UNIVERSITY

Exercise Book

Write Your Mission

DM

CSE Math

Book

① 7th edition

The foundation

1.1 Propositional logic

* discrete mathematics

1 2 3 4 . . . 5125

* Logic

* Proposition (T/F) - (1/0)

*

C.W
5.3.22

Discrete Math

Propositional Equivalence

De Morgan's laws

$\neg (p \wedge q) \equiv \neg p \vee \neg q$

$\neg (p \vee q) \equiv \neg p \wedge \neg q$

* Equivalence Laws

* Identity : $p \wedge T \equiv p$ $p \vee F \equiv p$
* Domination : $p \vee T \equiv T$ $p \wedge F \equiv F$
* Idempotent : $p \vee p \equiv p$ $p \wedge p \equiv p$
* Double negation : $\neg\neg p \equiv p$
* Commutative :- $p \vee q \equiv q \vee p$, $p \wedge q \equiv q \wedge p$
* Associative : $(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

* Distributive :- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

* De Morgan's :-

$$\neg (p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg (p \vee q) \equiv \neg p \wedge \neg q$$

* Absorption :-

$$p \vee (p \wedge q) \equiv p$$

$$p \wedge (p \vee q) \equiv p$$

* Trivial tautology / contradiction :-

$p \vee \neg p \equiv T \qquad p \wedge \neg p \equiv F$

P.T.O

* Exclusive - Or :-

$p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$

$p \oplus q \equiv (p \vee q) \wedge \neg (p \wedge q)$

* Implies : $p \to q \equiv \neg p \vee q$

* Biconditional :

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$p \leftrightarrow q \equiv \neg (p \oplus q)$

* Show that $\neg (p \to q)$ and $p \wedge \neg q$ are logically equivalant :-

$\neg (p \to q)$ [Expand definition of $\to$]

$= \neg(\neg p \vee q)$ [De Morgan's Law]

$= \neg (\neg p) \wedge \neg q$ [Double Negation]

$= p \wedge \neg q$

P.T.O

* $(p \land \neg q) \to (p \oplus r) \equiv \neg p \lor q \lor \neg r$

$(p \land \neg q) \to (p \oplus r)$ [$\longrightarrow$]

$\equiv \neg(p \land \neg q) \lor (p \oplus r)$ [$\oplus$]

$\equiv \neg(p \land \neg q) \lor ((p \lor r) \land \neg(p \land r))$ [De Morgan]

$\equiv (\neg p \lor q) \lor ((p \lor r) \land \neg(p \land r))$

* Predicate

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DM-

C-1

Universe of Discourse (U.D.)

$P(x) = x+1 > x$

$(0+1>0) \wedge (1+1>1) \wedge (2+1>2) \wedge \ldots$

Quantifier Expressions

* "$\forall$" is the ForAll or universal quantifier

$\forall x\, P(x)$ means <u>for all</u> x in the domain $P(x)$.

* "$\exists$" is the EXISTS or existential quantifier

$\exists x\, P(x)$ means <u>there exists</u> an x in the domain (that is, 1 or more) such that $P(x)$.

$\neg$ (Not)

$\wedge$ (And)

$\vee$ (OR)

$\oplus$ (XOR

$\rightarrow$ Implies

$\leftrightarrow$ IFF

~~1×0 = 0~~

0×1 = 0

0×0 = 0

NOT

p	¬p
T	F
F	T

AND

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

OR

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Pr.o

~~x-or~~

p	q	$p \oplus q$	p "or" q	$p \to q$
T	T	F	?	T
T	F	T	T	F
F	T	T	T	T
F	F	F	F	T

De Morgan law

p	q	$p \land q$	$\neg p$	$\neg q$	$\neg p \lor \neg q$	$\neg(p \land q)$
T	T	T	F	F	F	F
T	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T

$\cancel{\neg(p \to q)}$

$\cancel{\equiv \neg(}$

1	0	$\Rightarrow$	1
0	1	$\Rightarrow$	1
0	0	$\Rightarrow$	0
1	1	=	~~1~~ 0

$\neg(p \to q)$ $\boxed{p \wedge \neg q}$

$\equiv \neg(\neg p \vee q)$ [$\to$]

$\equiv \neg(\neg p) \wedge (\neg q)$ [De Morgans law]

$\equiv p \wedge \neg q$ [Double negation]

$\cancel{\neg(q \to \neg p)}$

$\cancel{\equiv \neg(\neg q) \vee \neg p}$

$\cancel{\equiv q \vee \neg p}$

$\equiv$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$(q \to \neg p)$	$\neg(q \to \neg p)$
T	T	F	F	T	F	T
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

P-2

$(a \leftrightarrow b) \rightarrow (a \wedge b) = f$

a	b	$a \leftrightarrow b$	$a \wedge b$	f
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	T	F	F

This truth table contains an f in the last row. Thus, f is not a tautology.

P.T.O

P-2

$a \lor b \rightarrow c = p$ | $r = p \rightarrow q$

$a \land b \rightarrow c = q$ | $s = q \rightarrow p$

a	b	c	$a \lor b$	$a \land b$	p	q	r	s
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	F	T
T	F	T	T	F	T	T	T	T
T	F	F	T	F	F	T	T	F
F	T	T	T	F	T	T	T	T
F	T	F	T	F	F	T	T	F
F	F	T	F	F	T	T	T	T
F	F	F	F	F	T	T	T	T

Here r is a tautology and s is not. Therefore $(a \lor b \rightarrow c) \Rightarrow (a \land b \rightarrow c)$ but its converse is not true.

~~P 3~~

* $p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$

$s = p \to (q \wedge r)$ | $u = s \leftrightarrow t$

$t = (p \to q) \wedge (p \to r)$

p	q	r	$p \to q$	$p \to r$	$q \wedge r$	s	t	u
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	F	F	F	T
T	F	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T	T
F	F	T	T	T	F	T	T	T
F	F	F	T	T	F	T	T	T

$\longleftrightarrow$ ⇒ *দুইটা F থাকলে T হবে।
*একটা F " F "।

P-4

$\longrightarrow$ ⇒ *Land এর একটা main matter
*দুইটা F থাকলে T হবে।

$p \rightarrow (q \vee r)$

$\Leftrightarrow \neg p \vee (q \vee r)$ [Defination →]

$\Leftrightarrow (\neg p \vee q) \vee r$ [Assosiative]

$\Leftrightarrow \neg\neg(\neg p \vee q) \vee r$ [double Negation]

$\Leftrightarrow \neg(\neg\neg p \wedge \neg q) \vee r$ [De morgan]

$\Rightarrow \neg(p \wedge \neg q) \vee r$ [Double Negation]

$\Rightarrow (p \wedge \neg q) \rightarrow r$ [Logical Equivalance]

P.T.O

*

$$(p \land (\lnot r \lor q \lor \lnot q)) \lor ((r \lor t \lor \lnot r) \land \lnot q)$$

$$\Rightarrow (p \land (\lnot r \lor T)) \lor ((t \lor T) \land \lnot q)$$

$$\Rightarrow (p \land T) \lor (T \land \lnot q)$$

$$\Rightarrow p \lor \lnot q$$

C.W
8.3.22

D.M

* Quantifiers with Restricted Domains.

* $\forall x\,(x>0 \rightarrow P(x))$

* $\exists x\,(x>0 \land P(x))$

C.W 12-3-22

L-5

DM

* Nesting of Quantifiers

* $\exists y L(x,y) =$
"There is someone whom x likes."

(A statement with 1 free variable x – not a proposition)

* Then, $\forall x (\exists y L(x,y)) =$

"Everyone has someone whom they like."

(A proposition with 0 free variable)

P.T.O

$\forall x \, \exists y \, P(x, y)$

$\equiv \exists y \, P(1, y) \wedge \exists y \, P(2, y) \wedge \exists y \, P(3, y)$

$\equiv \left[P(1,1) \vee P(1,2), \; P(1,3)\right] \wedge$

$\left[P(2,1), \vee P(2,2), \; P(2,3)\right] \wedge$

$\left[P(3,1), \vee P(3,2), \vee P(3,3)\right]$

Md. Sadman Haque

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\# $(p \land \neg q) \rightarrow (p \oplus r)$

$\equiv \neg(p \land \neg q) \lor ((p \land \neg r) \lor (\neg p \land r))$

[Def of $\rightarrow$ and $\oplus$]

$\equiv (\neg p \lor q) \lor ((p \land \neg r) \lor (\neg p \land r))$

[De Morgan]

$\equiv (q \lor \neg p) \lor ((p \land \neg r) \lor (\neg p \land r))$

[Commutative]

$\equiv q \lor (\neg p \lor ((p \land \neg r) \lor (\neg p \land r)))$

[Associative]

$\equiv q \lor ((\neg p \lor (p \land \neg r)) \lor (\neg p \lor (\neg p \land r)))$

[Distributive]

$\equiv q \lor ((\neg p \lor p) \land (\neg p \lor \neg r)) \lor \neg p$

[Absorption]

P.T.O.

Sadman

$\equiv q \vee (T \wedge (\neg p \vee \neg r)) \vee \neg p$

[Negation]

$\equiv q \vee (\neg p \vee \neg r) \vee \neg p$

[Identity]

$\equiv q \vee (\neg r \vee \neg p) \vee \neg p$ [~~Associative~~ Commutative]

$\equiv q \vee \neg r \vee (\neg p \vee \neg p)$ [Associative]

$\equiv q \vee \neg r \vee \neg p$ [Idempotent]

$\equiv \neg p \vee q \vee \neg r$ [Commutative]

(Showed)

C.W
15.3.22

L.5

DM

Nested Quantifiers: Example

Let Domain = $\{1, 2, 3\}$

$\forall x \exists y \, P(x, y)$

$\equiv \exists y \, P(1, y) \wedge \exists y \, P(2, y) \wedge$
$\exists y \, P(3, y)$

$\equiv [P(1,1) \vee P(1,2) \vee P(1,3)] \wedge$
$[P(2,1) \vee P(2,2) \vee P(2,3)] \wedge$
$[P(3,1) \vee P(3,2) \vee P(3,3)]$

P.T.O

(1) ~~(...)~~ → Free

~~(1)~~ → (1, 2, 3, 4, 5)

(1)

$\forall x (\exists y \, P(x, y))$ = (1, ... n) →

Everyone (x) has some one (y) to rely on.

$\exists y (\forall x \, P(x, y))$

= There's a poor overburdened soul (y) whom everyone (x) relies upon (including himself)!

$\exists x (\forall y \, P(x, y))$

= There's some needy person (x) who relies upon everybody (y) (including himself.)

$\forall y (\exists x \, P(x, y))$

= Everyone (y) has someone (x) who relies upon them.

P.T.O.

$\forall x (\forall y P(x, y))$

= Everyone relies upon everybody.
(x, y) (x, y)

Negating Nested Quantifiers:-

$\neg \forall x \exists y (P(x, y) \wedge \exists z P(x, y, z))$

Quantifiers ⇒

Proposition ⇒ প্রস্তাব, বিবৃতি, বর্ণনা
প্রতিজ্ঞা, উপস্থাপন

Tautology ⇒ অনুলাপ, পুনরুক্তি, বহুউক্তি

Predicate ⇒ সম্বন্ধ (বিধেয়, গুণ, ধর্ম)

19.3.22

L-4

PM

$\forall x > 0$, $p(x)$ is shorthand for,

"For all x that are greater than zero, $p(x)$"

$= \forall x (x > 0 \to p(x))$

$\exists x > 0$ $p(x)$,

"There is an x greater than Zero, such that $p(x)$."

$= \exists x (x > 0 \land p(x))$

Every student in this class has studied calculus.

$= \forall x (s(x) \longrightarrow c(x))$

Some students in this class has visited Mexico.

$= \exists x (s(x) \land M(x))$

P.T.O

Every student in this class has visited either Canada or Mexico.

$= \forall x\,(C(x) \vee M(x))$

Negations of Quantifiers.

"Not every student in the class has studied calculus."

$\Rightarrow \neg \forall x\, P(x) \equiv \exists x\, \neg P(x)$

"There is no student in the class who has taken a course in calculus."

$\Rightarrow \neg \exists x\, P(x) \equiv \forall x\, \neg P(x)$

P.T.O

* Negation of Quantifiers :-

De Morgan's laws;

(1) $\neg \forall x \, P(x) \equiv \exists x \neg p(x)$

(2) $\neg \exists x \, p(x) \equiv \forall x \neg p(x)$

Negations : Exmple

* $\neg \forall x \, (x^2 > x)$

$\Rightarrow \exists x \neg (x^2 > x)$

$\Rightarrow \exists x \, (x^2 \leq x)$

* $\exists x \, (x^2 = 2)$

$\Rightarrow \forall x \neg (x^2 = 2)$

$\Rightarrow \forall x \, (x^2 \neq 2)$

P.T.O.

$\neg \forall x (P(x) \to Q(x))$

$\Rightarrow \exists x \neg (P(x) \to Q(x))$ (2nd)

$\Rightarrow \exists x \neg (\neg P(x) \vee Q(x))$ (Def of $\to$)

$\Rightarrow \exists x (\neg(\neg P(x)) \wedge (\neg Q(x))$ (De morgan)

$\Rightarrow \exists x (P(x) \wedge \neg Q(x))$ (Main)

Table-1	Quantifiers	
Statement	True ?	False ?
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false
$\exists x P(x)$	There is an x for which $P(x)$ is true	$P(x)$ is false for every x

Table-2 De Morgans laws for Quantifiers

Negation	Equivalent statement	When is Negation True?	False
$\neg \exists x P(x)$	$\forall x \neg p(x)$	For every x, $p(x)$ is false.	There is an x for which $p(x)$ is true
$\neg \forall x P(x)$	$\exists x \neg p(x)$	There is an x for which $p(x)$ is false	$p(x)$ is true for every x.

P.T.O

L.5

Nesting of Quantifiers

* $L(x, y)$ = "x likes y"

* $\exists y \, L(x, y)$ =

"There is someone whom x likes."

* $\forall x (\exists y \, L(x, y))$ =

"Everyone has someone whom they like."

P.T.O

Table : Quantifications of Two Variables		
Statement	True ?	When False
$\forall x \forall y\, P(x,y)$ $\forall y \forall x\, P(x,y)$	$P(x,y)$ is true for every pair x, y.	There is a pair x, y for which $P(x,y)$ is false.
$\forall x \exists y\, P(x,y)$	For every x there is a y for which $P(x,y)$ ~~which~~ P is true.	There is an x such that $P(x,y)$ is false for every y.
$\exists x \forall y\, P(x,y)$	true	False
$\exists x \exists y\, P(x,y)$ $\exists y \exists x\, P(x,y)$	There is a pair x, y for which $P(x,y)$ is true.	$P(x,y)$ is false for every pair x, y.
	P.T.O.	

$\neg \forall x \exists y \, (P(x,y) \wedge \exists z \; R(x,y,z))$

$\Rightarrow \exists x \neg \exists y \; (P(x,y) \wedge \exists z \; R(x,y,z))$

$\Rightarrow \exists x \; \forall y \neg (P(x,y) \wedge \exists z \; R(x,y,z))$

$\Rightarrow \exists x \; \forall y \; (\neg P(x,y) \vee \neg \exists z \; R(x,y,z))$

$\Rightarrow \exists x \; \forall y \; (\neg P(x,y) \vee \forall z \neg R(x,y,z))$

Equivalance Laws

* $\forall x \forall y \; P(x,y) \equiv \forall y \; \forall x \, P(x,y)$

  $\exists x \exists y \, P(x,y) \equiv \exists y \exists x \; P(x,y)$

* $\forall x (P(x) \wedge Q(x)) \equiv (\forall x \, P(x)) \wedge (\forall x \, Q(x))$

P.T.O

* $\exists x (P(x) \vee Q(x)) \equiv (\exists x \, P(x)) \vee (\exists x \, Q(x))$

P.T.O

Proof

$\forall x\,(P(x) \wedge Q(x)) \equiv (\forall x\,P(x)) \wedge (\forall x\,Q(x))$

$\forall x\,(P(x) \wedge Q(x)) \equiv \forall x\,P(x) \wedge \forall x\,Q(x)$

Proof:-

Suppose $\forall x\,(P(x) \wedge Q(x))$ is true. Then for all a in the domain $P(a) \wedge Q(a)$ is true. Hence, both $P(a)$ is true and $Q(a)$ is true. Since $P(a)$ is true for all a in the domain, $\forall x\,P(x)$ is true. Since $Q(a)$ is true for all a in the domain, $\forall x\,Q(x)$ is true. Hence $\forall x\,(P(x) \wedge Q(x))$ is true.

Suppose that $\forall x\,(P(x) \wedge Q(x))$ is true. It follows that $\forall x\,P(x)$ is true., and that $\forall x\,Q(x)$ is

P.T.O.

CamScanner

true. Hence, for each element a in the domain $P(a)$ is true, and $Q(a)$ is true. Hence $P(a) \wedge Q(a)$ is true for each element a in the domain.

Therefore, by definition, $\forall x\, (P(x) \wedge Q(x))$ is true.

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* Rules of Inference

Valid Arguments (1)

If I work all night, then I get tired.

P.T.O

p : I work all night

q : I get tired.

$\Rightarrow$

$p \to q$ premise 1 $\Rightarrow$ (then / implies)

p premise 2 $\Rightarrow$ (reason)

$\therefore q$ conclusion $\Rightarrow$ (End result)

(ns)

p = I'm a business man

q : I'm ~~rich~~ rich

$p \to q$ premise 1

p premise 2

q conclusion

4.5

Predicates

1.

Let $P(x)$ denote the statement "$x>3$". What are the truth value of $P(4)$ and $P(2)$

$\Rightarrow P(x) = x > 3$

$P(4) = 4 > 3 = (T)$

$P(2) = 2 > 3 = (F)$

2.

Let $Q(x, y)$ denote the statement "$x = y+3$". What are the truth value of the propositions $Q(1,2)$ and $Q(3,0)$

$\Rightarrow Q(x, y) \Rightarrow x = y + 3$

$Q(1,2) \Rightarrow 1 = 5 \Rightarrow (F)$

$Q(3,0) \Rightarrow 3 = 3 \Rightarrow (T)$

Quantifiers

1.

Let $Q(x)$ be the statement "$x < 2$", what is the truth value of the quantification $\forall x\, Q(x)$, where the domain consists of all real numbers?

$\Rightarrow \forall x\, Q(x) = F$

$Q(x) = x < 2$

$Q(1) = 1 < 2$

$Q(2) = 2 < 2$ X

$Q(3) = 3 < 2$ X

2.

What is the truth value of $\forall x\, P(x)$, where $P(x)$ is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4?

$\Rightarrow P(x) = x^2 < 10$

domain, $x = \{0, 1, 2, 3, 4\}$

$\Rightarrow 4^2 < 10$ X

$\therefore \forall x\, P(x) = F$

3

What is the truth value of $\exists x P(x)$, where $P(x)$ is the statement "$x^2 > 10$" and the domain consists of the positive integers not exceeding 4?

$\Rightarrow \exists x P(x) = x^2 > 10$

$= (T)$

$x = \{0, 1, 2, 3, 4\}$

$x^2 > 10$ ✗

$0^2 > 10$ ✗

$2^2 > 10$ ✗

$3^2 > 10$ ✗

$4^2 > 10$ ✓

P.T.O

For every person x, if x is a student in the class, then x has visited Mexico or x has visited Canada.

$\Rightarrow \quad \forall x \left(S(x) \rightarrow M(x) \vee C(x) \right)$

Every mail message larger than one megabyte will be compressed.

$S(m, y)$ = m message larger than y megabyte.

$C(m)$ = m is message that will be compressed.

$\forall m \left(S(m, y) \rightarrow C(m) \right)$

P.T.O

English → Logical expression

$C(x) \rightarrow$ x has a cat

$D(x) \rightarrow$ x has a dog

$F(x) \rightarrow$ x has a Ferret

Domain → all student in your class.

(a) All students in your class has a cat, a dog or a ferret.

$\Rightarrow \forall x \, (C(x) \vee D(x) \vee F(x))$

(b) Some student in your class has a cat and a ~~dog~~ Feret (but not a dog)

$\Rightarrow \exists x \, (C(x) \wedge F(x) \wedge \neg D(x))$

P.T.O

c) No student in your class has a cat, a dog and a ferret

$\Rightarrow \exists x \neg (C(x) \wedge D(x) \wedge F(x))$

C(x) is, "x is a comedian."

F(x) is, " x is funny "

domain $\rightarrow$ all people

$\Rightarrow \exists x \; (C(x) \rightarrow F(x))$

$\Rightarrow$ There is exist 1 x, x is a comedian then x is funny.

$\Rightarrow$ Some comedian are funny.

P.T.O

* $\exists x\,(C(x) \land F(x))$

$\Rightarrow$ Some person is a funny comedian.

C.W
21.3.22

DM

Proof Terminology

Lemma ⇒ [illegible]

Corollary ⇒ অনুসিদ্ধান্ত

Theorem ⇒ উপপাদ্য

Conjecture ⇒ অনুমান

Proof Method

* For providing a statement p alone.

* Proof by Contradiction (indirect proof)

Assume $\neg p$, and prove $\neg p \rightarrow F$

p = $3n+2$ is odd

q = n is odd

$\neg q \longrightarrow \neg p$

$n = 2k$ (even)

$3n+2$

$\Rightarrow 3(2k)+2$

$\Rightarrow 6k+2$

$\Rightarrow 2(3k+1)$

$\Rightarrow 2j$

$2j$ is even.

$x+1>x$

$-1+1>-1$

$\Rightarrow 0>-1$

* Rational number

*

4,5

Exercise

bm

16 $x^2>10$

$\Rightarrow 1>10$ ✗

$\Rightarrow 2^2>10 \Rightarrow 4>10$ ✗

$\Rightarrow 3^2>10 \Rightarrow 9>10$ ✗

$\Rightarrow 4^2>10 \Rightarrow 16>10$ ✓

$\therefore$ (T)

P.T.O

21

$\neg \forall x (x^2 > x)$

$\Rightarrow \exists x \neg (x^2 > x)$

$\Rightarrow \exists x (x^2 \leq x)$

$\neg \exists x (x^2 = x)$

$\Rightarrow \forall x \neg (x^2 = x)$

22

$\neg \forall x (P(x) \rightarrow Q(x))$

$\Rightarrow \exists x \neg (P(x) \rightarrow Q(x))$

$\Rightarrow \exists x (P(x) \land \neg Q(x))$

$\Rightarrow$

* $\exists x\,(P(x) \wedge Q(x)) \not\equiv \exists x\, P(x) \wedge \exists x\, Q(x)$

Suppoce that the hypothesis $\exists x\,(P(x) \wedge Q(x))$ is true. That means there will be one x for which $P(x)$ and $Q(x)$ is true.

It also possible that for ~~...~~ there ~~is~~ exist one x both $P(x)$ and $Q(x)$ are true. But any ~~case~~ case, there exist one x, must satisfy $P(x)$ and $Q(x)$ or both, since the hypothesis is true.

The conclusion (RHS) is true when

P.T.O

when the ~~dis~~ conjunction is true.

As is clear from all above reasoning that $p(x)$ is true for one value of x and $Q(x)$ for one.

Thus both $\exists x\, P(x)$ and $\exists x\, Q(x)$ are false, since neither of them are true for ~~one~~ any one value of x.

In the case where $p(x)$ and $Q(x)$ hold one value of x, than this equivalence is true, but otherwise it is false.

P.T.O

So, $\exists x P(x) \wedge \exists x Q(x) \equiv F$.

According to our assumption, the hypothesis is true. but our conclusion turned out to be false.

So

$\exists x (P(x) \wedge Q(x)) \Rightarrow \exists x P(x) \wedge \exists x Q(x)$

is false.

So

$\exists x (P(x) \wedge Q(x)) \not\equiv \exists x P(x) \wedge \exists x Q(x)$

The two expressions do not have the same truth value for this case.

CW

DM

Set

The empty set

$\phi = \{ \ \}$ $\in$ = element of

* U set

* Subset / Superset

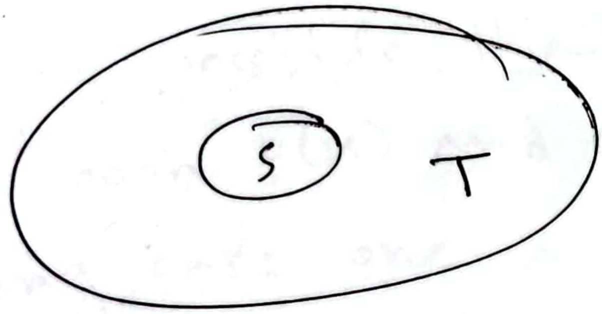
subset $\leftarrow T \subseteq S \rightarrow$ superset

$T \supseteq S$

Superset (Proper)

$S \subset T$

$\{1, 2\} \subset \{1, 2, 3\}$



Let,

$S = \{x \mid x \subseteq \{1, 2, 3\}\}$

$\Rightarrow S \Rightarrow \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

P.T.O

$1 \neq \{1\} \neq \{\{1\}\}$

* <u>Power set</u>

Md. Sadman Haque
ID: 011221592

* $\exists x(P(x) \land Q(x)) \not\equiv \exists x P(x) \land \exists x Q(x)$

Prof:

Suppose that the hypothesis $\exists x(P(x) \land Q(x))$ is true. That means there will be one x for which $P(x)$ and $Q(x)$ is true. It also possible that for there exist one x both $P(x)$ and $Q(x)$ are true. But any case one x must statisfy $P(x)$ and $Q(x)$ ~~&~~ both, since the hypothesis is true.

The conclusion (RHS) is true when the conjunction is true. As is clear from all above reasoning that $P(x)$ is true for one value of x and $Q(x)$ for one.

P.T.O.

Thus both $\exists x\, P(x)$ and $\exists x\, Q(x)$ are false, since neither of them are true for any one value of x. In the case where $p(x)$ and $Q(x)$ hold one value of x than this equivalance is true. But otherwise it is false.

So, $\exists x\, P(x) \wedge \exists x\, Q(x) \equiv F$.

According to our assumption, the hypothesis is true. But our conclusion turned out to be false.

So the two expressions do not have the same truth value for this case.

$\exists x\,(P(x) \wedge Q(x)) \Rightarrow \exists x\, P(x) \wedge \exists x\, Q(x)$

is false.

P.T.O.

Since one conditional is false, the complete biconditional is false,

And finally we can say that,

$$\exists x \, (P(x) \land Q(x)) \not\equiv \exists x \, P(x) \land \exists x \, Q(x)$$

(Proved)

MIP

DM

Fall 21

Ans to the ques no: 1 (a)

People feel stressed when they have a lot on their plate.

(q when p)

$\therefore$ If people have a lot on their plate, then they feel stressed.

Inverse $(\neg p \rightarrow \neg q)$

If people don't have a lot one their plate, then they don't feel stressed.

Converse $(q \rightarrow p)$

If people feel stressed, then they have a lot on their plate.

P.T.O

Contra positive $(\neg q \rightarrow \neg p)$

If people don't feel stressed, then they don't have a lot on their plate.

$s = (\neg p \leftrightarrow \neg q)$

$t = (q \leftrightarrow r)$ $u = s \leftrightarrow t$

(d)

p	q	r	¬p	¬q	s	t	u
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

c

$(p \to q) \to r$

$\equiv (\neg p \vee q) \to r$ (Def $\to$)

$\equiv \neg(\neg p \vee q) \vee r$ (Def $\to$)

$\equiv (p \wedge \neg q) \vee r$ (De morgan law)

$\equiv r \vee (p \wedge \neg q)$ (Commutative)

$\equiv (r \vee p) \wedge (r \vee \neg q)$ (Distributive)

$\equiv$ — — — $\wedge (\neg q \vee r)$ (Commutative)

$\equiv (\neg r \to p) \wedge (q \to r)$ (Def of $\to$)

Ans

Ans to the ques no : 2 (a)

(i) $\exists x (T(x) \wedge L(x))$

(ii) $\forall x (T(x) \rightarrow D(x))$

(iii) $\exists x \exists y (S(x, y) \wedge (D(x) \wedge T(x) \wedge L(y)))$

(b)

(i) for all x there exist ~~that~~ some y where x equal to y.

(ii) For ~~some x~~ there exist some x for all y where x and y equal to 0

P.T.O

Suppose

3.

(i) $A \subset B$

$U = \{1, 2, 3, 4, 5\}$

$A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4\}$

$B' = U - B$

$= \{5\}$

$A' = U - A$

$= \{4, 5\}$

$\therefore B' \subset A'$

This is true.

Ans

(ii) $B - A = \theta$

$B - A = \{1, 2, 3, 4\} - \{1, 2, 3\} = \{4\}$

$\therefore B - A \neq \theta$

$\therefore$ This is false.

5(d)

$p \rightarrow$ One rational

other one irrational

$q \rightarrow$ product irrational

$p \Rightarrow$ true $\qquad \neg q$: True

$a = r_1 \qquad \therefore ab = r_2$

$b = i_1 \qquad r_1 i_1 = r_2$

$$i_1 = \frac{r_2}{r_1}$$

Sping 20

1

(a)

$$f(x) = x^3 \qquad g(x) = \frac{(x^2+1)}{(x^2+2)}$$

$\therefore$ $f \circ g$

$$= f(g(x))$$

$$= f\left(\frac{x^2+1}{x^2+2}\right)$$

$$= \frac{x^3(x^2+1)}{x^2+2}$$

$$= \frac{x^5+x^3}{x^2+2}$$

$$g \circ f = g(f(x))$$

$$= g\, x^3$$

$$= \left\{\frac{(x^2+1)}{(x^2+2)}\right\}^3$$

Ans

P.T.O

3/

1(a)

$\neg p \rightarrow (q \rightarrow r)$

$\equiv p \vee (q \rightarrow r)$ (Def of $\rightarrow$)

$\equiv p \vee (\neg q \vee r)$ (Def of $\rightarrow$)

$\equiv p \vee (r \vee \neg q)$ (Commutative)

$\equiv (p \vee r) \vee \neg q$ (Associative ~~Commutative~~)

$\equiv \neg q \vee (p \vee r)$ (Commutative)

$\equiv q \rightarrow (p \vee r)$ (Def of $\rightarrow$)

P.T.O

~~c~~

If he studies hard, he will pass

$$\underbrace{\text{If he studies hard}}_{p}, \underbrace{\text{he will pass}}_{q}$$

$\therefore$ converse $= (q \rightarrow p)$

contrapositive $= (\neg q \rightarrow \neg p)$

inverse $= (\neg p \rightarrow \neg q)$

* He will not pass if he doesn't study hard.

Summer 20

b

$n = 1, 2, 3$

$n = 1,$

$\therefore n^2 + 5 = 1^2 + 5$

$= 1 + 5 = 6 = \text{even}$

$n = 1 = \text{odd}$ (True)

$n = 2$

$n^2 + 5 = 2^2 + 5 = 9 = \text{odd}$

$n = 2 = \text{even}$

(Not wanted)

$N = 3,\ n^2 + 5 = 9 + 5 = 14 = \text{even}$

$\therefore n = 3 = \text{odd}.$

(True)

* ~~Contrapositio~~

5(c)

$p = 7n+4$ is even

$q = n$ is even

Contrapositive

if n is odd, then $7n+4$ is odd.

$\neg q \rightarrow \neg p$

$n = 2k+1$

$\therefore 7n+4$

$= 7(2k+1)+4$

$= 14k+7+4$

$= 14k+11$

$= 14k+10+1$

$= 2(7k+5)+1$

$= 2j+1$

so $2j+1$ is odd. k any intiger.

P.T.O

$\therefore (\neg\, n \text{ is even}) \rightarrow (\neg\, 7n+4 \text{ is even})$

$\Rightarrow (7n+4 \text{ is even}) \rightarrow (n \text{ is even})$ is true

(Proved)

$a+b = \dfrac{p}{q}$

$\Rightarrow a^2+2ab+b^2 = \dfrac{p^2}{q^2}$

$\Rightarrow q^2(a^2+2ab+b^2) = p^2$

$\Rightarrow p = \sqrt{q^2(a^2+2ab+b^2)}$

$= 77$

$a = 5$

$b = 6$

$q = 7$

So 77 is a rational number.

5.

(v)

Suppose,

p = n is multiple of 3

q = $2n+3$ is multiple of 3

Assume p is true,

$\cancel{n = a+b+c}$

$\therefore \cancel{q = 2n+3}$

$\cancel{= 2(a+b+c)+3}$

$\cancel{= 2a+2b+2c}$

P.T.O

~~$n = 9m^2 + 6m + 1$ (Suppose)~~

$\therefore$ ~~$q = 2n + 3$~~

~~$= 2(9m^2 + 6m + 1) + 3$~~

~~$= 18m^2 + 12m + 2 + 3$~~

~~$= 18m^2 + 12m + 5$~~

$\therefore$ Both p & q become multiple of three.

So this True

(Proved)

$p = n = 3, 6, 9 \ldots$

$q = 2n+3$
$= 2 \cdot 3 + 3$
$= 6 + 3$
$= 9$

$= 2 \cdot 6 + 3$
$= 12 + 3$
$= 15$

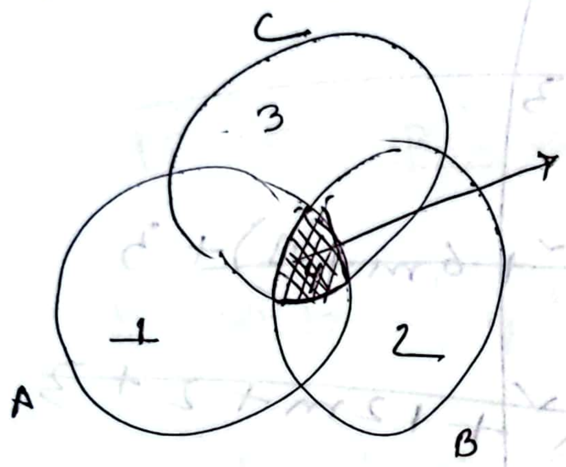
$2n+3$
$= 2 \cdot 9 + 3$
$= 18 + 3$
$= 21$

$\therefore$ true

$U - B$

$U - A$

$\cancel{(B' \cup A')} \cap C$



A

$A = \{1, 4\}$

$B = \{2, 4\}$

$C \ \{3, 4\}$

$(A \cap C) = \{1\ 4\} \cap \{3, 4\} = 4$

$\cup$

$(A \cap B) = \{4\}$

$\{4\}$

~~Fun~~ bm

Function Practice

$f(x) = 3x + 2$

$g(x) = 4 - 5x$

$(f+g)(x)$

$= f(x) + g(x)$

$= 3x + 2 + 4 - 5x$

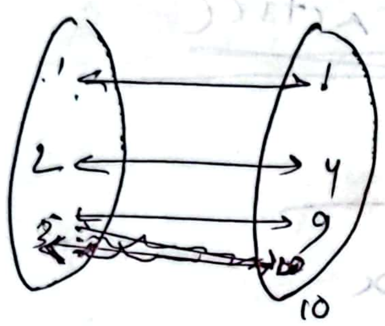
$= 6 - 2x$

$(f-g)(x)$

$= f(x) - g(x)$

$= 3x + 2 - 4 + 5x$

$= 8x - 2$



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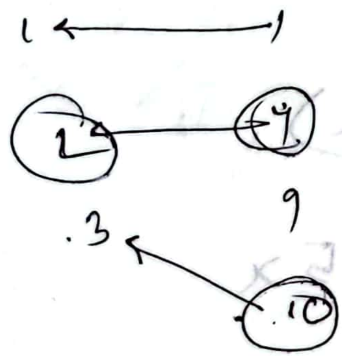


(1, 1), (2, 4), (3, 9), (3, 10)

$A = \{1, 2, 3\}$: $x \longrightarrow Y$

$B = \{1, 4, 9, 10\}$

1.3

24

$p \to (q \vee r)$

$\Rightarrow \neg p \vee (q \vee r)$ (Def of $\to$)

$\Rightarrow \cancel{(\neg p \vee q) \vee (\neg p \vee r)}$

$\Rightarrow (\neg p \vee \neg p) \vee (q \vee r)$ idempotent

$\Rightarrow (\neg p \vee q) \vee (\neg p \vee r)$ (associative & commutative)

$\Rightarrow (p \to q) \vee (p \to r)$ (Def of $\to$)

25)

$(p \to r) \vee (q \to r)$

$\equiv (\neg p \vee r) \vee (\neg q \vee r)$ (Def $\to$)

$\equiv (\neg p \vee \neg q) \vee (r \vee r)$

(Associative & Commutative)

$\equiv (\neg p \vee \neg q) \vee r$ (idempotent)

$\equiv \neg(p \wedge q) \vee r$ (de morgan)

$\equiv (p \wedge q) \to r$ (Def $\to$)

A

2C

$\neg p \rightarrow (q \rightarrow r)$

$\Rightarrow$

$\Rightarrow p \vee (q \rightarrow r)$ (Def $\rightarrow$)

$\Rightarrow p \vee \neg q \vee r$ (Def of $\rightarrow$)

$\Rightarrow \neg q \vee (p \vee r)$ (Associative & commutative)

$\Rightarrow q \rightarrow (p \vee r)$ (Def of $\rightarrow$)

P.T.O

27

$p \leftrightarrow q$

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\equiv \{p \vee (\neg p \wedge \neg q)\} \wedge \{q \vee (\neg p \wedge \neg q)\}$

$\equiv \{(p \vee \neg p) \wedge (p \vee \neg q)\} \wedge \{(q \vee \neg p) \wedge (q \vee \neg q)\}$

$\equiv \{T \wedge (p \vee \neg q)\} \wedge \{(q \vee \neg p) \wedge T\}$

$\equiv (p \vee \neg q) \wedge (q \vee \neg p)$

$\equiv (q \rightarrow p) \wedge (p \rightarrow q)$

$\equiv$

* $p \leftrightarrow q \Rightarrow$ True

$\neg p \leftrightarrow \neg q \Rightarrow$ True

$\neg p \Rightarrow$ False

$\neg q$ = False

This false & false become true

so.

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

---

* $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

$A \times P(A) = \{$

$A \times B = \{1, 2\} \times \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

$= \{(1, \phi), (2, \phi), (1, \{1\}), (2, \{1\}),$

$\cancel{(1\{2\}), (2, \{2\}), (1, \{1, 2\})},$

$\cancel{(2, \{1, 2\})},$

$\cancel{(1, 2, \phi), (1, 2, \{1\}), (1, 2, \{2\})},$

$\cancel{(1, 2, \{1, 2\})},$

$P(A) = 2^2 = 4$

$\therefore A \times B = 2^4 = 16$

$P(A \times B) = 2^{16} = 65536$

$P(P(A \times B)) = 2^{65536}$

a, b

$\{\cancel{1,2}\} \times \{1, 2, 3\}$

$= \cancel{(1,1), (1,2), (1,3)}$

$= \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

$P(A) = 2^2 = 4$

$A \times P(A) = 2 \times 4 = 8$

$P(A \times P(A)) = 2^8 = 256$

$\therefore P(P(A)) = 2^{256}$

~~Contraposition ⇒ ¬q → ¬p~~

(i) $\forall x \exists y \left((x+1)^2 = y \right)$

$x = 1, 2, 3, 4$

$(x+1)^2 = y$

$\Rightarrow (1+1)^2 = y$

$\Rightarrow 2^2 = y \Rightarrow y = 4$

$\Rightarrow x = 1, y = 4$

False.

~~(ii) $(-x+1)^2 = x^2$ $\quad x = 1, 2, 3$~~

$\Rightarrow (-1+1)^2 = x^2$

$\Rightarrow x = 0$

$(-2+1)^2 = x$

$\Rightarrow x^2 = (-1)^2 = 1$

* $\exists x\, ((-x+1)^2 = x^2)$ $x = 0$

$(-0+1)^2 = 0^2$

$\Rightarrow 1^2 = 0$

$\Rightarrow 1 = 0$

This also False.

~~# $\neg \forall x (x^3 > 0)$~~

~~False. No x^3 is greater than 0.~~

~~This is false.~~ False.

Because $0^3 = 0$

* $\neg \forall x (x^3 > 0)$

$\rightarrow \exists x \neg (x^3 > 0$

$\Rightarrow \exists x (x^3 \leq 0)$

$\therefore$ This is true.

CamScanner

contraposition

$\neg q \rightarrow \neg p$

$p \Rightarrow n^3-1$ even

$q = n$ odd

$\therefore \neg q \Rightarrow n^3-1$ odd

$n =$ even

n^3-1

$\Rightarrow \cancel{(2k)^3 - 1}$

$\Rightarrow \cancel{16k^3 - 2}$

$\Rightarrow \cancel{8k^3 - 1}$

$\Rightarrow 8k^3 - 1$

$\Rightarrow (2h-1)(4h^2+2h+1)$

$\cancel{(2h-1)^3 + 3 \cdot 2h \cdot 1 (2h-1)}$

$2j+2$

$2h-2 \neq 0$

$a-b \neq 0$

$n = 2k$

$\Rightarrow (2k+1)^3 - 1$

$\Rightarrow 8k^3$

$\Rightarrow 4h^2+2h+2$

$\Rightarrow 2(2h^2+h)+1$

$\Rightarrow 2j+1$

* n even

$3n+2$ even

$n = 2k$

$3n+2$

$\Rightarrow 3 \cdot 2k + 2$

$\Rightarrow 6k+2$

$\Rightarrow$ ~~6k+2~~

$\Rightarrow 2(3k+2)$ $[3k+2 = j]$

$\Rightarrow 2j$

$\therefore$ $3n+2$ even

$x = 1$

$y = 2$

$z^2 > 0$

$\Rightarrow$

$\exists x \forall y$

$x^y y^x = 2$

$\Rightarrow 1^2 \cdot 2^1 = 2$

$= 2 = 2$

$x = 1$

$y = 1, 2, 3$

$p = n^2 + 5$ even

$q = n$ is ~~[illegible]~~ odd

$\boxed{n = 2k+1}$

$\therefore n^2 + 5$

$= (2k+1)^2 + 5$

$= 4k^2 + 4k + 1 + 5$

$= 4k^2 + 4k + 6$

$= 2(2k^2 + 2k + 3)$

$= 2j \quad [j = 2k^2 + 2k + 3]$

$\therefore n^2 + 5$ is even when n is odd.

(Proved)

(1) $p \Rightarrow x$ is even

$q = y$ is odd.

$\therefore$ $y = 2k+1$

$x = 2k$

$\therefore xy = (2k+1)2k$

$= 4k^2 + 2k$

$= 2(2k^2 + k)$

$= 2j$ $[j = 2k^2 + k]$

$\therefore$ x is even, y is odd, xy is even.

(Proved)

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	T

p	q	$p \rightarrow q$
T	T	T
T	F	F is only false case
F	T	T
F	F	T

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

tautology ⇒ True

Contradiction ⇒ False

Contingency ⇒ Nothing

$\neg \forall x \, \exists y \, (P(x,y) \land \exists z \, R(x,y,z))$

$\Rightarrow \exists x \, \neg \exists y \, (P(x,y) \land \exists z \, R(x,y,z))$

$\Rightarrow \exists x \, \forall y \, \neg (P(x,y) \land \exists z \, R(x,y,z))$

$\Rightarrow \exists x \, \forall y \, (\neg P(x,y) \lor \neg \exists z \, R(x,y,z))$

$\Rightarrow \exists x \, \forall y \, (\neg P(x,y) \lor \forall z \, \neg R(x,y,z))$

A

P.T.O

* Contradiction

Assume $\neg p$

Prove $\neg p \rightarrow F$

* Proof methods

Trival proof :→ Prove q by itself

* $\sqrt{2}$ is irrational

Suppose,

$\sqrt{2} = \frac{p}{q}$

$\Rightarrow 2 = \frac{p^2}{q^2}$

$\Rightarrow 2q = \frac{p^2}{q}$

But $2q \neq \frac{p^2}{q}$ [Because this can't be possible.]

$\therefore \sqrt{2} \neq \frac{p}{q}$ so $\sqrt{2}$ is a irrational Number.

$\{1,4\} - \{2,10\}$

1

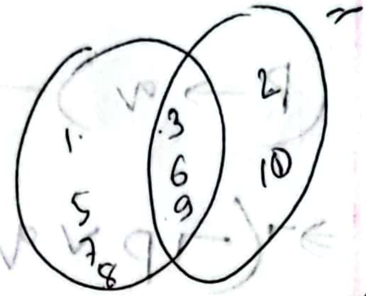
$A - B$

$= A - B$

$\{1,2,3\} - 4$

$1,2,3$

$P(S) = \{\{\}, \{\}, \{\}\}$



$A \times B = \{(), (), ()\}$

$1, 3, 5, 6, 7, 8, 9$ —

$2, 3, 6, 9, 10$

$A - B = \{1, 5, 7, 8\}$

$B - A = \{2, 10\}$

~~A - B = B - A~~

$A \cap B = \{3, 6, 9\}$

~~A ∪ B = {1, 2, 5, 7, 8, 10}~~

$A \cup B = \{1, 2, 3, 5, 6, 7, 8, 9, 16\}$

$\therefore A =$

$(p \to q) \to r$

$\Rightarrow \neg(\neg p \vee q) \vee r$ [Def of $\to$]

$\Rightarrow (p \wedge \neg q) \vee r$ [De Morgan]

$\Rightarrow r \vee (p \wedge \neg q)$ [Commutative]

$\Rightarrow (r \vee p) \wedge (r \vee \neg q)$ [Distributive]

$\Rightarrow (\neg r \to p) \wedge (q \to r)$ [Conditional]

l. $\exists x\,(T(x) \land L(x))$

n. $\forall x\,(T(x) \rightarrow D(x))$

m. $\exists x\,(S(x, y) \land D(x) \land T(x) \land L(y))$

$$f(x) = \frac{x-1}{x+1}$$

$$f(a) = \frac{a-1}{a+1}$$

$$f(b) = \frac{b-1}{b+1}$$

$$f(a) = f(b)$$

$$\Rightarrow \frac{a-1}{a+1} = \frac{b-1}{b+1}$$

$$\Rightarrow (a-1)(b+1) = (b-1)(a+1)$$

$$\Rightarrow ab + a - b - 1 = ab + b - a - 1$$

$$\Rightarrow a = b$$

So this is one to one, ~~and invertable~~.

P.T.O

$f(x) = y$

$\Rightarrow \dfrac{x-1}{x+1} = y$

$\Rightarrow x-1 = y(x+1)$

$\Rightarrow x-1 = xy + y$

$\Rightarrow x = xy + y + 1$

$\Rightarrow x - xy = y + 1$

$\Rightarrow x(1-y) = y + 1$

$\Rightarrow x = \dfrac{y+1}{1-y}$

$\therefore f(x) = \dfrac{\dfrac{(y+1)}{1-y} - 1}{\dfrac{y+1}{1-y} + 1}$

$= \dfrac{\dfrac{y+1-1+y}{1-y}}{\dfrac{y+1+1-y}{1-y}}$

$$= \frac{2y}{2}$$

$$= y$$

$$\therefore f(x) = y$$

So this is a one to one and on to function.

So this is invertable.

C.W

Induction

DM

Problem

* $1+2+3+4+ \dots n = \frac{n(n+1)}{2}$

$p(n) = 1+2+3+4+ \dots + n = \frac{n(n+1)}{2}$

Basis: $P(1)$ is true. became -

$L.H.S = 1$

$R.H.S = \frac{1(1+1)}{2} = 1$

Induction:

$p(k) \longrightarrow p(k+1)$

Let, $p(k)$ be true. where k is a positive intigers.

$\therefore 1+2+3+ \dots + k = \frac{k(k+1)}{2}$ .... (1)

P.T.O

~~Where~~ we have to prove that $P(k+1)$ is true.

$$1+2+3+\dots+(k+1) = \frac{(k+1)(k+2)}{2}$$

Adding $(k+1)$ to both side of (1)

$$1+2+3+\dots+k+(k+1) = \frac{k(k+1)}{2}+(k+2)$$

$$= \frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$\therefore$ $P(k+1)$ is also true.

$\therefore$ By mathematical induction $P(n)$ is true for all positive integers n.

* $1+2+2^2+2^3+\ldots+2^n = 2^{n+1}-1$

<u>Basis:</u>

$2^0+2^1+2^2+2^3+\ldots 2^n$

$p(0)$ is true. because

$L.H.S = 1$

$R.H.S = \dfrac{2^{0+1}-1}{\cancel{}}$

$= 2-1 = 1$

<u>Induction:</u>

$p(k) \longrightarrow p(k+1)$

Let $p(k)$ be true where k is a nonnegative integer.

$\therefore 1+2+2^2+\ldots+2^k = 2^{k+1}-1$ — (1)

<u>P.T.O</u>

We have to prove that $P(k+1)$ is true.

$$1+2+2^2+\ldots 2^{(k+1)} = 2^{k+2}-1$$

Adding 2^{k+1} both side of (1)

$$1+2+2^2+2^k+2^{k+1} = 2^{k+2}-1+2^{k+1}$$

$$= 2\cdot 2^{k+1}-1$$

$$= 2^{1+k+1}-1$$

$$= 2^{k+2}-1$$

$\therefore$ $P(k+1)$ is true.

$$1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n+1)}{2}$$

Basis: P(1) is true.

$$L.H.S = 1^2 = 1$$

$$R.H.S = (-1)^{1-1} \cdot \frac{1(1+1)}{2}$$

$$= (-1)^0 \cdot \frac{1 \times 2}{2} = 1$$

Induction:

$P(k) \longrightarrow P(k+1)$

Let P(k) be true. Where k is a positive integer.

P.T.O

$$\therefore 1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{k-1} k^2$$

$$= (-1)^{k-1} \frac{k(k+1)}{2}$$

Prove $P(k+2)$ is true

$$1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^k (k+1)^2$$

$$(-1)^k \frac{(k+2)(k+2)}{2}$$

Adding $(-1)^k (k+1)^2$

$$\therefore 1^2 - 2^2 + 3^2 - 4^2 + \dots (-1)^{k-1} k^2 + (-2)^k (k+1)^2$$

$$= (-1)^{k-1} \frac{k(k+1)}{2} + (-1)^k (k+2)^2$$

$$= (-2)^{k-2} \frac{k(k+2)}{2} + (-1)^{k-1} (-1) (k+1)^2$$

P.T.O.

$$= (-1)^{k-1}(k+1)\left[\frac{k}{2} + (-1)(k+1)\right]$$

$$= (-1)^{k-1}(k+1)\left[\frac{k}{2} - (k+1)\right]$$

$$= (-1)^{k-1}(k+1)\left[\frac{k}{2} - (k+1)\right]$$

$$= (-1)^{k-1}(k+1)\frac{k-2k-2}{2}$$

$$= (-1)^{k-1}(k+1)\frac{-k-2}{2}$$

$$= (-1)^{k-1}(k+1)\frac{(-1)(k+2)}{2}$$

$$= (-1)^{k}\frac{(k+1)(k+2)}{2}$$

one to one / onto

$f(x) = x^3 + 1$

$f(a) = f(b)$

$\Rightarrow a^3 + 1 = b^3 + 1$

$\Rightarrow a^3 = b^3$

$\Rightarrow a = b$

onto:-

$x^3 + 1 = y$

$\Rightarrow x^3 = y - 1$

$\therefore f(x) = x^3 + 1$

$= y - 1 + 1$

$= y$

A

$f(x) = |x| + 5$

$f(a) = f(b)$

$|a| + 5 = |b| + 5$

$\Rightarrow |a| = |b|$

$\Rightarrow a = b$

Onto:

$f(x) = y$

$\Rightarrow |x| + 5 = y$

$\Rightarrow |x| = y - 5$

$\therefore f(x) = |x| + 5$

$= y - 5 + 5$

$= y$

CW
18. 4. 22

Counting

DM

Product rule
Sum rule

Product rule

$$\prod_{i=1}^{m} n_m$$

* There are twelve empty rooms in the office of Acme Softwares, inc. Alif and Laila joins the company. How many ways can they be assigned a room each from these twelve?

⇒ Room for Alif = 12
" " Laila = 11

$\therefore$ total $= 12 \times 11 = 132$

Ans

* What is the total number of bit strings of length 7?

⇒ ~~Exm~~

Each of the 7 cells can be filled up with a bit 2 ways.

$\therefore$ total ⇒ $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^7$

$= 128$

Sum Rule

$$\sum_{i=1}^{m} n_i$$

* How many strings of lowercase Letters are there of length four or less?

⇒

string of length $1 \Rightarrow n_1$

$\square \rightarrow 26$

〃 〃 〃 $2 \Rightarrow n_2$

$\square\square \rightarrow 26^2$

〃 〃 〃 $3 \Rightarrow n_3$

$\square\square\square \rightarrow 26^3$

〃 〃 〃 $4 \Rightarrow n_4$

$\square\square\square\square \rightarrow 26^4$

$n_1 + n_2 + n_3 + n_4 = 26 + 26^2 + 26^3 + 26^4$

$=$

Pigeonhole Principal

*

$$2 + 6 + 12 + \dots + n(n+1)$$

Base: $\Rightarrow \longrightarrow$

$$L.H.S = 1 \times (1+1)$$

$$= 1 \times 2$$

$$= 2$$

$$R.H.S = \frac{1(1+1)(1+2)}{3}$$

$$= \frac{1 \times 2 \times 3}{3}$$

$$= 2$$