University

Name : S[illegible]mal

School/College : Calculus and linear algebra

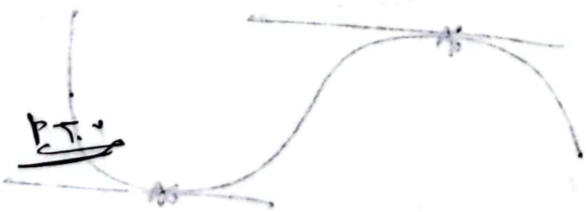
Subject : .... Roll: ....

Section: .... Year: ....

C.W
27.1.23

Graph to study

* Linear equation / function
* Quadratic "
* Cubic "
* exponential "
* Logarithmic "
* Sin & cos function



* Differentiation & Integration

$$A \xrightarrow{\text{Differentiate}} B$$

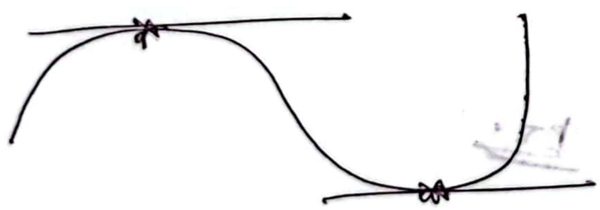
$$A \xleftarrow[\text{Intregrate}]{} B$$

$$\begin{tikzcd} y \arrow[r, "D"] & F(x) \arrow[l, bend left, "In"] \end{tikzcd}$$

* $\dfrac{dy}{dx} = F(x)$

$$y = \int F(x)\, dx$$

Turning point / stationary point of a function:-



* turning point

* At turning point, $\dfrac{dy}{dx} = 0$

PT.

Integration Rules:-

* $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$

* $\int (ax+b)^n = \frac{(ax+b)^{n+1}}{(n+1)\times a} + C$

* $\int e^x \, dx = e^x + C$

* $\int e^{ax+b} = \frac{e^{ax+b}}{a} + C$

* $\int \frac{1}{x} \, dx = \ln x + C$

* $\int \frac{1}{ax+b} \, dx = \frac{\ln (ax+b)}{a} + C$

* $\int \frac{1}{x^2+a^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$

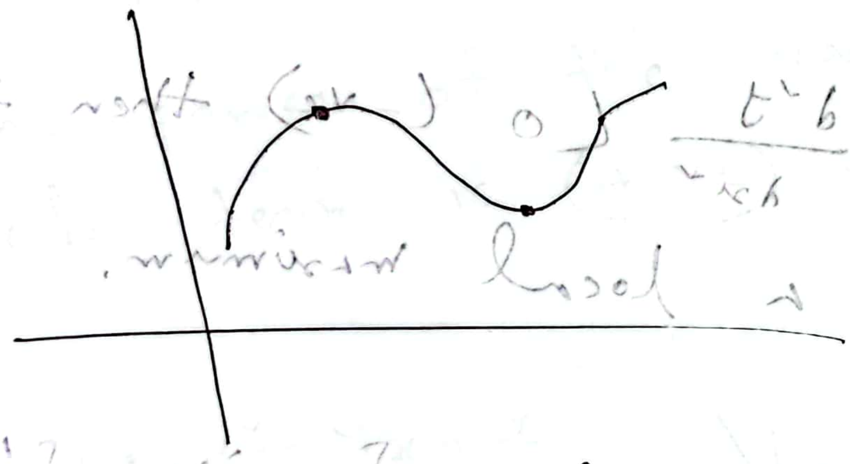
* $\int uv \, dx = u \int v \, dx - \int \left[\frac{d}{dx} u \cdot \int v \, dx \right] dx$

LIATE

- log / ln
- Inverse
- Algebra
- Trigonometric
- e^x

Turning / stationary point(s), nature of turning point and increasing and decreasing function :-

given that $y = f(x)$



At turning / stationary point $\frac{dy}{dx} = 0$

[First derivative test]

P.T.O.

Nature of turning point(s)

[2nd derivative test]

(i) If $\frac{d^2y}{dx^2} > 0$ (+ve) then there exists a local minimum.

(ii) If $\frac{d^2y}{dx^2} < 0$ (-ve) then there exists a local maximum.

for increasing funcition $f'(x) = \frac{dy}{dx} > 0$

a decreasing function, $f'(x) = \frac{dy}{dx} < 0$

$y = x^2 - 4x + 7$

<u>Polynomial</u>

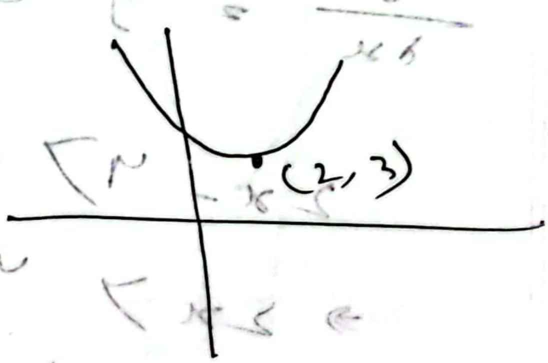
$y = ax + b \longrightarrow$ linear

$y = ax^2 + bx + c \longrightarrow$ Quadratic

$y = ax^3 + bx^2 + cx + d \longrightarrow$ Cubic

The power of the variable in each term is (+ve) integer.

* $y = x^2 - 4x + 7$



$\frac{dy}{dx} = 2x - 4 = 0$

$\Rightarrow 2x = 4$

$\Rightarrow x = 2$

and, $x = 2$, $y = 4 - 8 + 7 = 3$

$$\frac{dy}{dx} = 2x - 4$$

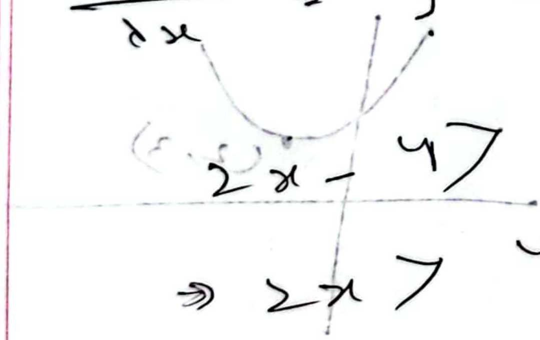
$$\frac{d^2y}{dx^2} = 2 \quad [\text{2nd derivative}]$$

$$2 > 0 \quad [+ve]$$

$\therefore$ $(2,3)$ is a local Minimum point.

For increasing function

$$\frac{dy}{dx} = f'(x) > 0$$



$$2x - 4 > 0$$

$$\Rightarrow 2x > 4$$

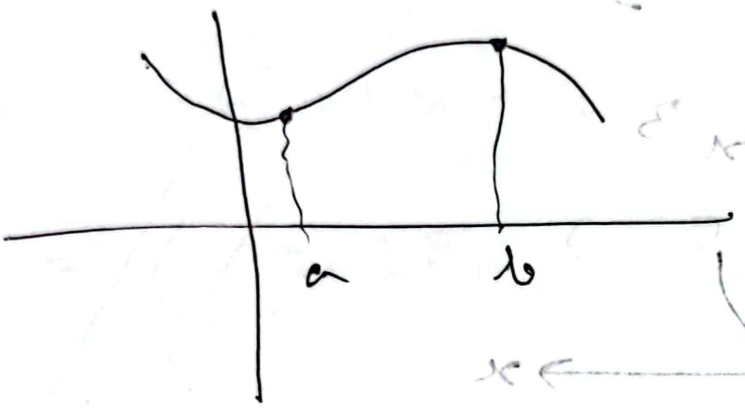
$$\Rightarrow x > 2$$

For decreasing function :-

$$\frac{dy}{dx} < 0 \quad \therefore \quad 2x - 4 < 0$$

$$\Rightarrow x < 2$$

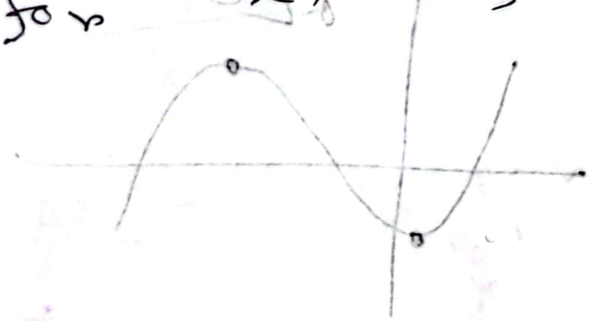
* $y = f(x)$



for $x < a$; $f(x)$ is decreasing

for $a < x < b$; $f(x)$ is increasing

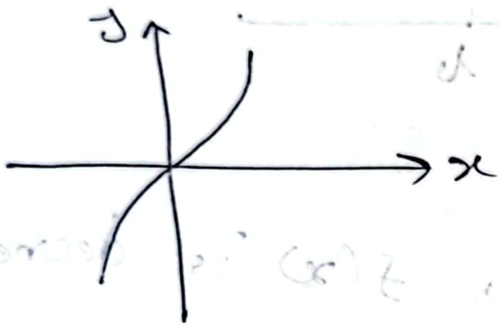
for $x > b$; $f(x)$ is decreasing



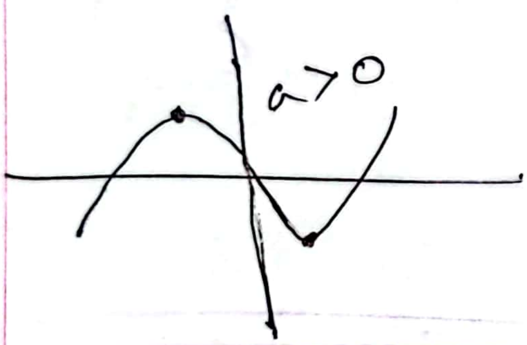
P.T.O

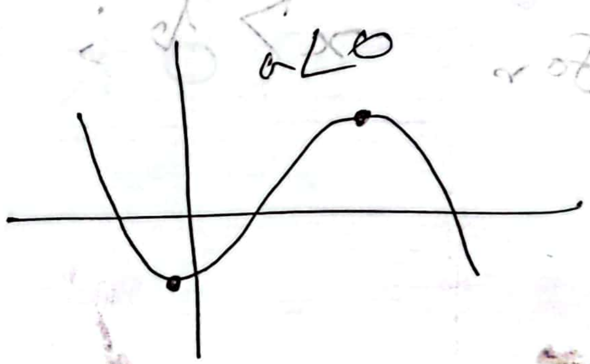
* graph of quadratic and cubic function :-

* $y = x^3$



$y = ax^3 + bx^2 + cx + d$





P.T.O

$$y = \frac{1}{3}x^3 - 3x^2 + 5x + 1$$

$$\frac{1}{3}x^3 - 3x^2 + 5x + 1 = 0$$

$$\Rightarrow x = (-0.18,\ 6.69)$$

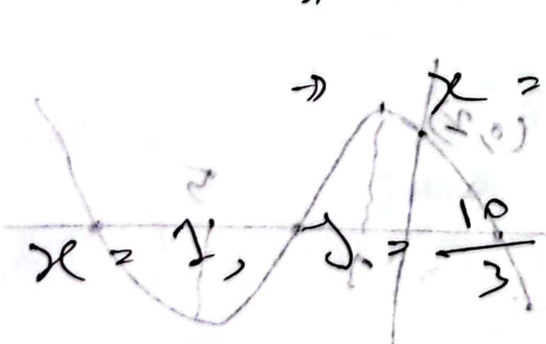
$$y = 1$$

$$\frac{dy}{dx} = x^2 - 6x + 5$$

At turning point $\frac{dy}{dx} = 0$

$$\Rightarrow x^2 - 6x + 5 = 0$$

$$\Rightarrow x = (1, 5)$$



at $x = 1,\ y = \frac{10}{3}$ $\left(1, \frac{10}{3}\right)$

at, $x = 5,\ y = -\frac{22}{3}$ $\left(5, -\frac{22}{3}\right)$

P.T.O.

Nature of the turning points:-

$$\frac{dy}{dx} = x^2 - 6x + 5$$

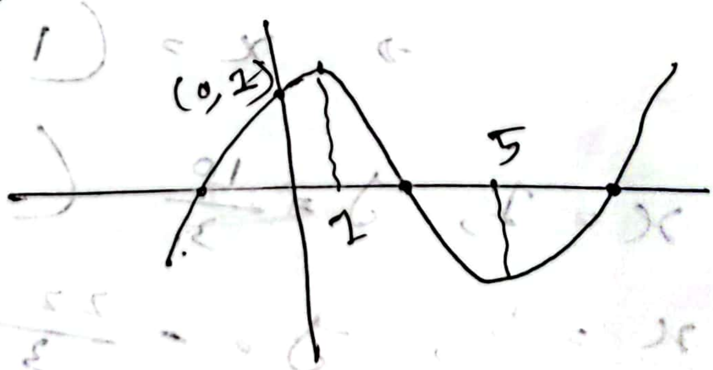
$$\frac{d^2y}{dx^2} = 2x - 6$$

at, $x = 1$, $\frac{d^2y}{dx^2} = 2 - 6 = -4 < 0$

$(1, \frac{10}{3})$ is a local Maximum point,

at, $x = 5$, $\frac{d^2y}{dx^2} = 10 - 6 = 4 > 0$

$(5, -\frac{22}{3})$ is a local minimum point.



P.T.O

* for $x < 1$; $f(x)$ is increasing

* for $1 < x < 5$; $f(x)$ is decreasing

* for $x > 5$; $f(x)$ is increasing

$y = x^3 - 3x^2 + 2x$

$y = (x-2)^2(-3x-6x)$

H.W
1.2.23

Calculus 2

* $y = x^3 - 3x^2 + 2x$

$$\frac{dy}{dx} = 3x^2 - 6x + 2$$

$$3x^2 - 6x + 2 = 0$$

$$\Rightarrow x = (1.577,\ 0.423)$$

$x = 1.577$, $y = -\cancel{1.?}\ 0.38$

$x = 0.423$, $y = +0.38$

Nature of turning points:-

$$\frac{dy}{dx} = 3x^2 - 6x + 2$$

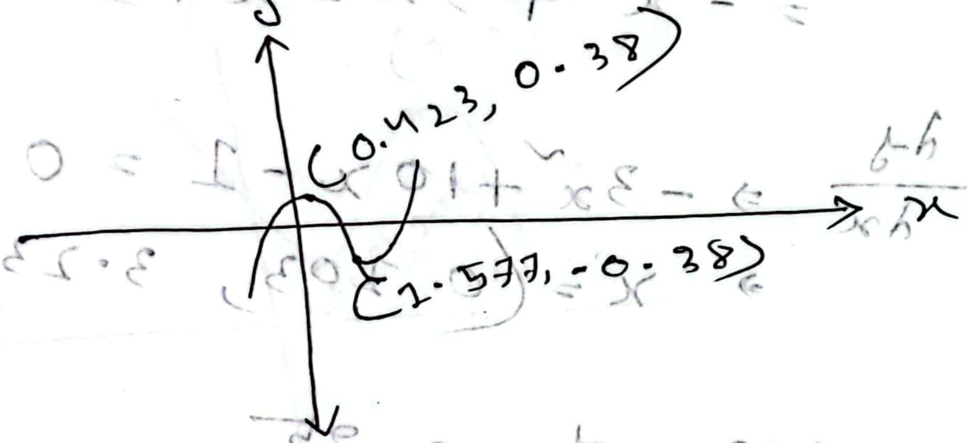
$$\frac{d^2y}{dx^2} = 6x - 6$$

$x = 1.577$, $\frac{d^2y}{dx^2} = 3.462 > 0$

so $(1.577, -0.38)$ is local minimum point.

$x = 0.423$, $\frac{d^2y}{dx^2} = -3.462 < 0$

so, $(0.423, 0.38)$ is local maximum point.



for $x < 0.423$, f(x) is increasing

for $0.423 < x < 1.577$, f(x) is decreasing

for $x > 1.577$, f(x) is increasing.

* $y = (x-1)^2 (3-x)$

$= (x^2 - 2x + 1)(3 - x)$

$= 3x^2 - 6x + 3 - x^3 + 2x^2 - x$

$= -x^3 + 5x^2 - x + 3$

$\frac{dy}{dx} \Rightarrow -3x^2 + 10x - 1 = 0$

$\Rightarrow x = (0.103, 3.23)$

$x = 0.103$, $y = 2.95$

$x = 3.23$, $y = 18.24$

Nature of turning points :-

$\frac{dy}{dx} = -3x^2 + 10x - 1$

$\frac{d^2y}{dx^2} = -6x + 10$

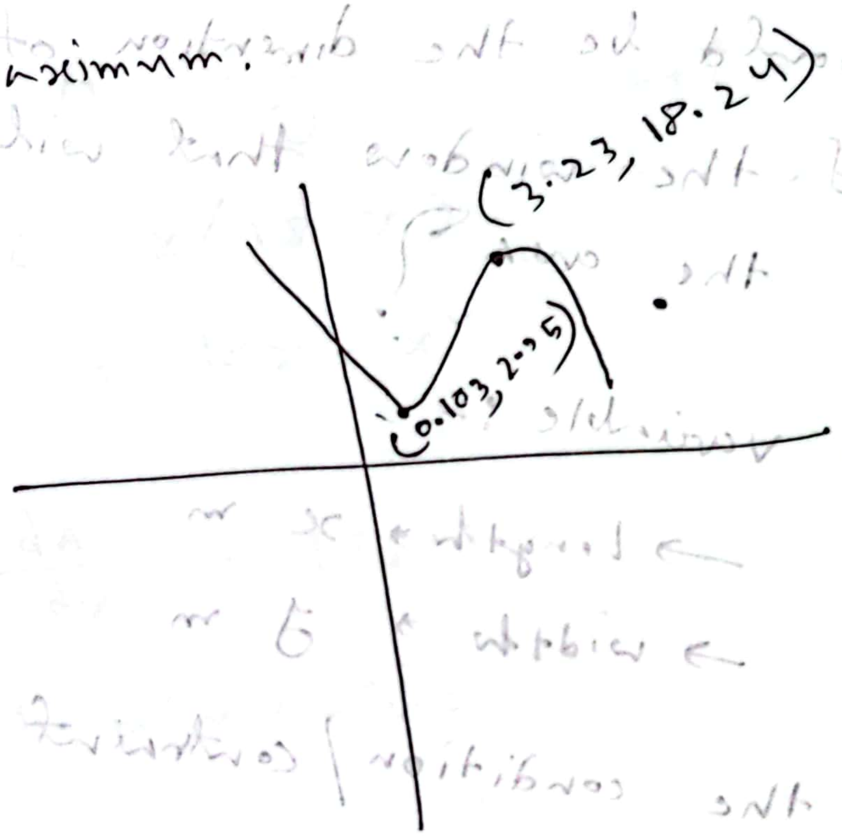
$x = 0.103$, $\frac{d^2y}{dx^2} = 9.382 > 0$

$(0.103, 2.95)$ point is local minimum.

P.T.O

$x = 3.23$, $\frac{d^2y}{dx^2} = -9.38 < 0$

$(3.23, 18.24)$ point is local maximum.



* $x < 0.103$, $f(x)$ is decreasing

* $0.103 < x < 3.23$, $f(x)$ is increasing.

* $x > 3.23$, $f(x)$ is decreasing

CW 5.2.23

Calculus

* Optimization (Maximum or Minimum):-

———————— 36m

what should be the dimention of the frame of the window that will maximise the area?

* Assign variable :-

→ Length → x m

→ width → y m

* Express the condition / contraint in the terms of variable :-

$$2x + 2y = 36$$

$$\Rightarrow x + y = 18$$

P.T.O.

* Express & simplify the objective function in terms of chosen variable :-

$A = xy$

$x + y = 18$

$\Rightarrow y = 18 - x$

$A = x(18 - x)$

$= 18x - x^2$

$\frac{dA}{dx} = 18 - 2x$

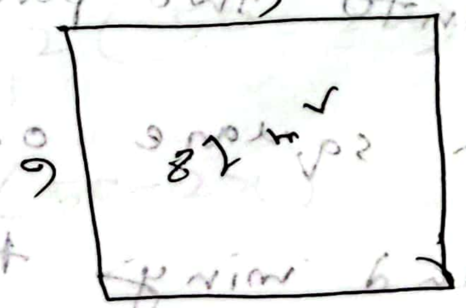
* At max / min :-

$\frac{dA}{dx} = 0$

$\Rightarrow 18 - 2x = 0$

$\Rightarrow x = 9$

$y = 9$



$\therefore (x, y) = (9, 9)$

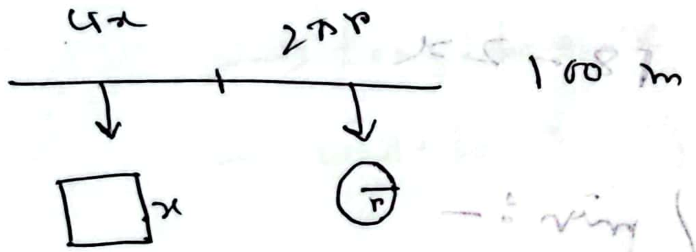
P.T.O.

$$\frac{dA}{dx} = 18 - 2x$$

$$\frac{d^2A}{dx^2} = -2 < 0$$

$\therefore$ 9m & 9m will maximize the area.

#



A string of length 100 m is devided into two parts. Using one part a square of side x m is form and using the other side a ~~circle~~ circle of radius r is formed.

(i) Express r in terms of x.

(ii) Find the value of x that will minimize the sum of the areas of the square & circle.

(i) $4x + 2\pi r = 100$

$\Rightarrow r = \dfrac{50-2x}{\pi}$

(ii) $A = x^2 + \pi r^2$

$A = x^2 + \dfrac{(50-2x)^2}{\pi}$

$\dfrac{dA}{dx} = 2x + \dfrac{1}{\pi} 2(50-2x)(-2)$

$= 2x - \dfrac{4}{\pi}(50-2x)$

P.T.O

At max:-

$$\frac{dA}{dx} = 0$$

$$\Rightarrow 2x - \frac{4}{\pi}(50 - 2x) = 0$$

$$\Rightarrow 2x - \frac{200}{\pi} + \frac{8}{\pi}x = 0$$

$$\Rightarrow x\left(2 + \frac{8}{\pi}\right) = \frac{200}{\pi}$$

$$\Rightarrow x\left(\frac{2\pi + 8}{\pi}\right) = \frac{200}{\pi}$$

$$\Rightarrow x(2\pi + 8) = 200$$

$$\Rightarrow x = \frac{200}{2\pi + 8}$$

$$\Rightarrow x = 14.0$$

$$\frac{dA}{dx} = 2x - \frac{4}{\pi}(50 - 2x)$$

$$\Rightarrow \frac{d^2A}{dx^2} = 2 - \frac{4}{\pi}(-2)$$

$$= 2 + \frac{8}{\pi} > 0$$

$\therefore$ $x = \frac{100}{\pi + 4}$ will minimize the area.

The sum of two numbers is 124. Find the number that will maximize their product.

find the number that will minimize the sum of a number and it's reciprocal. [The number is positive

P.T.O.

(1) $x + y = 124$

$\Rightarrow y = 124 - x$

$A = xy$

$= x(124 - x)$

$= 124x - x^2$

$\frac{dA}{dx} = 124 - 2x$

at max :-

$\frac{dA}{dx} = 0$

$124 - 2x = 0$

$\Rightarrow -2x = -124$

$\Rightarrow x = 62$

$\therefore y = 62$

$(x, y) = (62, 62)$

$$\frac{d^2A}{dx^2} = -2 < 0$$

local maximum available.

(ii) $x = 62$

$y = 62$

$$\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$$

$$= \frac{62+62}{62 \times 62}$$

$$= \frac{1}{31}$$

$x + \frac{1}{x}$

$y = x + \frac{1}{x}$

$y = x + (x)^{-1}$

$$\frac{dy}{dx} = 1 + -x^{-2}$$

$$= 1 - \frac{1}{x^2}$$

At max/min:-

$$1 - \frac{1}{x^2} = 0$$

$$\Rightarrow \frac{1}{x^2} = 1$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm 1 \Rightarrow x = 1 \text{ [positive number]}$$

$$\frac{d^2y}{dx^2} = -(x)^{-2}$$

$$= 2x^{-3}$$

$$= \frac{2}{x^3}$$

$$= 2 > 0$$

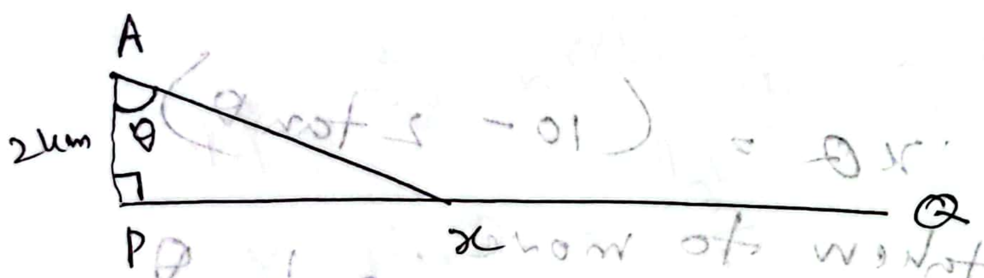
CW

4 (i) $\frac{d}{d\theta}\left(\frac{1}{\cos\theta}\right) = \frac{d}{d\theta}(\cos\theta)^{-1}$

$= (-1)(\cos\theta)^{-2} \times (-\sin\theta)$

$= \frac{\sin\theta}{\cos^2\theta}$

(ii)



$\cos\theta = \frac{2}{Ax}$

$\Rightarrow Ax = \frac{2}{\cos\theta}$

$\Rightarrow Ax = 2\sec\theta$

P.T.O.

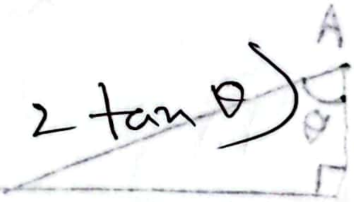
time taken to move from

A to x $= \dfrac{2\sec\theta}{3}$

Again, $\tan\theta = \dfrac{Px}{2}$

$Px = 2\tan\theta$

$\therefore \; xQ = (10 - 2\tan\theta)$



Time taken to move

x to Q $= \dfrac{10 - 2\tan\theta}{5}$

$= 2 - \dfrac{2}{5}\tan\theta$

Total time $= \dfrac{2\sec\theta}{3} + 2 - \dfrac{2}{5}\tan\theta$

$= \dfrac{2}{3}\dfrac{1}{\cos\theta} + 2 - \dfrac{2}{5}\dfrac{\sin\theta}{\cos\theta}$

$= 2.5331\ s$

$$\frac{dT}{d\theta} = \frac{2}{3}\sec\theta\tan\theta - \frac{2}{5}\sec^2\theta$$

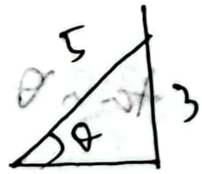
$$= \frac{2}{3}\cdot\frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta} - \frac{2}{5}\,\frac{1}{\cos^2\theta}$$

$$= \frac{2\sin\theta}{3\cos^2\theta} - \frac{2}{5\cos^2\theta} = 0$$

$$\Rightarrow \frac{2}{3}\sin\theta - \frac{2}{5} = 0$$

$$\Rightarrow \sin\theta = \frac{3}{5}$$

$$\Rightarrow \theta = 36.87$$



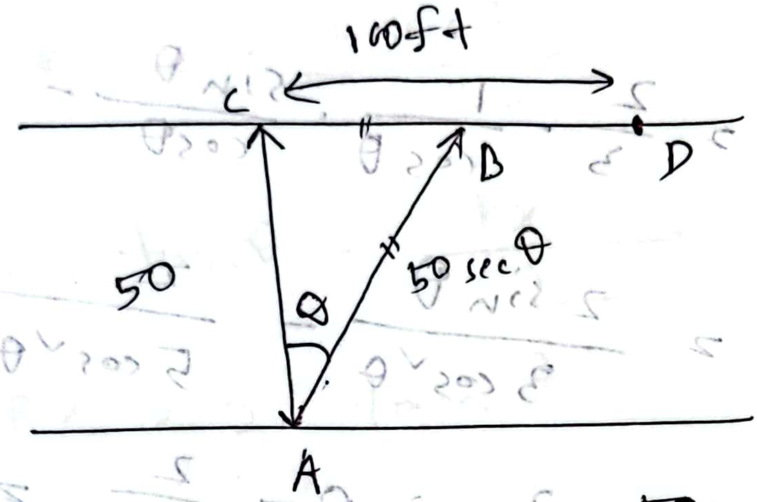
$$\tan\theta = \frac{3}{4}$$

$$\therefore px = 2\tan\theta$$

$$= 2\times\frac{3}{4}$$

$$= 1.5$$

Exm: 1



$\cancel{\cos\theta = \frac{50}{AB}}$ $\quad \cos\theta = \frac{50}{AB}$

$\Rightarrow AB = 50 \sec\theta, \; t = \frac{50\sec\theta}{5}$

$\tan\theta = \frac{BC}{50}$

$\Rightarrow BC = 50 \tan\theta$

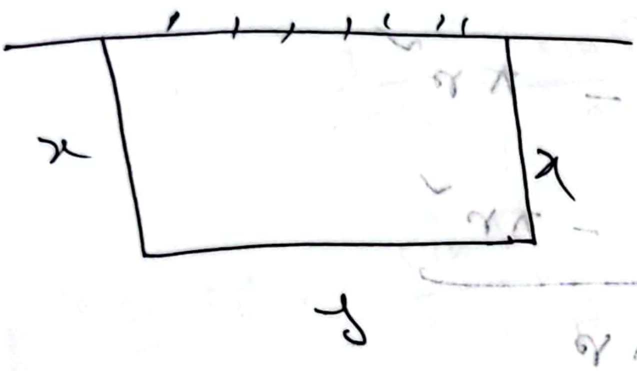
$BD = 100 - 50\tan\theta, \; t = \frac{100 - 50\tan\theta}{15}$

$$\text{total} = \frac{50\sec\theta}{5} + \frac{100 - 50\tan\theta}{15}$$

$$= 10\sec\theta + \frac{20}{3} - \frac{10}{3}\cancel{\sec}\tan\theta$$

$$= 10 \cdot \frac{1}{\cos\theta} + \frac{20}{3} - \frac{10\sin\theta}{3\cos\theta}$$

Exm: 2



$xy = 75 \Rightarrow y = \frac{75}{x}$

$C = (2x+y) \times 8 + 4y$

$= 16x + 8y + 4y$

$= 16x + 12y$

$= 16x + 12\left(\frac{75}{x}\right)$

$= 16x + \frac{900}{x}$

$\frac{dC}{dx} = 16 - \frac{900}{x^2}$

$16 - \frac{900}{x^2} = 0$

$\Rightarrow \frac{900}{x^2} = 16$

$\Rightarrow \frac{1}{x^2} = \frac{16}{900}$

$\Rightarrow x^2 = 56.25$

$\Rightarrow x = 7.5$

$\therefore y = 10$ Ans

P.T.O

3.

$\pi r^2 + \pi r L = 120 \, m^2$

$\pi r L = 120 - \pi r^2$

$\therefore L = \dfrac{120 - \pi r^2}{\pi r}$

$V = \frac{1}{2} \pi r^2 L$

$= \frac{1}{2} \pi r^2 \left(\dfrac{120 - \pi r^2}{\pi r} \right)$

$= \frac{1}{2} r (120 - \pi r^2)$

$= 60 r - \frac{1}{2} \pi r^3 = \cancel{194.083} = 142.79$

$\dfrac{dV}{dr} = 60 - \frac{1}{2} \pi \cdot 3 r^2 = 0$

$\Rightarrow \frac{1}{2} \pi 3 r^2 = 60$

$\Rightarrow r^2 = \dfrac{60}{\frac{3}{2} \pi} = 12.73$

$\Rightarrow r = 3.578$

$\dfrac{d^2 V}{dr^2} = -\frac{1}{2} \pi 6 r$

$= -33.65 < 0 \therefore$ maximum

$$\frac{dT}{d\theta} = 10 \sec\theta \tan\theta - \frac{10}{3} \sec^2\theta$$

$$= 10 \cdot \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta} - \frac{10}{3} \cdot \frac{1}{\cos^2\theta}$$

$$= 10 \cdot \frac{\sin\theta}{\cos^2\theta} - \frac{10}{3} \frac{1}{\cos^2\theta}$$

$$\Rightarrow 10 \sin\theta - \frac{10}{3} = 0$$

$$\Rightarrow 10 \sin\theta = \frac{10}{3}$$

$$\Rightarrow \sin\theta = \frac{10}{30}$$

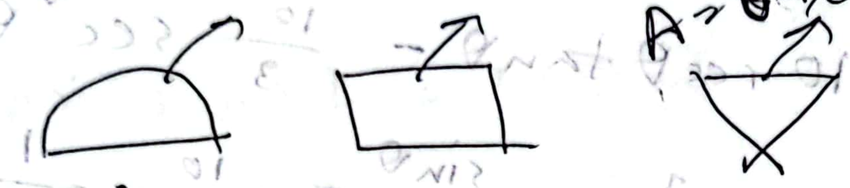
$$\Rightarrow \theta = \sin^{-1}\left(\frac{10}{30}\right)$$

$$= 19.4712$$

total time = 16.0948 sec

$$s = \frac{a+b+c}{2}$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

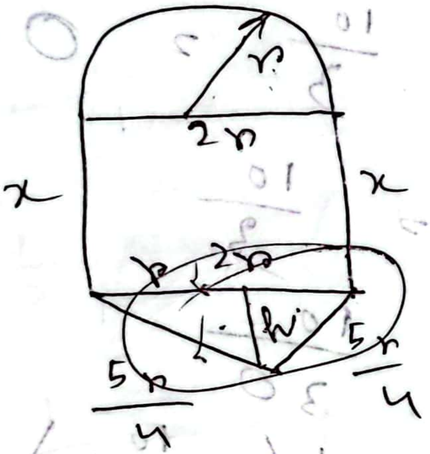


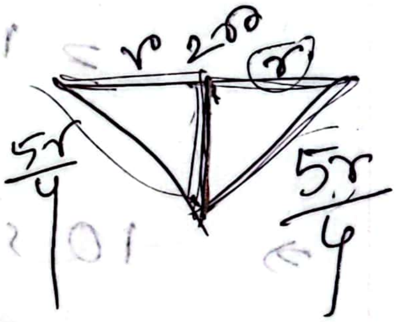
$$\left(\frac{5r}{4}\right)^2 =$$

9r6

$\frac{5r}{2}$

$s =$





$$\frac{1}{2} r \frac{5r}{4}$$

$$\frac{5r^2}{8} + \frac{5r^2}{8}$$

$$= \frac{5r^2}{8}$$

$$r^2 = 8$$

$$\frac{1}{2}\pi r^2 + (2rx) + \cancel{2rx\left(\frac{5r}{4}\right)}$$

$$\frac{\frac{5r}{4} + \frac{5r}{4} + 2r}{2}$$

$$= \frac{\frac{5r+5r+8r}{4}}{2}$$

$$= \frac{18r}{8} = \frac{9r}{4} = s$$

$$\frac{1}{2}\pi r^2 + 2rx + \frac{3r^2}{4} = 125$$

$$\Rightarrow 2rx = 125 - \frac{3r^2}{4} - \frac{1}{2}\pi r^2$$

$$\Rightarrow x = \frac{125}{2r} - \frac{\frac{3r^2}{4}}{2r} - \frac{\frac{1}{2}\pi r^2}{2r}$$

$$= \frac{125}{2r} - \frac{3r}{8} - \frac{\pi r}{4}$$

$$= \frac{50}{r} - \frac{3r}{8} - \frac{\pi r}{4}$$

$$\cancel{\frac{1}{2}\pi r^2 + 2rx + \frac{3r^2}{4} = A}$$

$$\cancel{A = \frac{1}{2}\pi r^2 + 2r\left(\frac{125}{2} - \frac{5r}{4} - \frac{\pi r}{2}\right)}$$

$$125 = \frac{1}{2}\cdot 2\pi r + 2x + 2\cdot\frac{5r}{4}$$

$$125 = \pi r + 2x + \frac{5r}{2}$$

$$\Rightarrow 2x = 125 - \frac{5r}{2} - \pi r$$

$$\Rightarrow x = \frac{125}{2} - \frac{5r}{4} - \frac{\pi r}{2}$$

$$A = \frac{1}{2}\pi r^2 + 2r\left(\frac{125}{2} - \frac{5r}{4} - \frac{\pi r}{2}\right) + \frac{3r^2}{4}$$

$$= \frac{1}{2}\pi r^2 + 125r - \frac{5r^2}{2} - \pi r^2 + \frac{3r^2}{4}$$

$$= 125r - \frac{1}{2}\pi r^2 - \frac{7}{4}r^2$$

$$\frac{dA}{dx} = 125 - \pi r - \frac{7}{2} r$$

$$125 - \pi r - \frac{7}{2} r = 0$$

$$\Rightarrow \pi r + \frac{7}{2} r = 125$$

$$\Rightarrow r\left(\pi + \frac{7}{2}\right) = 125$$

$$\Rightarrow r = \frac{125}{\left(\pi + \frac{7}{2}\right)} = 18.82 \text{ cm}$$

$$= \cancel{18.82 \text{ m}}$$

$$= \frac{18.82}{100} \text{ m}$$

$$= 0.1882 \text{ m}$$

$$A = 23.1407 \text{ m}^2$$

A

$\Rightarrow$ $\cancel{\frac{1}{}}$ $2\cancel{} + \cancel{}\, \wedge r^{\vee} + \wedge rL$

GW 12.2.23

C2

Chain Rule / Connected rate of change:-

$$y = [f(x)]^{n}$$

$$\frac{dy}{dx} = n[f(x)]^{n-1} \cdot \frac{d}{dx}(f(x))$$

$y \to x \to t$

$y \to f(x),\ x \to f(t)$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

P.T.O

A

* $y = (3x^2 - 1)^5$

$\Rightarrow 5(3x^2-1)^4 \cdot \frac{d}{dx}(3x^2-1)$

$\Rightarrow 5(3x^2-1)^4 \cdot 6x$

$= 30x(3x^2-1)^4$

* $y = \sin^5(4x-1)$

$\frac{dy}{dx} = 5\sin^4(4x-1) \cdot \cos(4x-1) \cdot 4$

$\Rightarrow$

power $\rightarrow$ Ratio $\rightarrow$ Angle

P.T.O

$y = \sqrt{\tan(7\theta + 1)}$

$$\frac{dy}{d\theta} = \left(\tan(7\theta + 1)\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\tan(7\theta + 1)^{-\frac{1}{2}} \sec^2(7\theta + 1) \cdot 7$$

$$= \frac{7}{2}\tan(7\theta + 1)^{-\frac{1}{2}} \sec^2(7\theta + 1)$$

$u = e^{2\sin^3(3\theta - 1)}$

$$\frac{du}{d\theta} = e^{2\sin^3(3\theta - 1)} \cdot 6\sin^2(3\theta - 1) \cdot \cos(3\theta - 1) \cdot 3$$

$$= 18 e^{2\sin(3\theta - 2)} \cdot \sin^2(3\theta - 1) \cdot \cos(3\theta - 1)$$

PTO

* $y = \ln\left(e^{2x}+1\right)$

$$\frac{dy}{dx} = \frac{1}{e^{2x}+1} \cdot e^{2x} \cdot 2$$

$$= \frac{2e^{2x}}{e^{2x}+1}$$

* $y = \ln\left(\sin(2x+1)\right)$

$$\frac{dy}{dx} = \frac{1}{\sin(2x+2)} \cdot \cos(2x+2) \cdot 2$$

$$= \frac{2\cos(2x+2)}{\sin(2x+1)}$$

* The rate of change of radius of a circle 0.25 cms^{-2}. find the rate of change of area of the circle when radius is 2 cm.

$\Rightarrow A = \pi r^2$ $\frac{dr}{dt} = 0.25\ cms^{-2}$

$\frac{dA}{dr} = 2\pi r$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

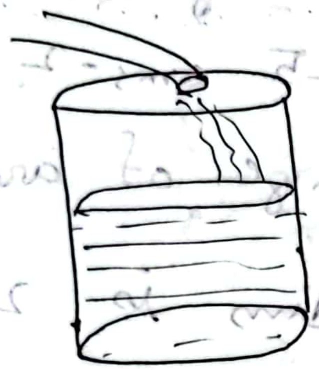
$\frac{dA}{dt} = 2\pi r \times 0.25$

$= 2\pi \times 2 \times 0.25$

$= \pi\ cm^2 s^{-1}$

PT.

$$V = \frac{4}{3} h^3$$



$$\frac{dv}{dt} = 7.5\ cm^3/s$$

Find the rate of change of the height when the volume of the cylinder is $25\ cm^3$.

$$\Rightarrow \frac{dh}{dt} = \frac{dh}{dv} \cdot \frac{dv}{dt}$$

$$\frac{dh}{dv} = \left(\frac{3v}{4}\right)^{\frac{1}{3}}$$

$$= \frac{1}{3}\left(\frac{3v}{4}\right)^{\frac{1}{3}-1} \cdot \left(\frac{1}{4} \cdot 3\right)$$

$$= \frac{1}{3}\left(\frac{3v}{4}\right)^{-\frac{2}{3}} \cdot \frac{3}{4}$$

$$= \frac{1}{4}\left(\frac{3v}{4}\right)^{-\frac{2}{3}}$$

$\Rightarrow \frac{dh}{dt}$ $V \rightarrow h \rightarrow t$

$$\frac{dv}{dt} = \frac{dv}{dh} \times \frac{dh}{dt}$$

$$25 = \frac{4}{3} h^3$$

$$\Rightarrow h^3 = \frac{25 \times 3}{4}$$

$$\Rightarrow h = \sqrt[3]{\frac{75}{4}}$$

$$z = e^{x+y} \sin(xy)$$

$$\frac{dz}{dx} = e^{x+y} \frac{d}{dx} \sin(xy) + \sin(xy) \frac{d}{dx}(e^{x+y})$$

$$= e^{x+y} \cdot \cos(xy) \cdot y + \sin(xy)\, e^{x+y} \frac{d}{dx}(x+y)$$

* Partial differentiation :-

$S = f(M, P, E \ldots)$

$\frac{\partial y}{\partial x}$ = Partial diff of y w.r.to x.

$\frac{\partial S}{\partial M}$ = change of S with respect to M considering P & E constant.

→ Partial diff of S with respect to M.

* When a variable is a function of more than one independent variable then the differentiation of the given dependent variable with respect to one independent variable (keeping other constant) is known as partial diff.

$u = f(x, y, z)$

$\frac{\partial u}{\partial x}$

$\frac{\partial u}{\partial y}$

$\frac{\partial u}{\partial z}$

partial differentiation

$y = f(x) \rightarrow$ explicit function

$f(x, y) = 0 \rightarrow$ implicit "

* $z = x^3 + 2xy + y^2$

$\frac{\partial z}{\partial x} = 3x^2 + 2y.1 + 0$

$= 3x^2 + 2y$

$\frac{\partial z}{\partial y} = 0 + 2x.1 + 2y$

$= 2x + 2y$

GW 15.2.23

C2

Partial differentiation :-

$$z = f(x, y)$$

The differentiation of z with respect to x, holding y fixed is known as partial diff of z with respect to x. and denoted by $\frac{\partial z}{\partial x} = f_x(x, y)$

$\frac{\partial z}{\partial y} = f_y(x, y) \Rightarrow$ Partial diff of z with respect to y.

* $f(x, y) = x^3 + 3xy^2 - y^2 + 10$

$f_x(x, y) = 3x^2 + 3y^2$

$f_y(x, y) = 6xy - 2y$

P.T.O

* $z = f(x, y) = 2x^2y^3 + 2y + 4x - y$ *

$f_x = 4xy^3 + 4$

$f_y = 6x^2y^2 + 2$

$f_x(-1, 2) = -28$

$f_y(2, 3) = 56$

Notation and higher order partial derivatives :-

$u = f(x, y, z)$

$\frac{\partial u}{\partial x} = f_x(x, y, z)$ *

$\frac{\partial^2 u}{\partial x^2} = f_{xx}(x, y, z) \rightarrow$ 2nd derivative w.r.to x *

P.T.O.

* $\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = f_{xy}(x, y, z)$

→ First x derivative than y.

* $\frac{\partial^3 u}{\partial x \partial y \partial z} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial z}\right)\right)$

$= f_{zyx}$

→ First z then y Last x

$u = f(x, y)$ is continuous over a given domain than $\boxed{f_{xy} = f_{yx}}$

P.T.O.

$$\boxed{\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}}$$

$f(x,y) = x^3 y^2 + 3xy - 2y^3$

(i) $f_{xx} = 3x^2 y^2 + 3y$

$= 6xy^2$

(ii) $f_{xy} = \cancel{2x^3 y + 3x}$

$= 3x^2 y^2 + 3y$

$= 6x^2 y + 3$

(iii) $f_{yx} = 2x^3 y + 3x - 3y^2$

$= 6x^2 y + 3$

(iv) $f_{yy} = 2x^3 y + 3x - 3y^2$

$= 2x^3 - 6y$

* Differentiation of implicit function :-

$f(x, y) = x^3 + 6xy - y^2 + 10$

$x^2 + y^2 - 3xy + 10 = 0$

Find $\frac{dy}{dx} = ?$

* Method I ⇒

$y = \frac{dy}{dx}$ $\qquad e^y = e^y \cdot \frac{dy}{dx}$

$y^2 = 2y \frac{dy}{dx}$ $\qquad \sin y = \cos y \cdot \frac{dy}{dx}$

$\ln y = \frac{1}{y} \cdot \frac{dy}{dx}$

y diff with r.s to x.

P.T.O.

Solve:

$x^2 + y^2 - 3xy + 10 = 0$

Diff with respect to x:

$2x + 2y \cdot \frac{dy}{dx} - 3\left[x\frac{dy}{dx} + y \cdot 1\right] = 0$

$\Rightarrow 2x + 2y\frac{dy}{dx} - 3x\frac{dy}{dx} - 3y = 0$

$\Rightarrow 2y\frac{dy}{dx} - 3x\frac{dy}{dx} = 3y - 2x$

$\Rightarrow \frac{dy}{dx}(2y - 3x) = (3y - 2x)$

$\Rightarrow \frac{dy}{dx} = \frac{3y - 2x}{2y - 3x}$ Ans

P.T.O.

Method 2:

$$\boxed{\frac{\partial y}{\partial x} = \frac{-f_x}{f_y}}$$ formula

Solve:

$f(x, y) = x^2 + y^2 - 3xy + 10$

$f_x = 2x - 3y$

$f_y = 2y - 3x$

$$\therefore \frac{dy}{dx} = \frac{-(2x-3y)}{2y-3x}$$

$$= \frac{3y-2x}{2y-3x}$$

P.T.O

* $2x^3 - y^2 + 3x + 2y + 27 = 0$

$$\frac{dy}{dx} = \frac{-f_x}{f_y}$$

$$f_x = 6x^2 + 3$$

$$f_y = -2y + 2$$

$$\frac{dy}{dx} = \frac{-6x^2 - 3}{2 - 2y}$$

$\therefore (-2, 2) = \frac{9}{2}$

* Inflaction ~~point~~

Mid

* Slope & Connectivity :⇒ (31²)

4.1.1, 4.1.2, 4.1.4, 4.1.5

Exm :- (1-5)

Ex :- 4.1 : 9, 10, (15-20)

* Analysis of function II : Relative Extrema & Polynomials :⇒ (31⁴)

{4.2 (1, 2, 3, 4)}

Exm :- (1-8)

Ex :⇒ 4.2 ⇒ 33-54

* Partial Derivatives :- (x,6)

13.3 Exm :- (1-5), (10-14)

Ex :. 1-13, 25-52, 85-95, 95-104

clear

Chain Rule : (7, 8)

13.5. (1, 2, 3, 4, 5)

Exm :- 1 - 8

Ex :- (1-10) (17-34) (41-44) (50-54)

Linear Algebra :- (9, 10, 11)

2.1 ⇒ Exm : (1-7)

Ex :- (1-23) odd

2.2 ⇒ Exm :- (1-7)

Ex :- (1-37) odd, 43, 45

$y=(2x+1)^3$

$\Rightarrow y = 8x^3 + 3(2x)^2 \cdot 1 + 3 \cdot 2x \cdot 1^2 + 1^3$

$= 8x^3 + 12x^2 + 6x + 1$

$y_1 = 24x^2 + 24x + 6 = 0$

$\Rightarrow 4x^2 + 4x + 1 = 0$

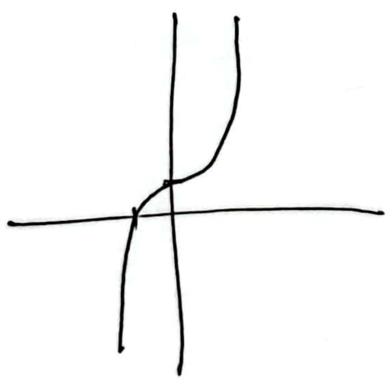
$\Rightarrow x = -\frac{1}{2}$

$\left(-\frac{1}{2}, 0\right)$ turning point

$\therefore y = 0$

$y_2 = 8x + 4$

$y_2\Big|_{-\frac{1}{2}} = 0$, Neither



19.2.23

* Chain rule :-

$x = f(t), \quad y = f(t)$

$z = f(x, y)$

$$
\begin{tikzcd}[row sep=small]
 & x \arrow[r] & t \\
Z \arrow[ur] \arrow[dr] & & \\
 & y \arrow[r] & t
\end{tikzcd}
$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \times \frac{dx}{dt} + \frac{\partial z}{\partial y} \times \frac{dy}{dt}$$

* $z = x^2 y + 5$

$x = t^2 + 1$

$y = \sqrt{t}$

$\frac{dz}{dt} = ?$

P.T.O.

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \times \frac{dx}{dt} + \frac{\partial z}{\partial y} \times \frac{dy}{dt}$$

$$= 2xy \times 2t + x^2 \times \frac{1}{2\sqrt{t}}$$

$$= 2(t^2+1)\sqrt{t} \times 2t + (t^2+1)^2 \times \frac{1}{2\sqrt{t}}$$

Ans

$u = f(x, y, z)$ and $x = f(t)$,

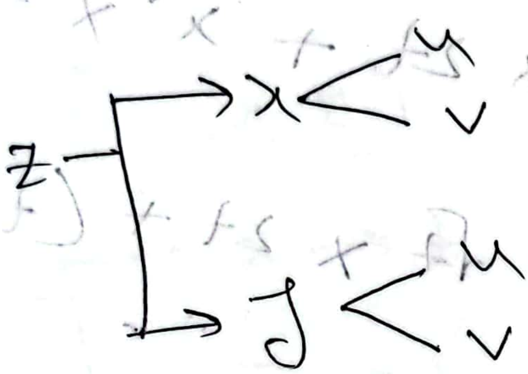
$y = g(t)$, $z = h(t)$

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \times \frac{dx}{dt} + \frac{\partial u}{\partial y} \times \frac{dy}{dt} + \frac{\partial u}{\partial z} \times \frac{dz}{dt}$$

$$u \begin{cases} \to x \to t \\ \to y \to t \\ \to z \to t \end{cases}$$

P.T.O

If $z = f(x, y)$ and $x = f(u, v)$, $y = f(u, v)$



$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \times \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \times \frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \times \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \times \frac{\partial y}{\partial v}$$

13.5

12

$$Z = 8x^2y - 2x + 3y \; ; \quad x = uv$$
$$y = u - v$$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \times \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \times \frac{\partial y}{\partial u}$$

$$= (16xy - 2) \times v + (8x^2 + 3) \cdot 1$$

$$= \{16(uv)\cdot(u-v) - 2\} \times v \; (8(uv)^2 + 3) \times 1$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \times \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \times \frac{\partial y}{\partial v}$$

$$= (16xy - 2) \times u + (8x^2 + 3) \times 1$$

$$= \{16(uv)(u-v) - 2\} \times u + (8(uv)^2 + 3) \times 1$$

CW 20.2.23

Maximum and Minimum of function of two variables:-

$z = f(x,y)$ be a function of two independent variable x & y.

At turning / stationary point

$$\frac{\partial z}{\partial x} = f_x(x,y) = 0 \text{ and also}$$

$$\frac{\partial z}{\partial y} = f_y(x,y) = 0$$

[first derivative test]

2nd derivative test:-

$$\frac{\partial^2 z}{\partial x^2} = f_{xx}(x,y), \quad \frac{\partial^2 z}{\partial y^2} = f_{yy}(x,y).$$

$$\frac{\partial^2 z}{\partial x \partial y} = f_{yx} = f_{xy} = \frac{\partial^2 z}{\partial y \partial x}$$

* $f_{xx} f_{yy} - (f_{xy})^2 = D$

(i) if $D > 0$ & $f_{xx} > 0$ for (x_0, y_0) then there exists a relative minimum.

(ii) if $D > 0$ & $f_{xx} < 0$; then there exists a relative Maximum.

(iii) if $D < 0$; then (x_0, y_0) is a saddle point.

(iv) if $D = 0$, no conclusion can be drawn.

P.T.O.

Exm:7

Locate all relative extrema and saddle points of

$f(x,y) = 3x^2 - 2xy + y^2 - 18y$

$f_x(x,y) = 6x - 2y = 0$

$\Rightarrow y = 3x$ —— ①

$f_y(x,y) = -2x + 2y - 8 = 0$

$\Rightarrow x - y = -4$ —— ⑪

for ① and ⑪,

$x - 3x = -4$

$\Rightarrow x = 2, \quad y = 6$

$\therefore (2, 6).$

$f_{xx} = 6$ #

$f_{yy} = 2$

$f_{xy} = -2$

$$D = f_{xx} f_{yy} - (f_{xy})^2$$

$$= 12 - (2)^2$$

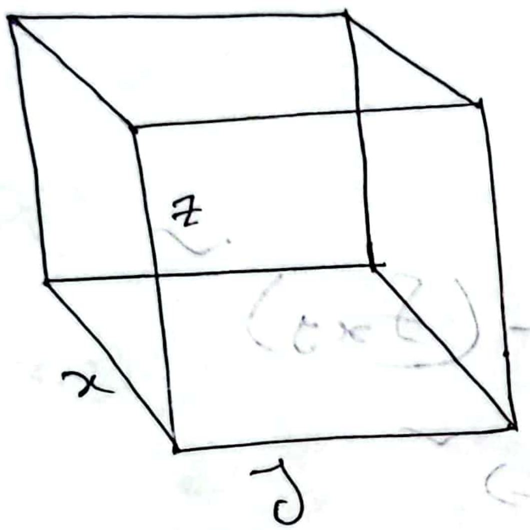
$$= 12 - 4$$

$$= 8 > 0$$

since $D > 0$, $f_{xx} > 0$ at $(2, 6)$

relative minimum.

#



volume of the box = 32 cm^3

Find the dimension which will be required for minimize at construction.

$\Rightarrow xyz = 32$, $z = \dfrac{32}{xy}$

$S = xy + 2xy + 2yz$

P.T.O

$S = xy + 2\cdot x \cdot \frac{32}{xy} + 2\cdot y \cdot \frac{32}{xy}$

$= xy + \frac{64}{y} + \frac{64}{x}$

$S_x = y - \frac{64}{x^2}$

$S_y = x - \frac{64}{y^2}$

$y - \frac{64}{x^2} = 0$

$\Rightarrow \frac{64}{x^2} = y$

$\Rightarrow x^2 = \frac{64}{y}$

$\Rightarrow \cancel{x^2 = \frac{64}{y}}$

$\Rightarrow y = \frac{64}{x^2}$

$\therefore y = 4$

$x - \frac{64}{y^2} = 0$

$\Rightarrow x - \frac{64}{\left(\frac{64}{x^2}\right)^2} = 0$

$\Rightarrow x - \frac{64}{\frac{4096}{x^4}} = 0$

$\Rightarrow x - \frac{64x^4}{4096} = 0$

$\Rightarrow \frac{4096x - 64x^4}{4096} = 0$

$\Rightarrow 4096x - 64x^4 = 0$

$\Rightarrow 64x(64 - x^3) = 0$

$\Rightarrow \cancel{x^4 = 64}$

$\Rightarrow x = 0, 4$

$f_{xx} = \frac{128}{x^3} = \frac{128}{64} = 2 > 0$

$f_{yy} = \frac{128}{y^3} = \frac{128}{64} = 2 > 0$

$f_{xy} = 1$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$= 2 \times 2 - (1)^2$

$= 3 > 0$

$z = \frac{32}{xy} = \frac{32}{4 \times 4} = 2$

$\therefore x = 4$, $y = 4$ and $z = 2$ will be required to have the minimum volume.

Differential equation :-

Any equation that includes one or more terms of differentiation is known as differential equation.

- → ordinary (ODE)
- → partial (PDE)

$$* \quad \frac{dy}{dx} = 2xy$$

$$* \quad x\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + 5 = 0$$

} (ODE)

$$* \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + 10 = 0 \text{ — (PDE)}$$

P.T.O

*

$$\frac{dA}{dt} \propto t$$

$$\frac{dA}{dt} = kt$$

C.W 22.2.23

Differential Equation:-

$$x^2 \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 0$$

Order of a differential Equation:-

differential equation is the order of the highest derivative.

$$\frac{d^3 y}{dx^3} + (x+1)\frac{dy}{dx} + xe^x = 0$$

$\frac{d^3 y}{dx^3}$ → 3rd order

$\frac{dy}{dx}$ → 1st order

[3rd order equation]

Degree of differential equation:-

→ power of the heighest derivatives to which it is raised.

$$x\left(\frac{d^2 y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2x - 5 = 0$$

→ [2nd degree differential equation]

Solution of a differential equation :-

$y = \sin x$

$\frac{dy}{dx} = \cos x$

$\frac{d^2y}{dx^2} = -\sin x$

$-\sin x + \sin x = 0$

$y = \sin x$ is the solution of this differential equation.

The solution of a differential equation is the function that satisfied the given differential equation.

$\Rightarrow \frac{d^2x}{dt^2} \propto x$

$x \rightarrow \frac{dx}{dt} = v$

$\frac{dv}{dt} = \frac{d^2x}{dt^2} = a$

$\frac{d^2x}{dt^2} = kx$

P.T.O

solution of a diff equation:-

- → General solution (GS)
- → Particular solution (PS)

* Any solution that includes an arbitrary constant is known as G.S.

→ $y = x^2 + c$

$y = x^2 + 2$

[P.S]

Linear & non-Linear differential equation:-

* $(x^2+1)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = 5$ [linear]

* $(x+y)\dfrac{d^3y}{dx^3} + y\dfrac{dy}{dx} = x^2+1$ [Non]

* $\sqrt{y}\,\dfrac{dy}{dx} + (x+y)\dfrac{d^2y}{dx^2} = 10$ [Non]

P.T.O.

* $a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{(n-1)}(x)\frac{dy}{dx} + a_n = 0$

where $a_0, a_1, \dots a_n \rightarrow$ are the coefficients and function of x

$\rightarrow$ [linear]

* otherwise all non-linear.

Techniques of solving differential equation:-

(1) $\frac{dy}{dx} = f(x)$

$y = \int f(x)\,dx$

* $\frac{dy}{dx} = 2x+5$ [$y(1) = 3$]

$y = \int 2x+5 = x^2 + 5x + c$

$x = 1, \quad y = 3$

$3 = 1 + 5 + c$

$\Rightarrow c = -3$

$\therefore y = x^2 + 5x - 3$

Given that $y = f(x)$ and the gradient of y is $-3x^2 + 4x + 1$ and the curve passes through the point $(-5, 7)$. Find y in terms of x

$y = \int -3x^2 + 4x + 1$

$= -\frac{3x^3}{3} + \frac{4x^2}{2} + x + c$

$y = -\frac{3x^3}{3} + 2x^2 + x + c$

$\Rightarrow 7 = -\frac{3 \times (-5)^3}{3} + 2 \times (-5)^2 + (-5) + c$

$\Rightarrow 7 = 170 + c$

$\Rightarrow c = -163$

$\therefore y = -\frac{3x^3}{3} + 2x^2 + x - 163$

$\therefore y = -x^3 + 2x^2 + x - 163$ Ans

Variable separable :-

$$\frac{dy}{dx} = f(x) \times g(y)$$

or, $f(y)\,dy = f(x)\,dx$

$$\frac{dy}{dx} = f(x)\, f\, y$$

$$F(y)\,dy = F(x)\,dx$$

$$\Rightarrow \int F(y)\,dy = \int F(x)\,dx$$

$$\therefore \frac{dy}{dx} = (x^2+1)\sqrt{y}$$

$$\frac{dy}{\sqrt{y}} = (x^2+1)\,dx$$

$$\Rightarrow \int y^{-\frac{1}{2}}\,dy = \int (x^2+1)\,dx$$

$$\Rightarrow \frac{y^{\frac{1}{2}}}{\frac{1}{2}} = \frac{1}{3}x^3 + x + c$$

$$\Rightarrow 2\sqrt{y} = \frac{1}{3}x^3 + x + c$$

* $(1+y)\,dx = x\,dy$

$\Rightarrow \dfrac{(1+y)}{dy} = \dfrac{x}{dx}$

$\Rightarrow \dfrac{dy}{(1+y)} = \dfrac{dx}{x}$

$= \cancel{\int (1+y)^{-1}\,dy = \int \frac{1}{x}\,dx}$

$= \cancel{\frac{(1+y)^{0}}{0}} =$

$= \int \dfrac{1}{(1+y)}\,dy = \int \dfrac{1}{x}\,dx$

$= \ln(1+y) = \ln(x) + \ln c$

$\Rightarrow \ln(1+y) = \ln(cx)$

$\Rightarrow 1+y = cx$

$\Rightarrow y = cx - 1$

C.W

C_2

Diff Eq

$A \to r \to t$

$$\frac{dA}{dt} \propto V$$

$$\Rightarrow \frac{dA}{dt} = k \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{dA}{dr} \times \frac{dr}{dt} = \frac{4k}{3}\pi r^3$$

$$\Rightarrow 8\pi r \times \frac{dr}{dt} = \frac{4k}{3}\pi r^3$$

$$\Rightarrow \frac{dr}{dt} = \frac{4k\pi r^3}{3 \cdot 8\pi r}$$

$$\Rightarrow \frac{dr}{dt} = \frac{k r^2}{6}$$

$$\Rightarrow 2 = \frac{k}{6} \times 5$$

$$\Rightarrow k = \frac{12}{25}$$

$$\therefore \frac{dr}{dt} = \frac{12\, r^2}{25 \times 6}$$

$$= 0.08\, r^2$$

$$\frac{dr}{dt} = 0.08\, r^2$$

$$\cancel{\int 0.08\, r^2}$$

$$\int \frac{dr}{r^2} = \int 0.08\, dt$$

$$\int \frac{1}{r^2}\, dr = 0.08\, t$$

$$\Rightarrow \int r^{-2}\, dr = 0.08\, t$$

$$\Rightarrow -r^{-1} = 0.08\, t + C$$

$$\Rightarrow -\frac{1}{r} = 0.08t + C$$

$$\Rightarrow -\frac{1}{5} = C \quad \therefore C = -0.2$$

$$\Rightarrow -\frac{1}{r} = 0.087t - 0.2$$

$$\Rightarrow \frac{1}{r} = 0.2 - 0.087t$$

$$\Rightarrow r = \frac{1}{0.2 - 0.087t}$$

(iii)

$$0.2 - 0.087t > 0$$

$$\Rightarrow 0.087t < 0.2$$

$$\Rightarrow t < \frac{0.2}{0.08}$$

$$\Rightarrow 0 < t < 2.5$$

Q 18

(a) $\frac{dv}{dt} \propto -v$

$\frac{dv}{dt} = -kv$

(b) $t = 0, \quad V = \$10,000$

$\frac{dv}{dt} = -kv$

$\Rightarrow \int \frac{dv}{v} = \int -k\,dt$

$\Rightarrow \ln v = -kt + C$

$\Rightarrow V = e^{-kt + c}$

$\Rightarrow V = e^{-kt} \cdot e^{c}$

$\Rightarrow 10{,}000 = e^{0} \cdot e^{c}$

$\Rightarrow e^{c} = 10000$

$\therefore V = 10{,}000\, e^{-kt}$

$t = 36, \quad V = 5000$

$\Rightarrow 5000 = 10{,}000 \times e^{-36k}$

$\Rightarrow e^{-36k} = \frac{1}{2}$

$\Rightarrow -36k = \ln\left(\frac{1}{2}\right)$

$\Rightarrow k = \frac{-0.693}{-36}$

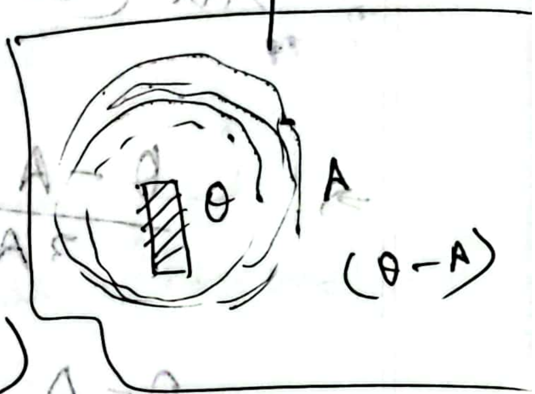
$= 0.01925$

$\therefore V = 10{,}000\, e^{-0.01925t}$

$$V = 10000\, e^{-0.01925 \times 24}$$

$$= 6300.22$$

$t = 24$ m



Q1

(1) $$\frac{d\theta}{dt} \propto (\theta - A)$$

$$\Rightarrow \frac{d\theta}{dt} = -k(\theta - A)$$

$$\int \frac{d\theta}{\theta - A} = \int -k\,dt$$

$$\Rightarrow \cancel{\ln(\theta - A) = -kt}$$

$$\Rightarrow \ln(\theta - A) = -kt + c$$

$$\Rightarrow \ln(4A - A) = 0 + c$$

$$\Rightarrow \ln 3A = c$$

$$\Rightarrow \ln(\theta - A) = -kt + \ln 3A$$

$$\Rightarrow \ln\left(\frac{\theta - A}{3A}\right) = -kt$$

$$\Rightarrow \frac{\theta - A}{3A} = e^{-kt}$$

$$\Rightarrow \theta - A = 3Ae^{-kt}$$

$$\Rightarrow \theta = A + 3Ae^{-kt} \quad (1)$$

(ii) $3A = A + 3Ae^{-kt}$

$$\Rightarrow 2A = 3Ae^{-kt}$$

$$\Rightarrow \cancel{\frac{2}{3} = e^{-k}}$$

$$\Rightarrow -kt = \ln \frac{2}{3}$$

$$\Rightarrow kt = \ln \frac{3}{2}$$

Linear ~~vs~~ First order differential equation :-

$$\boxed{\frac{dy}{dx} + P(x)\,y = Q(x)}$$

$\to$ First order linear

[Integration Factor $= e^{\int P(x)\,dx}$]

$$x^2 \frac{dy}{dx} + (2x+1)\,y = e^{x}$$

$$* \quad x \frac{dy}{dx} - 4y = x^{5}e^{x}$$

$$\Rightarrow \frac{dy}{dx} - \frac{4}{x}y = x^{4}e^{x}$$ [Standard form] of

$$\theta = A\left(1 + 3e^{-kt}\right)$$

$$= A\left(1 + 3e^{-\ln\frac{3}{2}\times 2}\right)$$

$$= \frac{7}{3}A$$

$$I.F = e^{\int -\frac{4}{x}\,dx}$$

$$= e^{-4\ln x}$$

$$= e^{\ln x^{-4}}$$

$$= x^{-4}$$

$$\Rightarrow \cancel{x^{-3}\frac{dy}{dx} - 4x^{-4}y = e^{x}}$$

$$x^{-4}\frac{dy}{dx} - 4x^{-5}y = x^{0}e^{x}$$

$$\Rightarrow \frac{d}{dx}\left[x^{-4}y\right] = e^{x}$$

$$x^{-4}y = \int e^{x}\,dx$$

$$x^{-4}y = e^{x} + c$$

$$\Rightarrow y = x^{4}e^{x} + cx^{4}$$

C.W

C_2

~~$\frac{dy}{dx}$~~

* $(x^2+9)\frac{dy}{dx}+xy=0$

$\Rightarrow \frac{dy}{dx}+\left(\frac{x}{x^2+9}\right)y=0$

$$I.F = e^{\int \frac{x}{x^2+9}dx}$$

$$\boxed{* \int \frac{f'(x)}{f(x)}dx = \ln f(x)}$$

$$= e^{\frac{1}{2}\int \frac{2x}{x^2+9}dx}$$

$$= e^{\frac{1}{2}\ln(x^2+9)}$$

$$= e^{\ln\sqrt{x^2+9}}$$

$$= \sqrt{x^2+9}$$

$$\sqrt{x^2+9}\,\frac{dy}{dx}+\left(\frac{x}{\sqrt{x^2+9}}\right)y=0$$

$$\Rightarrow \frac{d}{dx}\left(\sqrt{x^2+9}\times y\right)=0$$

$$\Rightarrow \sqrt{x^2+9}\cdot y = C$$

$$\therefore y=\frac{C}{\sqrt{x^2+9}}$$

$x\frac{dy}{dx} + y = 2x$, $y(1) = 0$

$\Rightarrow \frac{dy}{dx} + \frac{1}{x} y = 2$

$\Rightarrow I.F = e^{\int \frac{1}{x} dx}$

$= e^{\ln x}$

$= x$

$x\frac{dy}{dx} + y = 2x$

$\Rightarrow \frac{d}{dx}(xy) = 2x$

$\Rightarrow xy = x^2 + c$

$\Rightarrow 0 = 1 + c$

$\Rightarrow c = -1$

$xy = x^2 - 1$

$\Rightarrow y = x - \frac{1}{x}$

* $\frac{dy}{dx} = y + e^{x}$ (ii)

* $\frac{dx}{dy} = x + y.$

* $y' = 2y + x(e^{3x} - e^{2x}),$

$y(0) = 2$

* $\frac{dy}{dx} - y = e^{x}$

$$I.F = e^{\int -1\, dx}$$

$$= e^{-x}$$

$$e^{-x}\frac{dy}{dx} - e^{-x}y = 1$$

$$\Rightarrow \frac{d}{dx}(e^{-x}\cdot y) = 1$$

$$\Rightarrow e^{-x}y = x + C$$

$$\Rightarrow y = xe^{x} + Ce^{x}$$

(ii) $\frac{dx}{dy} - x = y$

$\therefore I.F = e^{\int -1\, dy}$

$= e^{-y}$

$\Rightarrow e^{-y}\frac{dx}{dy} - xe^{-y} = ye^{-y}$

$\Rightarrow \frac{d}{dy}(e^{-y}\cdot x) = ye^{-y}$

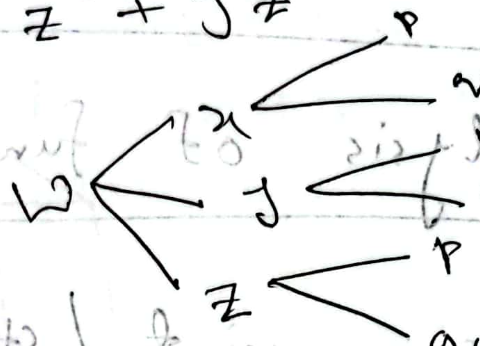
$\Rightarrow xe^{-y} = \int ye^{-y}\, dy$

Assignment 2

Q*

$$w = x^2 y - 3x^2 z^2 + yz$$

$x = p^2 q$



$y = p - q$

$z = q^3 p$

$$\frac{\partial w}{\partial p} = \frac{\partial w}{\partial x} \times \frac{\partial x}{\partial p} + \frac{\partial w}{\partial y} \times \frac{\partial y}{\partial p} + \frac{\partial w}{\partial z} \times \frac{\partial z}{\partial p}$$

GW

* Newton's law of cooling

* Analysis of function:

* Turning point / stationary point

$\frac{dy}{dx} = 0$

Max, $\frac{d^2y}{dx^2} < 0$

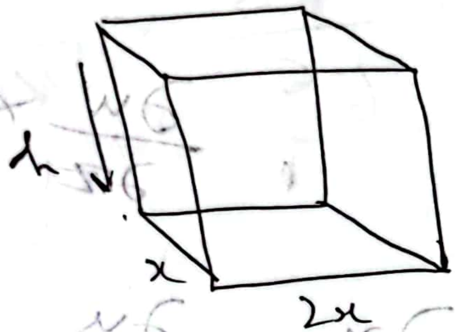
Min, $\frac{d^2y}{dx^2} > 0$

$f'(x) > 0$ [increasing]

$f'(x) < 0$ [decreasing]

* optimization!

4)



$$A = 2(x \times 2x + x \times h + 2x \times h)$$

$$= 2(2x^2 + 3xh) = 120$$

$$\Rightarrow 4x^2 + 6xh = 120$$

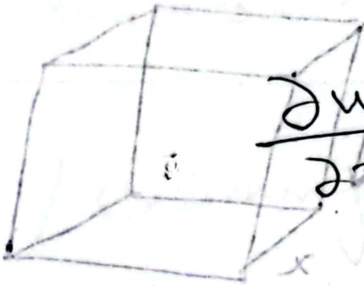
$$\Rightarrow h = \frac{60 - 2x^2}{3x}$$

$$V = x \times 2x \times h$$

$$= 2x^2\left(\frac{60-2x^2}{3x}\right)$$

$$= 40x - \frac{4x^3}{3}$$

$$\frac{\partial u}{\partial p} = \frac{\partial u}{\partial x} \times \frac{\partial x}{\partial p} + \frac{\partial u}{\partial y} \times \frac{\partial y}{\partial p} + \frac{\partial u}{\partial z} \times \frac{\partial z}{\partial p}$$



$$\frac{\partial u}{\partial Q} = \frac{\partial u}{\partial x} \times \frac{\partial x}{\partial Q} + \frac{\partial u}{\partial y} \times \frac{\partial y}{\partial Q} + \frac{\partial u}{\partial z} \times \frac{\partial z}{\partial Q}$$

$$\frac{\partial w}{\partial t} = \left(\frac{\partial w}{\partial u} \times \frac{\partial u}{\partial x} \times \frac{dx}{dt}\right) + \left(\frac{\partial w}{\partial v} \times \frac{\partial v}{\partial x} \times \frac{dx}{dt}\right) +$$

$$\left(\frac{\partial w}{\partial u} \times \frac{\partial u}{\partial y} \times \frac{\partial y}{\partial t}\right) +$$

$$\left(\frac{\partial w}{\partial v} \times \frac{\partial v}{\partial y} \times \frac{dy}{dt}\right)$$

Work sheet 3

Ans to the ques no: 1

$$f'(x) = 2x^{-\frac{1}{3}} - x^{+\frac{1}{3}}$$

$$f(x) = \frac{2x^{\frac{2}{3}}}{\frac{2}{3}} - \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + C$$

$$\Rightarrow 5 = \frac{2\cdot(8)^{\frac{2}{3}}}{\frac{2}{3}} - \frac{(8)^{\frac{4}{3}}}{\frac{4}{3}} + C$$

$$\Rightarrow 5 = 0 + C$$

$$\Rightarrow C = 5$$

$$y = \frac{2x^{\frac{2}{3}}}{\frac{2}{3}} - \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + 5$$

Ans

P.T.O

Ans to the ques no: 2 (d.)

$$\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$$

$$\frac{dy}{dx} = \frac{6x^3}{3} - 4 \cdot \frac{x^{-2}}{-2}$$

$$= 2x^3 + \frac{2}{x^2}$$

$$y = \frac{2x^4}{4} + 2 \cdot \frac{x^{-1}}{-1}$$

$$= \frac{x^4}{2} + 2 \cdot \frac{x^{-1}}{-1}$$

$$\Rightarrow \frac{x^4}{2} - \frac{2}{x}$$

(1) $x = -1$, $\frac{d^2y}{dx^2} = 6 \times (-1)^2 - \frac{4}{(-1)^3}$

$= 10 > 0$

The function has local minimum.

(b)

$$\frac{dy}{dt} = 5 \text{ unit/s}$$

$$\frac{dy}{dx}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\Rightarrow \cancel{\left(2x^3 + \frac{2}{x^2}\right) =}$$

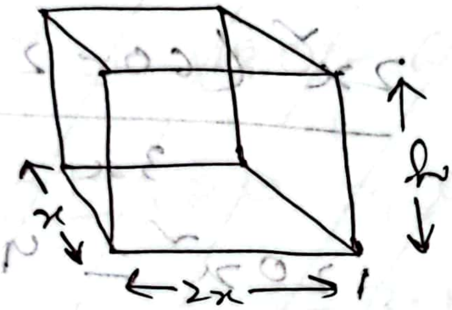
$$\Rightarrow 5 = \left(2x^3 + \frac{2}{x^2}\right) \times \frac{dx}{dt}$$

$$\Rightarrow 5 = 4 \times \frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = 1.25 \text{ unit/s}$$

Ans

Ans. to the ques no: 4 (ii)



$a = x$

$b = 2x$

$c = h$

(i)

$$2(ab + bc + ca) = 120$$

$$\Rightarrow 2(2x^2 + 2xh + xh) = 120$$

$$\Rightarrow \cancel{4x^2 + 4xh + 2xh}$$

$$\Rightarrow 2x^2 + 2xh + xh = 60$$

$$\Rightarrow 2xh + xh = 60 - 2x^2$$

$$\Rightarrow 3xh = 60 - 2x^2$$

$$\Rightarrow h = \frac{60 - 2x^2}{3x}$$

$$\Rightarrow \cancel{\frac{4x}{3} = \frac{60 - 2x^2}{3x}}$$

P.T.O

(ii)

$$V = 2x^2 h = 2x^2\left(\frac{60 - 2x^2}{3x}\right)$$

$$= \frac{2x^2(60 - 2x^2)}{3x}$$

$$= \frac{120x^2 - 4x^4}{3x}$$

$$= 40x - \frac{4x^3}{3}$$

(iii)

$$\frac{dv}{dx} = 40 - \frac{12x^2}{3}$$

$$= 40 - 4x^2$$

$\cancel{x = 3.16}$

$$40 - 4x^2 = 0$$

$$\Rightarrow x = (3.16, -3.16)$$

{x value can't be negative}

~~[x value can't be negative]~~

$$\frac{4x}{3} = \frac{60 - 12x^2}{3x}$$

$\Rightarrow 12x^2 = 180 - 6x^2$

$\Rightarrow 18x^2 - 180 = 0$

$\Rightarrow x = 3.16, -3.16$

$h = \frac{4x}{3}$

~~ধরে~~

$40 - 4x^2 = 0$

same ধরে

নিলে,

$$\frac{d^2v}{dx^2} = -8x$$

$= -25.28 < 0$

$x = 3.16, \ \frac{d^2v}{dx^2} < 0$ local maximum

~~$x = -3.16, \ \frac{d^2v}{dx^2} = 6.28 > 0$~~

$$V = 40 \times 3.16 - \frac{4 \times (3.16)^3}{3}$$

$= 84.33 \ cm^3$

Ans to the ques no: 5

$$f_x(1,3) = 6x^2y^2 + 4$$

$$= 6 \times 1 \times (3)^2 + 4$$

$$= 58 \text{ Ans}$$

$$f_y(1,3) = 4x^3y + 2$$

$$= 4(1)^3 \times 3 + 2$$

$$= 14 \text{ Ans}$$

Ans to the ques no: 6

(a) $f_x = 2xy$

$$= 2 \times (1) \times (-2)$$

$$= -4 \text{ Ans}$$

(b) $f_y = x^2 + 15y^2$

$$= 1^2 + 15 \times (-2)^2 = 60 \text{ Ans}$$

Ans to the qus no: 7 (a)

(a) $z = x^2 - y^2 + 2xy$

$$\frac{\partial z}{\partial x} = 2x + 2y$$

$$\frac{\partial^2 z}{\partial x^2} = 2$$

$$\frac{\partial z}{\partial y} = -2y + 2x$$

$$\frac{\partial^2 z}{\partial y^2} = -2$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 2 - 2 = 0$$

(b) $\frac{\partial z}{\partial x} = e^{x} \sin y + e^{y}(-\sin x)$

$$\frac{\partial^2 z}{\partial x^2} = e^{x} \sin y - e^{y} \cos x$$

$$\frac{\partial z}{\partial y} = e^{x} \cos y + e^{y} \cos x$$

$$\frac{\partial^2 z}{\partial y^2} = e^{x}(-\sin y) + e^{y} \cos x$$

$$= -e^{x} \sin y + e^{y} \cos x$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^{x} \sin y - e^{y} \cos x - e^{x} \sin y + e^{y} \cos x$$

$$= 0$$

Ans

Ans to the ques no: 8

$$\begin{tikzcd} & x \arrow[r] & t \\ z \arrow[ur] \arrow[dr] \arrow[drr, bend right] & & \\ & y \arrow[r] & t \end{tikzcd}$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \times \frac{dx}{dt} + \frac{\partial z}{\partial y} \times \frac{dy}{dt}$$

$$= 2xy \times 2t + x^2 \times 3t^2$$

$$= 4xyt + 3t^2x^2$$

$$z = x^2 y = \cancel{x^2 \frac{dy}{dt} + y \frac{dx^2}{dt}}$$

$$\cancel{= (t^2) \cdot t}$$

$$\cancel{= x^2 \, 3t^2 + y \, 4t^3}$$

$$= x^2 \frac{d}{dt}(y) + y \frac{d}{dt}(x^2)$$

$$= x^2 \, 3t^2 + y \, 4t^3$$

$$\Rightarrow x^2 \, 3t^2 + 4t^2 \cdot t \, y$$

$$\Rightarrow x^2 \, 3t^2 + 4xyt \quad [t^2 = x]$$

Ans to the qusAno: 9

(a) $f(x,y) = y^2 + xy + 3y + 2x + 3$

$f_x = y + 2 = 0$

$f_y = 2y + x + 3 = 0$

$(x, y) = (1, -2)$

$f_{xx} = 0$

$f_{yy} = 2$

$f_{xy} = 1$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$= 0 - (1)^2$

$= -1 < 0$

Local maximum available.

(d) $f(x,y) = x^2 + xy - 2y - 2x + 1$

$f_x = 2x + y - 2 = 0$

$f_y = x - 2 = 0$

$(x, y) = (2, -2)$

$f_{xx} = 2$

$f_{yy} = 0$

$f_{xy} = 1$

$D = 0 - (1)^2 = -1 < 0$

This function has local maximum value.

3

$$(3x^2 + x + 3x + 1)\frac{dy}{dx} = y$$

$$\int \frac{dy}{y} = \int \frac{1}{(3x^2+4x+1)}\,dx$$

$$\Rightarrow \ln y = \int \frac{1}{(x+1)(3x+1)}\,dx$$

$$= \int \frac{3}{2(3x+1)} - \frac{1}{2(x+1)}\,dx$$

$$= \frac{3}{2}\int \frac{1}{(3x+1)}\,dx - \frac{1}{2}\int \frac{1}{x+1}\,dx$$

$$= \frac{3}{2}\times\frac{1}{3}\int \frac{1}{u}\,du - \frac{1}{2}\ln(x+1)$$

$$= \frac{3}{2}\times\frac{1}{3}\ln(3x+1) - \frac{1}{2}\ln(x+1)$$

$$= \frac{\ln(3x+1) - \ln(x+1)}{2}$$

$u = 3x+1$

$\frac{du}{dx} = 3$

$du = 3\,dx$

$[x = 1, y = \boxed{2}]$

$$\ln y = \left(\frac{\ln(3x+1) - \ln(x+1)}{2}\right) + C$$

$$\Rightarrow \ln y - \left(\frac{\ln(3x+1) - \ln(x+1)}{2}\right) = C$$

$$\Rightarrow \cancel{C = \ldots \ldots = -0.1438}$$

$$\cancel{\int_{x=3} e^{x+y} \cdot \cos(2x+1)}$$

$$\Rightarrow \ln(2) - \left(\frac{\ln(4) - \ln(3)}{2}\right) = C$$

$$\Rightarrow 0 - 0.144 = C$$

$\Rightarrow -0.144$ $\qquad y = 1.37$

$x=3$

Some basic Rule

Circle:

Area, $A = \pi r^2$, Length, $L = 2\pi r$

Volume, $V = \frac{4}{3}\pi r^3$

Rectangle:

Length, $L = 2(l + w)$

Area, $A = l \times w$

Volume, $V = l \times w \times h$

l = length
w = width
h = height

Triangle:

$A = \frac{1}{2} \times b \times h$

$s = \frac{a+b+c}{2}$, $A = \sqrt{s(s-a)(s-b)(s-c)}$

triangular prism: $V = A \times h$ [A = Area]

" pyramid: $V = \frac{1}{3} A \times h$

CamScanner

Cylinder:

Literal area, $L = 2\pi r h$

Total surface area, $A = 2\pi r (r+h)$

Volume, $V = \pi r^2 h$

$$3x^2 + 2\frac{dy}{dx} - 2y\frac{dy}{dx} + 3 + 2\frac{dy}{dx} = 0$$

$$\Rightarrow 2\frac{dy}{dx} - 2y\frac{dy}{dx} + 2\frac{dy}{dx} = -3x^2$$

$$\Rightarrow \frac{dy}{dx}(2 - 2y + 2) = -3x^2$$

$$\Rightarrow \frac{dy}{dx}(-2y) = -3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{2y}$$

$$3x^{\frac{1}{2}} - \frac{3}{x^{\frac{1}{2}}} = 0$$

$$\Rightarrow \frac{3x - 3}{x^{\frac{1}{2}}} = 0$$

$$\Rightarrow 3x = 3$$

$$\Rightarrow x = \frac{3}{3} = 1$$

(c) $$\frac{d^2y}{dx^2} = \frac{3}{2}x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}}$$

$$x = 1, \quad \frac{d^2y}{dx^2} = 3 > 0$$

$$\frac{dx}{dt} = k(100 - x)$$

$$\Rightarrow \int \frac{1}{(100-x)}\, dx = \int k\, dt$$

$$= -\ln(100-x) = kt + C$$

$$\Rightarrow \ln(100-x) = -kt - C$$

$$\Rightarrow 100 - x = e^{-kt-C}$$

$$\Rightarrow +x = -e^{-kt - C} + 100$$

$$100 - x = e^{-kt} \cdot e^{-C}$$

$$\Rightarrow e^{-C} = \frac{100 - 25}{e^{-0}}$$

$$= 75$$

$$\Rightarrow -C = \ln(75) = 4.32$$

$$\Rightarrow C = -4.32$$

$t = 180 \quad x = 85$

$\Rightarrow -\ln(100-85) = k \times 180 + -4.32$

$\Rightarrow -2.71 + 4.232 = k \times 180$

$\Rightarrow \dfrac{1.61}{180} = k$

$\Rightarrow k = 8.94 \times 10^{-3}$

$x = e^{-kt + c} + 100$

$= -e^{-\{(8.94 \times 10^{-3}) \times 200\} + 4.32} + 100$

$\Rightarrow 87.42$

$$x^3 + 2xy - y^2 + 3x + 2y + 7 = 0$$

$$f_x = 3x^2 + 2y + 3$$

$$f_y = \cancel{\quad} \; 2x - 2y + 2$$

$$\frac{dy}{dx} = \frac{-f_x}{f_y}$$

$$= \frac{-(3x^2 + 2y + 3)}{2x - 2y + 2}$$

$$\frac{dv}{dr} = 96\pi r - \pi r^3$$

$$\frac{d^2v}{dr} = 96\pi - 3\pi r^2$$

$$\frac{d^2v}{dr^2} = -6\pi r$$

$$= -6\pi r$$

[এ খানের যে value
আসবে r এর
এই বসাইলে 0 থেকে
ছোট মান আসবে।]

এই mean.

$x^6 - 3x^2$

$x^6 = 3x^2$

$\sqrt{\frac{x^6}{3}}$

C.W 19.08.2023

Calculus 2

* Differential Equation :-

* Variable Separable
* Linear first order diff

* Exact differential equation :

A differential equation of the form $M(x,y)\,\cancel{diff}\,dx +$ $N(x,y)\,dy = 0$ will be exact differential equation.

if $\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$

where, $\frac{\delta}{\delta x}[f(x,y)] = M(x,y)$

$\frac{\delta}{\delta y}[f(x,y)] = N(x,y)$

and we are supposed to find the $f(x,y)=0$

$$\frac{\delta M}{\delta y} = \frac{\delta}{\delta y}\left[\frac{\delta}{\delta x}(f(x,y))\right] = \frac{\delta^2}{\delta y\,\delta x}(f(x,y))$$

$$\frac{\delta N}{\delta x} = \frac{\delta}{\delta x}\left[\frac{\delta}{\delta y}(f(x,y))\right] = \frac{\delta^2}{\delta x\,\delta y}(f(x,y))$$

* $x^2y^3\,dx + x^3y^2\,dy = 0$

$M(x,y) = x^2y^3$ $\quad \frac{\delta M}{\delta y} = 3x^2y^2$

$N(x,y) = x^3y^2$ $\quad \therefore \frac{\delta N}{\delta x} = 3x^2y^2$

$\therefore$ This is an exact differential equation.

$$\therefore \frac{\delta}{\delta x}[f(x,y)] = M = x^2y^3$$

$$\therefore f(x,y) = \int x^2y^3\,dx + g(y)$$

$$\therefore f(x,y) = \frac{1}{3}x^3y^3 + g(y)$$

$$\frac{\delta}{\delta y}[f(x,y)] = x^3y^2 + g'(y) = N = x^3y^2$$

$$\therefore g'(y) = 0 \quad \therefore g(y) = c$$

$$\therefore f(x,y) = \frac{1}{3}x^3y^3 + c$$

* $2xy\,dx + (x^2-1)\,dy = 0$

$M(x,y) = 2xy$ $\quad \therefore \frac{\delta M}{\delta y} = 2x$

$N(x,y) = x^2-1$ $\quad \therefore \frac{\delta N}{\delta x} = 2x$

since $\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$; This is an exact diff. equation

$$\frac{\delta}{\delta x} f(x,y) = 2xy$$

$$\therefore f(x,y) = \int 2xy \, dx + g(y)$$

$$\therefore f(x,y) = x^2y + g(y)$$

$$\frac{\delta f}{\delta y} = x^2 - 1$$

$$x^2 + g'(y) = x^2 - 1$$

$$\Rightarrow g'(y) = -1$$

$$\therefore g(y) = -y + c$$

$$\therefore f(x,y) = x^2y - y + c$$

Or, $\frac{\delta}{\delta y} f(x,y) = x^2 - 1$

$$\therefore f(x,y) = x^2y - y + g(x)$$

$\downarrow$

* Solving Homogeneous differential equation : *

If a function f(x,y); possesses the following property than it will be called homogeneous differential equation

i.e $f(tx, ty) = t^m f(x,y)$; m is a constant

$$(x^2+y^2)\,dx + (x^2 - xy)\,dy = 0$$

This is not exact diff. equation.

$(x^2+y^2)\,dx + (x^2-xy)\,dy = 0 \rightarrow$ Homogeneous [ঘাতও same]

x^2+y^2	$t^2x^2 - tx.ty$
$t^2x^2 + t^2y^2$	$t^2x^2 - t^2xy$
$\Rightarrow t^2(x^2+y^2)$	$= t^2(x^2-xy)$

* A substitution $y = vx$ can convert a homogeneous equation into a differential equation of variable seperable.

$$(x^2+y^2)\,dx + (x^2-xy)\,dy = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x^2+y^2}{x^2-xy}$$

$y = vx$

$\frac{dy}{dx} = v.1 + x\frac{dv}{dx}$

$$\therefore v + x\frac{dv}{dx} = -\frac{x^2+v^2x^2}{x^2-vx^2}$$

$$\Rightarrow v + x\frac{dv}{dx} = -\frac{x^2(1+v^2)}{x^2(1-v)}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{v-1} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2-v^2+v}{v-1}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v+1}{v-1}$$

$$\therefore \frac{v-1}{v+1}\,dv = \frac{1}{x}\,dx$$

$$\therefore \int \frac{v-1}{v+1}\, dv = \int \frac{1}{x}\, dx$$

$$\Rightarrow \int \frac{v+1-2}{v+1}\, dv = \ln x + c$$

$y = vx$

$v = \frac{y}{x}$

$$\Rightarrow \int \left(1 - \frac{2}{v+1}\right) dv = \ln x + c$$

$$\Rightarrow v - 2\ln(v+1) = \ln x + c$$

$\cancel{\therefore \frac{y}{x} - 2\ln(v+1) = \ln x +}$

$$\therefore \frac{y}{x} - 2\ln\left(\frac{y}{x}+1\right) = \ln x + c$$

$$\Rightarrow \frac{y}{x} = \ln\left(\frac{y+x}{x}\right)^2 + \ln x + c$$

$$\Rightarrow \frac{y}{x} = \ln\left\{x\left(\frac{y+x}{x}\right)^2\right\} + c$$

* $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ or, $xy\,dx - (x^2+y^2)\,dy = 0$

$$\therefore v + x\frac{dv}{dx} = \frac{x \cdot vx}{x^2+(vx)^2}$$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2 v}{x^2+v^2x^2}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2 v}{x^2(1+v^2)}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v}{1+v^2} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v - v - v^3}{1+v^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-v^3}{1+v^2}$$

$$\Rightarrow \int \frac{1+v^2}{-v^3}\,dv = \int \frac{dx}{x}$$

$$\Rightarrow \int -v^{-3} + \frac{1}{v}\,dv = \ln x + c$$

$$\Rightarrow \frac{-v^{-2}}{-2} + \ln v = \ln x + c$$

$$\Rightarrow \frac{1}{2}\cdot\frac{1}{v^2} + \ln v = \ln x + c$$

$$\Rightarrow \frac{1}{2}\cdot\frac{x^2}{y^2} + \ln(y/x) - \ln x = c$$

$$\Rightarrow \frac{1}{2}\cdot\frac{x^2}{y^2} + \ln(y/x) = c$$

* Second order linear homogeneous equation with constant coefficients:

$$3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 0$$

GW

* 2nd Order diff equation :-

Any equation of the form

$$P_1 \frac{d^n y}{dx^n} + P_2 \frac{d^{n-1} y}{dx^{n-1}} + \dots + P_{n-1} \frac{dy}{dx} + P_n y = f(x)$$

where $P_1, P_2, \dots P_n$ are either constant or function of x, is known as n^{th} order linear differential equation.

~~* 2nd order line~~

$$\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = f(x)$$ is a second order diff equation with constant coefficient.

* $f(x) = 0 \longrightarrow$ Homogeneous.

P.T.O.

* $\frac{d^2y}{dx^2}$

$y = \sin x$ $\quad \therefore y'' + y = 0$

$\frac{dy}{dx} = \cos x$

$\frac{d^2y}{dx^2} = -\sin x$

* $c_1 \frac{d^2y}{dx^2} + c_2 \frac{dy}{dx} + c_3 y = 0$

Corresponding Auxiliary equation :-

$c_1 m^2 + c_2 m + c_3 = 0$

Solving $m \rightarrow (m_1, m_2)$

if m_1 & m_2 are real & distinct (বাস্তব ও অসমান)

then $y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

A.E (real & distinct)

* $m = -1, 2, -3$

$\therefore y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-3x}$

* $m = -2, -1, 4, 5$

$y = c_1 e^{-2x} + c_2 e^{-x} + c_3 e^{4x} + c_4 e^{5x}$

* If roots are repeated :

* $m = 2, 2$

$y = (c_1 + c_2 x) e^{2x}$

* $m = -3, -3, -3$

$y = (c_1 + c_2 + c_3 x^2) e^{-3x}$

* $m \rightarrow 1, -2, -2$

$y = c_1 e^{x} + (c_2 + c_3 x) e^{-2x}$

PTO

(iv) $m = -1, -1, 3, 3$

$$y = (c_1 + c_2 x)e^{-x} + (c_3 + c_4 x)e^{3x}$$

* If roots are imaginary

$\alpha \pm i\beta$

$$y = e^{\alpha x}\left[c_1 \cos\beta x + c_2 \sin\beta x\right]$$

* Exm: $2 \pm 3i$

$$y = e^{2x}\left[c_1 \cos 3x + c_2 \sin 3x\right]$$

Sadman

* $\frac{d^2y}{dx^2} - 13\frac{dy}{dx} + 12y = 0$

AE

$m^2 - 13m + 12 = 0$

$\Rightarrow m = (1, 12)$

$\therefore$ Solution: $y = c_1e^{x} + c_2e^{12x}$

* $\frac{d^3x}{dt^3} + 6\frac{d^2x}{dt^2} + 11\frac{dx}{dt} + 6x = 0$

$y^3 + 6y^2 + 11y + 6 = 0$

$m = -3, -1, -2$

$\therefore$ Sol $\Rightarrow y = c_1e^{-3x} + c_2e^{-x} + c_3e^{-2x}$

$$\frac{d^4x}{dt^4} - \frac{d^3x}{dt^3} - \frac{dx}{dt} + 1x = 0$$

AF

$$m^4 - m^3 - m + 1 = 0$$

$$\Rightarrow m^3(m-1) - 1(m-1) = 0$$

$$\Rightarrow (m-1)(m^3-1) = 0$$

$m - 1 = 0$

$\Rightarrow m = 1$

$m^3 - 1 = 0$

$(m-1)(m^2+m+1) = 0$

$m = 1,\ m^2+m+1 = 0$

$$\Rightarrow m = \frac{-1 \pm \sqrt{1-4}}{2 \cdot 1} = -\frac{1}{2} \pm \frac{\sqrt{3}i}{2}$$

$$m = 1, 1, -\frac{1}{2} \pm \frac{\sqrt{3}i}{2}$$

IT.

Sol^n

$$x = (c_1 + c_2 t)\, e^{t} + e^{-\frac{1}{2}t} \left[c_3 \cos\frac{\sqrt{3}}{2}t + c_4 \sin\frac{\sqrt{3}}{2}t \right]$$

$$\boxed{\cancel{y'' - 2y' + y = e^{x}}}$$

~~General~~

* $y'' - 2y' + y = 0$, $y(0) = 5$, $y'(0) = 10$.

$m^2 - 2m + 1 = 0$

$\Rightarrow (m-1)^2 = 0$

$\Rightarrow m = (1, 1)$

$\therefore y = (c_1 + c_2 x)\, e^{x}$

$y(0) = 5$

$$y' = (c_1 + c_2 x) e^x + e^x \cdot c_2$$

$\Rightarrow x = 0,\ y = 5$

$$5 = (c_1 + 0) \cdot 1$$

$\Rightarrow c_1 = 5$

$x = 0,\ y' = 10$

$$\Rightarrow 10 = (5 + 0) \cdot 1 + 1 \cdot c_2$$

$$\Rightarrow 10 = 5 + c_2$$

$\Rightarrow c_2 = 5$

$$\therefore y = (5 + 5x) e^x$$

C.W 29.3.23

Diff eq

Friday (7 - 8) pm (Extra) Class

* $y'' + c_1 y' + c_2 y = f(x)$

$y'' + c_1 y' + c_2 y = 0$

* If $f(x)$ & $g(x)$ are the solution of the above diff eq.
then the linear combination of the solutions also the solution of the above ordinary Diff eq.

* $\sin x$ & $\cos x$ are the solution.
$\therefore$ $c_1 \sin x + c_2 \cos x$ is also the solution.

P.T.O.

* If e^{x}, e^{-x} are the solution

$\therefore c_1 e^{x} + c_2 e^{-x}$ is also the solution.

$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin x$

General solution:

= complementary function + Particular Integral

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

$\Rightarrow m^2 + m + 1 = 0$

$$m = \frac{-1 \pm \sqrt{1-4.1.1}}{2}$$

$$= \frac{-1 \pm \sqrt{3}\, i}{2}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$\therefore C.F = e^{-\frac{1}{2}x}\left[C_1 \sin\frac{\sqrt{3}}{2}x + C_2\cos\frac{\sqrt{3}}{2}x\right]$

$P.I = y = A\sin x + B\cos x$

$y' = A\cos x - B\sin x$

$y'' = -A\sin x - B\cos x$

$(-A\sin x - B\cos x) + (A\cos x - B\sin x) + (A\sin x + B\cos x) = \sin x$

$\sin(-A - B + A) + \cos x(-B + A + B) = \sin x$

$\Rightarrow -B\sin x + A\cos x = \sin x$

$-B = 1$ | $A = 0$

$\therefore B = -1$

$\boxed{\therefore P.I = -\cos x}$

Equating the coffisicent of like terms.

C.W

Matrix

Linear

* Diagonal Matrix :-

মুখ্য কর্ণ বাদে বাকি সব 0.

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* Identity Matrix:-

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

CW 21.4.23

linear

Solve this diff. equation

$$\frac{d^2y}{dx^2} + y = e^{2x}$$

$A.E \Rightarrow m^2 + 1 = 0$

$\Rightarrow m^2 = -1$

$\Rightarrow m = \pm\sqrt{-1}$

$= \pm i$

$G.S = C.F + P.I$

$CF = c_1 \cos x + c_2 \sin x$

$y = Ae^{2x}$

$y' = 2Ae^{2x}$

$y'' = 4Ae^{2x}$

$4Ae^{2x} + Ae^{2x} = e^{2x}$

$\Rightarrow 5Ae^{2x} = e^{2x}$

(operator method)

$$\boxed{\begin{aligned} 3y'' + 2y' + 5y &= f(x) \\ (3D^2 + 2D + 5)y &= f(x) \end{aligned}}$$

$\Rightarrow 5A = 1$

$\Rightarrow A = \frac{1}{5}$

$\therefore P.I = \frac{1}{5} e^{2x}$

$\therefore G.S = (C_1 \cos x + C_2 \sin x) + \frac{1}{5} e^{2x}$

* Finding P.I using operator:-

$y = Ae^{2x}$

$y'' + y = e^{2x}$

$\Rightarrow (D^2 + 1)y = e^{2x}$

$\Rightarrow y = \frac{e^{2x}}{(D^2 + 1)}$

$\Rightarrow y = \frac{e^{2x}}{2^2 + 1} = \frac{e^{2x}}{5}$

* $y''' + 3y'' + 2y = e^{-2x}$

$\Rightarrow (D^3 + 3D^2 + 2)y = e^{-2x}$

$\Rightarrow y = \dfrac{e^{-2x}}{D^3 + 3D^2 + 2}$

$= \dfrac{e^{-2x}}{6}$

$(-2)^3 + 3\cdot(2)^2 + 3$

$= -8 + 12 + 3$

$= 6$

* $3y'' + y' - 2y = e^{-x}$

$(3D^2 + D - 2)y = e^{-x}$

$\Rightarrow y = \dfrac{e^{-x}}{3D^2 + D - 2}$

$3 - 1 - 2$

$= 3 - 3$

$= 0$

$= xe^{-x} \times \dfrac{1}{6D+1}$

$a = -2$

$= xe^{-x} \times \dfrac{1}{6a+1} = xe^{-x} \times \dfrac{1}{-5}$

Rules

* $\dfrac{e^{ax}}{f(D)} = \dfrac{e^{ax}}{f(a)}$; $f(a) \neq 0$

* $\dfrac{e^{ax}}{f(D)}$, where $f(a) = 0$

$= xe^{ax} \times \dfrac{1}{f'(a)}$, $[f'(a) \neq 0]$

* $\dfrac{\sin ax}{f(D)}$ or $\dfrac{\cos ax}{f(D)}$ $[f(-a^2) \neq 0]$

$= \dfrac{1}{f(-a^2)} \sin ax = \dfrac{1}{f(-a^2)} \cos ax$

$[f'(a) = 0]$

$= x^2 e^{ax} \cdot \dfrac{1}{f''(a)}$ $[f''(a) \neq 0]$

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 5y = 3e^{3x}$$

$$(D^2 - 3D + 5)y = 3e^{3x}$$

$$y = \frac{3e^{3x}}{(D^2-3D+5)}$$

$$= \frac{3e^{3x}}{3^2 - 9 + 5}$$

$$= \frac{3e^{3x}}{9 - 9 + 5}$$

$$= \frac{3e^{3x}}{5}$$

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x}$$

$$\Rightarrow (D^2 - 3D + 2)y = e^{2x}$$

$$\Rightarrow y = \frac{e^{2x}}{D^2 - 3D + 2}$$

$$= xe^{2x} \times \frac{1}{f'(D)}$$

$$= xe^{2x} \times \frac{1}{2D-3}$$

$$= xe^{2x} \times \frac{1}{2\cdot 2-3} = xe^{2x} \times \frac{1}{4-3}$$

$$= xe^{2x}$$

$$\frac{d^2y}{dx^2} + 4y = \cos 3x$$

$$\Rightarrow (D^2+4)y = \cos 3x$$

$$\Rightarrow y = \frac{\cos 3x}{D^2+4}$$

$f(D) = D^2 + 4$

$f(a) = -a^2 + 4$

$= -3^2 + 4$

$= -9 + 4$

$= -5$

$$= -\frac{1}{5}\cos 3x$$

$f(D^2) = f(-a^2)$

* $\frac{d^2y}{dx^2} + y' + 5y = \sin 3x$

$(D^2 + D + 5)y = \sin 3x$

$\therefore y = \frac{\sin 3x}{D^2 + D + 5}$ $\quad D^2 = -a^2 = -(3^2) = -9$

$= \frac{\sin 3x}{-9 + D + 5}$

$\frac{1}{D-4} \sin 3x$

$\frac{D+4}{D^2-16} \sin 3x$

$= \frac{D+4}{-9-16} \sin 3x$ [D = Differentiation]

$= -\frac{1}{25} (D+4) \sin 3x$

$= -\frac{1}{25} (3\cos 3x + 4 \sin 3x)$

C.W 3.4.23

Linear

* $\frac{\sin ax}{f(D^2)}$

$= \frac{\sin(ax)}{f(-a^2)}$ $f(-a^2) \neq 0$

if, $f(-a^2) = 0$,

$= x \frac{\sin(ax)}{f'(D^2)}$ $[f'(D^2) \neq 0]$

* $\boxed{\frac{1}{D} = \int dx}$

* $(2D+1)(\cos 3x)$

$= 2(-\sin 3x \cdot 3) + \cos 3x$

$= -6\sin 3x + \cos 3x$

* $\dfrac{g(x)}{f(D)}$, $g(x)$ is polynomial *

* $(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3 + \dots$

* $(1+D)^{-2} = 1 + (-2)D + \dfrac{(-2)(-3)}{2!}D^2 + \boxed{\dfrac{(-2)(-3)(-4)}{3!}D^3} + \dots$ *

* $(y' + 2y) = x^2 + 1$

$\Rightarrow (D+2)y = x^2 + 1$

$\Rightarrow y = \dfrac{x^2+1}{D+2}$

$= \dfrac{1}{D+2}(x^2+2)$

$= \dfrac{1}{2\left(\frac{D}{2}+1\right)}(x^2+2)$

$$= \frac{1}{2}\left(1+\frac{D}{2}\right)^{-1}(x^2+1)$$

$$= \frac{1}{2}\left[1+(-1)\frac{D}{2}+\frac{(-1)(-2)}{2!}\left(\frac{D^2}{4}\right)+\cdots\right](x^2+1)$$

$$= \frac{1}{2}\left(1-\frac{D}{2}+\frac{1}{4}D^2-\cdots\right)(x^2+1)$$

$$= \frac{1}{2}\left[x^2-\frac{1}{2}\cdot 2x+\frac{1}{4}\cdot 2+1\right]$$

$$= \frac{1}{2}\left(x^2-x+\frac{3}{2}\right)$$

P.I: $y = \frac{1}{2}x^2 - \frac{x}{2} + \frac{3}{4}$

$$y''' + 2y'' + y' = e^{2x} + x^2 + x$$

$$\Rightarrow (D^3 + 2D^2 + D)y = e^{2x} + x^2 + x$$

$$m^3 + 2m^2 + m = 0$$

$$\Rightarrow m(m^2 + 2m + 1) = 0$$

$$m = 0, \quad m^2 + 2m + 1 = 0$$

$$\Rightarrow m(-1, -1)$$

$$C.F = C_1 e^{0} + (C_2 + C_3 x)e^{-x}$$

$$\rightarrow D(D+1)^2 y = e^{2x} + x^2 + x$$

$$\Rightarrow y = \frac{e^{2x}}{D(D+1)^2} + \frac{x^2 + x}{D(D+1)^2}$$

$$\frac{e^{2x}}{D(D+1)^2} = \frac{e^{2x}}{2\times 9} = \frac{1}{18}e^{2x}$$

$$\frac{x^2+x}{D(D+1)^2}$$

$$= (x^2+x)\frac{1}{D}(D+1)^{-2}$$

$$= \frac{1}{D}(1+D)^{-2}(x^2+x)$$

$$= \frac{1}{D}(1-2D+3D^2-4D^3+\dots)(x^2+x)$$

$$= \left(\frac{1}{D}-2+3D-4D^2+\dots\right)(x^2+x)$$

$$= \frac{x^3}{3} - 2x^2 + 6x - 8 + \frac{x^2}{2} - 2x + 3$$

$$= \frac{x^3}{3} - 2x^2 + 4x + \frac{x^2}{2} - 5$$

$$= \frac{x^3}{3} - \frac{3x^2}{2} + 4x - 5$$

$$y = c_1 + (c_2 + c_3 x) e^{-x} + \frac{x^3}{3} - \frac{3x^2}{2} + 4x - 5 + \frac{1}{18} e^{2x}$$

$x = 0, \quad y =$

$x = 0, \quad y' =$

$x = 0, \quad y'' =$

$y(0) =$

$y'(0) =$

$y''(0) =$

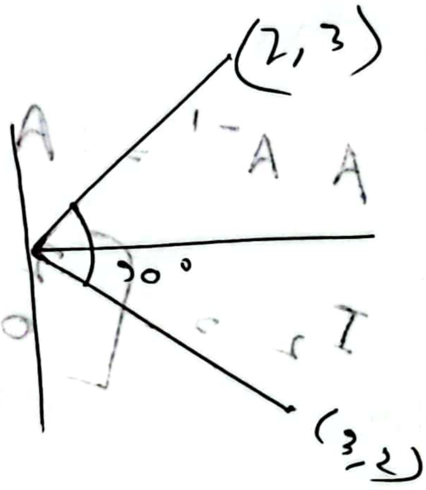
GW 5.4.23

Linear

* A^{-1}
* solving equation

* $90°$ check

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$



$$= \begin{bmatrix} 0+3 \\ -2+0 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

-ve

+ve

* Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

=

I.T.

* Inverse matrix of a matrix is denoted by A^{-1}

$A A^{-1} = A^{-1} A = I$

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4(A)

(i) School C 20 ~~to~~ position secure

করে।

school D 20 " " "

" ।

(ii) (b) total points

(iii)

6

(i)

(ii) total distance

CW 12.4.23

* System of linear equation.

A system of linear equation is a list of equations of same variable.

$2x + y = 5$ [linear equation with 2 variables]

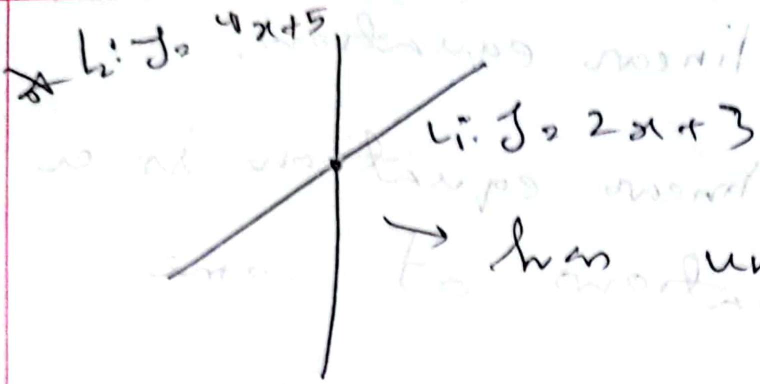
$3x - 2y + z = 10$ [3 variables]

*

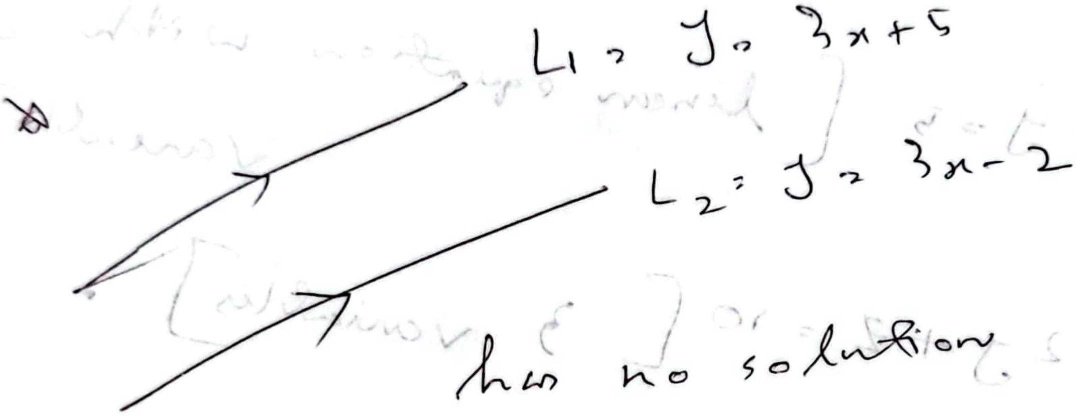
$$\left.\begin{aligned} 3x + 2y - z &= 10 \\ 4x - 2y + 7z &= 15 \\ -x + 3y + 2z &= 7 \end{aligned}\right\}$$

is an example of system of linear equation with 3 equations & 3 variables.

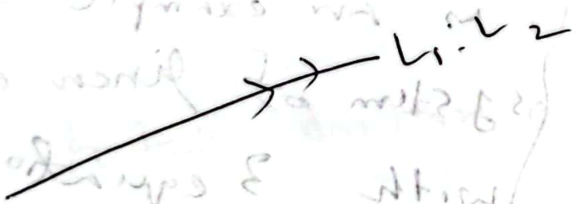
P.T.O.



→ has unique solution



has no solution



$L_1 : y \Rightarrow 2x + 3y = 10$

$L_2 : y \Rightarrow 3x + 4y = 15$

has infinite solution

* System of linear equation
  - Consistance
    - unique solution
    - Infinite solution
  - Inconsistance
    - has no solution

* $a_1 x + b_1 y = c_1$

$a_2 x + b_2 y = c_2$

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}, \quad B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$|A| = a_1 b_2 - b_1 a_2$$

$$|A| = 0$$

$$a_1 b_2 - b_1 a_2 = 0 \Rightarrow a_1 b_2 = b_1 a_2$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

* If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ [has no solution]

* $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ [has infinite solution]

* $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ [has unique solution]

---

* $a_{11}x_1 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + \ldots + a_{2n}x_n = b_2$

$\vdots$

$a_{m1}x_1 + \ldots + a_{mn}x_n = b_n$

where, (i) $x_1, x_2 \ldots x_n$ are variables

(ii) $a_{ij}, b_i \rightarrow$ constant

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$\downarrow$

coefficient matrix

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_n \end{array}\right]$$

$\hookrightarrow$ Augmented Matrix

P.T.O

* Solving system of Linear equation by Gaussian elimination method.

* $x + 2y + 3z = 1$

$x + 3y + 6z = 3$

$2x + 6y + 13z = 5$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 6 \\ 2 & 6 & 13 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$AX = B$

$X = BA^{-1}$

P.T.O.

$$x + 2y + 3z = 1$$

$$y + 3z = 2 \quad [R_2 = R_2 - R_1]$$

$$2y + 7z = 3 \quad [R_3 = R_3 - 2R_1]$$

$$x + 2y + 3z = 1$$

$$y + 3z = 2$$

$$z = -1 \quad [R_3 = R_3 - 2R_2]$$

$\therefore z = -1$

$\therefore x = -6,\ y = 5$

C.W
30-4-23

Gauss - Jordan elimination method :-

The Gauss - Jordan elimination method is something to Gaussian elimination method except that here each "pivot" is used to place "0" (zero)s both below and above it before working with the next pivot. Also one variation is that the pivot is to be unit.

$x - 2y + 3z + u = 2$

$x + y + 4z - u = 3$

$2x + 5y + 9z - 2u = 8$

P.T.O

Coefficient Matrix, $A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 1 & 1 & 4 & -1 \\ 2 & 5 & 9 & -2 \end{bmatrix}$

and the corresponding Augment

$$\text{Matrix} = \begin{bmatrix} 1 & -2 & 3 & 1 & 2 \\ 1 & 1 & 4 & -1 & 3 \\ 2 & 5 & 9 & -2 & 8 \end{bmatrix}$$

$\to$ pivots.

$$\sim \begin{bmatrix} 1 & -2 & 3 & 1 & 2 \\ 0 & 3 & 1 & -2 & 1 \\ 0 & 9 & 3 & -4 & 4 \end{bmatrix} \quad \begin{matrix} R_2' = R_2 - R_1 \\ R_3' = R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 3 & 1 & 2 \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ 0 & 9 & 3 & -4 & 4 \end{bmatrix} \quad R_2' = R_2 \div 3$$

[Treating as equation, not matrix]

$$\approx \begin{bmatrix} 1 & 0 & \frac{11}{3} & \frac{1}{3} & \frac{8}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix} \quad \begin{matrix} R_1' = R_1 + 2R_2 \\ R_3' = R_3 - 9 \times R_2 \end{matrix}$$

$\cong \quad 2u = 1 \Rightarrow u = \frac{1}{2}$

$\therefore y + \frac{11}{3} z - \frac{2}{3} u = \frac{1}{3}$

$\Rightarrow y + \frac{11}{3} z = \text{constant}$

$\therefore x + \frac{11}{3} z = \text{constant}$

Put $z = 0$; find x, y.

* Homogeneous System of linear Equations

A system of linear equation is said to be homogeneous if all the constant term are zero. Then a homogeneous system has the form $AX = 0$.

$a_{11}x_1 + a_{22}x_2 + \dots + a_{1n}x_n = 0$

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = 0$

$\vdots$

$a_{r_1}x_1 + a_{r_2}x_2 + \dots + a_{rn}x_n = 0$

Variable - n, Number of Eqn $\Rightarrow r$

P.T.O.

* If, $r = n$ then, the system has only zero solution, All variables = 0.

* If, $r < n$, then it has non-zero solution.

$x + y - z = 0$
$2x - 3y + z = 0$
$x - 4y + 2z = 0$

Solve to find if it has zero or non-zero soln.

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 2 & -3 & 1 & 0 \\ 1 & -4 & 2 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ 1 & -4 & 2 & 0 \\ 2 & -3 & 1 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -5 & 3 & 0 \\ 0 & -5 & 3 & 0 \end{bmatrix} \begin{array}{l} r_2' = r_2 - r_1 \\ r_3' = r_3 - 2r_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -5 & 3 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{bmatrix} \quad R_3 = R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -5 & 3 & 0 \end{bmatrix}$$

variable > eqn

has non-zero solution.

$0 \quad 1 \quad -3/5 \quad 0$

$x + y - z = 0$

$y - \frac{3}{5} z = 0$

$z = 1$

$y = 3/5$

* Eigen values & Eigen Vector:-

If, $AV = \lambda V$

Then, $V =$ Eigen vector

$\lambda =$ Eigen value

Eigen value & Eigen vectors

(1) $$\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix}$$

$$Ax = \lambda x$$

$$\Rightarrow Ax - \lambda x = 0$$

$$\Rightarrow x(A - \lambda I) = 0$$

for non-zero soln :-

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & 2-0 \\ 1-0 & -1-\lambda \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & 2 \\ 1 & -1-\lambda \end{bmatrix}$$

$$(3-\lambda)(-1-\lambda) - 2 = 0$$

$$\Rightarrow \cancel{-3+\lambda+\lambda-3\lambda-2 = 0}$$

$$\Rightarrow \cancel{\lambda^2-2\lambda-5 = 0}$$

$$\Rightarrow \cancel{\lambda =} \; -3-3\lambda+\lambda+\lambda^2-2 = 0$$

$$\Rightarrow \lambda^2-2\lambda-5 = 0$$

$$\Rightarrow \lambda = 1+\sqrt{6},\quad 1-\sqrt{6}$$

$\lambda = 1+\sqrt{6}$,

$$\begin{matrix} 3x+2y \\ x+-y \end{matrix} = \begin{matrix} (1+\sqrt{6})x \\ (1+\sqrt{6})y \end{matrix}$$

$$\Rightarrow \begin{matrix} 3x+2y-(1+\sqrt{6})x \\ x-y-(1+\sqrt{6})y \end{matrix} = 0$$

$$\Rightarrow \begin{matrix} (2-\sqrt{6})x+2y \\ \cancel{}\, x + 2-\sqrt{6}\, y \end{matrix} = 0 \qquad \begin{matrix} x = \\ y = \end{matrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

eigen vector

$\lambda = 1 - \sqrt{6}$

$3x + 2y = (1-\sqrt{6})x$

$x - y = (1-\sqrt{6})y$

$\Rightarrow (2+\sqrt{6})x + 2y = 0$

$x + (2+\sqrt{6})y = 0$

$\therefore x = 0, \; y = 0$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ eigen vector