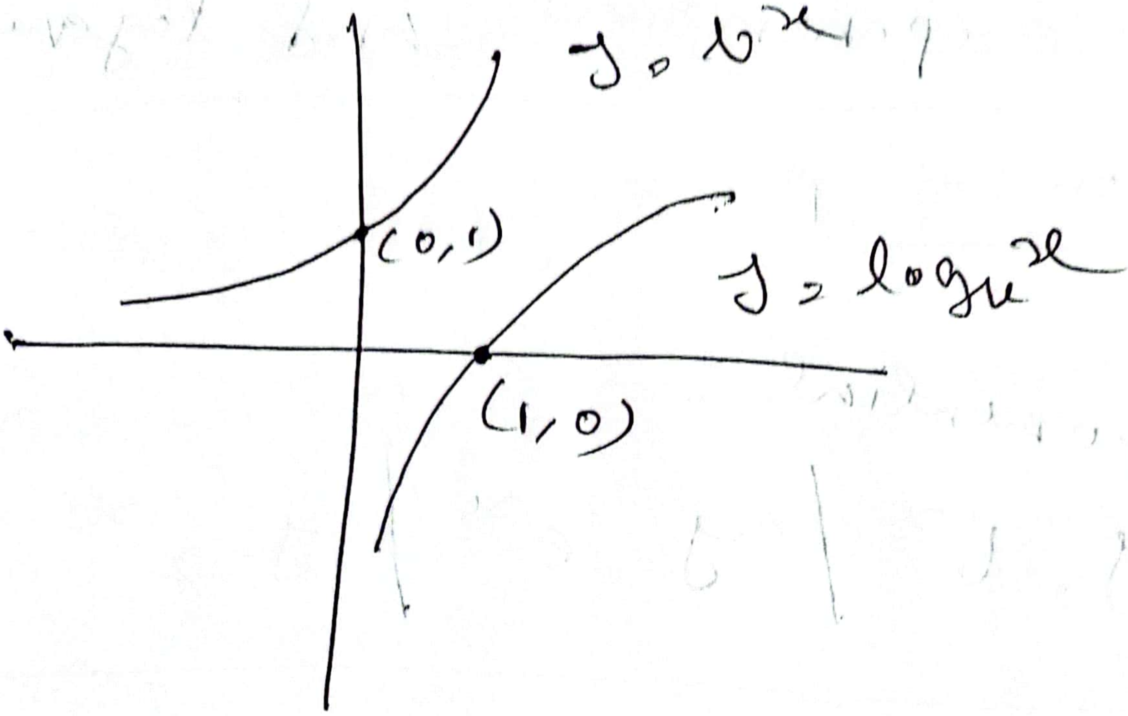
$b \neq 1$

$b = 1$, $y = 1^x$

$(x, y) = (0, 1)$

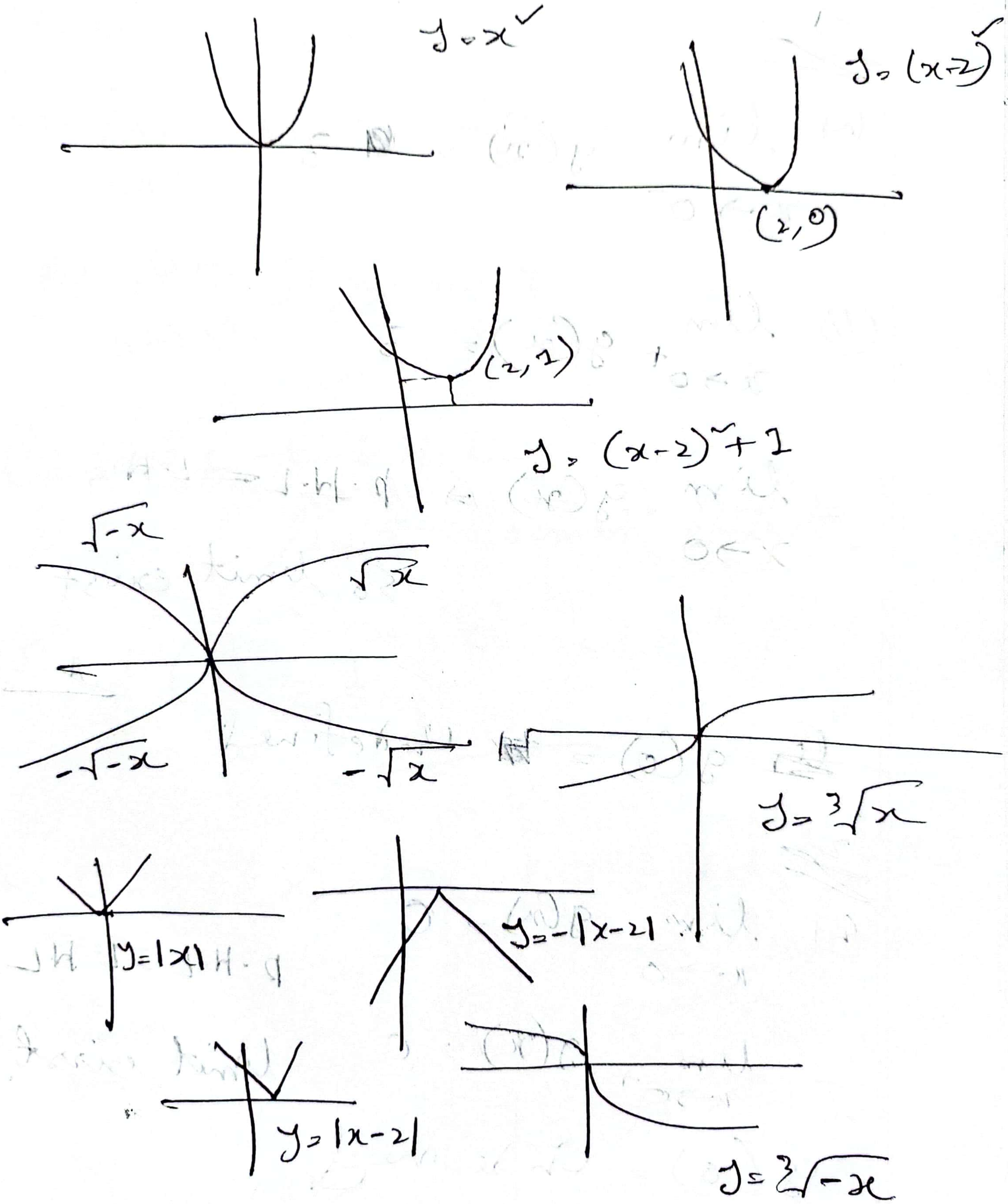
$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

PT.



$$y = \log_e x = \frac{\log x}{\log e} = \log x = \ln x$$

$\log e \rightarrow 1$


y = x²
y = (x−2)²
(2, 0)
(2, 1)
y = (x−2)² + 1
√−x
√x
−√−x
−√x
y = ∛x
y = |x|
y = −|x−2|
y = |x−2|
y = ∛−x

1

(a) $\lim_{x \to 0^-} g(x) = 3$

(b) $\lim_{x \to 0^+} g(x) = 3$

$\lim_{x \to 0} g(x) \Rightarrow$ R.H.L = L.H.S

So limit exist

A

$g(0) =$ Undefined

2

(a) $\lim_{x \to 0^-} g(x) = 0$

R.H.L = L.H.L

$\lim_{x \to 0^+} g(x) = 0$

$\therefore$ limit exist.

$g(0) =$ Undefined

3

(a) $\lim_{x \to 3^-} f(x) = -1$

(b) $\lim_{x \to 3^+} f(x) = 3$

(c) R.H.L $\neq$ L.H.L

limit doesn't exist.

(d) $f(3) = 1$

4

(a) $\lim_{x \to 2^-} f(x) = 2$

(b) $\lim_{x \to 2^+} f(x) = 0$

(c) R.H.L $\neq$ L.H.S

limit doesn't exist.

(d) $f(2) = 2$

P.T.O

5

(a) $\lim_{x \to -2^-} f(x) = 0$

(b) $\lim_{x \to -2^+} f(x) = 0$

(c) L.H.L = R.H.L (limit exists)

(d) $f(-2) = 3$

6

(a) $\lim_{x \to 0^-} g(x) = 1$

(b) $\lim_{x \to 0^+} g(x) = 1$

(c) L.H.S = R.H.L (limit exist)

(d) $g(x) = 0$

P.T.O

7

(a) $\lim_{x \to 3^-} f(x) = -\infty$

(b) $\lim_{x \to 3^+} f(x) = -\infty$

(c) L.H.L = R.H.L (limit exists)

(d) $f(3) = 1$

8

(a) $\lim_{x \to 4^-} \phi(x) = +\infty$

(b) $\lim_{x \to 4^+} \phi(x) = +\infty$

(c) limit exist

(d) Undefined

P.T.O

9

(a) $\lim_{x \to -2^-} f(x) = +\infty$

$\lim_{x \to -2^+} f(x) = +\infty$

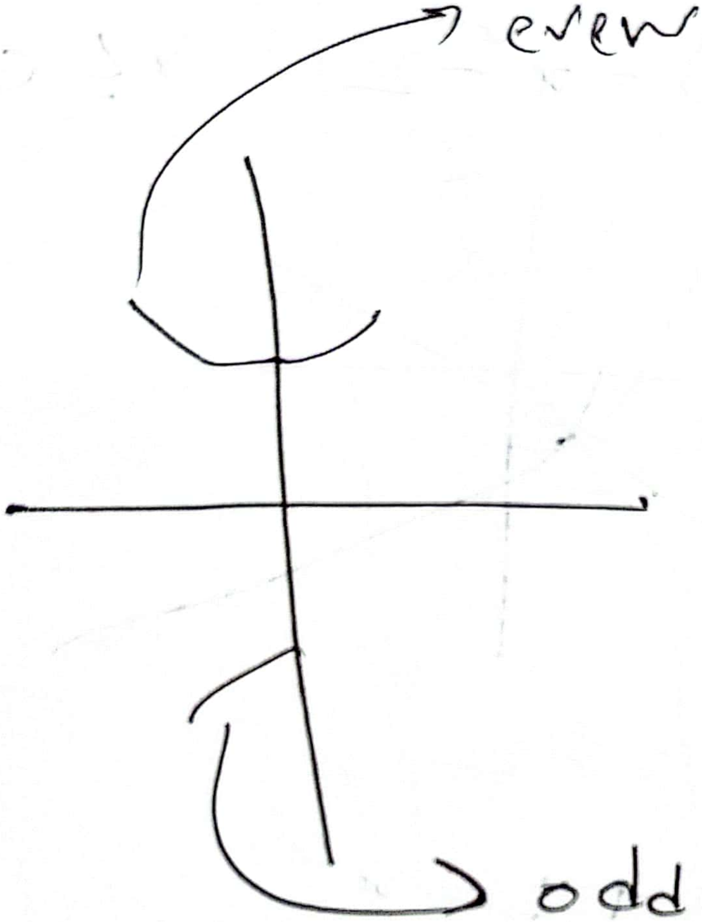
(b) $\lim_{x \to 0^-} f(x) = +\infty$

(c) $\lim_{x \to 0^+} f(x) = 2$

(d) $\lim_{x \to 2^-} f(x) = 2$

(e) $\lim_{x \to 2^+} f(x) = -\infty$

1

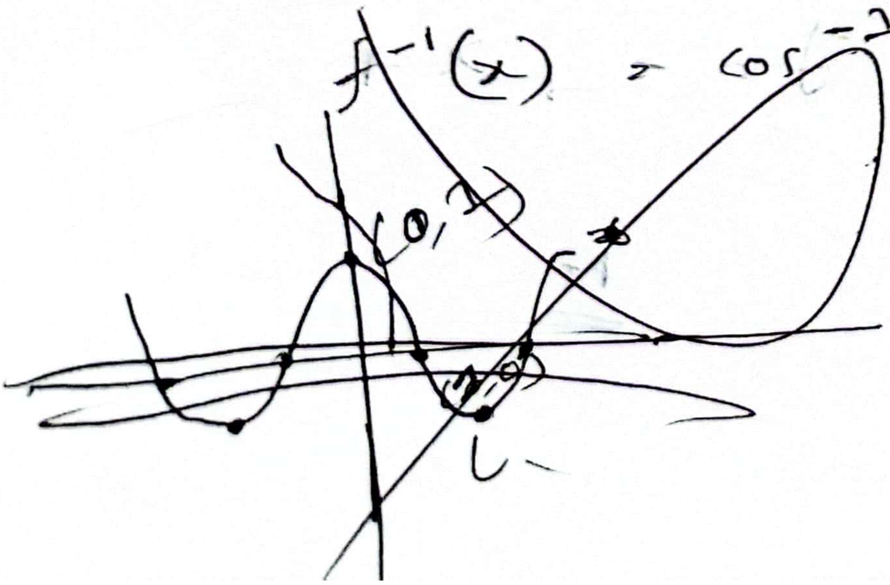


2

(1) $f(x) = \cos 2x$

$f^{-1}(x) = \cos^{-1} \frac{1}{2x}$ $\quad \frac{1}{\cos 2x}$

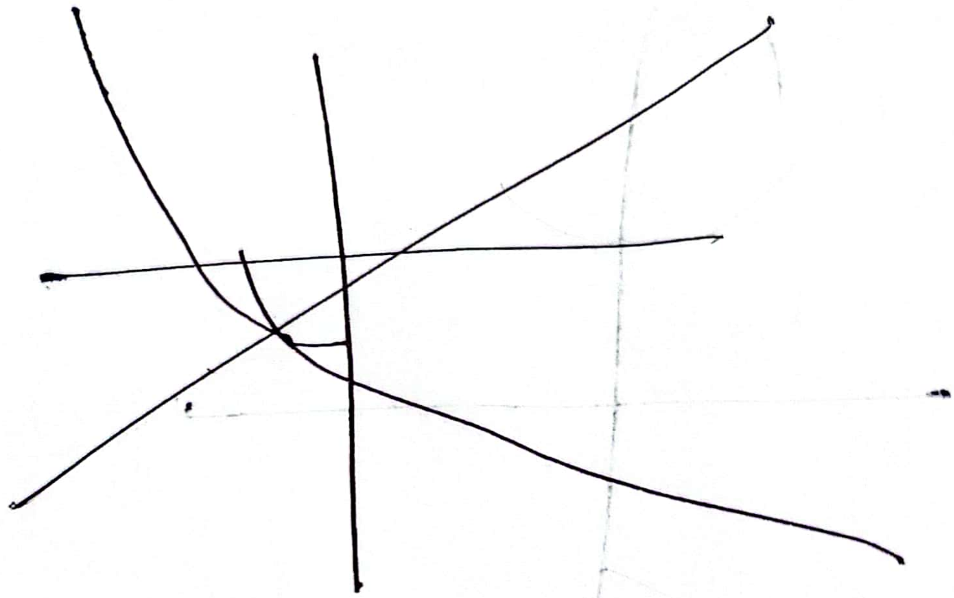
$\cos^{-1} \frac{1}{2x}$



PT!

(ii) $g(x) = x^2 - 2 \quad x \leq 0$

$\Rightarrow$



$y = x^2 - 2$

$\Rightarrow y + 2 = x^2$

$\Rightarrow x = \sqrt{y+2}$

$\Rightarrow y = \sqrt{x+2}$

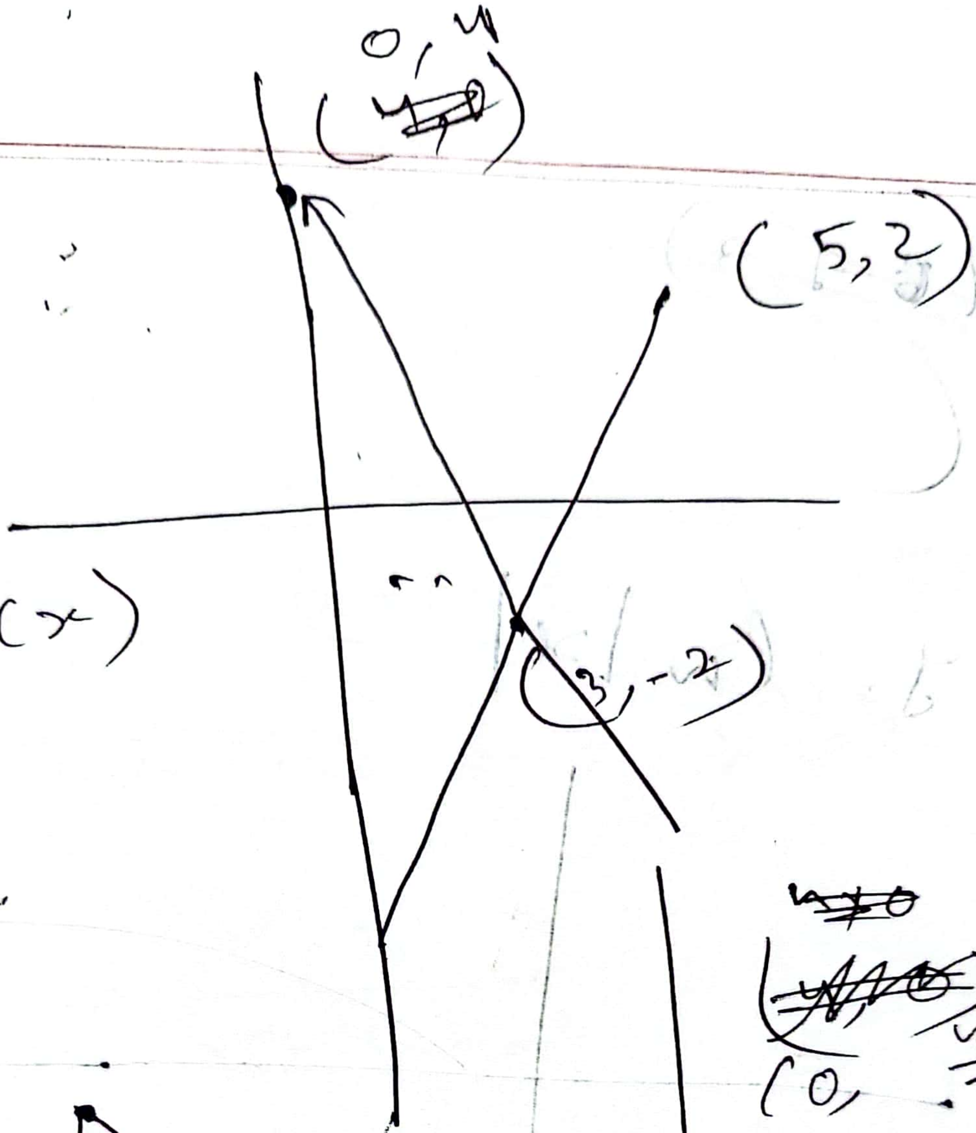
$\Rightarrow y = -\sqrt{x+2}$

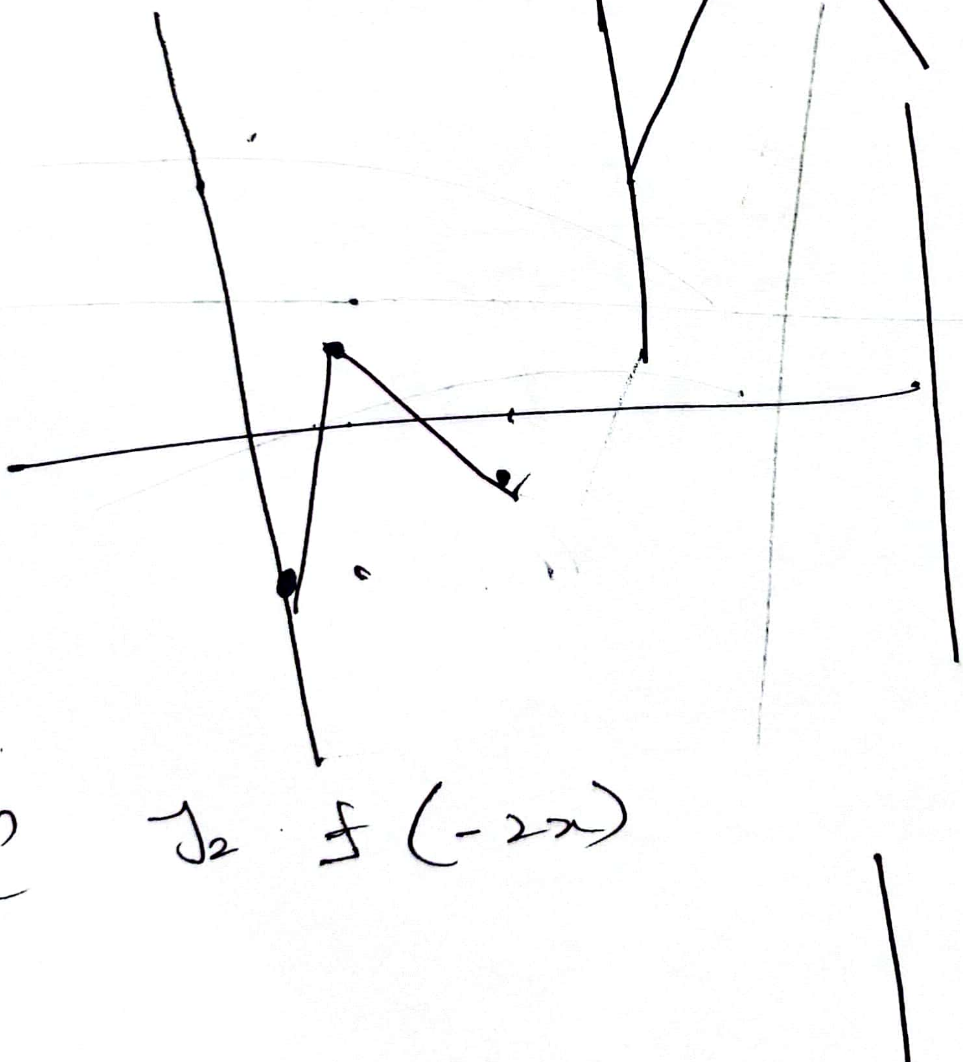
PT:

3

(i)

$y = -2f(x)$





$\left(0, \frac{4}{-2}\right) = (0, -2)$

$\Rightarrow \left(5, \frac{2}{-2}\right)$

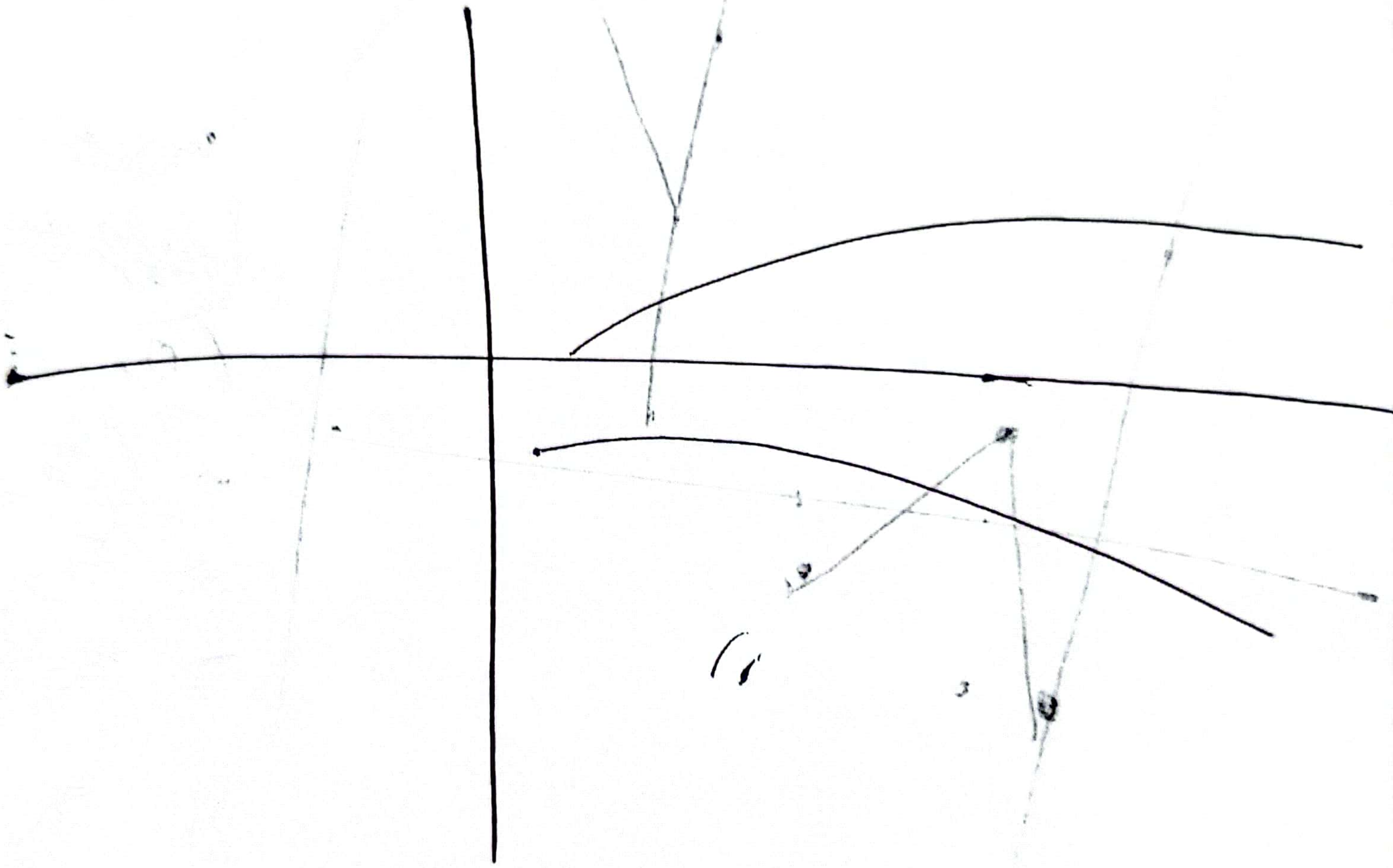
$\Rightarrow 5, -1$

$\Rightarrow 3, \left(-\frac{2}{-2}\right)$

$\Rightarrow 3, 1$

(ii) $y = f(-2x)$

$y = \ln|x|$



GW 15.11.22

Cal

4

(a) $f(x) = \dfrac{x+4}{x^2-9}$

$= \dfrac{x+4}{(x+3)(x-3)}$

Domain: $x \neq -3$ and $x \neq 3$

(b) $f(x) = \dfrac{x^2+1}{x^2+4x-21}$

f is undefined if $x^2+4x-21=0$

$\Rightarrow x^2+7x-3x-21=0$

$\Rightarrow (x+7)(x-3)=0$

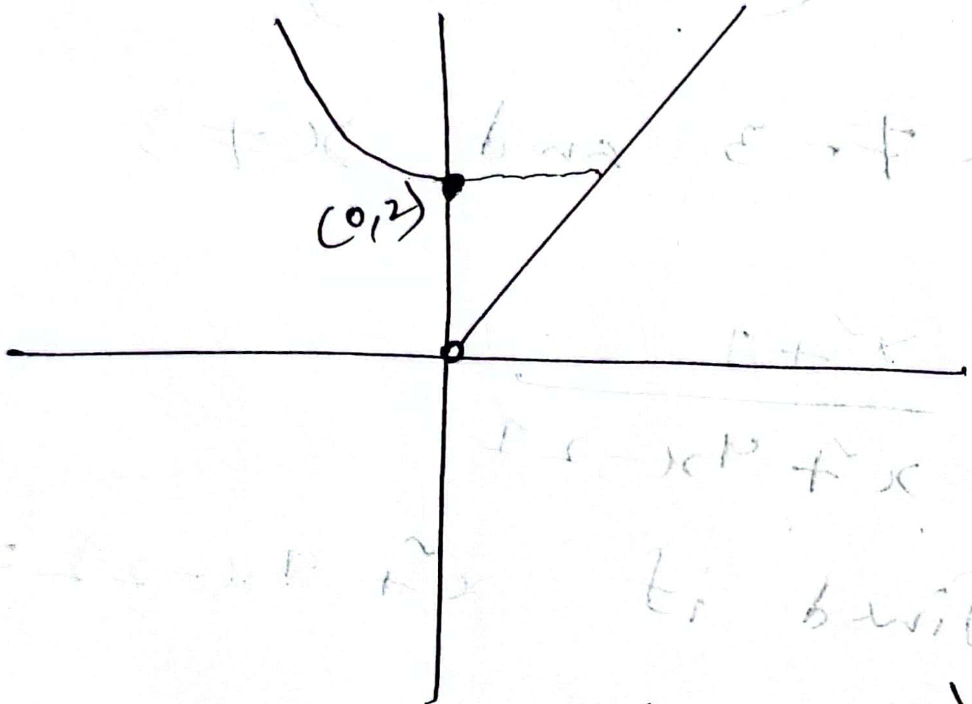
$\Rightarrow x = -7, 3$

Dom: $x \neq -7$ & $x \neq 3$.

P.T.O.

5

$$f(x) = \begin{cases} x^2 + 2; & x \leq 0 \\ x; & x > 0 \end{cases}$$



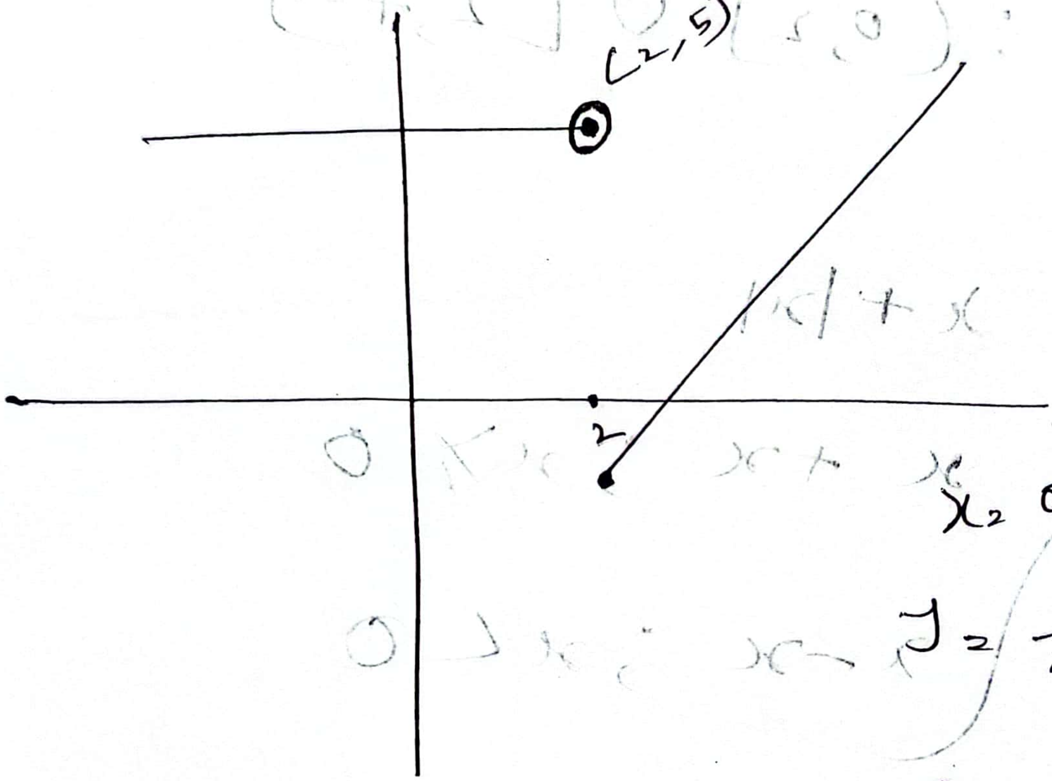
Dom $(-\infty, 0] \cup (0, +\infty)$

$\rightarrow (-\infty, +\infty)$

Range: $(0, 2] \cup [2, +\infty)$

b.T.O

(b) $y = \begin{cases} 5 & ; \; x < 2 \\ \frac{1}{2}x - 3 & ; \; x \geq 2 \end{cases}$



$x = 0, \; y = -3$

$y = \frac{1}{2}x - 3$

$\Rightarrow 0 = \frac{1}{2}x - 3$

$\Rightarrow \frac{1}{2}x = 3$

$\Rightarrow x = 6$

D: $(-\infty, 2) \cup [2, +\infty)$

R: $[-3, -2) \cup [-1, 3]$

Ans

* $y = |x| = \begin{cases} x \; ; \; x \geq 0 \\ -x \; ; \; x < 0 \end{cases}$

$D : (-\infty, 0] \cup (0, +\infty)$

Range : $(0, 2] \cup [2, +\infty)$

* $y = x + |x|$

$\begin{cases} x + x \; ; \; x \geq 0 \\ x - x \; ; \; x < 0 \end{cases}$

PT.

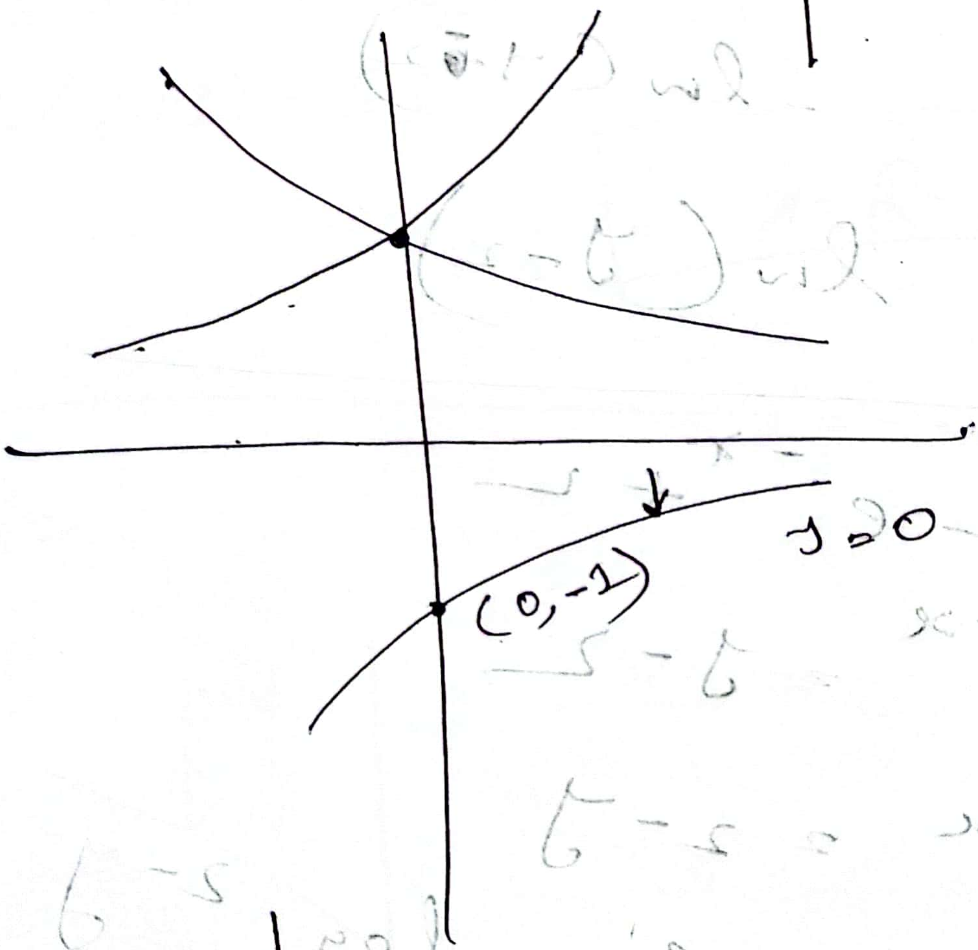
* $y = 2 - e^{-x}$

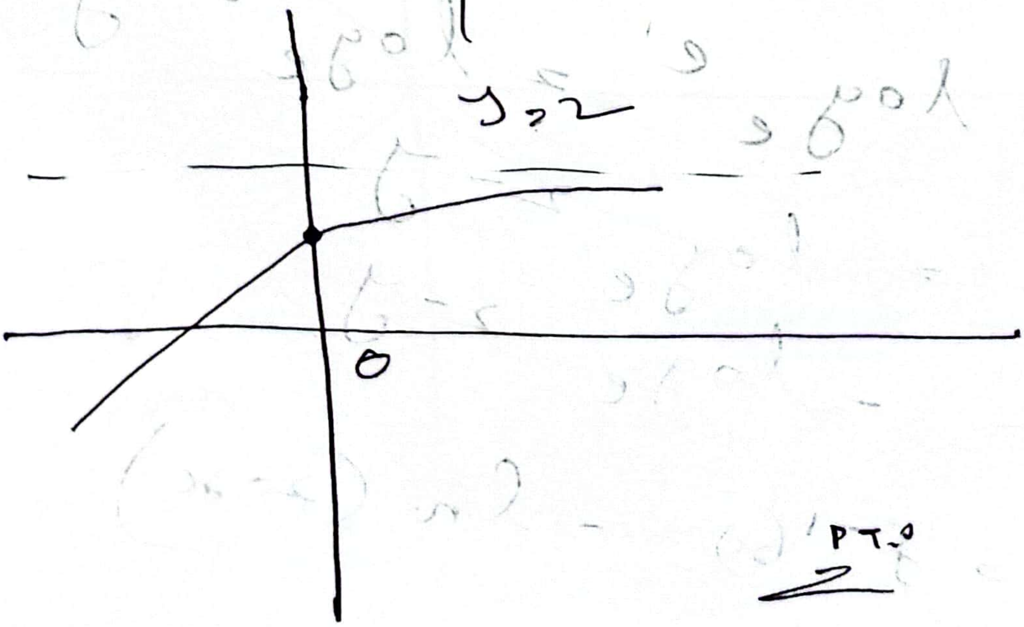
$x = 0,$

$y = -e^{-0} + 2$

$= (-1 + 2)$

$= 1$





P.T.O

$$y = 2 - e^{-x}$$

$$\Rightarrow -e^{-x} = y - 2$$

$$\Rightarrow -x = -\ln(y-2)$$

$$\Rightarrow x = \ln(y-2)$$

$$y = -e^{-x} + 2$$

$$\Rightarrow -e^{-x} = y - 2$$

$$e^{-x} = 2 - y$$

$$\Rightarrow -x \log_e e = \log_e 2 - y$$

$$\Rightarrow -x = \log_e 2 - y$$

$$\Rightarrow x = -\log_e 2 - y$$

$$\Rightarrow y = f^{-1}(x) = -\ln(2 - x)$$

$2 \cdot 2 \cdot x$

$x^2+4x-21 = (x+2)^2-25 = (x+2)^2-25 \neq 0$

$x^2+4x+4-25$

$\Rightarrow (x+2)^2-25$

$\Rightarrow x+2 = \pm 5$

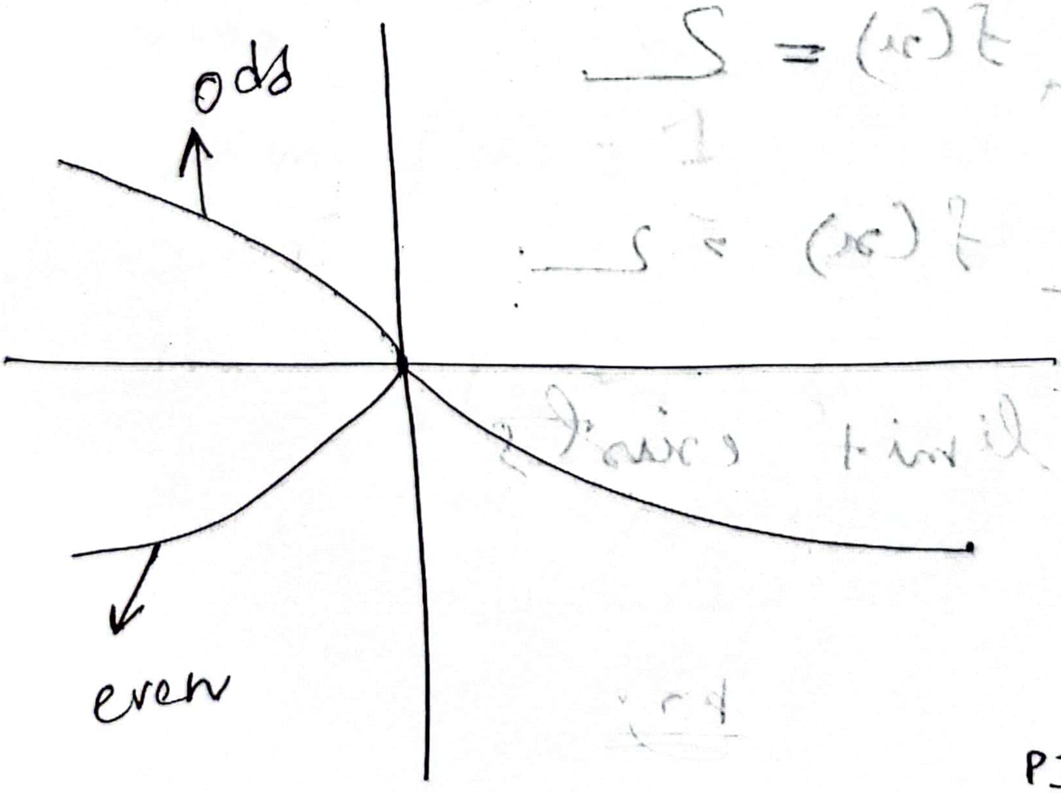
$\Rightarrow x = -2 \pm 5$

$= -7, +3$

$$\boxed{x = \mathbb{R} - \{3, -7\}}$$

---

ro



P.T.

18

$$f(x) = \begin{cases} x^2+1 & x \geq 0 \\ x^2+1 & x \leq 0 \end{cases}$$

19

(a) $\lim\limits_{x\to 1} f(x)$

$\lim\limits_{x\to 1^+} f(x) = 2$

$\lim\limits_{x\to 1^-} f(x) = 2$

limit exists

p.t.o

(b) $\lim_{x \to -2} f(x)$

$\lim_{x \to -2^+} f(x) = 1$

$\lim_{x \to -2^-} f(x) = 2$

limit doesn't exist.

(c) $\lim_{x \to 2} f(x)$

$\lim_{x \to 2^-} f(x) = 1$

$\lim_{x \to 2^+} f(x) = 3$

limit doesn't exist.

P.T.O.

1

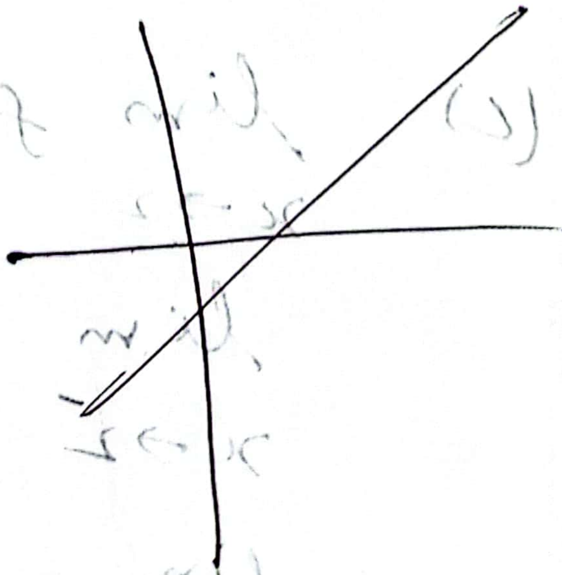
(a) $2x - 5y = 7$

$\Rightarrow -5y = 7 - 2x$

$\Rightarrow y = \dfrac{7-2x}{-5}$

$= \dfrac{-(2x-7)}{-5}$

$= \dfrac{2x-7}{5}$



$x = 0, \quad y = -\dfrac{7}{5}$

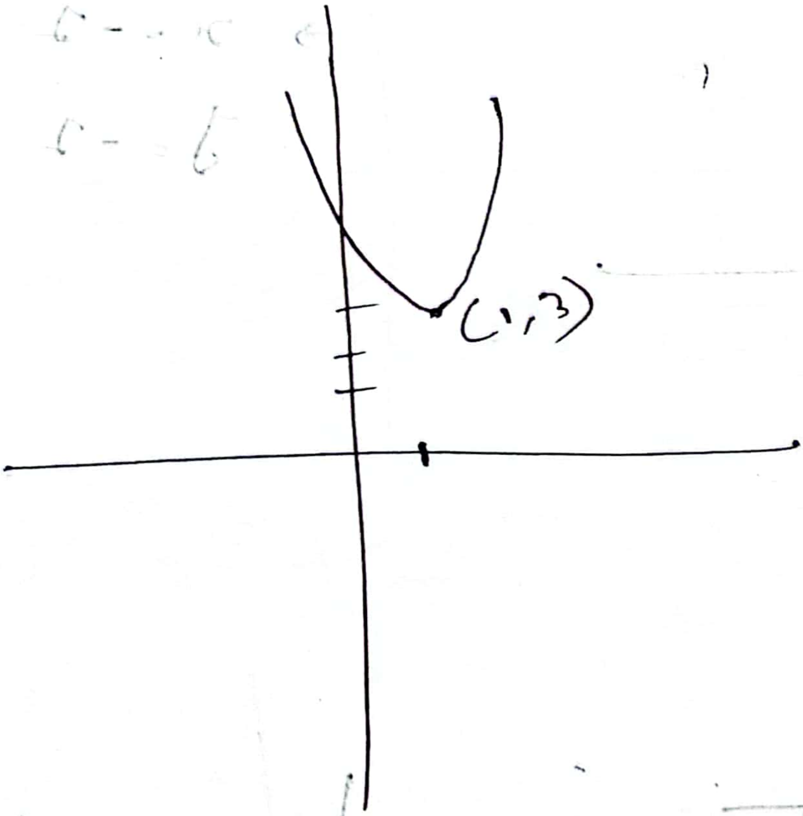
$x = -1, \quad y = -\dfrac{9}{5}$

$x = 1, \quad y = -1$

This equation defines y as a function of x.

ই/

(ঙ) $y = (x-1)^2 + 3$



$x - 1 = 0$

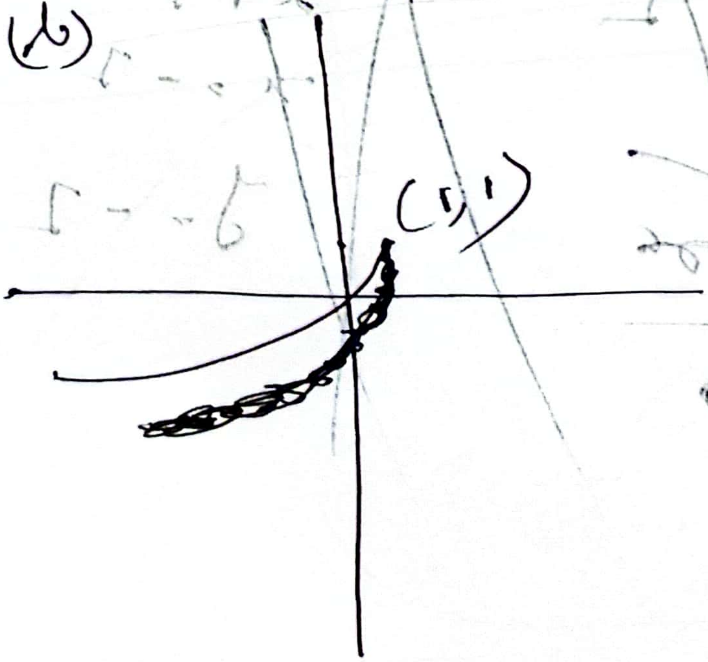
$x = 1$

D: $(-\infty, +\infty)$

R: $[3, +\infty)$

(ছ) $y = -\sqrt{1-x} + 1$

$\Rightarrow -\sqrt{-(x-1)} + 1$



$x - 1 = 0$

$x = 1$

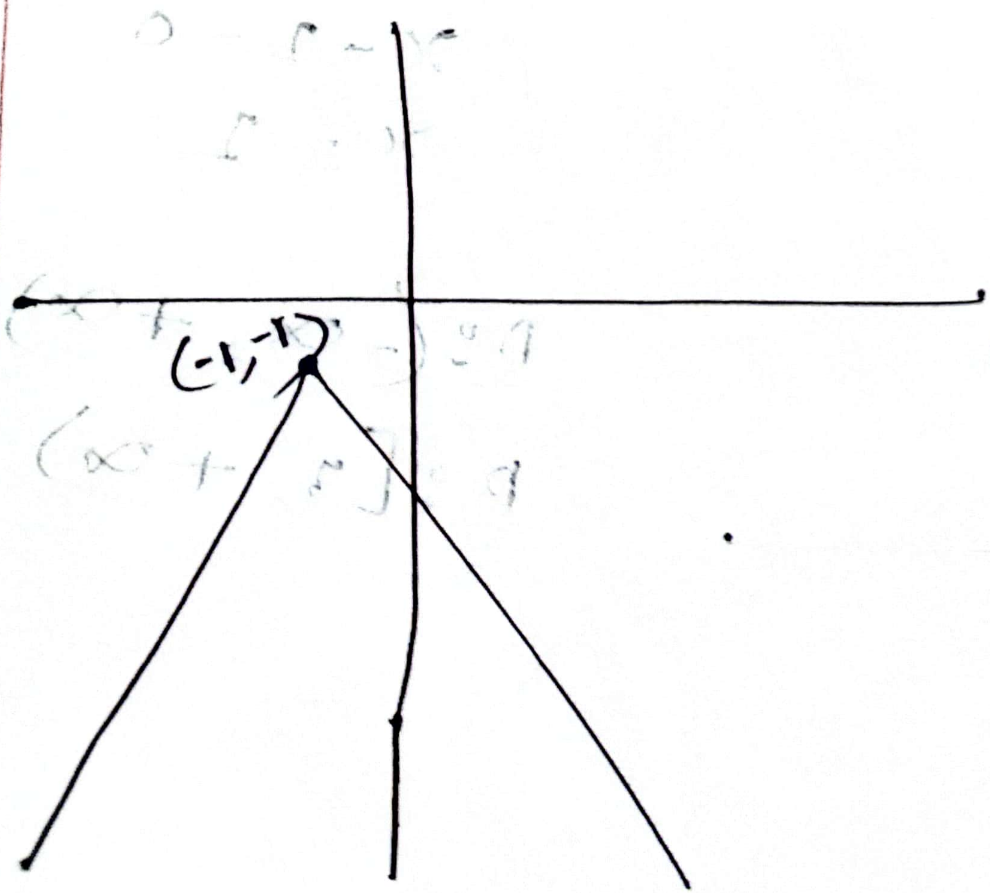
4/

$y = -|x+1| - 1$

$x + 1 = 0$

$\Rightarrow x = -1$

$y = -1$

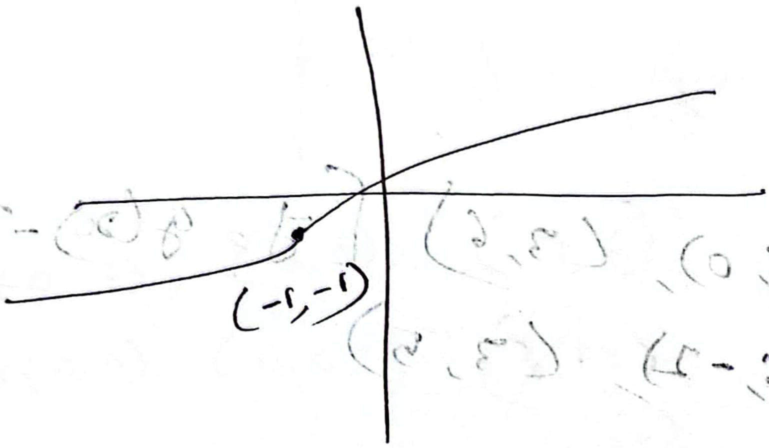


(d) $y = \sqrt[3]{x+1} - 1$

$x + 1 = 0$

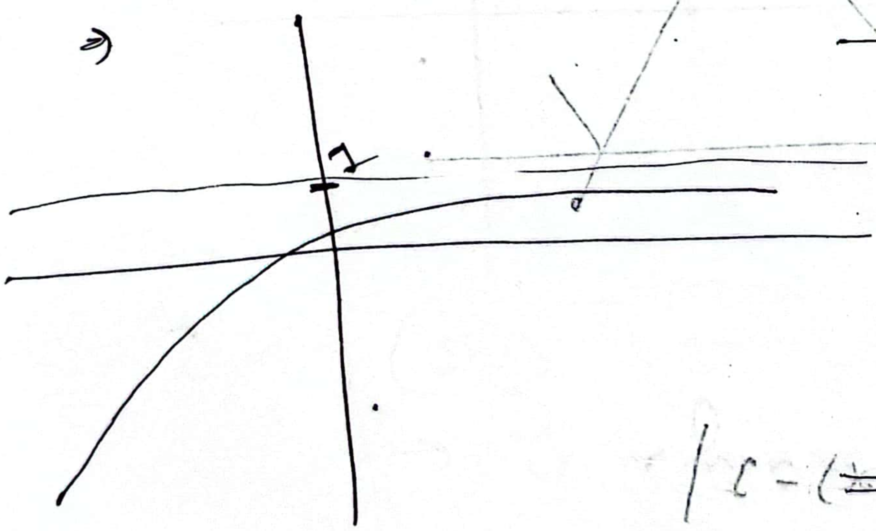
$\Rightarrow x = -1$

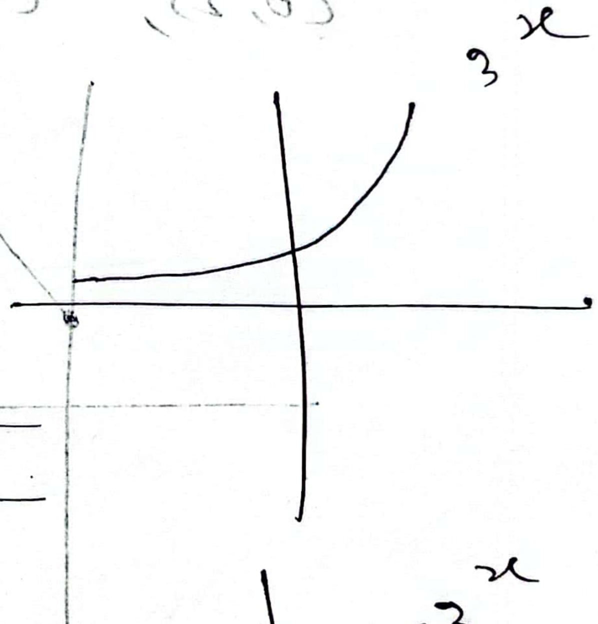
$y = -1$

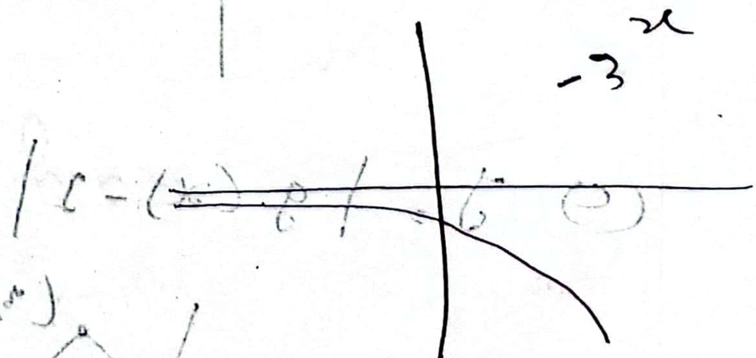


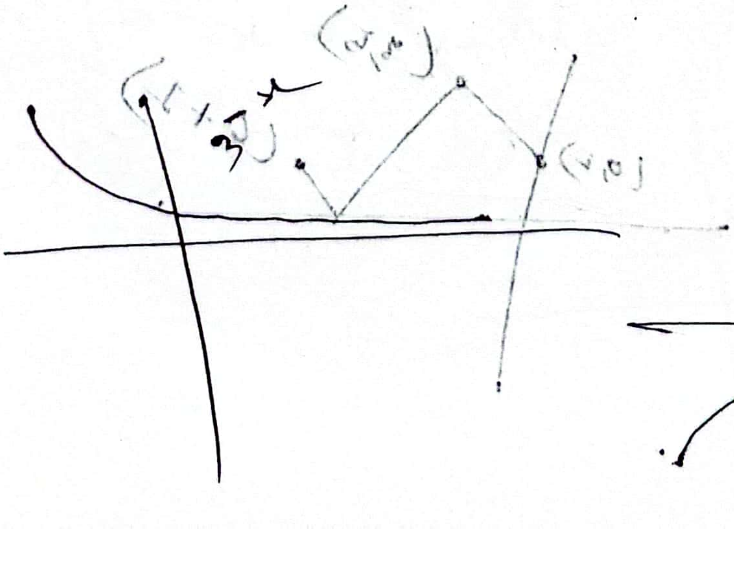
(e) $-3^{-x} + 1$

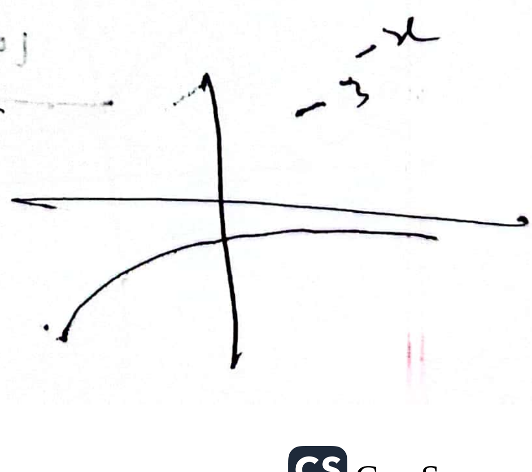
$\Rightarrow$











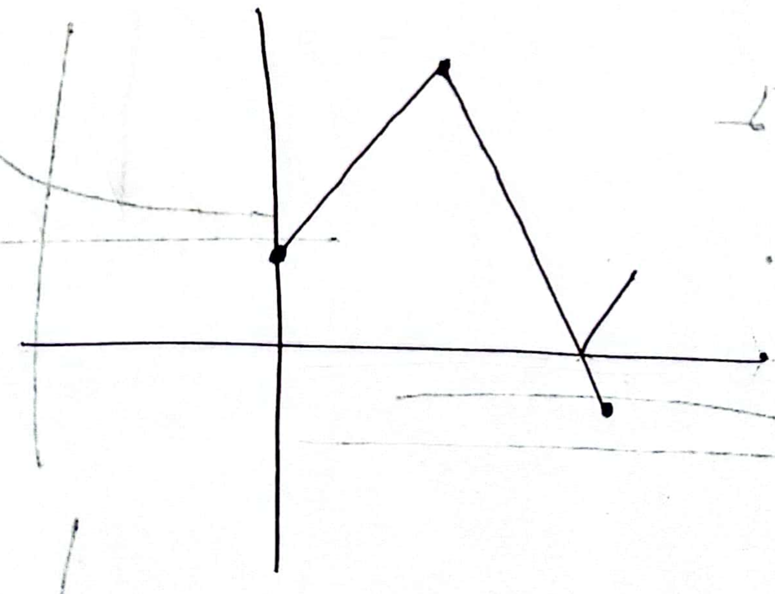
8/

(a) D : [0, 6]

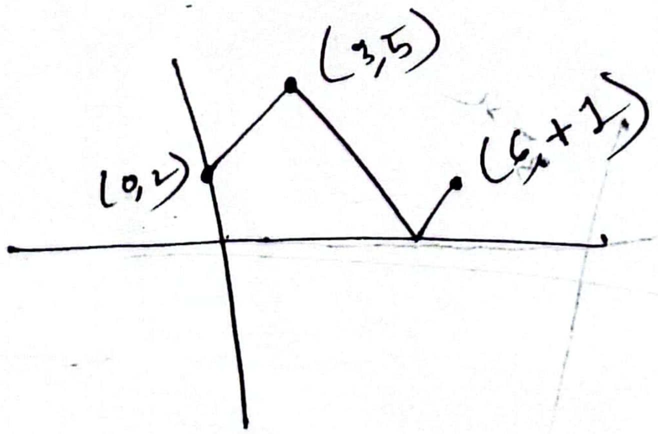
R : [0, 6]

(b) (0, 3), (6, 0), (3, 6) [$y = g(x) - 1$]

(0, 2), (6, -1), (3, 5)



(c) $y = |g(x) - 1|$



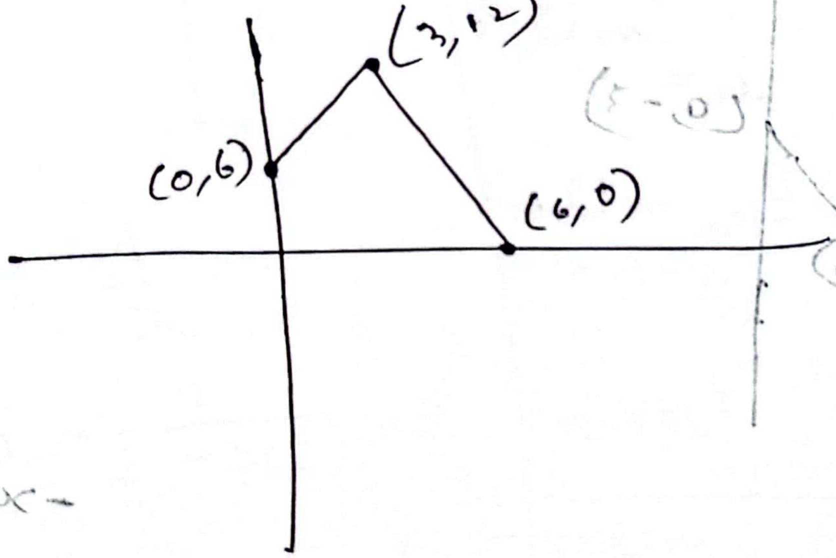
(d) $y = 2g(x)$

→ x change হবে না

→ y → (value × 2)

$(0, 3)$, $(6, 0)$, $(3, 6)$

→ $(0, 6)$, $(6, 0)$, $(3, 12)$



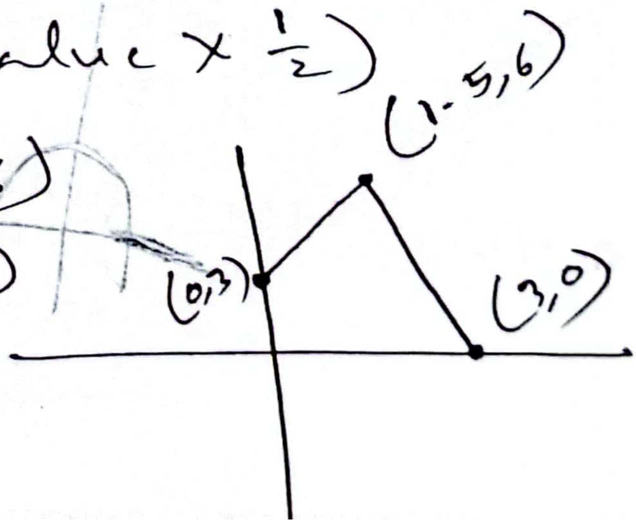
(e) $y = g(2x)$

→ y change হবে না

→ x → (value × $\frac{1}{2}$)

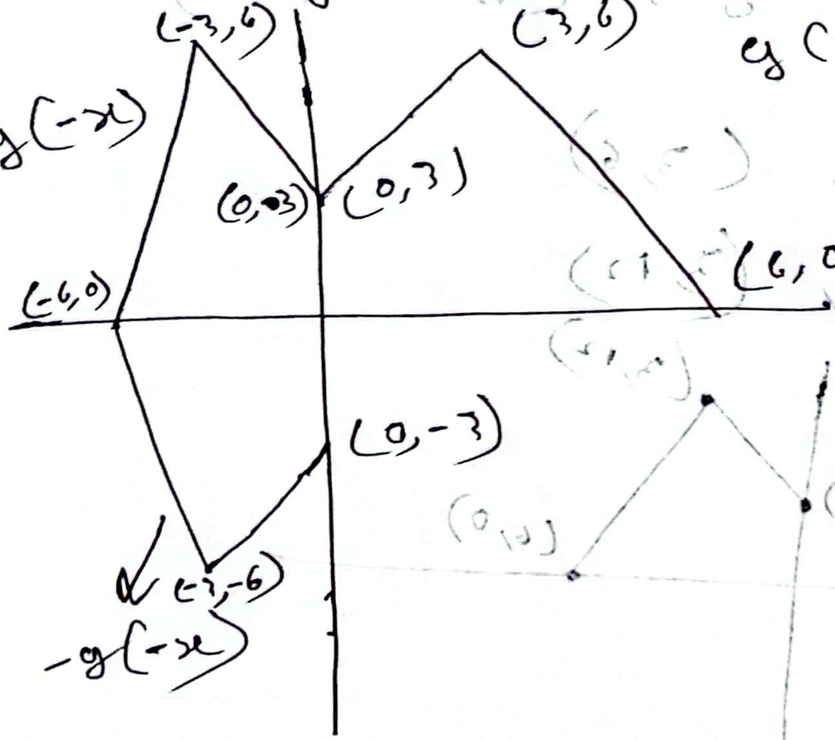
$(0, 3)$, $(3, 0)$, $\left(\frac{3}{2}, 6\right)$

$= (1.5, 6)$



(e, f, g) same

(h) $y = -g(-x)$

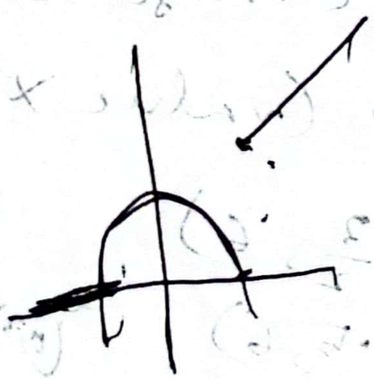


(i)

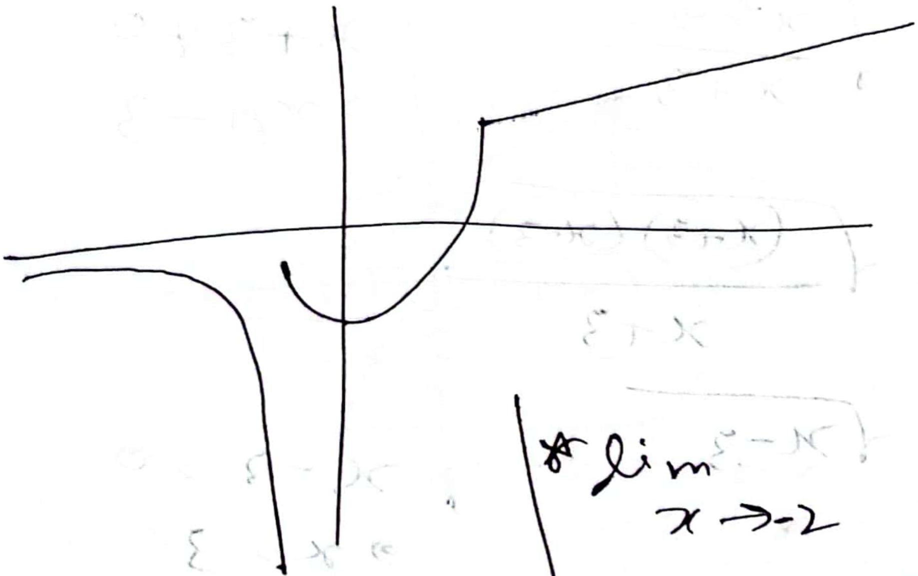
$-x-1=0$

$\Rightarrow -x = 1$

$\Rightarrow x = -1$



23



$f(x) = \frac{1}{x}$

$f(-x) = \frac{1}{-x}$

$f(x) \neq f(-x)$

$-f(x) = \frac{-1}{x} \neq f(-x)$

* $\lim_{x \to -2}$

$\lim_{x \to -2^-} f(x) = -\infty$

$\lim_{x \to -2^+} f(x) = -1$

limit नेइ

$\lim_{x \to 3^-} f(x) = 4$

$\lim_{x \to 3^+} f(x) = 4$

limit आछे

So this is neigther even nor odd.

I

1. $y = \sqrt{\dfrac{x^2-9}{x+3}}$

$x+3 \neq 0$

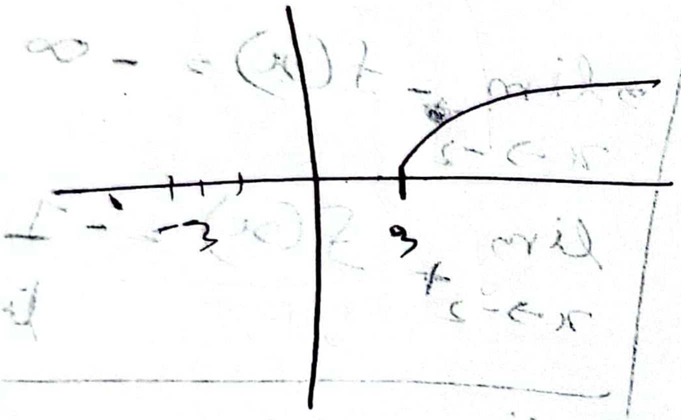
$\Rightarrow x \neq -3$

$= \sqrt{\dfrac{(x+3)(x-3)}{x+3}}$

$= \sqrt{x-3}$

$x-3=0$

$\Rightarrow x = 3$

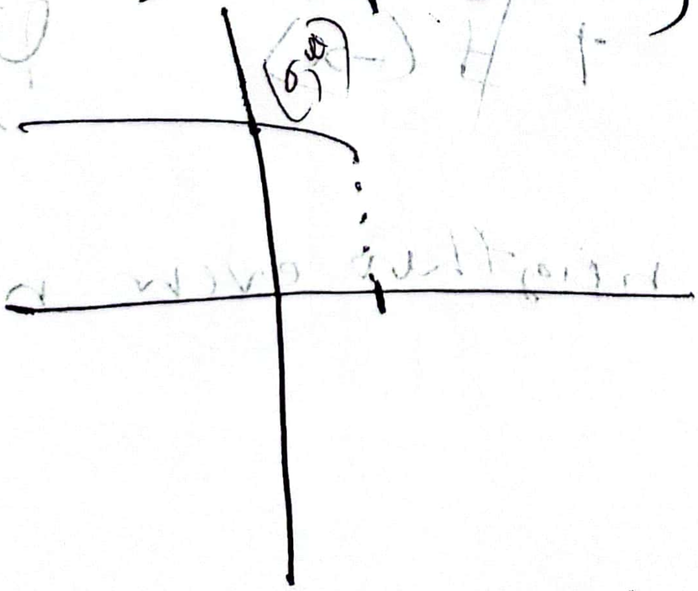


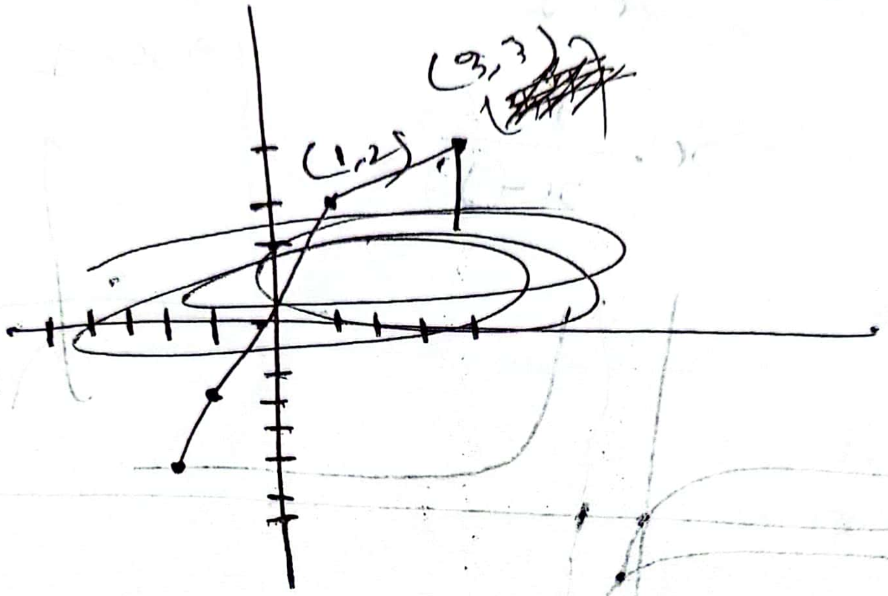
(ii) $y = 4 + \sqrt{2-x}$

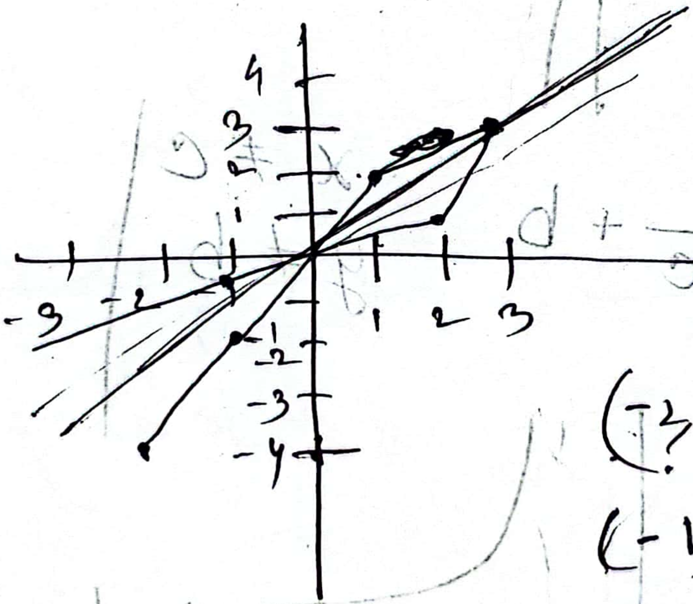
$= 4 + \sqrt{-(x-2)}$

$x-2=0$

$\Rightarrow x = 2$







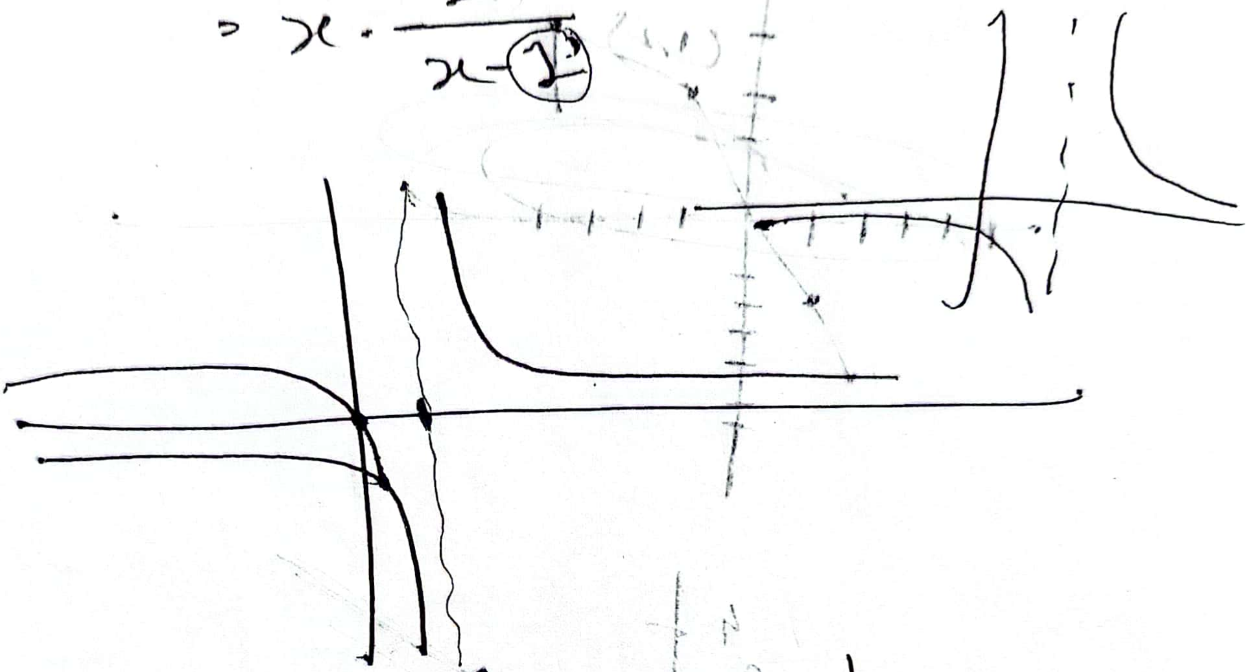
(-2, -4)	(-4, -2)
(-1, -2)	(-2, -1)
(1, 2)	(2, 1)
(3, 3)	(3, 3)
dom: Ran	Dom Ran

(iii) $y = \frac{x}{x-1}$

$x - 1 = 0$

$\Rightarrow x = 1$

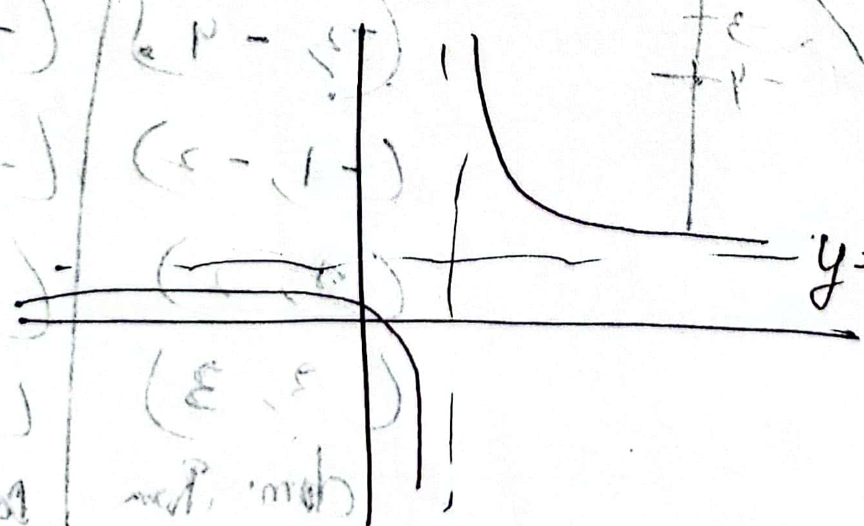
$= x \cdot \frac{1}{x-1}$



$y = \frac{1}{x+c} + b$

$x \neq c$

$y \neq b$

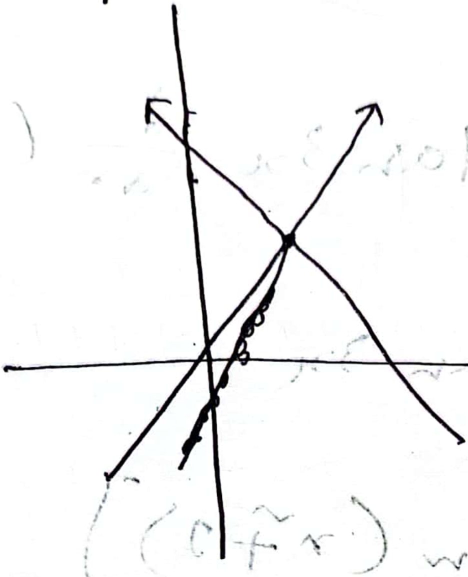


$y = b$

$x = c$

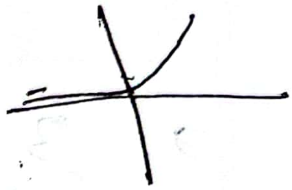
$f - \frac{y}{(4+x)}$

(iv) $y = 3 - |2x-1|$



$2x-1=0$

$\Rightarrow x = \frac{1}{2}$



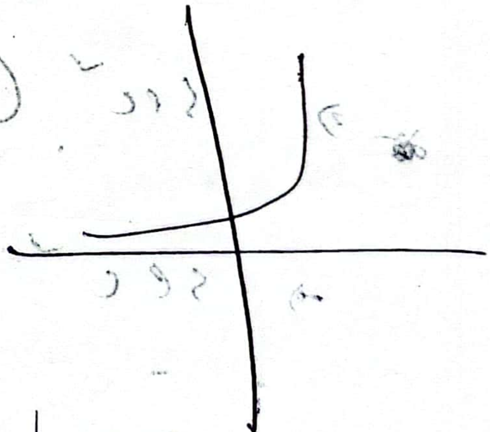
$\lim_{x \to 1}$

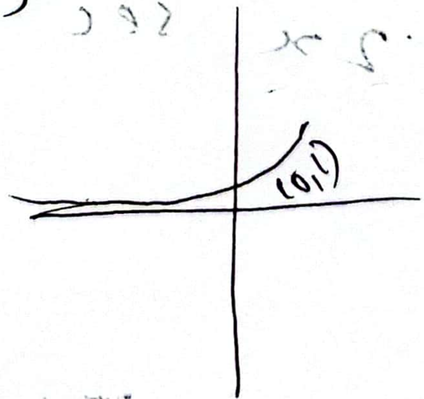
* $y = e^{x-3}$

$\Rightarrow \ln y = x-3$

$\Rightarrow x = \ln y + 3$

$\Rightarrow y = \ln x + 3$





* $\frac{d}{dx}(\sec 3x)$

$\Rightarrow \sec 3x \tan 3x \frac{d}{dx}(3x)$

$\Rightarrow 3 \sec 3x \tan 3x$

* $\frac{d}{dx}\left[\tan(x^2+1)\right]$

* $\Rightarrow \sec^2(x^2+1) \frac{d}{dx}(x^2+1)$

$\Rightarrow \sec^2(x^2+1) \cdot 2x$

$\Rightarrow 2x \sec^2(x^2+1)$

P.T.O.

$$\frac{d}{dx}\left(\sqrt{x^3+\operatorname{cosec} x}\right)$$

$$\Rightarrow \frac{d}{dx}\left(x^3+\operatorname{cosec} x\right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{1}{2}\left(x^3+\operatorname{cosec} x\right)^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{1}{2}\left(x^3+\operatorname{cosec} x\right)^{-\frac{1}{2}} \frac{d}{dx}\left(x^3+\operatorname{cosec} x\right)$$

$$\Rightarrow \frac{1}{2\sqrt{x^3+\operatorname{cosec}}}\left(3x^2-\operatorname{cosec} x \cot x\right)$$

$$\frac{d}{dx}\left[x^2 - x + 2\right]^{\frac{3}{4}}$$

$$\Rightarrow \frac{3}{4}\left(x^2 - x + 2\right)^{-\frac{1}{4}} \cdot \frac{d}{dx}\left[x^2 - x + 2\right]$$

$$\Rightarrow \frac{3}{4}\left(x^2 - x + 2\right)^{-\frac{1}{4}}\left(2x - 1\right)$$

A

Product rule:-

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Division rule:-

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

P.T.O.

* $x \cos x$

$\Rightarrow x(-\sin x) + \cos x$

$\Rightarrow \cos x . 1 + x(-\sin x)$

* $\frac{d}{dx}\left[(1 + x^5 \cot x)^{-8}\right]$

$\Rightarrow -8(1+x^5 \cot x)^{-8-1} \frac{d}{dx}(1+x^5 \cot x)$

$\Rightarrow -8(1+x^5 \cot x)^{-9}$

$(x^5(-\operatorname{cosec}^2 x) + \cot x . 5x^4)$

$$\Rightarrow -\frac{8(5x^4 \cot x - x^5 \operatorname{cosec}^2 x)}{(1 + x^5 \cot x)^9}$$

2.6 [7 – 49]

$$y = \frac{\sin x}{x^5}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\,\frac{\sin x}{x^5}$$

$$\Rightarrow \frac{dy}{dx} = \sin x \cdot x^{-5}$$

$$\Rightarrow \frac{dy}{dx} = \sin x\,\frac{d}{dx}\left(x^{-5}\right) + x^{-5}\,\frac{d}{dx}\left(\sin x\right)$$

$$= \sin x\left(-5x^{-5-1}\right) + x^{-5}\cos x$$

$$= \sin x\left(-5x^{-6}\right) + x^{-5}\cos x$$

$\Rightarrow$

(iii) $y = x^3 \cdot e^{x+1}$

$$\Rightarrow \frac{dy}{dx} = x^3 \frac{d}{dx}\left(e^{x+1}\right) + e^{x+1} \frac{d}{dx} x^3$$

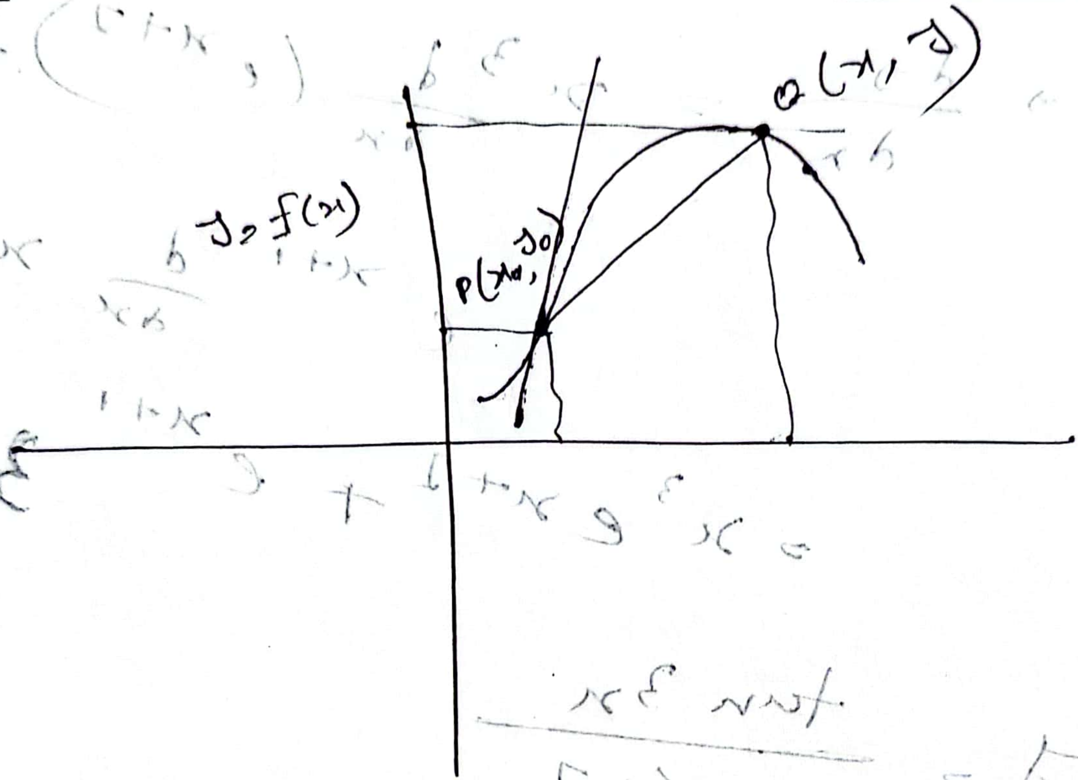
$$= x^3 e^{x+1} + e^{x+1} \, 3x^2$$

(ii) $y = \dfrac{\tan 3x}{x^2+1}$

$$= \tan 3x \left(x^2+1\right)^{-1}$$

Chapter - 2.1

Tangent line and rate of change



Slope of the secant line = m_{PQ} = average rate of change of y with respect to x. over the interval $[x_0, x]$ is = $r_{avg} = \frac{\Delta y}{\Delta x} = \frac{y - y_0}{x - x_0}$

$= \frac{f(x) - f(x_0)}{x - x_0}$

$= \frac{2-0}{1.30 - 1.20}$

$= \frac{2 \text{ km}}{10 \text{ minute}}$

P.T.O

Slope of the tangent line = m_{tan} = instantaneous rate of change of y with respect to x = derivative of

prime
→ not inverse

y with respect to x = $\frac{dy}{dx} = f'(x) =$

$$\lim_{x \to x_0} \frac{y - y_0}{x - x_0} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

x_0, $x = x_0 + h$

$$m_{avg} = \frac{f(x) - f(x_0)}{x - x_0}$$

$$= \frac{f(x_0 + h) - f(x_0)}{x_0 + h - x_0}$$

$$= \frac{f(x_0 + h) - f(x_0)}{h}$$

P.T.O

$$m_{ins} = \frac{dy}{dx} \quad \Big| \quad m_{tan} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$= \lim_{x_0+h \to x_0} \frac{f(x_0+h) - f(x_0)}{x_0 + h - x_0}$$

$$= \lim_{h \to x_0 - x_0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

* Formula for finding equation tangent line at is

$$y - y_0 = m_{tan}(x - x_0) \quad \text{—— (1)}$$

$$\Rightarrow y - y_0 = \frac{d}{dx}(x - x_0) \quad \text{—— (2)}$$

Chap - 2.1

11 - 18 (Ex)

(Slope of Secant line)

$y = x^2$, $x_0 = 0$, $x_1 = 1$

$\Rightarrow$ We know

$$m_{avg} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \frac{f(1) - f(0)}{1 - 0}$$

$$= \frac{1 - 0}{1}$$

$$= 1$$

$y = f(x) = x^2$

$f(1) = 1^2 = 1$

$f(0) = 0^2 = 0$

P.T.O

* tangent line instantaneous

$x = x_0$, $y = x^2$

$$m_{tan} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{x^2 - x_0^2}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{\cancel{(x - x_0)}(x + x_0)}{\cancel{x - x_0}}$$

$$= \lim_{x \to x_0} x + x_0$$

$$= (x_0 + x_0)$$

$$= 2x_0$$

P.T.O.

c

$$m_{tan} = 2x_0 \longrightarrow (1)$$

Here, we have

$$x = x_0 = 1$$

m_{tan} at $x_0 = 1$ is $= 2 \cdot 1$

$= 2$

(d) $x = 1$, $y = x^2$, $x = x_0 = 1$

$$y - y_0 = \left.\frac{dy}{dx}\right|_{x = x_0} (x - x_0)$$

$$\Rightarrow y - 1 = \left.\frac{dy}{dx}\right|_{x = x_0 = 1} (x - 1)$$

point is $(x_0, y_0) = (x_0, f(x_0))$

$= (1, f(1))$

$= (1, 1^2)$

$= (1, 1)$

$\Rightarrow y - 1 = 2(x - 1)$

$\Rightarrow y = 2x - 2 + 1$

$= 2x - 1$

(e) Draw the graph of the secant line over the interval $[0, 1]$,

$f = x^2$

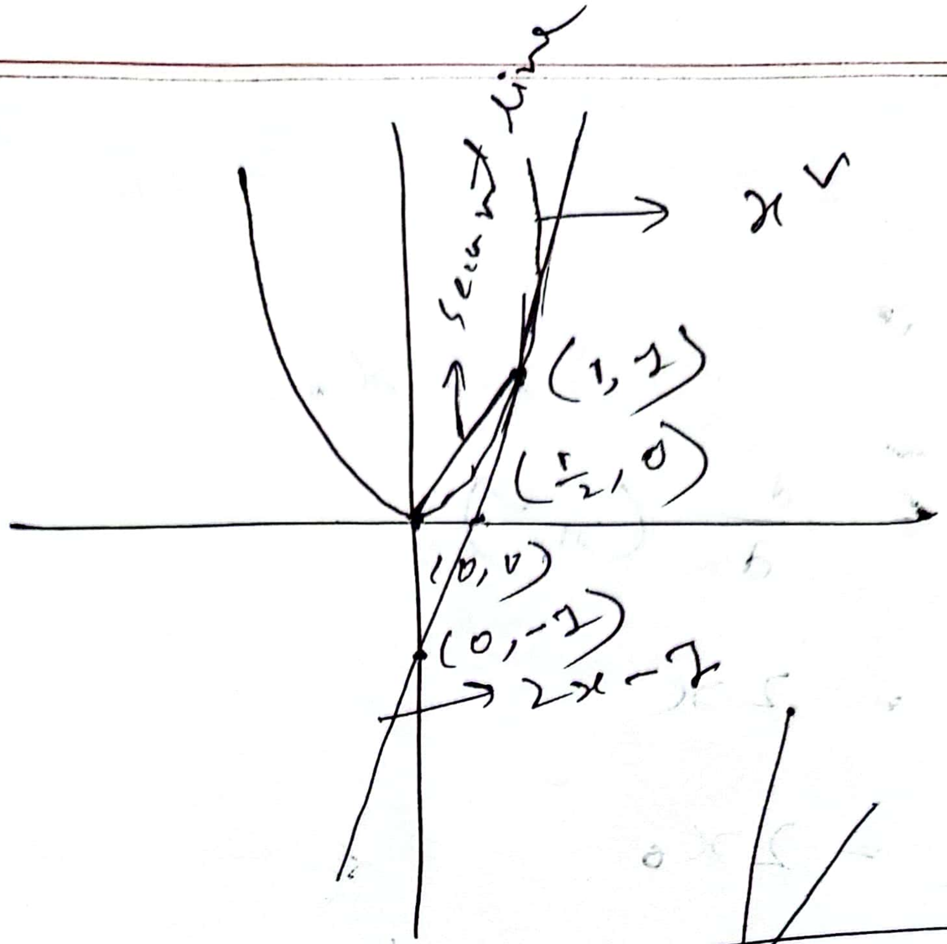
$\Rightarrow$ Joining point (x_0, y_0) & (x_1, y_1)

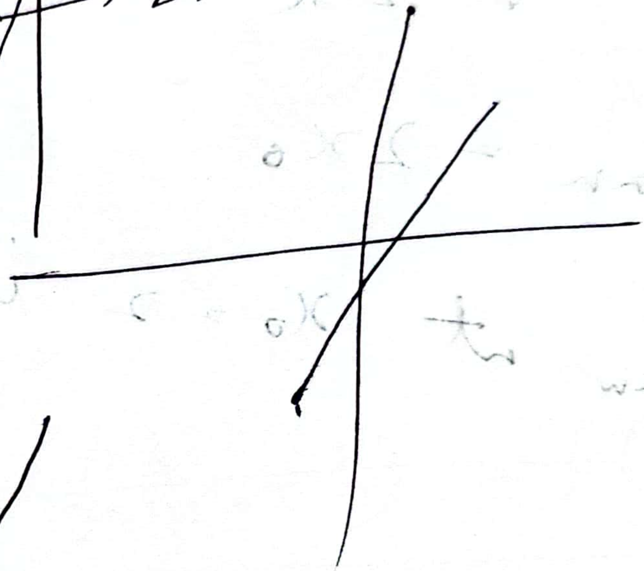
$(x_0, f(x_0))$ & $(x_1, f(x_1))$

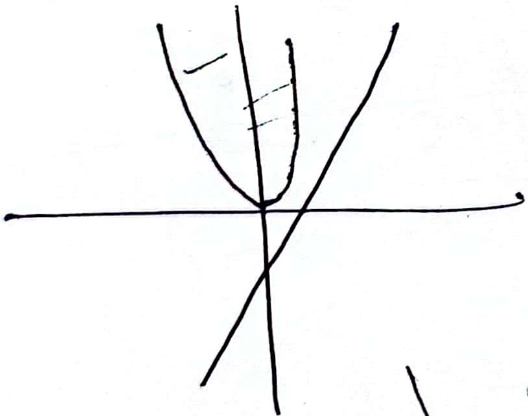
$(0, f(0)) = (0, 0)$

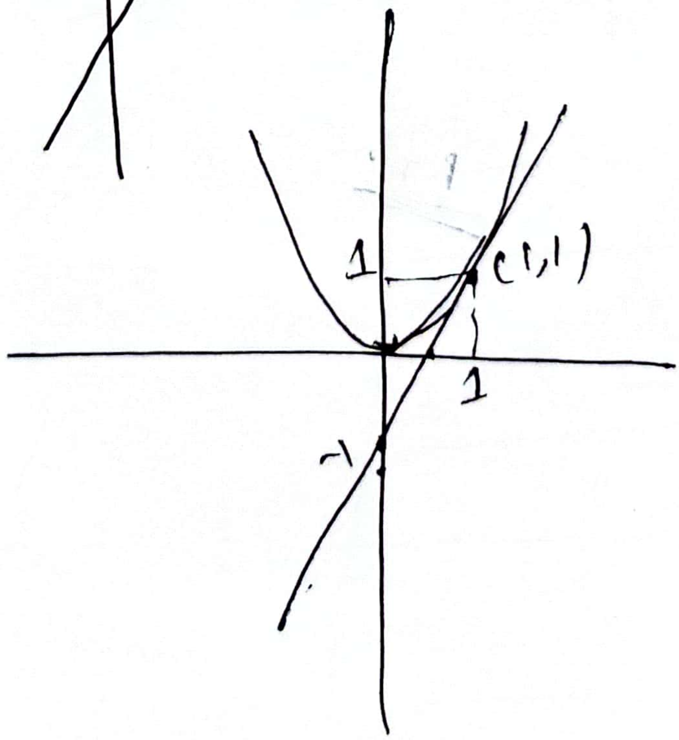
$(1, f(1)) = (1, 1)$

P.T.O


secant line
x²
(1, 1)
(½, 0)
(0, 0)
(0, −1)
2x − 1






1
(1,1)
1
-1

Ex: 1

$y = x^2$ $\quad$ $x = x_0$

$\therefore f'(x) = \frac{d}{dx}(x^2)$

$= 2x$

$\therefore m_{tan} = 2x_0$

$\therefore m_{tan}$ at $x_0 = 2$ is $= 2 \cdot 2 = 4$

P.T.O

$$y = \frac{2}{x}, \quad x = 2, \quad x = x_0$$

$$f'(x) = \frac{d}{dx}\left(\frac{2}{x}\right)$$

$$= \frac{d}{dx}\left(2 \cdot \frac{1}{x}\right)$$

$$= 2 \cdot -\frac{1}{x^2} + \frac{1}{x} \cdot 0$$

$$= -\frac{2}{x^2}$$

$$=$$

$$m_{\cancel{tan}} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2+h}{x_0} - \frac{2}{x_0}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2+h-2}{x_0}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{h}{x_0}}{h} \quad h$$

$y = \frac{2}{x}$, $x = 2$, $x = x_0$

$$m_{tan} = \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \frac{\frac{2}{x_0+h} - \frac{2}{x_0}}{h}$$

$$= \frac{\frac{2x_0 - 2(x_0+h)}{x_0(x_0+h)}}{h}$$

$$= \frac{\frac{2x_0 - 2x_0 - 2h}{x_0(x_0+h)}}{h}$$

$$= \frac{\frac{-2h}{x_0(x_0+h)}}{h}$$

$$= \frac{-2h}{x_0(x_0+h)}$$

2.1

Example : 7

5

$S = f(t) = 1 + 5t - 2t^2$

$\frac{d}{dx}(1 + 5t - 2t^2)$

$\Rightarrow 5 - 4t$

$\Rightarrow 5 - 4(2)$

$\Rightarrow 5 - 8 = -3$

CamScanner

Ex-4

$y = \sqrt{x}$ at $x_0 = 1, 4, 9$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x_0+h} - \sqrt{x_0}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x_0+h} - \sqrt{x_0})(\sqrt{x_0+h} + \sqrt{x_0})}{h(\sqrt{x_0+h} + \sqrt{x_0})}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x_0+h})^2 - (\sqrt{x_0})^2}{h(\sqrt{x_0+h} + \sqrt{x_0})}$$

$$= \lim_{h \to 0} \frac{x_0 + h - x_0}{h(\sqrt{x_0+h} + \sqrt{x_0})}$$

P.T.O.

$$= \frac{1}{\sqrt{x_0 + 0} + \sqrt{x_0}} = \frac{1}{2\sqrt{x}}$$

11

$f = 2x^2$

$f(x_0) = 2x_0^2$

$f(x_0+h) = 2(x_0+h)^2$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \lim_{h\to 0} \frac{2(x_0+h)^2 - 2x_0^2}{h}$$

$$= \lim_{h\to 0} \frac{2(x_0^2 + 2h\cdot x_0 + h^2) - 2x_0^2}{h}$$

$$= \lim_{h\to 0} \frac{2x_0^2 + 4hx_0 + h^2 - 2x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x_0 + h)}{\cancel{h}} \Big|_{h=0}$$

$= 4x_0$

12

$y = x^3$

$f(x) = x^3$

$f(x+h) = \cancel{x^3 + h} \; (x+h)^3$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + h - x^3}{h}$$

$$= \lim_{h \to 0} \frac{h}{h}$$

$= 0$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$= 3x^2 + 3xh + h^2 \,\Big|_{h=0}$

$= 3x^2$

13

$y = \frac{1}{x}$ $\quad$ $x = x_0$

$f(x+h) = \frac{1}{x+h}$

$f(x)$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x-x-h}{x(x+h)}}{h}$

$= \frac{\frac{-h}{x^2+xh}}{h}$

$= \frac{h^2}{x^2+xh}$

$= \lim_{h \to 0} \frac{\frac{x-x-h}{x(x+h)}}{h}$

$= \lim_{h \to 0} \frac{\frac{-h}{x(x+h)}}{h}$

$= \frac{h(x-x-h)}{x(x+h)}$

$= \lim_{h \to 0} \frac{-h^2}{x^2+xh}$

$= \frac{-h^2}{x(x+h)}$

$= \lim_{h \to 0} h^2 \cdot \frac{1}{x^2+xh} =$

$= -\frac{1}{x^2}$

$m_{tan} = -\frac{1}{x_0^2}$

14

$f(x) = \frac{1}{x^2}$, $f(x+h) = \frac{1}{(x+h)^2}$, $x = x_0$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x^2+2x \cdot h + h^2} - \frac{1}{x^2}}{h}$$

$$= \cancel{\lim_{h \to 0} \frac{h^2(2x+h)}{x^3 + 2x^2h + xh^2}}$$

$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \frac{\frac{x - x - h}{x(x+h)}}{h}$$

$$= \frac{-h}{h(x^2 + xh)}$$

$$= -\frac{1}{x^2}$$

$$\frac{\frac{1}{x^2+2xh+h^2} - \frac{1}{x^2}}{h}$$

$$= \frac{\frac{x^2 - x^2 - 2xh - h^2}{x^2(x^2+2xh+h^2)}}{h}$$

P.T.O.

$$= \frac{-2xh - h^2}{h(x^4 + 2x^3h + h^2)}$$

$$= \frac{h(-2x - h)}{h(x^4 + 2x^3h + h^2)}$$

$$= \frac{-2x - h}{x^4 + 2x^3h + h^2}$$

$$= \frac{-2x}{x^4}$$

$$= \frac{-2}{x^3}$$

$$= -2x^{-3}$$

A

HW

2.2

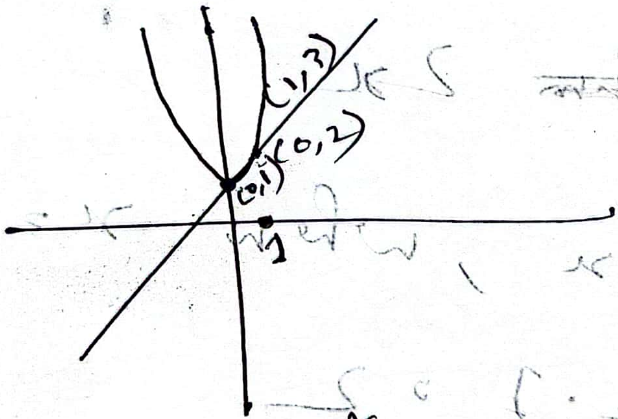
Calculus

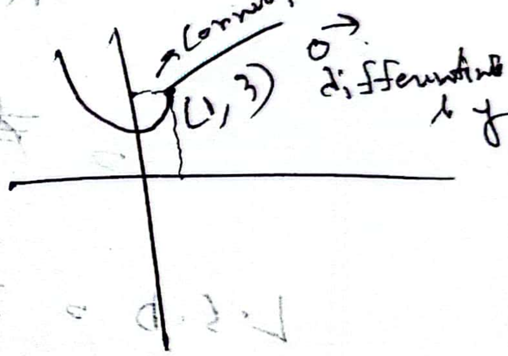
$0 \rightarrow$ undefined / not differentiable

Exercise : 47, 48

48

$$f(x) = \begin{cases} x^2+2 ; & x \le 1, \quad x \to 1^- \\ x+2 ; & x > 1 \quad x \to 1^+ \end{cases}$$





$\lim_{x \to 1^-} f(x) = x^2+2 = 1^2+2 = 3$

$\lim_{x \to 1^+} f(x) = 3$

P.T.O

$$L.S.D = \lim_{h\to 0^-} \frac{f(x+h)-f(x)}{h}$$

$h \to 0^-$

$f(x) = x^2+1$

$x = 1$

$f(x+h) = (x+h)^2+1$

$$= \lim_{h\to 0^-} \frac{x^2+2hx+2-x^2-2}{h}$$

$$= \lim_{h\to 0^+} \frac{h(2x+h)}{h}$$

$$= \cancel{\lim}\ 2x$$

$L.S.D = 2x$, when $x = 1$,

$L.S.D = 2\cdot 1 = 2$.

$$R.H.D = \lim_{h\to 0^+} \frac{f(x+h)-f(x)}{h}$$

$h \to 0^+$

$f(x) = x+2$

$f(x+h) = (x+h)+1$

$$= \lim_{h\to 0} \frac{(x+h)+\cancel{2}-x-\cancel{2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x}+h-\cancel{x}}{h} = \frac{h}{h} = 1$$

Exercise

47

$$f(x) = \begin{cases} x^2+1, & x \le 1, \quad x \to 1^- \\ 2x, & x > 1, \quad x \to 1^+ \end{cases}$$

$$\lim_{x \to 1^-} f(x) = 1^2+1 = 2$$

$$\lim_{x \to 1^+} f(x) = 2 \cdot 1 = 2$$

L.H.L = R.H.L

$\therefore$ Limit exist

$$L.S.D = \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 1 - x^2 - 1}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2 - 1 + 1}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 1 + 1}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h) - 1 + 1}{h} = 2x$$

L.S.D = $2x$ when, $x = 1$

$\therefore$ L.S.D = $2 \cdot 1 = 2$

R.H.D = $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{2(x+h) - 2x}{h}$

$= \lim_{h \to 0} \frac{2x + 2h - 2x}{h}$

$= \lim_{h \to 0} \frac{2h}{h}$

$= 2$

L.S.D = R.H.D

differentiable at $x = 1$

[redacted]

$\Rightarrow$ [redacted]

$$\frac{d}{dx}(\sqrt{x}) = (x)^{\frac{1}{2}}$$

$$\Rightarrow \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}}$$

$$\frac{1}{2} \cdot x^{-\frac{1}{2}}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

2.1 → Example → (1–7)

Exercise → (11–18)

2.2 → Continuity diff

(47, 48)

2 (i)

(a) $f(x) = x^3 - 3$ $\qquad x = x_0$

$f(x+h) = (x+h)^3 - 3$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - 3 - x^3 + 3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{3} - \cancel{x^3} + \cancel{3}}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2)}{h}$$

$= 3x^2$

$\therefore m_{\tan} = 3x_0^2$

(b) $f(x) = 2x - x^2$

$f(x+h) = 2(x+h) - (x+h)^2$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x} + 2h - \cancel{x^2} - 2xh - h^2 - \cancel{2x} + \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{2h - 2xh - h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2 - 2x - h)}{h}$$

$$= \lim_{h \to 0} 2 - 2x - h$$

$$= 2 - 2x$$

$m_{tan} = 2 - 2x_0$

(ii) if $x = x_0 = 1$

(a) $3 \cdot 1^2 = 3$

(b) $2 - 2 \cdot 1 = 0$

* $w = \tan(2x^3 - 3x + 5)$

$$\frac{dw}{dx} = \sec^2(2x^3 - 3x + 5) \cdot (6x^2 - 3)$$

* $x = t^2 - 2t + \frac{3}{t} - 4$

$$\frac{dx}{dt} = 2t - 2 - \frac{3}{t^2}$$

$$\frac{dw}{dt} = \frac{dw}{dx} \cdot \frac{dx}{dt}$$

$$= \sec^2 x(2x^3 - 3x + 5)(6x^2 - 3)$$

$$(2t - 2 - \frac{3}{t^2})$$

Ex. 1

$f(x) = x^2$ $\quad x_0 = 1$

$f(x+h) = (x+h)^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h}$$

$\therefore m_{tan} = 2x.$

$\therefore m_{tan} = 2$

$\therefore (x_0, y_0) = (1, 1)$

$y - 1 = 2(x - 1)$

$\Rightarrow y - 1 = 2x - 2$

$\Rightarrow y = 2x - 2 + 1$

$\Rightarrow y = 2x - 1$

* $y = \frac{2}{x}$

$x = x_0$

$x_0 = 2$

$y' = 2 \cdot (x^{-1})$

$= 2 \cdot (-1)\, x^{-1-1}$

$= 2 \cdot -x^{-2}$

$= -\frac{2}{x^2}$

$\therefore m_{tan} = -\frac{2}{2^2} = -\frac{2}{4} = -\frac{1}{2}$

$\left\{ x_0 = 2,\ \left(y_0 = \frac{2}{x_0} = \frac{2}{2} = 1 \right) \right\}$

$y - 1 = -\frac{1}{2}(x - 2)$

$\Rightarrow y - 1 = -\frac{1}{2}x + 1$

$\Rightarrow y = -\frac{1}{2}x + 1 + 1$

$\Rightarrow y = -\frac{1}{2}x + 2$

$\Rightarrow 2y = -x + 4$

Calculus Assignment

I

(1) (a) $s = f(t) = 3 - 5t - 2t^2$

~~$x_0 = -3, x = 2$~~

~~$V_{avg} = \frac{f(x) - f(x_0)}{x - x_0}$~~

~~$= \frac{-15 - 0}{2-(-3)} = \frac{-15}{5}$~~

~~$= -3$~~ Ans

(ii) $f'(t) = -5 - 4t$

$x_0 = -3, x = 2$

$V_{avg} = \frac{f'(x) - f'(x_0)}{x - x_0}$

$= \frac{-13 - 7}{5}$

$= \frac{-20}{5}$

$= -4$

Ans

(b) $s = f(t) = 7 - 3t^2 - 2t^3$, $x_0 = -3, x = 2$

$f'(t) = -6t - 6t^2$

$\therefore V_{avg} = \frac{f'(x) - f'(x_0)}{x - x_0} = \frac{-36 + 36}{5} = 0$

Ans

(11)

(a) $f(t) = 3 - 5t - 2t^2$

$f(t+h) = 3 - 5(t+h) - 2(t+h)^2$

$$\lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

$$= \lim_{h \to 0} \frac{3 - 5t - 5h - 2(t^2 + 2th + h^2) - 3 + 5t + 2t^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3} - \cancel{5t} - 5h - \cancel{2t^2} - 4th - 2h^2 - \cancel{3} + \cancel{5t} + \cancel{2t^2}}{h}$$

$$= \lim_{h \to 0} \frac{-5h - 4th - 2h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-5 - 4t - 2h)}{\cancel{h}}$$

$= -5 - 4t$

$\therefore m_{tan} = -5 - 4 \times (-2) = 3$

A

(b) $f(t) = 7 - 3t^2 - 2t^3$

$f(t+h) = 7 - 3(t+h)^2 - 2(t+h)^3$

$= 7 - 3(t^2 + 2th + h^2) - 2(t^3 + 3t^2h + 3th^2 + h^3)$

$= 7 - 3t^2 - 6th - 3h^2 - 2t^3 - 6t^2h - 6th^2 - 2h^3$

$$\lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{7} - \cancel{3t^2} - 6th - 3h^2 - \cancel{2t^3} - 6t^2h - 6th^2 - 2h^3 - \cancel{7} + \cancel{3t^2} + \cancel{2t^3}}{h}$$

$$= \lim_{h \to 0} \frac{-6th - 3h^2 - 6t^2h - 6th^2 - 2h^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-6t - 3h - 6t^2 - 6th - 2h^2)}{\cancel{h}}$$

$= -6t - 6t^2$

$\therefore m_{tan} = -6 \times (-2) - 6 \times (-2)^2 = -12$

2/

(1) (a) $f(x) = x^3 - 2$, $x = x_0$

$$f(x+h) = (x+h)^3 - 2$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{-2} \cancel{-x^3} \cancel{+2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$= 3x^2$$

$\therefore m_{tan} = 3x_0^2$

(b) $f(x) = 2x - x^2$

$$f(x+h) = 2(x+h) - (x+h)^2$$

$$= 2x + 2h - x^2 - 2xh - h^2$$

P.T.O.

$$\lim_{h\to 0} \frac{\cancel{2x}+2h-\cancel{x^2}-2xh-h^2-\cancel{2x}+\cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{h(2-2x-h)}{h}$$

$$= 2-2x$$

$$m_{tan} = 2-2x_0$$

(ii) (a) $x = x_0$

slope of the tangent line at $x_0 = 1$,

$3\times(1)^2$ $3\times 3 \cdot x$

(b) $x = x_0$

m_{tan} at $x_0 = 1$, $2-2\times(1)$

$= 2-2$

$= 0$

(iv) $x_0 = -1, \ x = 1$

(a) $r_{avg} = \dfrac{f(x) - f(x_0)}{x - x_0}$

$= \dfrac{-1-(-3)}{1+1}$

$= \dfrac{2}{2} = 1$ Ans

$y = mx + c$
$\Rightarrow -1 = 1 \times 1 + c$
$\Rightarrow -1 = 1 + c$
$\Rightarrow c = -2$

$y = x - 2$

$y - y_0 = m(x$
$y + 3 = 1(x$
$\Rightarrow y + 3 = x +$
$\Rightarrow y + 3 - x - 1$
$\Rightarrow y - x + 2$
$\Rightarrow x - y - 2$

(b) $r_{avg} = \dfrac{f(x) - f(x_0)}{x - x_0}$

$= \dfrac{\cancel{1+(-3)}\ 1+3}{1+1} = \dfrac{4}{2}$

$= \cancel{\ } = \cancel{\ }\ 2$ Ans

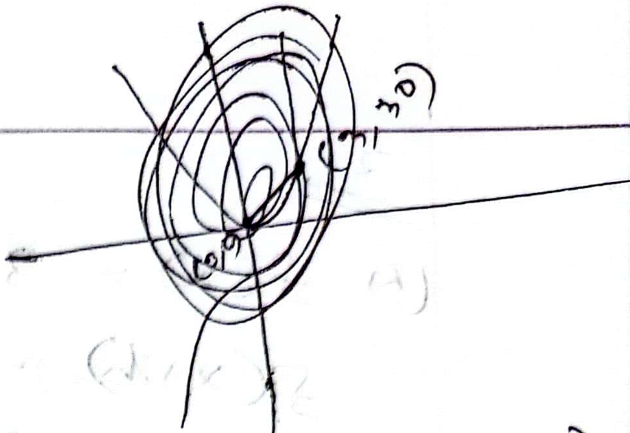
$y = mx + c$
$\therefore 1 = 2 \times 1 + c$
$\Rightarrow 1 - 2 = c$
$\Rightarrow c = -1$

$y = x - 1$ Ans

~~(iii) (iv)~~

~~(a) $f(x) = x^3 - 2$~~

~~$f'(x) = 3x$~~

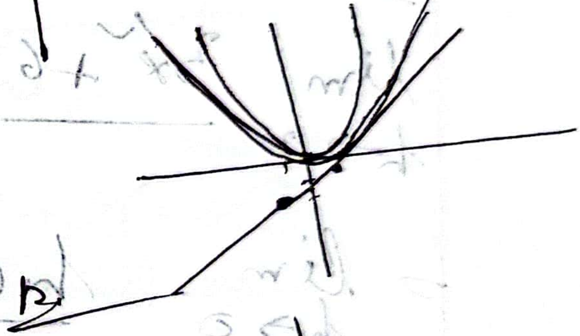


(a) $x_0 = -1, \; x = 1$

$y_0 = -3, \; y = -1$

$(x_0, y_0) = (-1, -3)$

$(x, y) = (1, -1)$



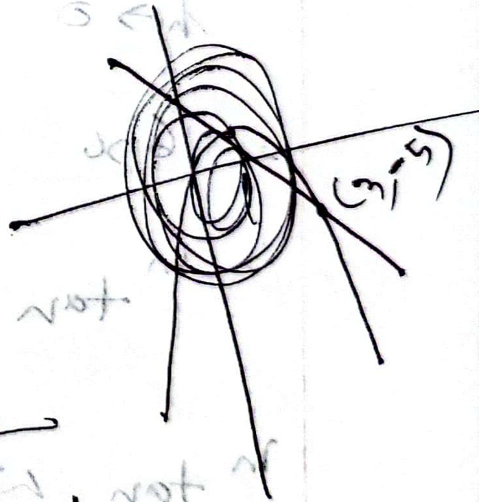
$y + 3 = x + 1$

$\Rightarrow y + 3 - x - 1 = 0$

$\Rightarrow x - y - 2 = 0$

(b) $x_0 = -1, \; x = 1$

$y_0 = -3, \; y = 1$



$y + 3 = 2(x+1)$

$\Rightarrow y = 2x + 2 - 3$

$= 2x - 1$

$f(x) = 2x + x^2$ same

$f'(x) = 2 + 2x$

P.T.O.

3

$$\boxed{x \to x_0}$$

(A) $f(x) = 3x^2$

$f(x+h) = 3(x+h)^2$

$= 3(x^2 + 2xh + h^2)$

$= 3x^2 + 6xh + 3h^2$

$$\lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2}}{h}$$

$$= \lim_{h \to 0} \frac{h(6x + 3h)}{h}$$

$= 6x$

$m_{tan} = 6x_0$

$y - y_0 = m(x - x_0)$

$y_0 = 3 \cdot (-1)^2$

$= 3 \cdot 1 = 3$

m_{tan} at $x_0 = -1$, $6 \times (-1)$

$y - 3 = -6(x+1)$

$\Rightarrow y - 3 = -6x - 6$ <u>T.A.T.O</u>

$\Rightarrow y - 3 + 6x + 6 = 0$

$\Rightarrow y + 6x + 3 = 0$

(b) $f(x) = 4 - x^3$

$f(x+h) = 4 - (x+h)^3$

$= 4 - (x^3 + 3x^2h + 3xh^2 + h^3)$

$= 4 - x^3 - 3x^2h - 3xh^2 - h^3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4} - \cancel{x^3} - 3x^2h - 3xh^2 - h^3 - \cancel{4} + \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} \frac{h(-3x^2 - 3xh - h^2)}{h}$$

$= -3x^2$

$m_{tan} = -3x_0^2$

m_{tan} at $x_0 = -1$

$= -3 \times (-1)^2$

$= -3$

$y - 5 = -3(x+1)$

$\Rightarrow y - 5 = -3x - 3$

$\Rightarrow y - 5 + 3x + 3 = 0$

$\Rightarrow 3x + y - 2 = 0$

Ans

Table : 5.2.1

(মুখস্থ)

* $\frac{d}{dx} \sin(x^2)$

$= \cos x^2 \frac{d}{dx} x^2$

$= \cos x^2 \, 2x$

* $\frac{d}{dt} \tan(4t^3 + t)$

$\Rightarrow \sec^2(4t^3+t) \frac{d}{dt}(4t^3+t)$

$\Rightarrow \sec^2(4t^3+t) \cdot 12t^2 + 1$

1 T.

* $y = \operatorname{cosec}\left(\sqrt{x^3+x}\right)$ *

$$\frac{dy}{dx} = -\operatorname{cosec}\left(\sqrt{x^3+x}\right) \cdot \cot\left(\sqrt{x^3+x}\right)$$

$$\frac{d}{dx}\left(\sqrt{x^3+x}\right)$$

$$= -\operatorname{cosec}\sqrt{x^3+x}\ \cot\sqrt{x^3+x} \cdot$$

$$\frac{1}{2\sqrt{x^3+x}}\ \frac{d}{dx}\left(x^3+x\right)$$

$$= -\operatorname{cosec}\sqrt{x^3+x}\ \cot\sqrt{x^3+x} \cdot \frac{1}{2\sqrt{x^3+x}}$$

$$\cdot\ 3x^2+1$$

Ans

$$* \quad \frac{d}{dx} \tan^2 4x$$

$$\Rightarrow \frac{d}{dx} \left[\tan (4x) \right]^2$$

$$\Rightarrow 2 \tan (4x) \frac{d}{dx} \tan 4x$$

$$= 2 \tan 4x \sec^2 x \frac{d}{dx} 4x$$

$$\Rightarrow 2 \tan 4x \sec^2 x \cdot 4$$

$$\Rightarrow 8 \tan 4x \sec^2 x$$

eg

$$\frac{d}{dx}\left(1+x^5\cot x\right)^{-8}$$

$$= -8\left(1+x^5\cot x\right) \frac{d}{dx}\left(1+x^5\cot x\right)$$

$$= -8\left(1+x^5\cot x\right)^{-9} \cdot x^5 \frac{d}{dx}\cot x + \cot x \frac{d}{dx}x^5$$

$$= -8\left(1+x^5\cot x\right)^{-9} \cdot x^5\left(-\csc^2 x\right) + \cot x \cdot 5x^4$$

* $y = \left(\frac{x}{(x+1)}\right)^{100}$

$$\frac{dy}{dx} = \left\{x \cdot (x+1)^{-1}\right\}^{100}$$

$$\frac{dy}{dx} = 100 \left\{x \cdot (x+1)^{-1}\right\}^{99} \cdot \frac{d}{dx} x \cdot (x+1)^{-1}$$

$$= 100 \left\{x \cdot (x+1)^{-1}\right\}^{99} \cdot x \frac{d}{dx}(x+1)^{-1} + (x+1)^{-1} \frac{d}{dx} x$$

$$= 100 \left\{x \cdot (x+1)^{-1}\right\}^{99} \cdot x \cdot \left\{-(x+1)^{-2}\right\} + (x+1)^{-1} \cdot 1$$

$y = x^3$

$f'(x) = 3x^2$

$* \ y = x^3 - x$

$f'(x) = 3x^2 - 1$

$x^3 - 5x^2 - x + 5 = 0$

$\Rightarrow 3x^2 - 10x - 1 = 0$

Assignment

4

(a) $\lim_{x \to -2} \frac{x^2 - x - 6}{x + 2}$

$= \lim_{x \to -2} \frac{2x - 1}{1}$

$= 2 \times (-2) - 1$

$= -4 - 1$

$= -5$

A

$|a| = -a$

$|a| = a,$

(b) $\lim_{x \to 3} \frac{|x-3|}{x-3}$

$\Rightarrow \lim_{x \to 3} \frac{-(x-3)}{x-3}$

$\Rightarrow \lim_{x \to 3} \frac{-1}{1}$

$\Rightarrow -1$

$\Rightarrow \lim_{x \to 3} \frac{x-3}{x-3}$

$= 1$

A

(c)

$$\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$$

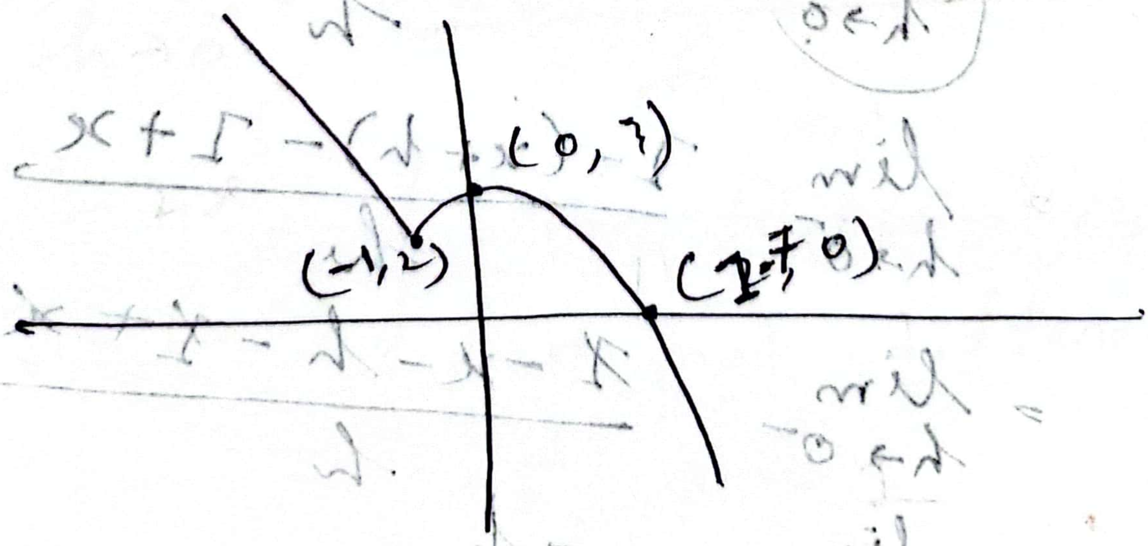
$$\Rightarrow \lim_{x \to 2} \frac{3x^2}{1}$$

$= 3 \cdot (2)^2$

$\Rightarrow 3 \cdot 4 \Rightarrow 12$

7

(a) $f(x) = \begin{cases} 1 - x, & \text{if } x < -1 \\ 3 - x^2 & \text{if } x \geq -1 \end{cases}$



(b) $f(x) = \begin{cases} 1-x, & x < -1, \quad x \to -1^- \\ 3-x^2, & x \geq -1, \quad x \to -1^+ \end{cases}$

$$\lim_{x \to -1^-} f(x) = 1-(-1) = 1+1 = 2$$

$$\lim_{x \to -1^+} f(x) = 3-(-1)^2 = 3-1 = 2$$

L.H.L = R.H.L

$\therefore$ Limit exist.

$$L.H.D = \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0^-} \frac{1-(x+h)-1+x}{h}$$

$$= \lim_{h \to 0^-} \frac{\cancel{1}-\cancel{x}-h-\cancel{1}+\cancel{x}}{h}$$

$$= \lim_{h \to 0^-} \frac{-h}{h}$$

$$= -1$$

$$R.H.D = \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0^+} \frac{3 - (x+h)^2 - 3 + x^2}{h}$$

$$= \lim_{h \to 0^+} \frac{3 - (x^2 + 2xh + h^2) - 3 + x^2}{h}$$

$$= \lim_{h \to 0^+} \frac{3 - x^2 - 2xh - h^2 - 3 + x^2}{h}$$

$$= \lim_{h \to 0^+} \frac{-2xh - h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(-2x - h)}{h}$$

$$= -2x$$

$$= -2(-2)$$

$$= 2$$

This function is continuous but not differentiable at $x = -1$

A

8.

$$\frac{dw}{dt} = \tan(2x^3 - 3x + 5)$$

$$x = t^2 - 2t + \frac{3}{t} - 4$$

$$tx = t^3 - 2t^2 + 3 - 4t$$

$$= t^3 - 2t^2 - 4t + 3$$

$$\boxed{\frac{dw}{dt} = \tan\left(2(t^3 - 2t^2 - 4t + 3)^3 - 3(t^3 - 2t^2 - 4t + 3) + 5\right)}$$

$= \cancel{\sec \ldots}$

$$= \sec^2\left(2(t^3 - 2t^2 - 4t + 3)^3 - 3t^3 + 6t^2 + 12t - 9 + 5\right)$$

$$\frac{d}{dt}\left(2(t^3 - 2t^2 - 4t + 3)^3 - 3t^3 + 6t^2 + 12t - 9 + 5\right)$$

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} \qquad \frac{d\,2}{dx} = 0$$

$$= \sec^2\left(2\cdot(t^3-2t^2-4t+3)^3 - 3t^3 + 6t^2+12t-9+5\right)$$

$$\frac{d}{dt}\left(2\frac{d}{dt}(t^3-2t^2-4t+3)^3 + (t^3-2t^2-4t+3)^3\frac{d}{dt}2 - \right.$$

$$\cancel{9t^2+12t+12)}$$

$$= \left(2\cdot 3(t^3-2t^2-4t+3)^2\cdot\frac{d}{dt}(t^3-2t^2-4t+3) - 9t^2+12t+12\right)$$

$$= \left(2\cdot 3(t^3-2t^2-4t+3)^2\cdot(3t^2-4t-4) - 9t^2+12t+12\right)$$

$$= \sec^2\left(2\cdot(t^3-2t^2-4t+3)^3 - 3t^3+6t^2+12t-4\right)\cdot$$
$$\left(2\cdot 3(t^3-2t^2-4t+3)^2\cdot(3t^2-4t-4) - 9t^2+12t+12\right)$$

$f(x_0) = y_0$ $\qquad$ $y - y_0 = m(x - x_0)$

11

$x = \sqrt{\pi}$

$y = \tan(4x^2)$

$x = x_0$

$f'(x) = \frac{d}{dx} \tan(4x^2)$

$= \sec^2(4x^2) \cdot 4 \cdot 2x$

$\sec^2(4x^2) \cdot \frac{d}{dx}(4x^2)$

$= \sec^2(4x^2) \cdot 8x$

$= \sec^2(4x^2) \cdot 8x$

$x_0 = \sqrt{\pi}$

$y_0 = \tan(4\sqrt{\pi}^2)$

$= \tan(4\pi)$

$= 0$

~~$f'(x)$ at $x = \sqrt{\pi}$~~

$x = \sqrt{\pi}$

$\sec^2(4(\sqrt{\pi})^2) \; 8\sqrt{\pi}$

$m_{\tan} = 8\sqrt{\pi}$

$\Rightarrow \sec^2 4\pi \cdot 8\sqrt{\pi}$

$= 1 \cdot 8\sqrt{\pi} = 8\sqrt{\pi}$

$\therefore y - y_0 = m_{\tan}(x - x_0)$

$\Rightarrow y - 0 = 8\sqrt{\pi}(x - \sqrt{\pi})$

$\Rightarrow y = 8\sqrt{\pi}(x - \sqrt{\pi})$

$t^2 - 2t + 1 + \frac{3}{t} - 5$. x^n $n-1$ t^{-1} x^n

$(t-1)^2 + \frac{3}{t} - 5$ $= n x^{n-1}$ $-1+1$ $(\cancel{n-1})\, x$

$$= \tan\left(2\left(t^2 - 2t + \frac{3}{t} - 4\right)^3 - 3\left(t^2 - 2t + \frac{3}{t} - 4 + 5\right)\right)$$

$$= \sec^2 \cancel{\times} \left(2 \text{ w.r.t. } \frac{d}{dt}(\,-\,) - 3\frac{d}{dt} + 5\right)$$

$$3 \cdot 2 \cdot \left(2t - 2 + 3 \cdot t^{-2}\right)^2 - 3 \cdot \left(2t - 2 - 3t^{-2} + 5\right)$$

$$\left(2t - 2 - 3t^{-2}\right)^2$$

$$\left(-3t^{-2}\right)^2 + (-2)^2 + (2t)^2 + 2 \cdot (3t^{-2}$$

$y = \tan(4x^2)$

w.r.t. x

$$w = \tan(2x^3 - 3x + 5)$$

$$\frac{dw}{dx} = \sec^2(2x^3 - 3x + 5)\,\frac{d}{dx}(2x^3 - 3x + 5)$$

$$= \sec^2(2x^3 - 3x + 5)\cdot(6x^2 - 3)$$

$$x = t^2 - 2t + \frac{3}{t} - 4$$

$$\Rightarrow x = t^2 - 2t + 3t^{-1} - 4$$

$$\frac{dx}{dt} = 2t - 2 + 3(-1)t^{-2} - 0$$

$$= \left(2t - 2 - \frac{3}{t^2}\right)$$

$$\frac{dw}{dt} = \frac{\frac{dw}{\cancel{dx}}}{\cancel{dx}} \times \frac{dx}{dt}$$

$$= \frac{dw}{dx} \times \frac{dx}{dt}.$$

$$= (6x^2 - 3)\sec^2(2x^3 - 3x + 5)\cdot\left(2t - 2 - \frac{3}{t^2}\right)$$

$f(x) = \operatorname{cosec}^3 \sqrt{x^5 - x^2 \cos x - 5}$

$f'(x) = \frac{d}{dx}\left\{ \operatorname{cosec}\left(\sqrt{x^5 - x^2 \cos x - 5}\right)\right\}^3$

$= 3\left\{\operatorname{cosec}\left(\sqrt{x^5 - x^2\cos x - 5}\right)\right\}^{3-1} \frac{d}{dx}\left(\operatorname{cosec}\sqrt{x^5 - x^2\cos x - 5}\right)$

$= 3\operatorname{cosec}^2\left(\sqrt{x^5 - x^2\cos x - 5}\right)\left\{-\operatorname{cosec}\sqrt{x^5 - x^2\cos x - 5}\right.$

$\left.\cot\sqrt{x^5 - x^2\cos x - 5}\right\} \frac{d}{dx}\sqrt{x^5 - x^2\cos x -}$

$= 3\operatorname{cosec}^2\left(\sqrt{x^5 - x^2\cos x - 5}\right)\left\{-\operatorname{cosec}\sqrt{x^5 - x^2\cos x - 5}\right.$

$\left.\cot\sqrt{x^5 - x^2\cos x - 5}\right\} \frac{1}{2\sqrt{x^5 - x^2\cos x - 5}}$

$\frac{d}{dx}\left(x^5 - x^2\cos x - 5\right)$

$= \text{''} \quad 5x^4 - \left(x^2 \cdot \frac{d}{dx}\cos x + \cos x \frac{d}{dx}x^2\right)$

$= \text{''} \quad 5x^4 - \left(x^2 \cdot (-\sin x) + \cos x \cdot 2x\right)$

$2 - (111, v)$, $5, 6$, ⊛, $\sqrt{2}$

$\cancel{f(x) = 2x}$

$\cancel{f(x^3) = x^3}$

12

$f'(x) = 7$

$f'(x^3) = 7$

$g(x) = (f(x))^3$

$g'(x) = 3(f(x))^2 \cdot f'(x)$

$g'(2) = 3 \cdot 1^2 \cdot 7$

$= 21$

$h(x) = f(x^3)$

$h'(x) = f'(x^3) \cdot 3x^2$

$h'(2) = 7\cancel{\cdot 3 \cdot 4 \cdot 3}\ 3(2)^2$

$= \cancel{7 \cdot 3 \cdot 4 \cdot 3} \cdot 12$

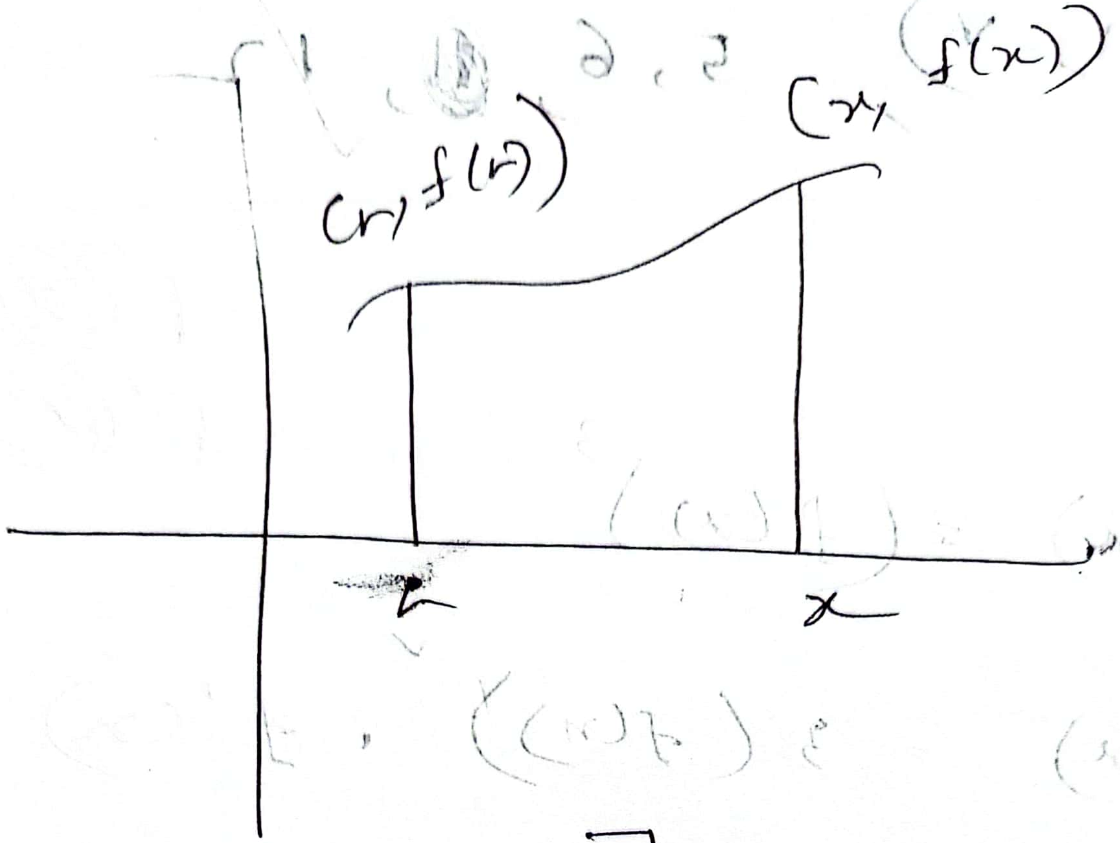
$= \cancel{4 \cdot 3 \cdot 4 \cdot 6}\ 84$

$f'(x) = 7$

$f'(x^3) = \cancel{7}^3$

$= \cancel{3 \cdot 4 \cdot 3}$

$= 7$



$$f(x) \longrightarrow [r, x]$$

$$A'(x) = f(x)$$

$$\downarrow$$

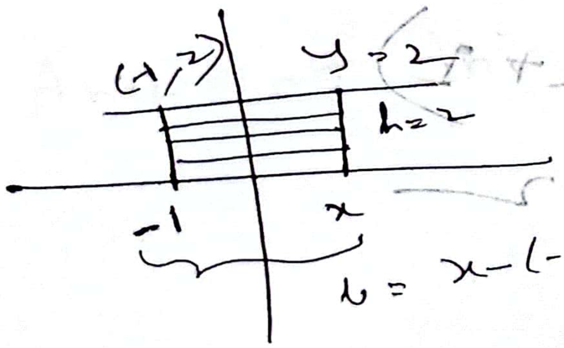
$$A(x) = \int_r^x f(x)\,dx$$

1

for each of the function f, find the area $A(x)$ between the graph of f and the interval $[a, x] = [-1, x]$ and also find the derivative $A'(x)$ of this area function.

(1) $f(x) = 2$, $\quad y = f(x) = 2$

$a = -1$, $f(a) = f(-1) = 2$



$(a, f(a)) = (-1, 2)$

$b = x - (-1) = x + 1$

$A(x) =$ area of rectangle $= b \cdot h$

$= \{x + 1\} \times 2$

$= 2x + 2$

$A'(x) = 2$

$$A(x) = \int_{-1}^{x} f(x)\, dx$$

$$= \int_{-1}^{x} 2\, dx$$

$$= 2\left[x\right]_{-1}^{x}$$

$$= 2\left[x - (-1)\right]$$

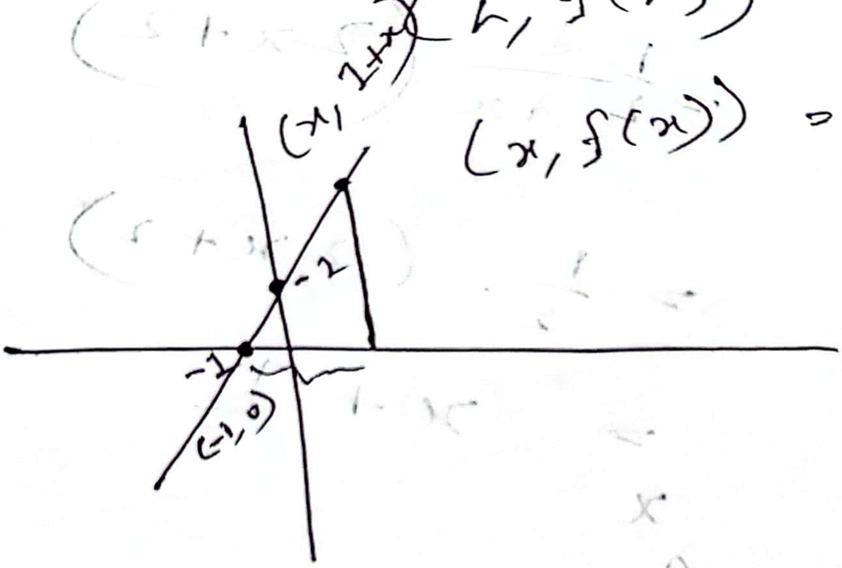
$$= 2(x+1)$$

$$= 2x + 2$$

$x+1=0$
$x=-1$

(b) $f(x)=x+1$, $f(A)=-1+1$
$=0$

$(A, f(A)) = (-1, 0)$
$(x, f(x)) = (x, x+1)$



Aner of triangle: $\frac{1}{2}\cdot b\cdot h$
$= \frac{1}{2}(x+1)(x+1)$
$= \frac{1}{2}(x+1)^2$

$A'(x) = \frac{1}{2}\cdot 2(x+1)$
$= x$

$$A''(x) = \frac{1}{2}(x+1)^2$$

$$= \frac{1}{2}(x^2+2x+1)$$

$$= \frac{1}{2}\frac{d}{dx}(2x+2)$$

$$= \frac{1}{2}\cdot(2x+2)$$

$$= x+1$$

max/mine ⇒ ঢাল 0 ↗ x অক্ষে

$$A(x) = \int^{x} f(x)$$

$$= \int_{-1}^{x} (x+1)^1 \, dx$$

$$= \int \frac{(x+1)^{1+1}}{1+1} \, dx$$

$$= \int_{-1}^{x} \frac{(x+1)^2}{2}\, dx$$

$$= \left[\frac{(x+1)^2}{2} \right]_{-1}^{x}$$

$$= \frac{(x+1)^2}{2} - \frac{(-1+1)^2}{2}$$

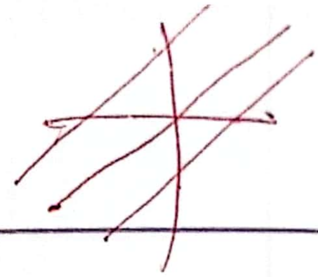
$$= \frac{(x+1)^2}{2}$$

$\int x\, dx$

$= \frac{x^{1+1}}{1+1}$

$\left[\frac{x^2}{2} + x\right]_{-1}^{x}$

$y = x+4$



$\to -2x$ $\qquad$ $x \to 0$

-1 $\qquad$ -2×0

$\to 0$

L.H.D = 0

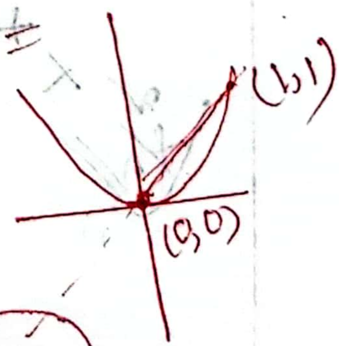
R.H.D = 1

5.1 + 5.2 + 5.5

$y = x^2$ $\qquad$ $[0, 1]$ (x_0, x_1) $\qquad$ $m_{avg} = \dfrac{f(1) - f(0)}{1-0}$

$(x_0, y_0) \to (x_1, y_1)$

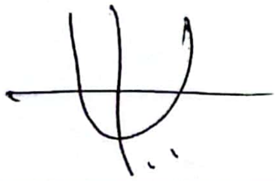
$(0,0) \to (1,1)$



Equation $\quad y - y_0 = m_{sec}(x - x_0)$

$y - 0 = \dfrac{1-0}{1-0}(x-0)$

$y = 1(x-0) = y = x$



* $f(x) = 2x + 3$, $[a, x] = [-1, x]$

$A(x) = \frac{1}{2}(a+b)\,d$ / or,

$A(x) = \int_{a}^{x} f(x)\,dx$

$= \int_{-1}^{x} f(x)\,dx$

* ~~$x=1$~~ find an equation, $\boxed{x=1}$, $y = x^2$

$x_0 = 1, y_0 = 1^2$

$\frac{dy}{dx} = 2x$

$\frac{dy}{dx}\Big|_{x=1} = 2$

$x_0 = 1, y_0 = 1$

$(x_0, y_0) = (1, 1)$

$y - y_0 = \frac{dy}{dx}\Big|_{x=1}(x - x_0)$

$y - 1 = 2(x-1)$

$y = 2x - 2 + 1 = 2x - 1$,

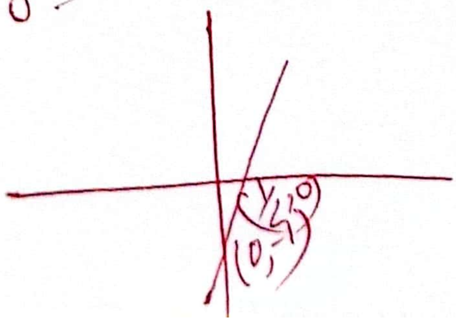
$x = 0$, $y = -1 \Rightarrow (0, -1)$

$y = 0$, $2x - 1 = 0$

$x = \frac{1}{2}$

$y = 2x$

$(x, y) = (\frac{1}{2}, 0)$



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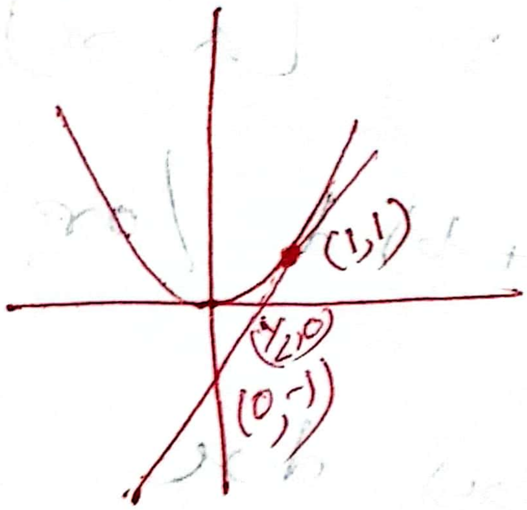


$y = -3x + 12$

$A'(x) = -3x + 12$

GW 18.12.22

use the antiderivative method to find the area function of $y = x^3$ over $[2, x]$, then find the area over $[2, 6]$.

$\Rightarrow A'(x) = f(x) = x^3$

$A(x) = \frac{x^4}{4} + C$

If, $x = 2$, $A(2) = \frac{16}{4} + C$

$= 4 + C$

$A(2) = 0$

$\rightarrow 0 = 4 + C$

$\Rightarrow C = -4$

$A(x) = \frac{x^4}{4} - 4$

P.T.O.

If, $x = 6$, than $A'(6) = \frac{6^4}{4} - 4$ $= 320$

$$\int_2^6 x^3\,dx = \left[\frac{x^4}{4}\right]_2^6$$

$$= \left(\frac{6^4}{4} - \frac{2^4}{4}\right)$$

$$= 320$$

5.2

Ex:

$$\int \frac{\cos x \, dx}{\sin^2 x}$$

let $\sin x = u$

$\Rightarrow \cos x \, dx = du$

$$= \int \frac{du}{u^2}$$

$$\int u^{-2} du$$

$$= \frac{u^{-2+1}}{-2+1} + c$$

$$= -\frac{1}{u} + c$$

$$= -\frac{1}{\sin x} + c$$

$$= -\operatorname{cosec} x + c$$

$$\int \frac{t^2 (1 - 2t^2)}{t^4} dt$$

$$= \int \frac{1 - 2t^2}{t^2} dt$$

$$= \int \left(\frac{1}{t^2} - 2 \right) dt$$

$$= \int (t^{-2} - 2) \, dt$$

$$= \frac{t^{-2+1}}{-2+1} - 2$$

$$= \frac{t^{-1}}{-1} - 2$$

$$= -\frac{1}{t} - 2 + c$$

P.T.O.

* $$\int \frac{x^2}{1+x^2}\,dx$$

$$= \int \frac{x^2+1-1}{1+x^2}\,dx$$

$$= \int \frac{x^2+1}{1+x^2}\,dx - \int \frac{1}{1+x^2}\,dx$$

$$= x - \tan^{-1}(x) + C$$

5.2

Exm : 5

$$\frac{dy}{dx} = x^2$$

$$\Rightarrow \int dy = \int x^2 \, dx$$

$$\Rightarrow y = \frac{x^3}{3} + C$$ the $[x, y] = [2, 1]$

$$\Rightarrow 1 = \frac{2^3}{3} + C$$

$$\Rightarrow 1 = \frac{8}{3} + C$$

$$\Rightarrow C = 1 - \frac{8}{3}$$

$$\Rightarrow C = \frac{-5}{3}$$

$$y = \frac{x^3}{3} - \frac{5}{3}$$

$$= \frac{1}{3}(x^3 - 5)$$

Ans

PT.

5.2 (43–46, [redacted]39,
53–55)

6.

$$\frac{dy}{dx} = \cos x$$

$$\Rightarrow \int dy = \int \cos x \, dx$$

$$\Rightarrow y = \sin x + c$$

Using the initial condition

$$y(0) = 1$$

$$\Rightarrow 1 = \sin(0) + c$$

$$= 0 + c$$

$$\Rightarrow c = 1$$

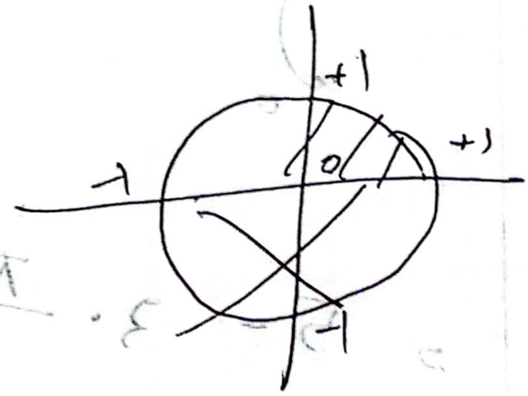
5.3 exercise [1, 6, 7, 8, 9, 10]

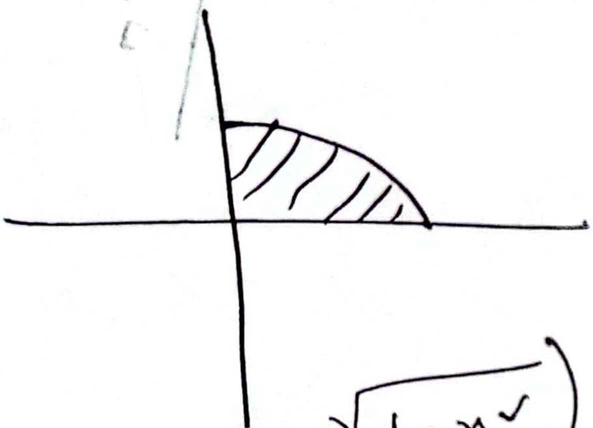
5.5

[Exm : 1 (a, b), 2]

Exm : 1

① $\int_0^1 \sqrt{1-x^2}\, dx$





$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4}\pi \cdot 1$

$= \frac{\pi}{4}$

$\sqrt{1-x^2}$

$= (1-x^2)^{\frac{1}{2}}$

$= \frac{(1-x^2)^{\frac{3}{2}}}{\frac{3}{2}} =$

Exm : 4

$\int_0^1 (5-3\sqrt{1-x^2})\, dx$

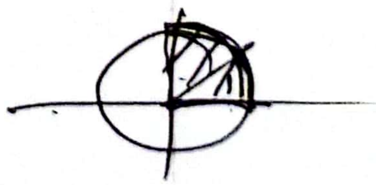
$\int_0^1 \sqrt{1-x^2}\, dx$

$\int_0^1 y\, dx$

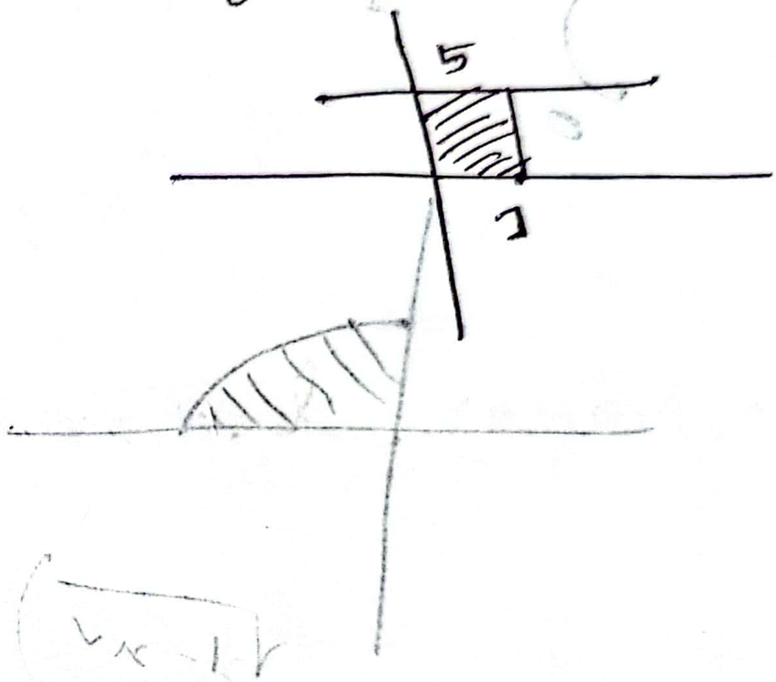
$y = \sqrt{1-x^2}$

$\Rightarrow y^2 = 1-x^2$

$\therefore x^2+y^2=1$



$$= \int_0^1 5\,dx - 3\int_0^1 \sqrt{1-x^3}\,dx$$



$$= 5 - 3\cdot\frac{\pi}{4}$$

$$= 5 - \frac{3\pi}{4}$$

15 (c)

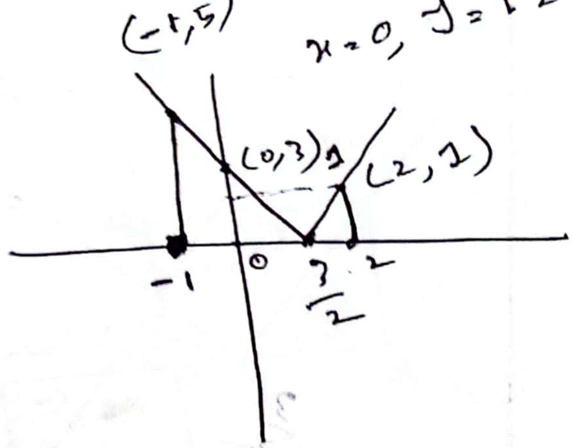
$$\int_{-1}^{2} |2x-3|\, dx$$

$3x-6=0$

$\Rightarrow 3x=6$

$\Rightarrow x=\frac{6}{3}=2$

$x=0,\ y=|2.0-3|$



$$f(x)=|2x-3|$$

$y=0,\ x=\frac{3}{2}$; $(a, f(a)) = (-1, 5)$

$$f(-1) = |2(-1)-3|$$
$$= |-5|$$
$$= 5$$

$(b, f(b)) = (2, f(2)) = (2, 1)$

$$A = \int_{-1}^{\frac{3}{2}} f(x)\,dx + \int_{\frac{3}{2}}^{2} f(x)\,dx$$

$$A = \frac{1}{2}\left(\frac{3}{2}+1\right)\cdot 5 + \frac{1}{2}\times\left(2-\frac{3}{2}\right)\cdot 1$$

$$= \frac{13}{2}$$

$$y = |2x-3| = \begin{cases} 2x-3 \ ; & 2x-3 \geq 0 \\ -(2x-3) \ ; & 2x-3 < 0 \end{cases}$$

$$= \begin{cases} x \geq \frac{3}{2} \rightarrow \left[\frac{3}{2}, 2\right] \\ x < \frac{3}{2} \rightarrow \left[-1, \frac{3}{2}\right] \end{cases}$$

$$A = \int_{-1}^{\frac{3}{2}} (3-2x)\,dx + \int_{\frac{3}{2}}^{2} (2x-3)\,dx$$

$$= \left[3x - \frac{2x^2}{2}\right]_{-1}^{\frac{3}{2}} + \left[\frac{2x^2}{2} - 3x\right]_{\frac{3}{2}}^{2}$$

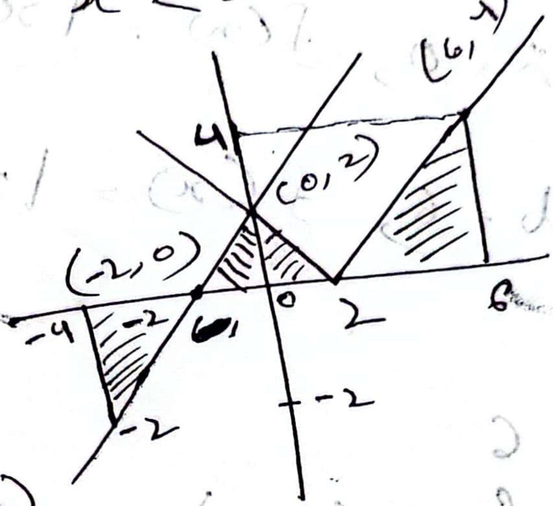
$-3x+12$

P.T.O.

Exercise → 17

$$f(x) = \begin{cases} |x-2| \ ; & x \geq 0 \\ x+2 \ ; & x < 0 \end{cases} \quad \Big| \ x = 0, \ y = 2$$

$y = x+2$



$x = 0, \ y = 2$

$y = 0, \ x = -2$

$\Rightarrow (0,2) \ \& \ (-2,0)$

(a) $\displaystyle\int_{-2}^{0} f(x) = \frac{1}{2} (0+2) \cdot 2 = 2$ Ans

(b) $\displaystyle\int_{-2}^{2} f(x)\, dx = \int_{-2}^{0} f(x)\, dx + \int_{0}^{2} (2-0) \cdot 2$

$= 2 + \frac{1}{2} \times (2-0) \cdot 2$

$= 4$

Ans

(c) $\int_0^6 f(x)\,dx$

$a = 0,\ f(x) = |x-2| = 2 \Rightarrow (0, 2)$

$b = 6,\ f(x) = |x-2| \Rightarrow f(6) = |6-2| = 4$

$\Rightarrow (6, 4)$

$$\int_0^6 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^6 f(x)\,dx$$

$$= \left\{\frac{1}{2} \times (2-0) \times 2\right\} + \left\{\frac{1}{2} \times (6-2) \times 4\right\}$$

$= 10$ Ans

(d) $\int_{-4}^{6} f(x)\,dx$

$a = -4$, $f(x) = x+2$, $f(-4) = -4+2$
$= -2$

$(a, f(a)) = (-4, -2)$

$(b, f(b)) = (6, 4)$

$= \int_{-4}^{-2} f(x)\,dx + \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{6} f(x)\,dx$

$= \left\{\frac{1}{2} \times (-2+4) \times (-2)\right\} + 2 + 10$

$= 10$

Ans

CW 24.12.22

Calculus

C.T ⇒ (2.6, 5.1) (27. 12.22)

$$\int \cos 5x \, dx$$

$\Rightarrow u = 5x$

$\Rightarrow \frac{dy}{dx} = 5$

$\Rightarrow dx = \frac{du}{5} = \frac{1}{5} \, du$

$\Rightarrow \frac{1}{5}\int \cos u \, du = \frac{1}{5} \sin u + C$

$= \frac{1}{5} \sin 5x + C$

pt.

* $$\int (x^2+1)^{50} \cdot 2x \, dx$$

let, $x^2+1=u$

$\Rightarrow 2x = \frac{du}{dx}$

$\Rightarrow 2x\,dx = du$

$$= \int u^{50} \cdot du$$

$$= \frac{u^{50+1}}{50+1} + C$$

$$= \frac{(x^2+1)^{51}}{51} + C$$

* $$\int \frac{dx}{\left(\frac{1}{3}x-5\right)^8} = \int \left(\frac{1}{3}x-5\right)^{-8} dx$$

Let, $u = \frac{1}{3}x-5$

$\Rightarrow \frac{du}{dx} = \frac{1}{3}$

$\Rightarrow dx = 3du$

$$= 3\int u^{-8} du$$

$$= 3\,\frac{u^7}{7} + C$$

$$= 3\,\frac{\left(\frac{1}{3}x-5\right)^7}{7} + C$$

I.T.

* $\int \sin^2 4x \cdot \cos 4x \, dx$

Let, $u = \sin 4x \Rightarrow \frac{dy}{dx} = \cos 4x \cdot 4$

$\Rightarrow \cos 4x \, dx = \frac{du}{4}$

$\Rightarrow \frac{1}{4} \int u^2 \, du$

$\Rightarrow \frac{1}{4} \cdot \frac{u^3}{3} + C$

$\Rightarrow \frac{1}{4} \cdot \frac{\sin^3 4x}{3} + C$

$\frac{1}{12} \sin^3 4x + C$

$\Rightarrow$

* $\int \cos^3 x \, dx$ $\quad \boxed{u = \sin x}$

$= \int \cos^2 x \cdot \cos x \, dx$

$= \int (1 - \sin^2 x) \cos x \, dx$

$= \int \cos x \, dx - \int \sin^2 x \cos x \, dx$

$= \int \cos x \, dx - \int u^2 \, du$

$= -\sin x - \frac{u^3}{3} + C$

$= -\sin x - \frac{\sin^3 x}{3} + C$

P.T.O

$$\int \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$$

Let, $-\sqrt{x} = u$

$$\Rightarrow -\frac{1}{2\sqrt{x}} = \frac{du}{dx}$$

$$\Rightarrow \frac{1}{\sqrt{x}}\,dx = -2\,du$$

$$= \int e^{u}(-2\,du)$$

$$= -2\int e^{u}\,du$$

$$= -2\,e^{-\sqrt{x}} + C$$

P.T.O

* $\int x^2 e^{-x^3}\,dx$

$\int b^x dx = \frac{b^x}{\ln b}+$

* $\int x \cdot 4^{-x^2}\,dx$

Let, $-x^2 = u$

$= \int 4^{-x^2} x\,dx$

$-2x = \frac{du}{dx}$

$\Rightarrow x\,dx = \frac{du}{-2}$

$= \int 4^u \frac{du}{-2}$

$= -\frac{1}{2}\int 4^u\,du$

$= -\frac{1}{2}\frac{4^{-x^2}}{\ln 4} + C$

* $\int x^2 \sqrt{x-1}\, dx$

Let $x-1 = u$

$\Rightarrow x = u+1$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow dx = du$

$= \int (u+1)^2 \sqrt{u}\, du$

$= \int (u^2+2u+1)\, u^{\frac{1}{2}}\, du$

$= \int \left(u^{2+\frac{1}{2}} + 2u^{1+\frac{1}{2}} + u^{\frac{1}{2}}\right) du$

PTO

$$= \int \left(\frac{u^{\frac{5}{2}+1}}{\frac{5}{2}+1} + 2 \cdot \frac{u^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right) + C$$

$$= \frac{(x-1)^{\frac{7}{2}}}{\frac{7}{2}} + \frac{2 \cdot (x-1)^{\frac{5}{2}}}{\frac{5}{2}} + \frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

PT.o

$= 0$

* $$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

if, $a = 1$

* $$\int \frac{dx}{1^2+x^2} = \frac{1}{1} \tan^{-1}\left(\frac{x}{1}\right) + C$$

$$\Rightarrow \int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$

* $$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

* $$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x) + C$$

* $$\int \frac{dx}{x\sqrt{x^2-a^2}} = \sec^{-1}\left|\frac{x}{a}\right| + C$$

P.T.O

* $$\int \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}\,dx$$

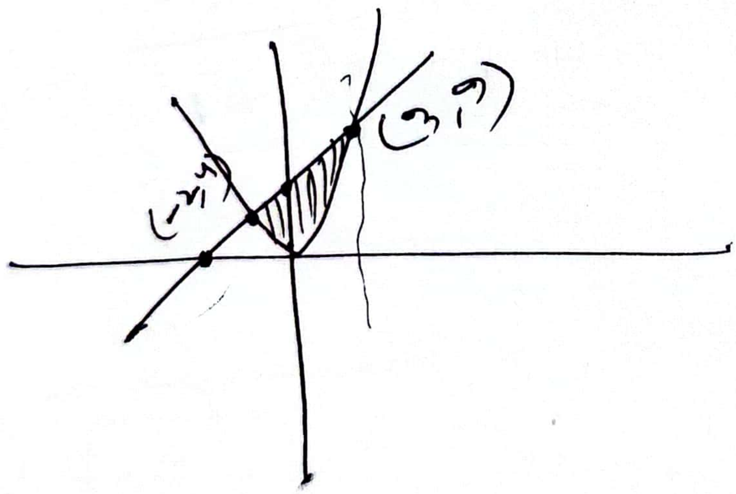
* $$\int \frac{t}{t^{4}+1}$$

* Find the area of the region that is enclosed between the curve $y = x^2$ & $y = x + 6$.

$\Rightarrow y = x + 6$

$x = 0, \; y = 6 \quad (x, y) = (0, 6)$

$y = 0, \; x = -6 \quad (x, y) = (-6, 0)$



$f(x) = x + 6$

$g(x) = x^2$

At the point,

$x^2 = x + 6$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x-3)(x+2) = 0$

$x = -2, \quad x = 3$

$y = x^2, \quad x = -2, \quad y = (-2)^2 = 4$

$x = 3, \quad y = (3)^2 = 9$

$(-2, 4)$

$(3, 9)$

$$A = \int_a^b [f(x) - g(x)] \, dx$$

$$= \int_{-2}^{3} [x + 6 - x^2] \, dx$$

$=$

P.T.O

$$= \left[\frac{x^{1+1}}{1+1} + 6x - \frac{x^{2+1}}{2+1} \right]_{-2}^{3}$$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3} \right]_{-2}^{3}$$

5.5 – (C.T)

5.3 (Class)

* Find the area of the region enclosed by $x = y^2$ & $y = x - 2$

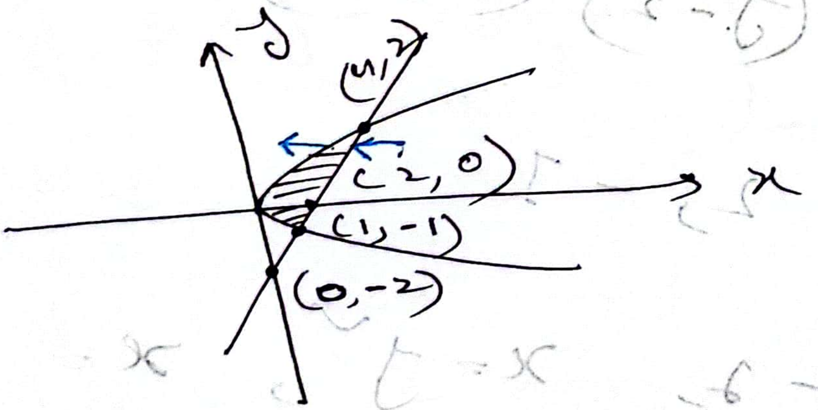
⇒ integrating with y

⇒ " with x

Ans:

$x = y^2$ $y = x - 2$

$\Rightarrow y = \pm\sqrt{x}$, $y = \sqrt{x}$, $y = -\sqrt{x}$



$y = 0, x = 2$

$x = 0, y = -2$

$(x, y) = (2, 0)$

$(x, y) = (0, -2)$

P.T.O

point of intersection:-

$x = y^2 \,,\quad x = y+2$

$y^2 = y+2$

$\Rightarrow y^2 - y - 2 = 0$

$\Rightarrow y^2 - 2y + y - 2 = 0$

$\Rightarrow (y-2)(y+1) = 0$

$y = 2, -1$

$y = -1, \quad x = y^2, \quad x = 1$

$y = 2, \quad x = y^2, \quad x = 4$

$\therefore (x, y) = (1, -1), \ (4, 2)$

[with respect to y]

$$A = \int_{-1}^{2} [w(y) - v(y)]\, dy$$

$$= \int_{-1}^{2} [y + 2 - y^2]\, dy$$

$$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2}$$

$=$

[5.1 + 5.5 + 5.6 + 6.1]

(27) $\displaystyle\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}}$

$= \left[\sec^{-1}|x|\right]_{\sqrt{2}}^{2}$

$= \sec^{-1}|(2)| - \sec^{-1}|(\sqrt{2})|$

$=$

* $\displaystyle\int uv\,dx = u\int v\,dx - \int\left[\frac{d}{dx}(u)\int v\,dx\right]dx$

① ② ③ ④ ⑤

L I A T E

E → e^x, e^{cx} [exponential]

T → $\sin x$, $\cos x$ [Trigonmetric]

A → Algebric [1, 2, ... x, y]

I → Inverse [$\sin^{-1}$, $\tan^{-1}$]

L → log ($\log x$, $\ln x$]

* $\int x \cos x \, dx$

$= x \int \cos x \, dx - \int \left[\frac{d}{dx}(x) \int \cos x \, dx \right] dx$

$= x \sin x - \int 1 \sin x$

$= x \sin x - (-\cos x) + C$

$= x \sin x + \cos x + C$

* $\int x^2 \cdot \cos x \, dx$

$= x^2 \int \cos x \, dx - \int \left[\frac{d}{dx}(x^2) \int \cos x \, dx \right] dx$

$= x^2 \sin x - 2 \int x \sin x \, dx$

$= x^2 \sin x - 2 \left[x \int \sin x \, dx - \int \left[\frac{d}{dx}(x) \int \sin x \, dx \right] dx \right.$

P.T.O.

$$= x^2 \sin x - 2x \cdot (-\cos x) - 2\int 1 \cdot (-\cos x)\, dx$$

$$= x^2 \sin x + 2x \cos x + 2 \sin x + C$$

A =

G.W 3.1.22

5.5

Cal.

Exm: 1

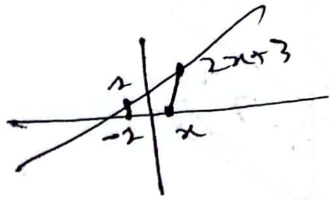
$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

$$\int \frac{dx}{x^2\sqrt{2^2-x^2}}$$

$$\int \frac{2\cos\theta\, d\theta}{2^2\sin^2\theta\sqrt{2^2-2^2\sin^2\theta}}$$

Let,

$x = a\sin\theta$



* $\int \ln x \, dx$

$2x+3$

$f(-1) = 1 \quad (-1, 1)$

$x, \; 2x+3$

$A = \frac{1}{2} \times (b + b') \cdot h$

$= \frac{1}{2} \times (2x+3+1) \cdot (x+1)$

$= \frac{1}{2} \times (2x+4)(x+1)$

$= \frac{1}{2} \times \frac{2x^2 + 2x + 4x + 4}{2}$

$= \frac{2(x^2+3x+2)}{2}$

$= x^2 + 3x + 2$

* $\int_0 \frac{x}{x^2+1} \, dx$

Let, $x^2 + 1 = u$

$\Rightarrow 2x + 0 = \frac{du}{dx}$

$\Rightarrow x \, dx = \frac{du}{2}$

$\int \frac{dx}{x} = \ln|x| + c$

P.T.O

$$\frac{1}{2}\int \frac{du}{u} = \ln|u| + C$$

$$= \ln(1+x^2) + C$$

$$\int_0^1 \frac{x}{1+x^2}\,dx$$

$$= \frac{1}{2}\int \frac{du}{u}$$

$$= \frac{1}{2}\left[\ln|u|\right]_1^2$$

$$= \frac{1}{2}\left(\ln|2| - \ln|1|\right)$$

$$= \frac{1}{2}\ln|2| - 0$$

$$= \frac{1}{2}\ln|2|$$

$$= \frac{1}{2}\ln|2|$$

A

$*\int (x+1) \cot (x^2+2x) \, dx$

$= \int \cot (x^2+2x) (x+1) \, dx$

let,

$u = x^2 + 2x$

$\frac{du}{dx} = 2x+2$

$\Rightarrow (2x+2) dx = du$

$\Rightarrow 2(x+1) dx = du$

$\Rightarrow (x+1) dx = \frac{du}{2}$

$\int \cot x \, dx$

$= \int \frac{\cos x}{\sin x} dx$

let,

$\sin x = u$

$\cos x \, dx = du$

$\int \frac{du}{u}$

$= \ln|u| + c$

$= \ln|\sin x| + c$

(5.3) $\{7.1\ (1-30)\}$

$$= \frac{1}{2}\int \cot(u)\,du$$

$$= \frac{1}{2}\ln|\sin u| + c$$

$$= \frac{1}{2}\ln|\sin(x^2+2x)| + c$$

$u = \ln x$

$v = 1$

$$* \int \ln x\,dx$$

$$= \int \ln x \cdot 1\,dx$$

$$= \ln x\int 1\,dx - \int\left[\frac{d}{dx}(\ln x)\int 1\cdot dx\right]dx$$

$$= \ln x - \int \frac{1}{x}\cdot x\,dx$$

$$= x\ln x - \int 1\cdot dx$$

$$= x\ln x - x + c$$

Ans

* $\int x^2 \ln x \, dx$

$= \ln x \int x^2 dx - \int [\frac{d}{dx}(\ln x) \int x^2 dx] dx$

$= \ln x \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$

$= \frac{x^3}{3} \ln x - \int \frac{x^2}{3} dx$

$= \frac{x^3}{3} \ln x - \frac{1}{3} \cdot \frac{x^3}{3} + C$

P.T.O

Final : 2.1, 2.2, 2.6
5.1, 5.2, 5.3 + 7.1
5.5, 5.6

* $\int \tan^{-1} x \, dx$ 6.1, 7.2

$= \int \tan^{-1} x \cdot 1 \, dx$

$= \tan^{-1} x \int 1 \, dx - \int \left[\frac{d}{dx} (\tan^{-1} x) \int 1 \, dx \right] dx$

$= \tan^{-1}(x) \cdot x - \int \frac{1}{1+x^2} x \, dx$

$= x \tan^{-1}(x) - \frac{1}{2} \ln (1+x^2) + c$ Ans

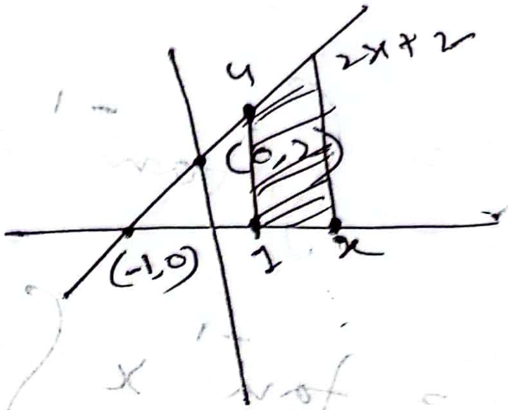
Calculus   5.1   Assy 4

17    $[a, x] = [1, x]$

$f(x) = 2x + 2$

$x = 0, \; y = 2$

$y = 0, \; 2x + 2 = 0$
$\Rightarrow x = -1$



$(x, y) = (0, 2), (-1, 0)$

$r = 1, \; f(r) = 2 \cdot 1 + 2 = 4, \; (r, f(r)) = (1, 4)$

$(x, f(x)) = (x, 2x + 2)$

$A(x) = \frac{1}{2} \times (2x + 2 + 4) \times (x - 1)$

$= \frac{1}{2} \times (2x + 6)(x - 1)$

$= \dfrac{2x^2 - 2x + 6x - 6}{2}$

$= \dfrac{2x^2 + 4x - 6}{2}$

$= x^2 + 2x - 3$

Ans.

18.

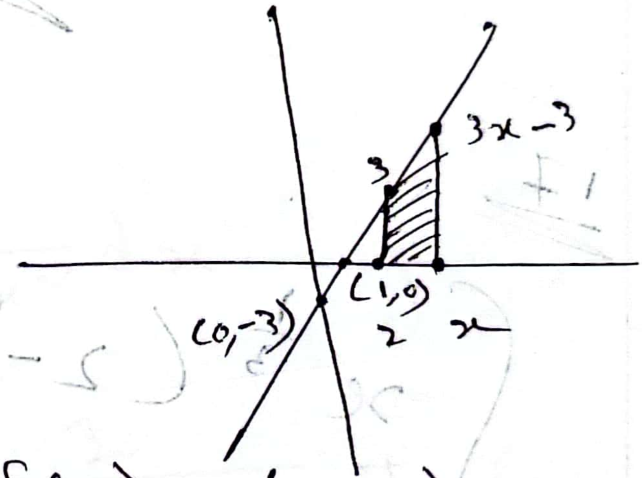
$f(x) = 3x - 3$; $[a, x] = [2, x]$

$x = 0$, $y = -3$, $(x, y) = (0, -3)$

$y = 0$, $3x - 3 = 0$, $(x, y) = (1, 0)$

$\Rightarrow 3x = 3$

$\Rightarrow x = 1$



$a = 2$, $f(a) = 3$, $(a, f(a)) = (2, 3)$

$(x, 3x - 3)$

$$A(x) = \frac{1}{2}(3x - 3 + 3) \cdot (x - 2)$$

$$= \frac{1}{2} 3x \cdot (x - 2)$$

$$= \frac{1}{2} 3x^2 - 6x$$

$$= \frac{3x^2}{2} - 3x$$

5.2

7

$$\frac{d}{dx}\left[\sin(2\sqrt{x})\right]$$

$$= \frac{\cos(2\sqrt{x})}{\sqrt{x}}$$

$$\frac{d}{dx}\, 2\sqrt{x}$$

$$= \cancel{2}\cdot\frac{1}{\cancel{2}\sqrt{x}}$$

$$= \frac{1}{\sqrt{x}}$$

A

17

$$\int x^{\frac{1}{3}}(2-x)^2\, dx$$

$$= \int (x-2)^2\, \sqrt[3]{x}\, dx$$

$$= \int \left(x^{\frac{7}{3}} - 4x^{\frac{4}{3}} + 4\sqrt[3]{x}\right) dx$$

$$= \frac{3x^{\frac{10}{3}}}{10} - \frac{12x^{\frac{7}{3}}}{7} + 3x^{\frac{4}{3}} + c$$

19

$$\int \frac{x^5+2x^2-1}{x^4}\,dx$$

$$= \int \left(x + \frac{2}{x^2} - \frac{1}{x^4}\right) dx$$

$$= \int x\,dx + 2\int \frac{1}{x^2}\,dx - \int \frac{1}{x^4}\,dx$$

$$= \frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3} + C$$

Ans

26

$$\int \operatorname{cosec} x\,(\sin x + \cot x)\,dx$$

$$= \int (\operatorname{cosec} x \cdot \sin x + \operatorname{cosec} x \cdot \cot x)\,dx$$

$$= \int \operatorname{cosec} x \cdot \sin x\,dx + \int \operatorname{cosec} x \cdot \cot x\,dx$$

$$= \int 1\,dx - \operatorname{cosec} x$$

$$= x - \operatorname{cosec} x + C$$ Ans

30

$$\int \left[\phi + \frac{2}{\sin^2 \phi} \right] d\phi$$

$$= \int \phi \, d\phi + \int \frac{2}{\sin^2 \phi} \, d\phi$$

$$= \frac{\phi^2}{2} + 2 \cdot \int \operatorname{cosec}^2 \phi \, d\phi$$

$$= \frac{\phi^2}{2} + 2(-\cot \phi)$$

$$= \frac{\phi^2}{2} - 2\cot \phi + C$$

Ans

31

$$\int (1 + \sin^2 \theta \operatorname{cosec} \theta) \, d\theta$$

$$= \theta + \int \sin(\theta) \, d\theta$$

$$= \theta - \cos \theta + C$$

Ans

$\sin^2\theta \cdot \frac{1}{\sin}$

$= \sin\theta$

5.3

23

$$\int \frac{dx}{\sqrt{1-4x^2}}$$

$u = 2x$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2} du$

$$= \frac{1}{2} \int \frac{du}{\sqrt{1-u^2}}$$

$$= \frac{\sin^{-1} u}{2}$$

$$= \frac{\sin^{-1}(2x)}{2} + C$$

Ans

26

$$\int \frac{x}{\sqrt{4-5x^2}} dx$$

$u = 4-5x^2$

$\frac{du}{dx} = -10x$

$dx = -\frac{1}{10x} du$

$$= -\frac{1}{10} \int \frac{1}{\sqrt{u}} du$$

$$= -\frac{\sqrt{u}}{5} = \frac{-\sqrt{4-5x^2}}{5} + C$$

Ans

28.

$$\int \frac{x^2+1}{\sqrt{x^3+3x}}\,dx$$

$u = x^3+3x$

$\frac{du}{dx} = 3x^2+3$

$dx = \frac{1}{3x^2+3}\,du$

$= \frac{1}{3(x^2+1)}\,du$

$$= \frac{1}{3}\int \frac{1}{\sqrt{u}}\,du$$

$$= \frac{2\sqrt{u}}{3}$$

$$= \frac{2\sqrt{x^3+3x}}{3} + C$$

A

35.

$$\int \frac{e^x}{1+e^{2x}}\,dx$$

$u = e^x$

$\frac{du}{dx} = e^x$

$dx = e^{-x}\,du$

$$= \int \frac{1}{u^2+1}\,du$$

$$= \tan^{-1}(u)$$

$$= \tan^{-1}(e^x) + C$$

A

37

$$\int \frac{\sin\left(\frac{5}{x}\right)}{x^2}\, dx$$

$u = \frac{5}{x}$

$\frac{du}{dx} = -\frac{5}{x^2}$

$dx = -\frac{x^2}{5}\, du$

$= -\frac{1}{5}\int \sin(u)\, du$

$= -\frac{1}{5}\cdot\left(-\cos\left(\frac{5}{x}\right)\right)$

$= \frac{\cos\left(\frac{5}{x}\right)}{5} + C$ Ans

40

$$\int \cos 2t \sin^5 2t \, dt$$

$u = \sin 2t$

$\frac{du}{dt} = 2\cos(2t)$

$dt = \frac{1}{2\cos(2t)}\, du$

$= \frac{1}{2}\int u^5\, du$

$= \frac{1}{2}\cdot\frac{u^6}{6}$

$\frac{u^6}{12} + C = \frac{\sin^6(2t)}{12} + C$ Ans

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45

$$\int \frac{\sec^2 x \, dx}{\sqrt{1-\tan^2 x}}$$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{1}{\sec^2 x} du$

$$= \int \frac{1}{\sqrt{1-u^2}} du$$

$$= \sin^{-1}(u)$$

$$= \sin^{-1}(\tan(x)) + C$$ A

47

$$\int \sec^3 2x \tan 2x \, dx$$

$\int$

$u = \sec^3(2x)$

$\frac{du}{dx} = 6\sec^3(2x)\tan(2x)$

$dx = \frac{1}{6\sec^3(2x)\tan(2x)} du$

$$= \frac{1}{6}\int 1 \, du$$

$$= \frac{u}{6} = \frac{\sec^3(2x)}{6} + C$$ A

52

$$\int \frac{e^{\sqrt{2y+1}}}{\sqrt{2y+1}}\, dy$$

$u = \sqrt{2y+1}$

$\frac{du}{dy} = \frac{1}{\sqrt{2y+1}}$

$dy = \sqrt{2y+1}\, du$

$$= \int e^{u}\, du$$

$$= e^{u}$$

$$= e^{\sqrt{2y+1}} + C$$

53

$$\int \frac{y}{\sqrt{2y+1}}\, dy$$

$u = 2y+1$

$\frac{du}{dy} = 2$

$dy = \frac{1}{2}\, du$

$$= \frac{1}{4} \int \frac{u-1}{\sqrt{u}}\, du$$

$$= \frac{1}{4}\int \sqrt{u}\,du - \int \frac{1}{\sqrt{u}}\,du$$

$$= \frac{1}{4}\left(\frac{2u^{\frac{3}{2}}}{3} - 2\sqrt{u}\right)$$

$$= \frac{u^{\frac{3}{2}}}{6} - \frac{\sqrt{u}}{2}$$

$$= \frac{(2x+1)^{\frac{3}{2}}}{6} - \frac{\sqrt{2x+1}}{2} + C$$

A

56.

$$\int \sec^4 3\theta \, d\theta \qquad u = 3\theta$$

$$= \frac{1}{3}\int \sec^4(u)\,du \qquad \frac{du}{d\theta} = 3$$

$$d\theta = \frac{1}{3}\,du$$

$$= \frac{1}{3}\int \sec^2(u)\left(\tan^2(u)+1\right) du$$

P.T.O

$$= \frac{1}{3}\int \sec^4(u)\,du$$

$$= \frac{\tan^3(3x)}{9} + \frac{\tan(3x)}{3} + C$$

Ans

$v = \tan(u)$

$$\frac{dv}{du} = \sec^2 u$$

$$du = \frac{1}{\sec^2} dv$$

$$\int (v^2+1)\,dv$$

$$= \int v^2 dv + \int 1\,dv$$

$$= \frac{v^3}{3} + v$$

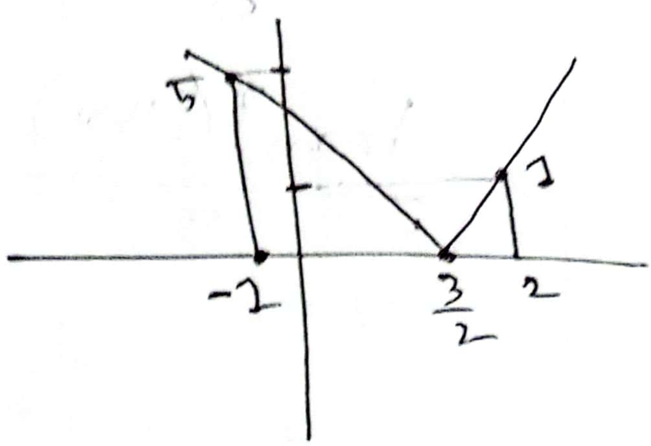
$$= \frac{\tan^3(u)}{3} + \tan(u)$$

P.T.O

5.5

15 (c)

$$\int_{-1}^{2} |2x-3| \cdot dx$$



$x = 2,\ f(x) = 1$

$x = -1,\ f(x) = 5$

$$A = \int_{-1}^{\frac{3}{2}} f(x) + \int_{\frac{3}{2}}^{2} f(x)$$

$$= \left\{\frac{1}{2} \times \left(\frac{3}{2}+1\right) \cdot 5\right\} + \left\{\frac{1}{2} \times \left(2-\frac{3}{2}\right) \cdot 1\right\}$$

$$= \frac{13}{2}$$

A

16

(b) $$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin x \, dx$$

$$= \left[-\cos x\right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

$$= \left(-\cos\frac{\pi}{3}\right) - \left(-\cos\left(-\frac{\pi}{3}\right)\right)$$

$$= 0$$

Ans

(d) $\int_0^2 \sqrt{4-x^2}\,dx$

$x = 2\sin(u)$

$u = \sin^{-1}\left(\frac{x}{2}\right)$

$dx = 2\cos(u)\,du$

$\sin\left(\sin^{-1}\left(\frac{x}{2}\right)\right) = \frac{x}{2}$

$\cos\left(\sin^{-1}\frac{x}{2}\right)$

$= \sqrt{1-\frac{x^2}{4}}$

$= \int_0^2 2\cos(u)\sqrt{4-4\sin^2(u)}\,du$

$= 4.\int_0^2 \cos^2(u)\,du$

$= 4.\ \frac{\cos(u)\ \sin(u)}{2} + \frac{1}{2}\int 1\,du$

$= 4.\ \frac{\cos(u)\ \sin(u)}{2} + \frac{u}{2}$

$= 2\cos(u)\ \sin(u) + 2u$

$= \left[\frac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$= 180 = \pi$

Ans

5.6

18

$$\int_1^4 \frac{1}{x\sqrt{x}}\,dx$$

$$= \left[-\frac{2}{\sqrt{x}}\right]_1^4$$

$$= \left(-\frac{2}{\sqrt{4}}\right) - \left(-\frac{2}{\sqrt{1}}\right)$$

$$= 1$$

Ans

$$\int \frac{1}{x\sqrt{x}}\,dx$$

$$= \int \frac{1}{x^{\frac{3}{2}}}\,dx$$

$$= -\frac{2}{\sqrt{x}} + C$$

28

$$\int_{-\sqrt{2}}^{-\frac{2}{\sqrt{3}}} \frac{dx}{x\sqrt{x^2-1}}$$

$$= \left[\tan^{-1}(\sqrt{x-1})\right]_{-\sqrt{2}}^{-\frac{2}{\sqrt{3}}}$$

$$= -30 = -\frac{\pi}{6}$$

Ans

$$u = \sqrt{x^2-1}$$

$$\frac{du}{dx} = \frac{x}{\sqrt{x^2-1}}$$

$$dx = \frac{\sqrt{x^2-1}}{x}\,du$$

$$\Rightarrow \int \frac{1}{u^2+1}\,du$$

$$= \tan^{-1}(\sqrt{x^2-1}) + C$$

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30

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(x + \frac{2}{\sin^2 x}\right) dx$$

$$= \left[\frac{x^2}{2} - 2\cot(x)\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$\int \frac{1}{\sin^2(x)} dx$$

$$= \int \operatorname{cosec}^2(x)\, dx$$

$$= -\cot(x)$$

A

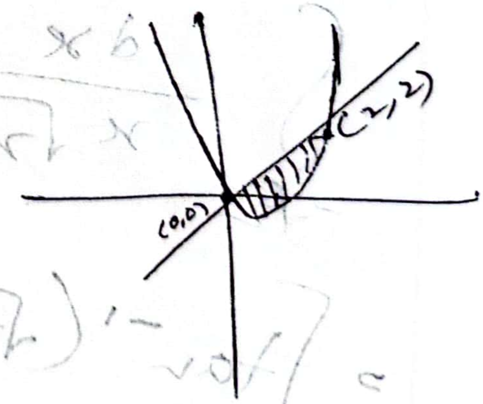
49

$y = x^2 - x \; ; \; [0, 2]$

$x = 0, \quad y = 0, \; (0, 0)$

$x = 2, \quad y = 2, \; (2, 2)$

$y = x$



$\cancel{x^2 - x = x}$

$\Rightarrow \cancel{x^2 - x - x = 0}$

$\Rightarrow \cancel{x^2 - 2x = 0}$

$$= \int_0^2 \left[x - (x^2 - x)\right]$$

$$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$= \frac{10}{3}$$

$$\int x - (x^2 - x)$$

$$= \int (3x - x^2)\,dx$$

$$= \frac{3x^2}{2} - \frac{x^3}{3} + C$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$$

$$x e^{-2x} dx$$

$u = x$

$v = e^{-2x}$

$$= u \int v\, dx - \int \left[\frac{d}{dx}(u) \int v \frac{d}{dx} \right] dx$$

$$= x \int e^{-2x} dx - \int \left[\frac{d}{dx}(x) \int e^{-2x} \frac{d}{dx} \right] dx$$

$$= x \cdot \left[-\frac{1}{2} e^{-2x} \right] - \int 1 \cdot \left(-2x \cdot \frac{1}{2} e^{-2x} \right) du$$

$$= x \cdot \left[-\frac{1}{2} e^{-2x} \right) - \int \left(-2x \cdot \frac{1}{2} e^{-2x} \right) du$$

$$= x \cdot \left[-\frac{1}{2} e^{-2x} \right) - x \cdot e^{-2x}$$

$$= \cancel{x \cdot e^{-2x} \left(\frac{1}{2} - 1 \right)}$$

$\int xe^{-2x}$

$$\int f g' = fg - \int f' g$$

$f = x \qquad g' = e^{-2x}$

$f' = 1 \qquad g = -\frac{1}{2} e^{-2x}$

$$x\left[-\frac{1}{2}e^{-2x}\right] - \int 1 \cdot \left(-\frac{1}{2}e^{-2x}\right)$$

$$= x\left[-\frac{1}{2}e^{-2x}\right] - \left(-\frac{1}{2}x\right)\left(-\frac{1}{2}e^{-2x}\right)$$

$$= x\left[-\frac{1}{2}e^{-2x}\right] - \frac{1}{4}xe^{-2x}$$

$$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}xe^{-2x}$$

$$\frac{1}{4}e^{-2x}(x+1)$$

$$\int xe^{-2x}\,dx$$

$$= x\int e^{-2x}\,dx - \int\left\{\frac{d}{dx}(x)\int e^{-2x}\,dx\right\}dx$$

$$= \frac{xe^{-2x}}{-2} - \int \frac{e^{-2x}}{-2}\,dx$$

$$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$$

$$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\cdot\frac{1}{-2}\cdot e^{-2x} + c$$

$$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + c$$

$$= -\frac{1}{2}e^{-2x}\left(x + \frac{1}{2}\right) + c$$

$$\boxed{x^2+y^2=9}$$

$$y^2=3^2-x^2$$

$$\sqrt{3^2-x^2}$$

$$\int_0^3 \left(2x+\sqrt{9-x^2}\right)dx$$

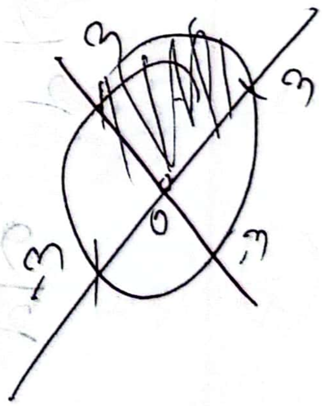
$$\frac{1}{4}\cdot\pi\cdot 9$$

$$\int_0^3 2x\,dx+\int_0^3 \sqrt{9-x^2}\,dx$$

$$=2\left[\frac{x^2}{2}\right]_0^3+\left[\frac{(9-x^2)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\cdot\left(\frac{1}{-2x}\right)\right]_0^3+c$$

$\cancel{x=3\sin\theta}$

$$=\int_0^3\left(2x+\sqrt{9-x^2}\right)dx$$



$$=\int_0^3 2x\,dx+\int_0^3\frac{1}{4}\pi\cdot 9$$

$$\int_0^3 \sqrt{9-x^2}\, dx$$

$x = 3\sin\theta$
$dx = 3\cos\theta\, d\theta$

$\sin^{-1} x/3$

$$\int_0^3 3\cos\theta \cdot 3\cos\theta\, d\theta$$

$$\frac{9}{2}\int_0^3 2\cos^2\theta\, d\theta$$

$$\frac{9}{2}\int_0^3 (\cos 2\theta + 1)\, d\theta$$

$$\frac{9}{2} \cancel{\int_0^3} \left[\frac{\sin 2\theta}{2} + \theta\right]_0^3$$

$$\frac{9}{2}\left[\frac{\sin 2\cdot(\sin^{-1} x/3)}{2} + \sin^{-1} x/3\right]_0^3$$

$$\frac{9}{2}\left(0 + \frac{\pi}{2} - 0 - 0\right)$$

$$\frac{\pi}{4} \cdot 9 \qquad \frac{1}{4} \cdot \pi \cdot 9$$

$$\cos\frac{5}{x} = u$$

$$\sin\frac{5}{x}\cdot\frac{5}{x} = \frac{du}{dx}$$

$$\int \frac{\cos\left(\frac{5}{x}\right)}{3x^2}\,dx$$

$$\cos\left(\frac{5}{x}\right) = u$$

$$-\sin\frac{5}{x} = \frac{du}{dx}$$

$$dx\left(-\sin\frac{5}{x}\right) = du$$

$$\frac{1}{3}\int \frac{-dP}{}$$

$$\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$\sin\left(\frac{5}{x}\right) = p$$

$$\frac{\cos\left(\frac{5}{x}\right)}{-x^2}\,dx = dp$$

$$-\frac{1}{3}p + C$$

$$\frac{\cos\frac{5}{x}}{x^2}\,dx = -dp$$

$$-\frac{1}{3}\sin\frac{5}{x} + C$$

$$\frac{5}{x} = u \qquad -\sin\left(\frac{5}{x}\right) = u$$

$$\frac{1}{9}\int \frac{\cos\left(\frac{5}{x}\right)}{x^2}\,dx \quad \cos(u)\,du$$

$$5\cdot x^{-1} = u \qquad \frac{5}{x} = u$$

$$5\cdot x^{-2} = u \qquad 5\cdot x^{-2} = \frac{du}{dx}$$

$$-\frac{5}{x^2} = \frac{du}{dx} \qquad -\frac{5}{x^2} = \frac{du}{dx}$$

$$dx\cdot 5 = du$$

$$dx\,\frac{5}{x^2} = du$$

$$dx = -\frac{x^2}{5}du$$

$$\int \frac{\cos\left(\frac{5}{x}\right)}{3x^2}\,dx$$

$$\int x^3 \sqrt{2+x^4}\,dx$$

$u = \cancel{\sqrt{}}\,2+x^4$

$\frac{du}{dx} = 4x^3$

$du = 4x^3\,dx$

$dx = \frac{1}{4x^3}\,du$

$$= \frac{1}{4}\int \cancel{\sqrt{2+x^4}}\ \sqrt{u}\,du$$

$$= \frac{1}{4}\left[\frac{u^{\frac{1}{2}+1}}{\frac{3}{2}}\right] + C$$

$$= 4 \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}}$$

$$= 4 \cdot u^{\frac{3}{2}} \times \frac{2}{3}$$

$$= \frac{8u^{\frac{3}{2}}}{3} = \frac{8(2+x^4)^{3/2}}{3}$$

Ans