

University



Name: Suman
School/College: Calculus and
Subject: linear algebra
Section: Roll:
Year:

W
22. 1.23

Graphs for study notes it will fit *

* Linear equation / function

$$y \in A$$

* Quadratic

$$y \in A$$

* Cubic

Dependent

* Exponential

$$y = \frac{b}{a^x}$$

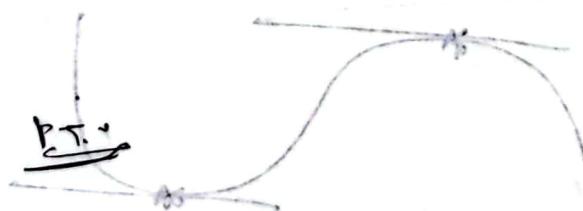
* Logarithmic

$$y = \log_a x$$

* Sin & cos functions

Ex: $y = \sin x$ or $y = \cos x$

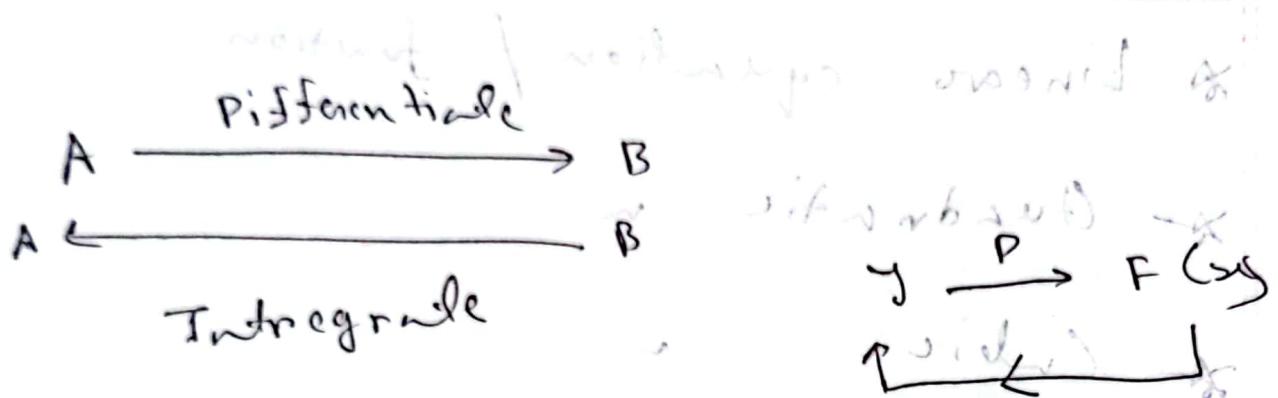
Ex: $y = \sin x$



$$y = \frac{b}{a^x}$$

Max

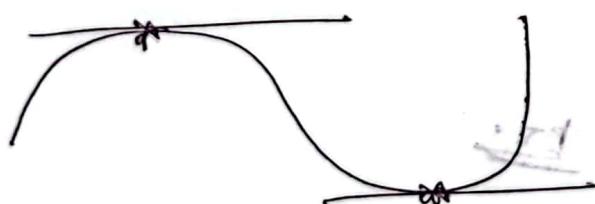
* Differentiation & Integration



* $\frac{dy}{dx} = F(x)$ Initial value \downarrow

$y = \int F(x) dx$ Initial value \downarrow

Turning point / stationary point of a function:-



* turning point

* At turning point, $\frac{dy}{dx} = 0$

P.T.
 $\cancel{\cancel{x}}$

Integration Rules :-

$$* \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$* \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1) \times n} + C$$

$$* \int e^{ax+b} dx = e^{ax+b} + C$$

$$* \int e^{ax+b} dx = \frac{e^{ax+b}}{a} + C$$

$$* \int \frac{1}{x} dx = \ln|x| + C$$

$$* \int \frac{1}{cx+d} dx = \frac{\ln(cx+d)}{c} + C$$

$$* \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$* \int u v dx = u \int v dx - \int \left[\frac{du}{dx} v \cdot \int v dx \right] dx$$

latter method

LIA TE

$$J + \frac{x}{x+1} = e^{\alpha x}$$

↓

$$J + \frac{(1+x)}{x(1+x)} = e^{\alpha x}$$

↓

$$J + \frac{1}{x} = e^{\alpha x}$$

↓

$$J = e^{\alpha x} - \frac{1}{x}$$

↓

$$\log J = \alpha x - \log x$$

Algebra
Inverse

Algebra

Algebra

$$J = e^{\alpha x} - \frac{1}{x} = e^{\alpha x} \left(1 - \frac{1}{x} \right)$$

(divides) wh.

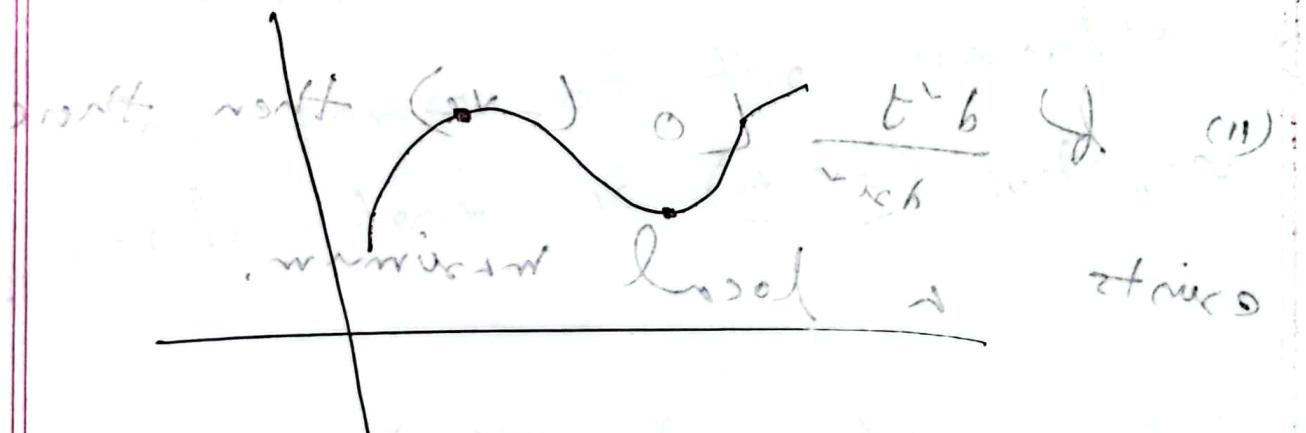
$$J = e^{\alpha x} \frac{1}{x} = e^{\alpha x} \frac{1}{\sinh x}$$

$$\frac{x}{\sinh x} = \frac{1}{e^{\alpha x}} \Rightarrow \sinh x = e^{\alpha x} \frac{1}{x}$$

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{\alpha x} - e^{-\alpha x}}{2} = \frac{e^{\alpha x}}{2} \left(1 - \frac{1}{e^{2\alpha x}} \right)$$

Turning / stationary points (i), nature of turning point and increasing and decreasing function :-

work with $f(x)$ of $\frac{t^6}{x^6}$ st. (i)
given that $f = f(x)$,
minimum local & strict



$\frac{df}{dx}(x)$ & turning / stationary point $\frac{df}{dx} = 0$
At turning

$\frac{d^2f}{dx^2}(x)$ & workout [First derivative test]

P.T.

L.C.T.

Nature of turning points

- finding local turning point by
- [2nd derivative test]
- without graphing the function

(i) If $\frac{d^2f}{dx^2} > 0$ (+ve) then there exists a local minimum.

(ii) If $\frac{d^2f}{dx^2} < 0$ (-ve) then there exists a local maximum.

for increasing function, $f'(x) > \frac{d}{dx} > 0$
by default + A

for decreasing function, $f'(x) > \frac{d}{dx} < 0$

PTP LTP

$$\# \quad f = x^2 - 4x + 7$$

$P = x^2 - \frac{6b}{x^b}$

Polynomial

Linear $y = ax + b \rightarrow f(x) = \frac{6b}{x^b}$

Quadratic $f(x) = x^2 + bx + c \rightarrow f(x) = x^2 + 4x + 7$

Cubic $f(x) = x^3 + bx^2 + cx + d$

The power of the variable in each term is (+ve) integers.

$$f(x) = x^2 - 4x + 7$$

$$\frac{dy}{dx} = 2x - 4 > 0 \quad \begin{matrix} x \\ \nearrow \\ 2 \end{matrix}$$

$$\Rightarrow 2x = 4 \quad \begin{matrix} x \\ \nearrow \\ 2 \end{matrix}$$

$$\Rightarrow x = 2$$

and $x = 2$, $f(x) = 4 - 8 + 7 = 3$

$$\therefore x = 2$$



$$\frac{dy}{dx} = 2x - 4$$

$$\frac{d^2y}{dx^2} = \text{marking } [2\text{nd derivative}]$$

order $\left(270^{\circ} \right)$ [+] side of

i.e. $(2, 3)$ is a local minimum point.

negative (< 0) function

$$\frac{dy}{dx} = f'(x) > 0$$

$$(2x - 4) > 0 \Rightarrow 2x - 4 > 0$$

$$\Rightarrow 2x > 4 \Rightarrow x > \frac{4}{2} = 2$$

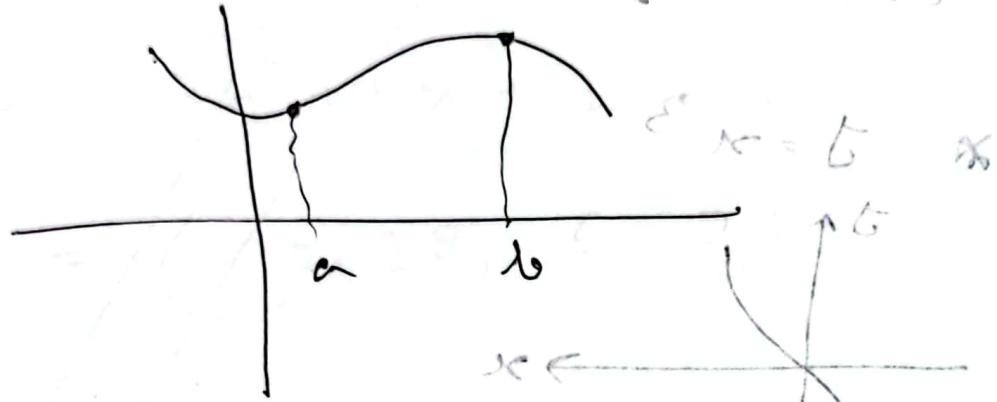
$$\Rightarrow x > 2 \quad \text{Max.}$$

for decreasing function :-

$$\frac{dy}{dx} = 2x - 4 < 0 \Rightarrow 2x - 4 < 0$$

$$\Rightarrow x < 2$$

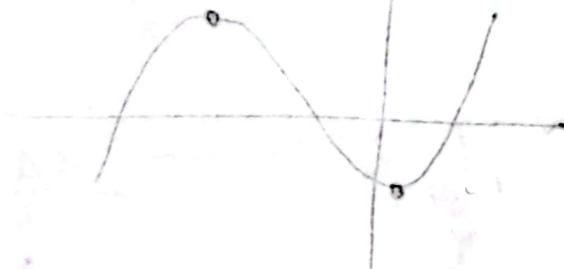
* If $f(x)$ steps to sharp ∞
→ with right sides



for ~~$x < a$~~ ; $f(x)$ is decreasing

for $a < x < b$; $f(x)$ is increasing

for $x > b$; $f(x)$ is decreasing

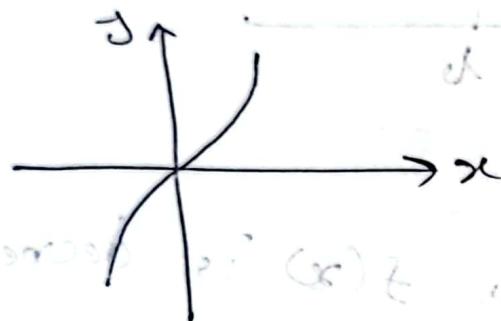


P.7.3



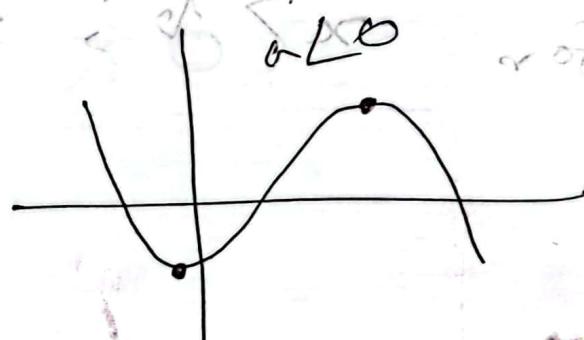
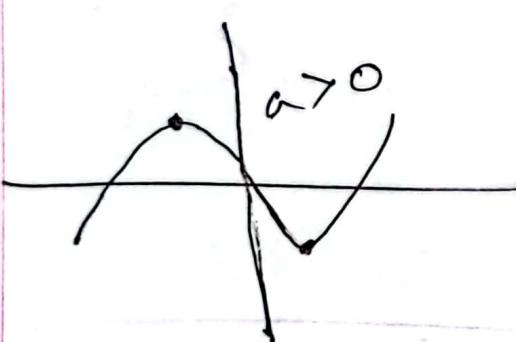
* graph of quartic and cubic function :-

* $y = x^3$



- general form of cubic function :-

$$y = ax^3 + bx^2 + cx + d$$



P.T.O

$$y = \frac{1}{3}x^3 - 3x^2 + 5x + 1$$

$$\frac{1}{3}x^3 - 3x^2 + 5x + 1 = 0$$

$$\Rightarrow x = \left(0.28, 6.69 \right)$$

at $x = 0.28$ local minima

$$\frac{dy}{dx} = x^2 - 6x + 5 \quad (\text{turning})$$

At turning point $\frac{dy}{dx} = 0$

min local $x^2 - 6x + 5 = 0$

$$\Rightarrow x = (1, 5)$$

at $x = 1, y = \frac{10}{3}$ $(1, \frac{10}{3})$

at $x = 5, y = -\frac{22}{3} (5, -\frac{22}{3})$

P.T.O.

Nature of the turning points:-

$$\frac{dy}{dx} = 2x^2 - 6x + 5$$

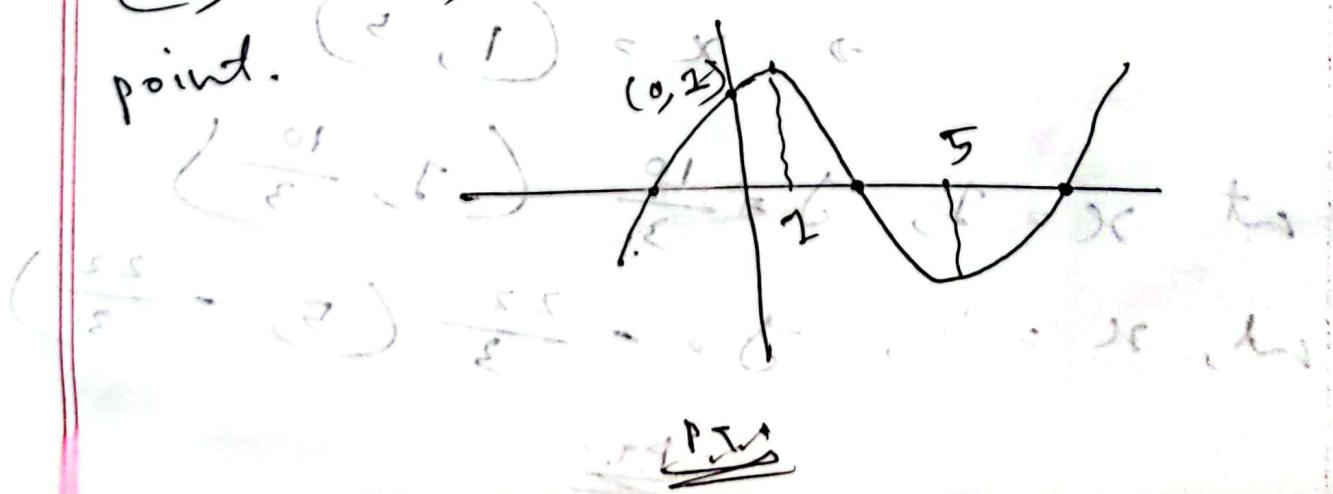
$$\frac{d^2y}{dx^2} = 2x^2 - 6 \rightarrow x = 0$$

$$\text{at, } x = 0, \frac{d^2y}{dx^2} \rightarrow 2 - 6 = -4 < 0$$

$(0, \frac{5}{2})$ is a local Maximum point.

$$\text{at, } x = \frac{5}{2}, \frac{d^2y}{dx^2} = 10 - 6 = 4 > 0$$

$(\frac{5}{2}, -\frac{25}{3})$ is a local minimum point.



* For $x < 2$; $f(x)$ is increasing

* for $2 \leq x \leq 5$; $f(x)$ is decreasing

* for $x > 5$; $f(x)$ is increasing

$$O = s + x^2 - xc^2$$

$$\# J = (s + x^2 - xc^2) - (s + x^2 - xc^2) = xc^2$$

$$\# J = (x-2)^2(-3+5x) \quad (x-2)(5x+3)$$

- turning point to switch

$$s + x^2 - xc^2 = \frac{6h}{xc^2}$$

$$2 - xc^2 = \frac{6h}{xc^2}$$

$$O \leq s + \frac{6h}{xc^2} \quad (x-2)(5x+3) > 0$$

maxima lies in $(s + 6h/c^2, 0)$ or
turning

~~H.W
J. 2. 23~~

Calculus 2

$$y = x^3 - 3x^2 + 2x \text{ not } *$$

$$\frac{dy}{dx} = 3x^2 - 6x + 2 \text{ not } *$$

$$3x^2 - 6x + 2 = 0$$

$$\Rightarrow x = (1.577, 0.423)$$

$$x = 1.577, y = -0.38$$

$$x = 0.423, y = +0.38$$

Nature of turning points:-

$$\frac{d^2y}{dx^2} = 6x - 6$$

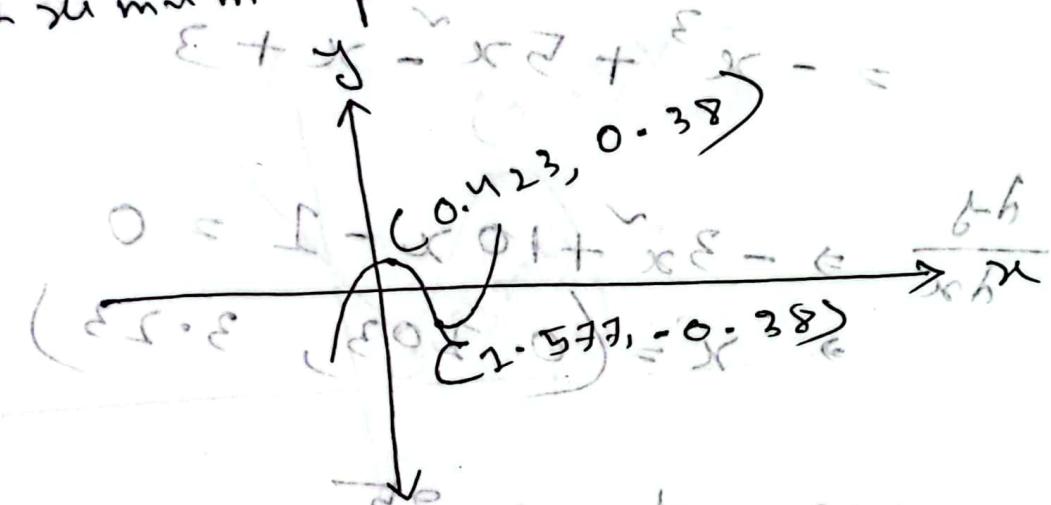
$$\frac{d^2y}{dx^2} = 6x - 6$$

$$x = 1.577, \frac{d^2y}{dx^2} = 3.462 > 0$$

so $(1.577, -0.38)$ is local minimum point.

$$x = 0.423, \frac{dy}{dx} = -3.462 < 0$$

$(x-\varepsilon) (1+x\varepsilon - x^2) \text{ local}$
 $\therefore (0.423, 0.38) \text{ is maximum point.}$



for $x < 0.423, f'(x) > 0$ so increasing

for $0.423 < x < 1.577, f'(x) < 0$ so decreasing

for $x > 1.577, f'(x) > 0$ so increasing.

$$0.1 + 0.2x = \frac{5x^2}{6}$$

$$0.1 + 0.2x = \frac{5x^2}{6} \quad (0.1 = x)$$

positive local extrema $(0.423, 0.38)$

Ans

$$J = (x-1)^{\tilde{v}} (3-x) \quad \text{E.S.N. } 0.001$$

$$\text{local} = (x^{\tilde{v}} - 2x + 1)(3-x)$$

$$= 3x^{\tilde{v}} - 6x + 3 - x^3 + 2x^{\tilde{v}} - 3x$$

$$= -x^3 + 5x^{\tilde{v}} - x + 3 \quad \text{minimum}$$

$$\frac{\partial J}{\partial x} \Rightarrow -3x^{\tilde{v}} + 10x - 1 = 0$$

$$\Rightarrow x = (0.103, 3.23)$$

$$x = 0.103, J = 2.95$$

$$x = 3.23, J = 18.24$$

Nature of turning points :-

$$\frac{\partial J}{\partial x} = -3x^{\tilde{v}} + 10x - 1$$

$$\frac{\partial^2 J}{\partial x^2} = -6x + 10$$

$$x = 0.103, \frac{\partial^2 J}{\partial x^2} = 9.382 > 0$$

(0.103, 2.95) point is local minimum.

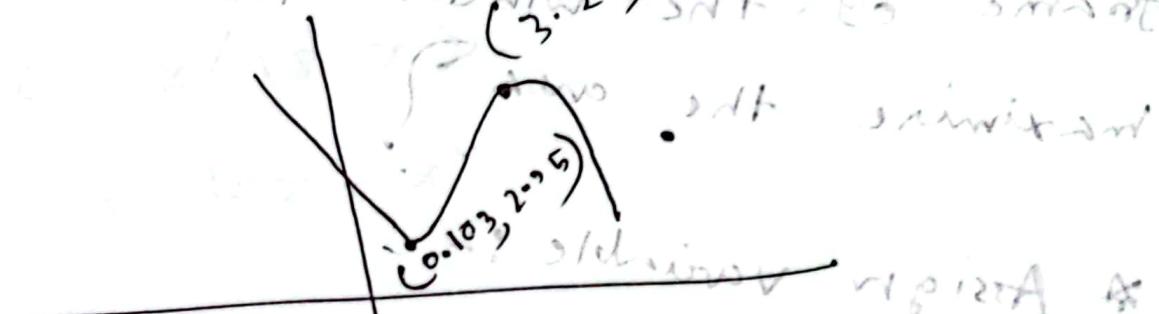
P.T.O.

$$x = 3.23, \frac{dy}{dx} = -9.38 < 0$$

$(3.23, 18.2)$ point is local

maximum wrt wrt all other points

This point is $(3.23, 18.2)$ wrt to all other points



$x = 0.103$ is local minima

$x = 3.23$ is local maxima

extreme values (minima) wrt wrt all except x

$x < 0.103, f(x)$ is decreasing

$0.103 < x < 3.23, f(x)$ is increasing

$x > 3.23, f(x)$ is decreasing

6x
5x 2.23

* Optimization (Maximum or Minimum) :-

~~length of the frame~~ $36 \text{ m} (5-8) \text{ (E.S.E)}$

what should be the dimension of the frame of the window that will maximise the area?

* Assign variable :-

→ Length $\rightarrow x \text{ m}$

→ width $\rightarrow y \text{ m}$

* Express the condition / constraint in the terms of variable :-

$$2x + 2y = 36$$

$$x + y = 18$$

b7.0

* Express & simplify the objective function in terms of chosen variable :-

$$A = xy$$

$$x + y = 18 \text{ m}$$

$$\rightarrow y = 18 - x$$

$$A = x(18 - x)$$

$$\rightarrow 18x - x^2$$

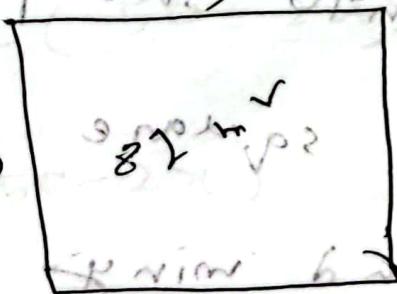
$$\frac{dA}{dx} = 18 - 2x$$

* A + max / min :- ①

Given $\frac{dA}{dx} = 0$ for profit to profit A

$\Rightarrow 18 - 2x = 0$. Then solve for x

$\Rightarrow x = 9$ shift to right



∴ A = 81 units

$\therefore (x, y) = (9, 9)$ will give the answer

P.T.O.

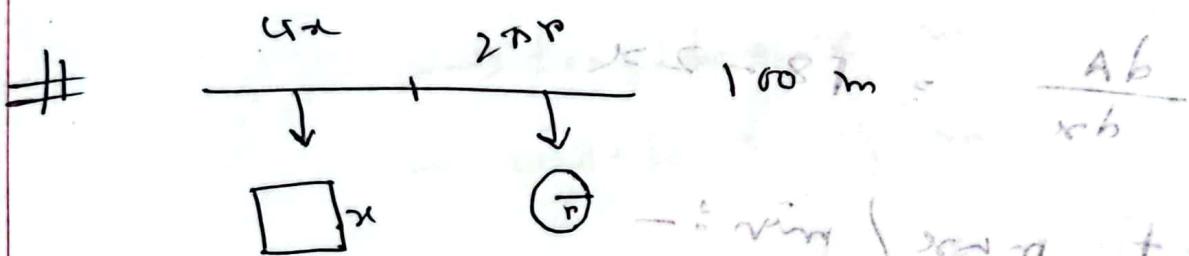
and also with length x

$$\frac{dA}{dx} = 18 - 2x$$

To make it zero

$$\frac{d^2A}{dx^2} = -2 < 0$$

$\therefore x = 9$ m will maximize the area.



A string of length 100 m is divided into two parts. Using one part a square of side $x \text{ m}$ is formed and using the other side a circle of radius r is formed.

(I) Express r in terms of x .

(II) Find the value of x that will minimize the sum of the area of the square & circle.

$$(I) 4x + \frac{\pi r^2}{\pi} = 100$$

$$\Rightarrow r = \frac{50 - 2x}{\pi}$$

$$(II) A = x^2 + \pi r^2$$

$$A = x^2 + \frac{(50 - 2x)^2}{\pi}$$

$$\frac{\partial A}{\partial x} = 2x + \frac{1}{\pi} 2(50 - 2x)(-2)$$

$$= 2x - \frac{4}{\pi}(50 - 2x)$$

P.T.O

$A + x$ (or) max :-

Put x to solve and get (1)

$$\frac{dA}{dx} = 0$$

Diff dA/dx w.r.t x & equate to zero

$$\Rightarrow 2x - \frac{4}{\pi} (50 - 2x) = 0 \quad \text{Equation}$$

$$\Rightarrow 2x - \frac{200}{\pi} + \frac{8}{\pi} x = 0 \quad x = 0 \quad (1)$$

$$\Rightarrow x \left(2 + \frac{8}{\pi} \right) = \frac{200}{\pi} \quad x = \frac{200}{\pi} \times \frac{1}{2 + \frac{8}{\pi}}$$

$$\Rightarrow x \left(\frac{2\pi + 8}{\pi} \right) = \frac{200}{\pi} \quad x = \frac{200}{2\pi + 8} \times \pi$$

$$\Rightarrow x \left(\frac{2\pi + 8}{\pi} \right) = 200 \quad x = \frac{200}{2\pi + 8} \times \pi = A \quad (1)$$

$$\Rightarrow x = \frac{200}{2\pi + 8} \quad x = \frac{200}{2\pi + 8} \times \pi = A$$

$$\Rightarrow x = 14.0 + 8.5 = \frac{A}{16.6}$$

$$(14.0 + 8.5) = \frac{A}{16.6}$$

Ans

$$\frac{dA}{dx} = 2x - 1 \frac{4}{\pi} (50 - 2x) \quad (1)$$

$$\Rightarrow \frac{d^2A}{dx^2} = 2 - 1 \frac{4}{\pi} \quad (-2)$$

$$\Rightarrow 2 + \frac{8}{\pi} > 0$$

$\therefore x = \frac{100}{\pi + 4x} \rightarrow x = 10$ will minimize the area.

The sum of two numbers is 124.

Find the number that will maximize their product.

$$P = x \frac{124-x}{x}$$

find the number that will minimize the sum of a number and it's reciprocal. [The number is positive]

P.T.O.

$$(1) \quad x + y = 124 - x = \frac{a+b}{x+b}$$

$$\Rightarrow y = 124 - x = \frac{a+b}{x+b} - a$$

$$A = x y = \frac{a+b}{x+b} x$$

$$= x(124 - x)$$

$$\text{for maximizing value } \frac{\partial A}{\partial x} = 124 - 2x - x^2$$

Now $\frac{\partial A}{\partial x} = 124 - 2x$ to max it will be

This point we have to find at max :-

$$\frac{\partial A}{\partial x} = 0 \quad \text{for optima value}$$

Now this point we have to find if

$$124 - 2x = 0 \quad \text{finding in } (x, y) = (62, 62)$$

$$\Rightarrow -2x = -124 \quad \text{value of } x = 62$$

$$\Rightarrow x = 62 \quad \text{value of } x = 62$$

$$\therefore y = 62$$

opt

$$\frac{\partial A}{\partial n^v} = -2 \angle 0 + x^2 t +$$

local minimum(m) - non-invertible.

(1) ~~$x = 62$~~

$$y = 62 \frac{1}{x} - 5$$

$$\frac{1}{x} + \frac{1}{y} = 0 \Rightarrow \frac{x+y}{xy} = 0 \Rightarrow \frac{62+62}{62 \times 62} = 0$$

$$\frac{1}{x} = \frac{1}{32} \Rightarrow x = 32$$

splitting function $y = x + 5 + \frac{1}{x}$

endpoints $x = 0, 62$

$$f(0) = 5 + \frac{1}{0}$$
 ~~$\approx \infty$~~

$$f(62) = 62 + 5 + \frac{1}{62} = 67.016$$

$$\# \quad J = x + \frac{1}{x^2} \quad S = -\frac{x^2 b}{m b}$$

\therefore find minima & maxima

$$\begin{aligned} \frac{\partial J}{\partial x} &= 1 - \frac{2}{x^2} \\ &= 1 - \frac{1}{x^2} = 0 \end{aligned}$$

A) max/min:-

$$1 - \frac{1}{x^2} = 0 \Rightarrow 1 = \frac{1}{x^2}$$

$$\Rightarrow \frac{1}{x^2} = 1$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm 1 \quad \text{positive number}$$

$$\begin{aligned} \frac{\partial^2 J}{\partial x^2} &= -\frac{2}{x^3} \\ &= -\frac{2}{x^3} \\ &= \frac{2}{x^3} \\ &= 2 > 0 \end{aligned}$$

6W

~~up*~~ $\frac{d}{d\theta} (\overline{\cos \theta}) \Rightarrow \frac{d}{d\theta} (\cos \theta)^{-1}$

(1)

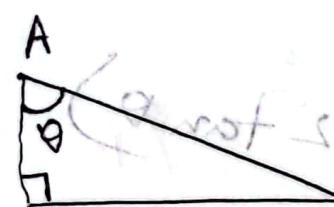
P_{out}

ϵ

$$\Rightarrow (-2)(\cos \theta)^{-2} \times (-\sin \theta)$$

$$\frac{xq}{s} = \frac{0.5 \sin \theta}{\cos^2 \theta \times \sin \theta}$$

$P_{\text{out}} = 5 \times q$

(11)  θ (angle is 0°)

$P_{\text{out}} = 0$ ~~now of width~~ θ of x

$$\cos \theta = \frac{2}{x}$$

$$P_{\text{out}} = \frac{A}{2} x$$

$$\Rightarrow A x = \frac{2}{\cos \theta}$$

$$\theta_{\text{out}} = c + \frac{\epsilon}{\cos \theta} = \text{with diff}$$

$$\Rightarrow A x = 2 \sec \theta$$

$$\theta_{\text{out}} = c + \frac{1}{\cos \theta} \frac{\epsilon}{\epsilon}$$

$$2 \frac{5.5}{2.5} = 2$$

time taken to move from

$$\text{Point A to } x = \frac{2 \sec \theta}{3}$$

$$\text{Again, } \tan \theta = \frac{P_x}{2}$$

$$P_x = 2 \tan \theta$$

$$\therefore x_Q = (10 - 2 \tan \theta)$$

Time taken to move

$$x \text{ to } Q = \frac{10 - 2 \tan \theta}{5} = \theta_{200}$$

$$\Rightarrow 2 - \frac{2}{5} \tan \theta$$

$$\text{Total time} = \frac{2 \sec \theta}{3} + 2 - \frac{2}{5} \tan \theta$$
$$\approx \frac{2}{3} \frac{1}{\cos \theta} + 2 - \frac{2}{5} \frac{\sin \theta}{\cos \theta}$$

$$= 2.533 \text{ s}$$

$$\frac{d\tau}{d\theta} = \frac{2}{3} \sec \theta \tan \theta - \frac{2}{5} \sec^2 \theta$$

$$\Rightarrow \frac{2}{3} \cdot \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} - \frac{2}{5} \cdot \frac{1}{\cos^2 \theta}$$

$$\Rightarrow \frac{2 \sin \theta}{3 \cos^2 \theta} - \frac{2}{5 \cos^2 \theta} = 0$$

$$\Rightarrow \frac{2}{3} \sin \theta - \frac{2}{5} = 0$$

$$\Rightarrow \tan \theta = \frac{3}{5}$$

$$\Rightarrow \theta = 36.87^\circ$$



$$\tan \theta = \frac{3}{4}$$

$$\therefore \theta = 2 \tan^{-1} \frac{3}{4}$$

$$\theta = \frac{2 \times 3}{5} = 1.2 \text{ radian}$$

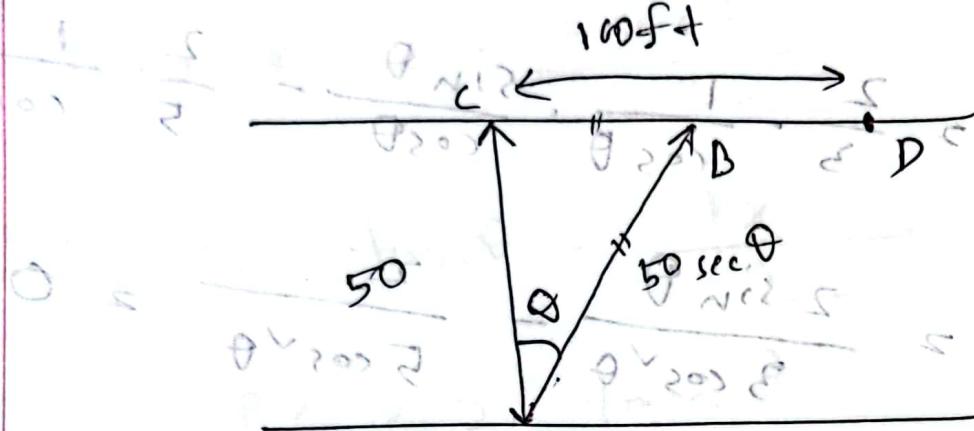
$$\theta = \frac{3}{5} = 0.6$$

$$\theta = \frac{0.6}{0.5} + \frac{1}{0.25} = 1.2$$

(Q)

~~Excm: 1~~

$$\frac{5}{\sin \theta} = 50 \sec \theta \Rightarrow \frac{5}{\sin \theta} = \frac{50 \sec \theta}{5}$$



$$\cos \theta = \frac{50}{AB}$$

$$\Rightarrow AB = 50 \sec \theta, t = \frac{50 \sec \theta}{5}$$

$$\tan \theta = \frac{BC}{50} = \theta$$

$$\Rightarrow BC = 50 \tan \theta$$

$$BD = 100 - 50 \tan \theta, t = \frac{100 - 50 \tan \theta}{15}$$

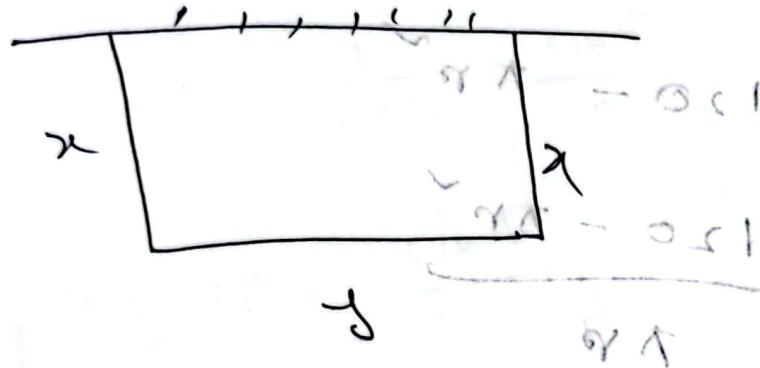
$$\text{total} = \frac{50 \sec \theta}{5} + \frac{100 - 50 \tan \theta}{15}$$

$$= 10 \sec \theta + \frac{20}{3} - \frac{10}{3} \sec \tan \theta$$

$$= 10 \cdot \frac{1}{\cos \theta} + \frac{20}{3} - \frac{10 \sin \theta}{3 \cos \theta}$$

~~Exm 1.2~~

$$\text{Total cost} = \text{Cost of A} + \text{Cost of B}$$



$$xy = 75 \rightarrow \text{Cost of A} = \frac{75}{x} \text{ and Cost of B} = \frac{75}{y}$$

$$\begin{aligned}
 C &= (2x+y) \times 8 + xy \\
 &= 16x + 8y + xy \\
 &= 16x + 112 \left(\frac{75}{x} \right)
 \end{aligned}$$

$$16 + \frac{900}{x} = 0$$

$$\text{Eff. CM} = -2 \cdot 16x + 12 \left(\frac{75}{x} \right)$$

$$\Rightarrow \frac{900}{x} = 16$$

$$0 = 16x + \frac{900}{x}$$

$$\Rightarrow \frac{900}{x^2} = \frac{16}{900}$$

$$0 = 16x + \frac{900}{x}$$

$$\Rightarrow x^2 = 56.25$$

$$\Rightarrow x = 7.5$$

$$\therefore y = 10$$

P.T.O.

minimum cost = 0.122488



3

$$\pi r^2 + \pi r L = 120 \text{ m}^2$$

$$\pi r L = 120 - \pi r^2$$

$$\therefore L = \frac{120 - \pi r^2}{\pi r}$$

$$V = \frac{1}{2} \pi r^2 L \leftarrow \text{for } L$$

$$= \frac{1}{2} \pi r^2 \left(\frac{120 - \pi r^2}{\pi r} \right)$$

$$= \frac{1}{2} r (120 - \pi r^2)$$

$$= 60r - \frac{1}{2} \pi r^3 \rightarrow 194.683$$

$$\frac{dV}{dr} = 60 - \frac{1}{2} \pi r^2 \rightarrow 0$$

$$\Rightarrow \frac{1}{2} \pi r^3 = 60$$

$$\Rightarrow r^3 = \frac{120}{\frac{3}{2} \pi} = 12.73$$

$$\Rightarrow r = 3.578$$

$$\frac{d^2V}{dr^2} = -\frac{1}{2} \pi r^2$$

$$= -33.65 < 0 \therefore \text{maximum}$$

$$\frac{\partial T}{\partial \theta} = 10 \sec \theta \tan \theta - \frac{10}{3} \sec^3 \theta$$

$$= 10 \cdot \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} - \frac{10}{3} \cdot \frac{1}{\cos^3 \theta}$$

$$10 \cdot \frac{\sin \theta}{\cos^2 \theta} - \frac{10}{3} \cdot \frac{1}{\cos^3 \theta}$$

$$10 \sin \theta - \frac{10}{3}$$

$$10 \sin \theta = \frac{10}{3}$$

$$\sin \theta = \frac{10}{30}$$

$$\theta = \sin^{-1} \left(\frac{10}{30} \right)$$

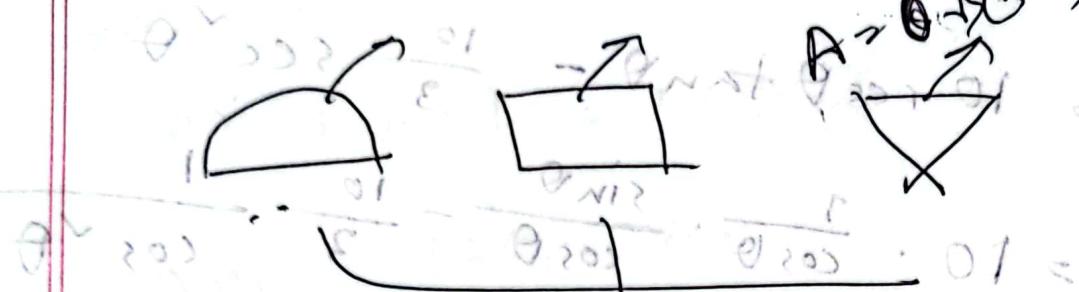
$$= 19.4712^\circ$$

~~$$\text{total time} = 16.0948 \text{ sec}$$~~

~~$$9.8 + \frac{rd}{n} + \frac{rd}{n}$$~~

~~$$T = \frac{m}{k}$$~~

$$\textcircled{5} = \frac{a+b+c}{2}$$



$$\begin{aligned} & \frac{5r}{2} \\ & \frac{5r}{4} \\ & \frac{5r}{4} + \frac{5r}{4} + 2r \\ & \frac{10r}{4} + \frac{5r}{4} \\ & = \frac{15r}{4} \\ & = \frac{15r}{8} \\ & = \frac{15r}{8} \times 2 = \frac{15r}{4} \end{aligned}$$

$$\frac{1}{2} \pi r^2 + (2r \times r) + \cancel{\frac{15r}{4}} = \cancel{\frac{15r}{4}}$$

~~$$0 = 8r \cancel{\frac{5r}{4}} + \cancel{\frac{5r}{4}} + 2r$$~~

with lot of

$$\frac{5r}{4} + \frac{5r}{4} + 8r$$

$$= \frac{18r}{8} = \frac{9r}{4} = 5$$

$$\frac{1}{2}\pi r^2 + 2\pi rx + \frac{3r^2}{4} = 125$$

$$\rightarrow 2\pi x = 125 - \frac{3r^2}{4} - \frac{1}{2}\pi r^2$$

$$\rightarrow x = \frac{125}{2\pi} - \frac{3r^2}{4\pi} - \frac{\pi r^2}{2\pi}$$

$$\frac{125}{2\pi} - \frac{3r^2}{8\pi} - \frac{\pi r^2}{4\pi}$$

~~$\frac{125}{2\pi} - \frac{3r^2}{8\pi} - \frac{\pi r^2}{4\pi}$~~

~~$\frac{50}{\pi} - \frac{3r^2}{8\pi} - \frac{\pi r^2}{4\pi}$~~

~~$\frac{50}{\pi} + (\pi r^2)$~~

~~$\frac{50}{\pi} + \pi r^2$~~

③

$\boxed{\pi r^2}$

200
200

$$\frac{1}{2}\pi r^2 + 2rx + \frac{3r^2}{4} = A$$

$$A = \frac{1}{2}\pi r^2 + 2r\left(\frac{125}{2} - \frac{5r}{4} - \frac{\pi r}{2}\right)$$

$$125 = \frac{1}{2} \cdot 2\pi r^2 + 2rx + 2 \cdot \frac{5r}{4}$$

$$125 = \pi r^2 + 2rx + \frac{5r}{2}$$

$$\Rightarrow 2rx = 125 - \frac{5r}{2} - \pi r^2$$

$$\Rightarrow x = \frac{125}{2} - \frac{5r}{4} - \frac{\pi r^2}{2}$$

$$A = \frac{1}{2}\pi r^2 + 2r\left(\frac{125}{2} - \frac{5r}{4} - \frac{\pi r^2}{2}\right) +$$

$$= \frac{1}{2}\pi r^2 + 125r - \frac{5r^2}{2} - \cancel{\frac{\pi r^3}{2}} + \frac{3r^2}{4}$$

$$= 125r - \frac{1}{4}\pi r^3 - \frac{7}{4}\pi r^2$$

$$\frac{dA}{dr} = \frac{125 - \pi r^2 - \frac{7}{2}r^2}{r^2} \quad A = \frac{\pi r^2}{2} + \frac{7}{2}r^2$$

$$125 - \pi r^2 - \frac{7}{2}r^2 = 0 \quad + \pi r^2 + \frac{1}{2}r^2 = 125$$

$$\Rightarrow \pi r^2 + \frac{7}{2}r^2 = 125$$

$$\Rightarrow r(\pi + \frac{7}{2}) = 125 \quad r = 25$$

$$\Rightarrow r = \frac{125}{\pi + \frac{7}{2}} = 18.82 \text{ cm}$$

$$+ \left(\frac{\pi r^2}{2} + \frac{7}{2}r^2 \right) m^2 = 0.2882 \text{ m}^2$$

$$A = 23.1407 \text{ m}^2$$

$$- \frac{r^2}{4} - 2r^2 + \pi r^2 =$$

$$- \frac{r^2}{4} - \pi r^2 + \frac{1}{2}r^2 =$$

A

$$\Rightarrow \cancel{\frac{1}{2} \pi r^2} + 2\cancel{\pi r} + \cancel{\pi rL}$$

6.2.2.2

C2

chain rule / connected rate of change:-

$$y = [f(x)]^n$$

$$\frac{dy}{dx} = n [f(x)]^{n-1} \cdot \frac{d}{dx} f(x) \quad *$$

$$n = (t - x)^N \cdot (t - x)^N \text{ (using)} \quad \frac{dt}{dx}$$

$$y \rightarrow x \rightarrow t$$

$$y \rightarrow f(x) \rightarrow t \rightarrow f(t)$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

6.7.9

F.T.

A

* $y = (3x^2 - 2)^5$

$\Rightarrow 5(3x^2 - 2)^4 \cdot \frac{d}{dx}(3x^2 - 2)$

(expand) $\Rightarrow 5(3x^2 - 2)^4 \cdot 6x$ | start with 6
 $= 30x(3x^2 - 2)^4$

$\boxed{[x]_0} = 6$

* $y = \sin^5(4x-2)$

$\frac{dy}{dx} = 5 \sin^4(4x-2) \cdot \cos(4x-2) \cdot 4$

\rightarrow + ← x ← t

power → ratio → angle $\boxed{[x]_0} \leftarrow t$

$$\frac{x^6}{6} \times \frac{6^5}{x^6} = \frac{6^5}{6!}$$

P.T.O.

✓ ✓

$$\# \quad J = \sqrt{\tan(7\theta + 2) \sin^2 \theta} \cdot 7$$

$$\begin{aligned} \frac{dJ}{d\theta} &= \left(\tan(7\theta + 2) \right)^{-\frac{1}{2}} \sec^2(7\theta + 2) \cdot 7 \\ &= \frac{1}{2} \tan(7\theta + 2)^{-\frac{1}{2}} \sec^2(7\theta + 2) \cdot 7 \\ &= \frac{7}{2} \tan(7\theta + 2)^{-\frac{1}{2}} \sec^2(7\theta + 2) \end{aligned}$$

$$\# \quad u = e^{2\sin^3(3\theta - 2)}$$

$$\begin{aligned} \frac{du}{d\theta} &= e^{2\sin^3(3\theta - 2)} \cdot \frac{6\sin^2(3\theta - 2) \cdot 3}{\cos(3\theta - 2)} \\ &= 18e^{2\sin^3(3\theta - 2)} \cdot \sin^2(3\theta - 2) \cdot \cos(3\theta - 2) \end{aligned}$$

P.T.O

$$* y = \ln(e^{2x} + 2) \quad \text{nr 4}$$

$$\frac{dy}{dx} = \frac{1}{e^{2x} + 2} \cdot e^{2x} \cdot 2$$

$$= \frac{2e^{2x}}{(e^{2x} + 2)^2} = \frac{2e^{2x}}{(e^{2x} + 2)^2}$$

$$* y = \ln(\sin(2x+2))$$

$$\frac{dy}{dx} = \frac{1}{\sin(2x+2)} \cdot \cos(2x+2) \cdot 2$$

$$= \frac{2 \cos(2x+2)}{\sin^2(2x+2)} = \frac{2 \cos(2x+2)}{\sin^2(2x+2)}$$

$$= \frac{2 \cos(2x+2)}{(1 - \cos^2(2x+2)) \sin(2x+2)} = \frac{2 \cos(2x+2)}{\sin(2x+2)(1 - \cos^2(2x+2))}$$

af 9

* The rate of change of radius of a circle 0.25 cms^{-2} . find the rate of change of area of the circle when radius is 2 cm .

$$\Rightarrow A = \pi r^2 \quad \frac{dA}{dt} = 0.25 \text{ cm}^{-2}$$

$\Rightarrow 2r$ is radius, $\frac{dr}{dt}$ is answer

$$\frac{\partial A}{\partial r} = 2\pi r \quad \frac{\partial A}{\partial t} = \frac{\partial A}{\partial r} \times \frac{\partial r}{\partial t}$$

$$\frac{\partial A}{\partial t} = (2\pi r) \times 0.25 \\ = 2\pi \times 2 \times 0.25 = \frac{4\pi}{5}$$

$$(E. 1) \quad \frac{\partial A}{\partial t} = \frac{4\pi}{5} \text{ cm}^{-2}$$

$$\frac{4\pi}{5} \cdot \frac{1}{2} \cdot \left(\frac{4\pi}{5}\right)^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{16\pi^2}{25}}} = \frac{1}{2} \cdot \frac{1}{\frac{4\pi}{5}} = \frac{5}{16\pi}$$

*
Water is being poured into a cylinder at $\frac{4}{3} \text{ cm}^3/\text{s}$.
If the height of water increases at $\frac{dh}{dt} = 7.5 \text{ cm/s}$, find the rate of change of the height when the volume of the cylinder is 25 cm^3 .



$$\begin{aligned}
 \frac{dh}{dt} &= \frac{A h}{\frac{dv}{dt}} \cdot \frac{dv}{dt} \\
 \Rightarrow \frac{dh}{dt} &= \frac{A h}{\frac{dv}{dt}} \cdot \frac{dv}{dt} \\
 \Rightarrow \frac{dh}{dt} &= \cancel{\frac{A h}{\frac{dv}{dt}}} \cdot \cancel{\frac{1}{\frac{dv}{dt}}} = \frac{A h}{v} \\
 &= \frac{1}{3} \left(\frac{3v}{4} \right)^{\frac{1}{3}-1} \cdot \cancel{A} \cdot \cancel{\left(\frac{1}{n} \cdot 3 \right)} \\
 &= \frac{1}{3} \left(\frac{3v}{4} \right)^{-\frac{2}{3}} \cdot \frac{3}{4} \\
 &= \frac{1}{4} \left(\frac{3v}{4} \right)^{-\frac{2}{3}}
 \end{aligned}$$

$$\rightarrow \frac{dh}{dt} \xrightarrow{\text{differentiate w.r.t. time}} h \rightarrow t$$

$$L \left(\dots - \frac{d^2v}{dt^2} \right) \Rightarrow \frac{dv}{dh} \times \frac{dh}{dt}$$

$$\text{Efficiency} = \frac{5G}{x^6}$$

$$\text{efficiency} \rightarrow \frac{4}{3} h^{26}$$

$$\text{Efficiency } M \text{ of fogger} \frac{25+3}{4}$$

$$\text{Efficiency} \rightarrow h^2$$

$$\text{Efficiency} \rightarrow h = \sqrt[3]{\frac{75}{4}}$$

horizontal component $\sin(xy)$ drives a wedge

drives transverse no want same

$\frac{\partial^2}{\partial z^2} = e^{xy} \frac{\partial^2}{\partial z^2} (\sin(xy) + \sin(xy) \frac{\partial^2}{\partial n^2} (e^{xy}))$

of fogger drives efficiency through e^{xy} which
is proportional to $\cos(xy) \cdot y + \sin(xy) e^{xy}$ which is $\frac{\partial}{\partial n} (\sin(xy))$

as we want $\frac{\partial}{\partial n} (\sin(xy))$ (transverse wedge
Efficiency)

* Partial differentiation

$$\frac{\partial s}{\partial M} = \frac{\partial s}{\partial f(M, P, E, \dots)}$$

$\frac{\partial s}{\partial M}$ → change of s with respect to M considering P, X, E constant.

$\frac{\partial y}{\partial x}$ = Partial diff of y w.r.t x .

→ Partial diff of s with respect to M .

* When a variable is a function of more than one independent variable then the differentiation of the given dependent variable with respect to one independent variable (keeping other constant) is known as partial diff.

$$u = f(x, y, z)$$

$$\frac{\partial u}{\partial x}$$

$$(t, x) \in \mathbb{R}^2$$

Want to

$$\frac{\partial u}{\partial y}$$

zero

$$\frac{\partial u}{\partial z}$$

so to avoid differentiation

practically differentiate out

$y = f(x) \rightarrow$ explicit function

$f(x, y) = 0 \rightarrow$ implicit eqn

$\frac{\partial f(x, y)}{\partial y} = 0 \rightarrow$ your choice

$\frac{\partial f(x, y)}{\partial z} = 0 \rightarrow$ 6th method

so this looks like

$$* z = x^3 + 2x^2y + y^2$$

$$\frac{\partial z}{\partial x} = 3x^2 + 2y^2 + 0$$

$$3x^2 + 2y^2 = (t, x) \in \mathbb{R}^2$$

$$\frac{\partial z}{\partial y} = 0 + 2x \cdot 2y + 2y = (t, x) \in \mathbb{R}^2$$

$$6xy + 2y = (t, x) \in \mathbb{R}^2$$

✓

CW
15.2.23

C2

#1 Partial differentiation :-

$$z = f(x, y)$$

The differentiation of z with respect to x , holding y fixed is known as partial diff. of z with respect to x . and denoted by $\frac{\partial z}{\partial x} = f_x(x, y)$

$\frac{\partial z}{\partial y} = f_y(x, y) \rightarrow$ Partial diff. of z with respect to y .

$$f(x, y) = x^3 + 3xy^2 - y^5 + 10$$

$$f_x(x, y) = 3x^2 + 3y^2$$

$$f_y(x, y) = 6xy - 5y^4$$

P.T.O.

$$f(x) = \frac{1}{6}x^6 + 2xy^3 + 2y + 4x - 2$$

$$f_x = 6x^5 + 2y^3 + 4$$

$$f_y = 6x^5y^2 + 2$$

$$f_x(-3, 2) = \frac{6 \cdot (-3)^5 + 2 \cdot 2^3}{6 \cdot 5 \cdot 4 \cdot 6}$$

$$f_x(-3, 2) = \frac{-561}{56}$$

Notation for higher order partial derivatives -

$$u = f(x, y, z) \text{ at } (t, x)$$

$t = t(x)$ want numbers - 3 variables

$$\frac{\partial u}{\partial x} = f_x(x, y, z)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} \right) = \frac{\partial}{\partial x} f_x(x, y, z) \rightarrow \text{2nd derivative w.r.t. } x$$

P.T.O.

$$*\frac{\partial}{\partial j} \left(\frac{\partial u}{\partial x} \right) = f_{xj}(x, y, z)$$

→ First x , derivative

then y & z right

$$*\frac{\partial^3 u}{\partial x \partial y \partial z} = \frac{\partial}{\partial x} \left(\frac{\partial}{\partial j} \left(\frac{\partial u}{\partial z} \right) \right)$$

$\swarrow = f_{zyx}(x, y, z)$

where \rightarrow first z
 then y written
 - last x written

$\partial u = f(x, y)$ is continuous over a
 given domain (x, y, z) f_{xj} = f_{jx}

P.T.O.

$$\frac{\partial^m u}{\partial j \partial x} = \frac{\partial^m u}{\partial x \partial y}$$

2.59

$$\# f(x, y) = 2x^3y + 3x^2y - 2xy^3 + 15$$

$$(1) \quad f_{xx} = \cancel{3x^2y^2} + 3y \quad \text{without } \cancel{3x^2y^2}$$

$$= 6x^2 - 6x^2 + \cancel{6x} = (6x^2) \cancel{+ 6x}$$

$$(1) \quad f_{xy} = \cancel{3x^2y^2} + 3y$$

$$= 3x^2y^2 + \cancel{3y} \quad \text{with } \cancel{3x^2y^2}$$

$$= 6x^2y + \cancel{3y} \quad \text{from } \cancel{3x^2y^2}$$

$$(1) \quad f_{y} = t_2, \quad 2x^3y + \frac{3x^2 - 3y^2}{t_2} = 6$$

$$\frac{6}{t_2} \cdot 6 = 6 \cdot 6x^2y + \frac{3}{t_2} \cancel{t_2} = 6$$

$$(W) \quad \frac{6}{t_2} = 6x^2y + 3x^2 - 3y^2$$

$$= 2x^3y - 6y^2 \quad \text{from } 6$$

~~15~~

* Differentiation of implicit function

function:-

$$e^x + e^{xy} = x^3 + 6xy - y^2 + 10 \quad (1)$$

$$x^3 + y^2 - 3xy + 10 = 0$$

Find $\frac{dy}{dx}$ ~~e^x + e^{xy}~~

* Method 2 ~~e^x + e^{xy}~~

$$ey = \frac{dy}{dx} + e^{xy} e^y \Rightarrow e^y = \frac{dy}{dx} + e^{xy} \frac{dy}{dx}$$

$$y' = 2y \frac{dy}{dx} \text{ using } y = \cos y \cdot \frac{dy}{dx}$$

$$e^x - x^2 + e^{xy} y' = \frac{1}{y} \cdot \frac{dy}{dx}$$

y diff with respect to x.

P.T.O.

~~solve~~

$$x^v + y^v - 3xy + 10 = 0 \quad \text{Equation}$$

Dif^r with respect to x

$$2x + 2y \cdot \frac{dy}{dx} - 3 \left[x \frac{dy}{dx} + y \cdot 1 \right] = 0$$

$$\Rightarrow 2x + 2y \frac{dy}{dx} - 3x \frac{dy}{dx} - 3y = 0$$

$$\Rightarrow 2y \frac{dy}{dx} - 3x \frac{dy}{dx} = 3y - 2x$$

$$\Rightarrow \frac{dy}{dx} (2y - 3x) = 3y - 2x$$

$$\Rightarrow \frac{dy}{dx} (2y - 3x) = 6(3y - 2x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{3y - 2x}{2y - 3x} = \frac{6y}{6x}$$

$$\frac{10x - 6y}{6x - 10y} = 1$$

~~P79~~

Method 2

$$\frac{dy}{dx} = \frac{\text{of } f_y \text{ or } f_x}{\text{of } f_x \text{ or } f_y}$$

~~formula~~

~~Solving~~

$$f(x, y) = x + y - 3xy + 10$$

$$f_x = 2x - 3y$$

$$f_y = 1 - 3x$$

$$\therefore \frac{dy}{dx} = \frac{2x - 3y}{1 - 3x}$$

$$= \frac{3y - 2x}{2y - 3x}$$

P.T.

$$2x^3 - y^2 + 3x + 2y + 2 = 0$$

$$\frac{dy}{dx} = \frac{-f_x}{f_y}$$

Fig. c) Find min & max

P.I.P P.C.P C.C.P C.C.P

$$f_x = 6x^2 + 3 \quad (x=0) \text{ is min}$$

$$f_y = -2y + 2 \quad (y=1) \text{ is max}$$

critical points not to singularity

$$\frac{dy}{dx} = \frac{-6x^2 - 3}{-2y^2} \quad \text{not sing}$$

$$\therefore (-2, 2) = \frac{9}{2} \quad (x=0) \text{ is max}$$

Fig d) - critical point

$$(x=0, y=0) \quad (x=1, y=1)$$

point 2e, 2f, 2g, 2h

* Inflection point :- ex 5

Mid

at

Chg

(*) slope & connectivity $\Rightarrow (m^2)$

~~Ex :- 4.1.1, 4.1.2, 4.1.4, 4.1.5~~

Ex :- (1-5)

Ex :- 4.1 : 9, 10, (15-20)

(*) Analysis of function II : Relative

Extrema & Polynomials $\Rightarrow (3^u)$

{4.2.(1, 2, 3, 4)}

Ex :- (1-8)

Ex :- 4.2 \Rightarrow 33-54

(*) Partial Derivatives :- (x^b)

13.3 Ex :- (1-5), (10-14)

Ex :- 1-13, 25-52, 85-95, 95-104

~~Chain Rule~~: (7, 8)

Ex:- $13 \cdot 5 \cdot (2, 2, 3, 4, 5)$

Exm :- 1 - 8

Ex:- $(1-10) (17-34) (41-44) (50-54)$

~~Linear Algebra~~ :- (9, 10, 11)

2. 1 \Rightarrow Exm :- (1-2)

Ex:- $(1-23)$ odd

2. 2 \Rightarrow Exm :- (2-7)

Ex:- $(2-37)$ odd, 43, 45

$$y = (2x+1)^3$$

$$\Rightarrow y = 8x^3 + 3(2x)^2 \cdot 1 + 3 \cdot 2x \cdot 1^2 + 1^3 \\ = 8x^3 + 12x^2 + 6x + 1$$

$$y_1 = 24x^2 + 24x + 6 = 0$$

$$\Rightarrow 4x^2 + 4x + 1 = 0$$

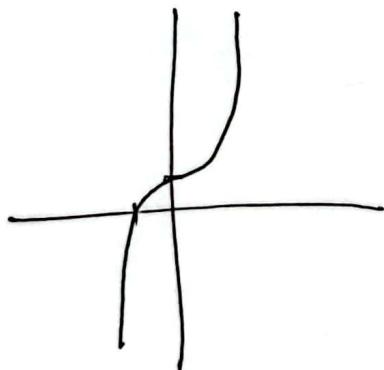
$$\Rightarrow x = -\frac{1}{2} \quad (-\frac{1}{2}, 0) \text{ turning point}$$

$$\therefore y = 0$$

$$y_1 \nearrow \begin{cases} \downarrow \\ \nearrow \end{cases} \nearrow$$

$$y_2 = 8x + 4$$

$$y_2 \Big|_{-\frac{1}{2}} = 0, \text{ Neither}$$



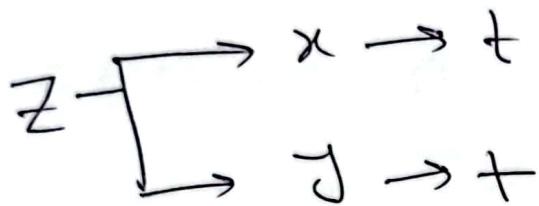
1.2.2
1.2.2.3

C2

* Chain rule :- $f = 3x^2 + 2x^3 - 1$

$$x = f(t) \rightarrow x^3 + 2x^2 - 1 = f(t)$$

$$z = f(x, t)$$



$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \times \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \times \frac{\partial y}{\partial t}$$

* $z = x^2 y + 5$

$$x = t^2 + 2$$

$$y = \sqrt{t}$$

$$\frac{\partial z}{\partial x} = \underbrace{?}_{P.T.}$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} + \frac{\partial z}{\partial t}$$

$$= 2x - y + 2t + x^2 \times \frac{1}{2\sqrt{t}}$$

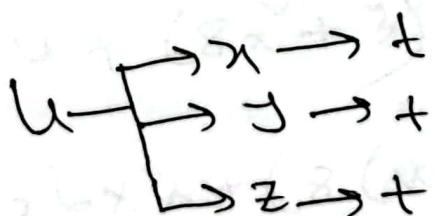
$$\Rightarrow 2(x+1)\sqrt{t} + 2t + (t+1) \times \frac{1}{2\sqrt{t}}$$

A

$$\frac{du}{dt} = f(x, y, z) \quad \text{and} \quad u = f(t)$$

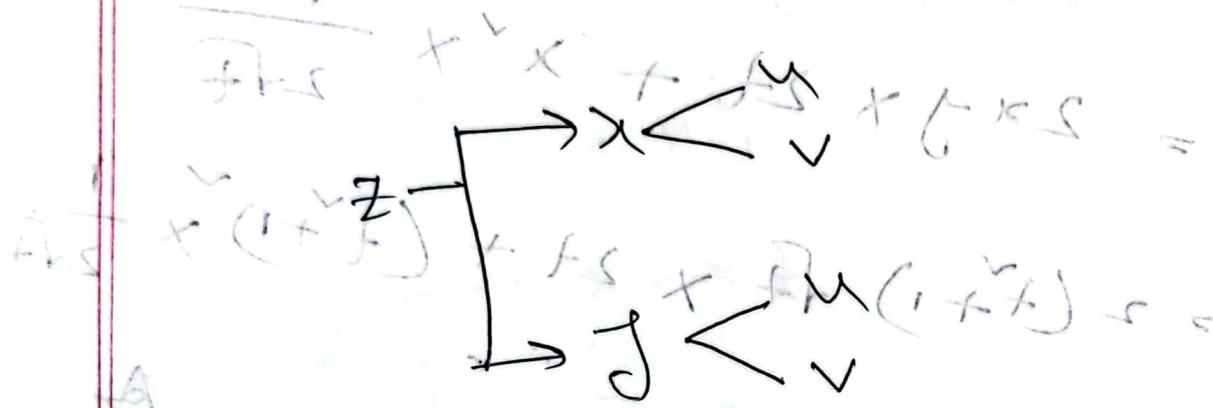
$$y = g(t), \quad z = h(t)$$

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \times \frac{dx}{dt} + \frac{\partial u}{\partial y} \times \frac{dy}{dt} + \frac{\partial u}{\partial z} \times \frac{dz}{dt}$$



P.S.

~~$\frac{\partial z}{\partial u} = f(x, y)$ and $\frac{\partial z}{\partial v} = f(u, v)$~~



$$\text{(e)} \frac{\partial z}{\partial u} = \frac{\partial z_0}{\partial x} \times \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \times \frac{\partial y}{\partial u}$$

$$\text{(f)} u = F, \text{ (g)} y = G$$

$$\frac{\partial z}{\partial u} = \frac{\partial z_0}{\partial x} \times \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \times \frac{\partial y}{\partial u}$$

$t \leftarrow x$
 $t \leftarrow c$
 $t \leftarrow s$

$\frac{\partial z}{\partial u}$

13.5

Want to minimize b/w maximizing it.

$$Z = 8xz^2 - 2x + 3y \quad ; \quad x = uv$$

where out to
 $y = u - v$

Want to minimize \Rightarrow L (Lagrange) function

$$\frac{\partial Z}{\partial u} = \frac{\partial Z}{\partial x} \times \frac{\partial x}{\partial u} + \frac{\partial Z}{\partial y} \times \frac{\partial y}{\partial u}$$

using formulae 1 approx + A

$$= (16yz^2 - 2) \times v + (8z^2 + 3) \times 1$$
$$= (16(uv)(u-v) - 2) \times v + (8(uv)^2 + 3) \times 1$$
$$= \{16(uv)(u-v) - 2\} \times v + (8(uv)^2 + 3) \times 1$$

$$[\text{not considering first}] \quad Lt = \frac{56}{56}$$

$$\frac{\partial Z}{\partial v} = \frac{\partial Z}{\partial x} \times \frac{\partial x}{\partial v} + \frac{\partial Z}{\partial y} \times \frac{\partial y}{\partial v}$$

$$= (16yz^2 - 2) \times u + (8z^2 + 3) \times 1$$
$$= (16(uv)(u-v) - 2) \times u + (8(uv)^2 + 3) \times 1$$

$$= \frac{16(uv)(u-v) - 2}{6} \times u + \frac{(8(uv)^2 + 3)}{6} \times 1$$
$$= 5vt = \frac{56}{56} \times 1 = \frac{56}{56}$$

6/1
20.3.23

C2

Maximum and Minimum of function
of two variables:-

$z = f(x, y)$ be a function of two
independent variable x & y .

At turning 1 stationary point

$$\frac{\partial z}{\partial x} = f_x(x, y) = 0 \text{ and also}$$

$$\frac{\partial z}{\partial y} = f_y(x, y) = 0.$$

[first derivative test]

$$2^{\text{nd}} \text{ derivative test } - \frac{\partial^2 z}{\partial x^2} = f_{xx}(x, y), \quad \frac{\partial^2 z}{\partial y^2} = f_{yy}(x, y).$$

$$\frac{\partial^2 z}{\partial x \partial y} = f_{xy} = \frac{\partial^2 z}{\partial y \partial x}$$

- ~~$f_{xx} f_{yy} - (f_{xy})^2 = D$~~
- for minima similar to $f_{xx} > 0$ for (x_0, y_0)
- (i) if $D > 0$ & $f_{xx} > 0$ then there exists a relative minimum.
 - (ii) if $D > 0$ & $f_{xx} < 0$; then there exists a relative maximum.
 - (iii) if $D < 0$; then (x_0, y_0) is a saddle point.
 - (iv) if $D = 0$, no conclusion can be drawn.

$$\begin{aligned} & \text{Let } x = x - x_0 \\ & \text{Let } y = y - y_0 \\ & \therefore (x, y) \end{aligned}$$

~~is a~~

Exm: 3

$$f(x, y) = 3x^2 - 2xy + y^2 - 18$$

Locate all relative extrema and saddle points of $f(x, y)$ (i)

$$f(x, y) = 3x^2 - 2xy + y^2 - 18 \text{ with}$$

$$f_x(x, y) = 6x - 2y = 0 \text{ (ii)}$$

$$\Rightarrow y = 3x \quad \text{(i)}$$

$$f_y(x, y) = -2x + 2y - 18 = 0 \quad \text{(iii)}$$

$$\Rightarrow x - y = -9 \quad \text{(ii)}$$

for (i) and (ii),

$$x - 3x = -y$$

$$\Rightarrow x = 2, \quad y = 6$$

$$\therefore (2, 6).$$

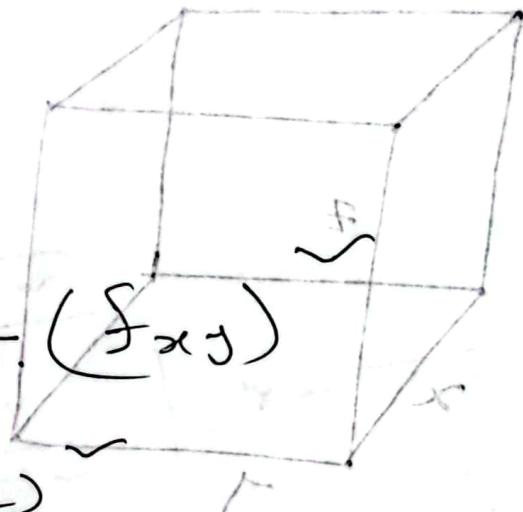
pt

$$f_{xx} = 6$$

$$f_{yy} = 2$$

$$f_{xy} = -2$$

$$D = f_{xx} f_{yy} - (f_{xy})^2$$



$$\epsilon = 12 - (2)$$

$\Rightarrow \epsilon = 10$ good opt. to solution

$\min = 870$ minimum with limit
to defining faculty point $(2, 6)$
since $D > 0$, $f_{xx} > 0$

relative minimum (without proof)

$$\frac{\partial f}{\partial x} = 6x + 2y - 2 = 0$$

$$6x + 6y + 2y - 2 = 0$$

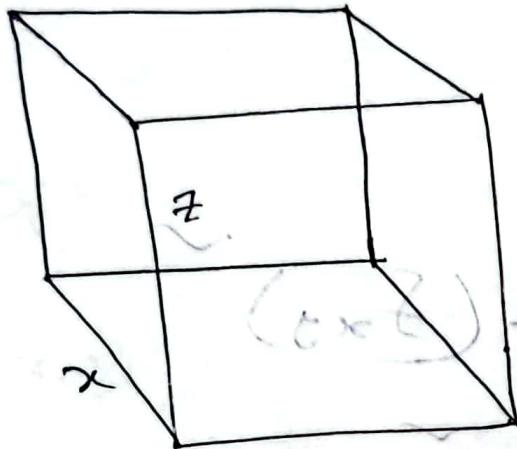
0.89

#

$$d = 1600 \text{ ft}$$

$$s = 60t$$

$$s = 60t$$



$$z = s t$$

volume of the box = $\frac{32}{s t} \text{ cm}^3$

Find the dimension on which will be required for minimize at construction.

$$\Rightarrow xyz = 32, z = \frac{32}{xy}$$

$$S = xy + 2xz + 2yz$$

P.T.O

$$0.5 = xy + 2 \cdot x \cdot \frac{32}{xy} + 2 \cdot y \cdot \frac{32}{xy}$$

$$0.5 = xy + \frac{64}{y} + \frac{64}{x} \Rightarrow 0.5 = 64/x + 64/y$$

$$f_x = y - \frac{64}{x^2}$$

$$f_y = x - \frac{64}{y^2} \quad (1)$$

$$y - \frac{64}{x^2} = 0$$

$$\Rightarrow \frac{64}{x^2} = y$$

$$\Rightarrow x^2 = \frac{64}{y}$$

~~$$\Rightarrow x = \frac{64}{y}$$~~

$$\Rightarrow y = \frac{64}{x^2}$$

$$\therefore y = 4$$

$$x = 64/y$$

$$x - \frac{64}{y} = 0$$

$$\Rightarrow x - \frac{64}{(64/x^2)} = 0$$

$$\Rightarrow x - \frac{64}{4096} = 0$$

$$\Rightarrow x - \frac{64x^4}{4096} = 0$$

$$\Rightarrow \frac{4096 - 64x^4}{4096} = 0$$

$$\Rightarrow 4096 - 64x^4 = 0$$

$$\Rightarrow 64x^4(64 - x^4) = 0$$

~~$$\Rightarrow x^4 = 64$$~~

$$\Rightarrow x = 0, 4$$

$$f_{xx} \cdot s_2 + \frac{128}{6x^3} \cdot x = \frac{128}{6x^2} \cdot x > 0$$

$$f_{yy} = \frac{128}{y^3} \cdot \frac{128}{64} = \frac{128}{64y^2} > 0$$

$$f_{xy} = 1$$

$$D = \frac{f_{xx}}{f_{yy}} - \frac{(f_{xy})^2}{f_{yy}}$$

$$(x_2) \frac{x}{2} \times 2$$

$$\frac{N^2}{N^2} - \frac{(f_{xy})^2}{f_{yy}}$$

$$D = \frac{N^2}{N^2} - b$$

$$\frac{32}{32} = \frac{32}{64 \times 4}$$

$$\frac{32}{64 \times 4} = \frac{1}{4}$$

$$\frac{Z = N^2 - x^2}{N^2} = \frac{32}{64} = \frac{1}{4}$$

$Z = \frac{1}{4}$ and $Z = 2$ will have the required to have the

minimum volume

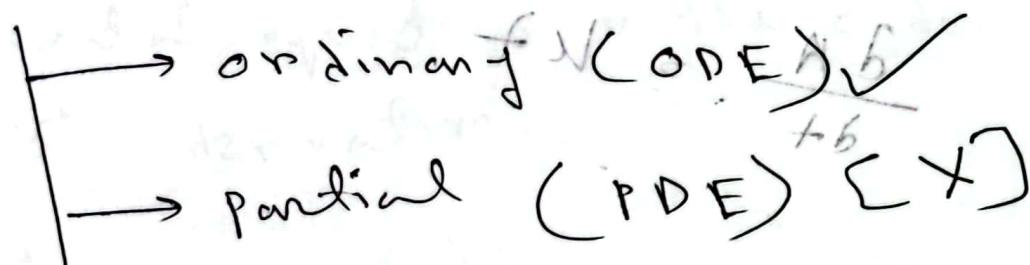
$$\frac{N^2}{N^2} = 6$$

$$N^2 = 6$$

$$N = 6 \therefore$$

Differential equation:-

Any equation that includes one or more terms of differentiation is known as differential equation.



* $\frac{dy}{dx} = 2xy$ (ODE)

* $x \frac{d^2y}{dx^2} - \left(\frac{dy}{dx} \right)^2 + 5 = 0$

* $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + 10 = 0$ (PDE)

P.T.O


 $\frac{dA}{dt} \rightarrow$ $\frac{dA}{dt} = \frac{1}{2} \pi r^2 \frac{dr}{dt}$
 $\frac{dA}{dt} = \frac{1}{2} \pi r^2 \times 2r \frac{dr}{dt}$
 $\frac{dA}{dt} = \pi r^3 \frac{dr}{dt}$
 $\frac{dA}{dt} = \pi r^3 \times 0.01$
 $\frac{dA}{dt} = 0.01 \pi r^3$

~~$\frac{dA}{dt} \rightarrow$ $\frac{dA}{dt} = \pi r^3 \times 0.01$
 $\frac{dA}{dt} = 0.01 \pi r^3$~~

$$(740) \left\{ \begin{array}{l} \frac{dV}{dt} = \frac{64}{100} \\ 0 = 2 + \left(\frac{64}{100} \right) - \frac{56}{100} x \end{array} \right.$$

$$0 = 2 + \left(\frac{64}{100} \right) - \frac{56}{100} x$$

$$(741) \quad 0 = 0.01 + \frac{0.64}{100} + \frac{0.56}{100} x$$

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2

Differential Equations - Definition/Order

$$x^2 \frac{d^3 y}{dx^3} + \frac{dy}{dx} + xy = 0$$

order with a non-zero coefficient of the highest derivative.

Order of a differential equation:-

differential equation is the order of the highest derivative.

$$\frac{d^3 y}{dx^3} + (x+1) \frac{dy}{dx} + xy = 0$$

3rd order

[3rd order equation]

Degree of differential equation:-

power of the highest derivatives to which it is raised.

$$x \left(\frac{d^2 y}{dx^2} \right)^2 + \left(\frac{dy}{dx} \right)^3 + 2x - 5 = 0$$

[2nd degree differential equation]

Solution of a differential equation :-

$$y = \sin x - \cos x$$

~~is a straight line~~ $\frac{dy}{dx} = \cos x$

which satisfies the differential equation.

$\frac{d^2y}{dx^2} = -\sin x$

The solution of a differential equation is the function that satisfies the given differential equation.

$$\left[\text{Ex. } \frac{d^2x}{dx^2} = kx \right] \rightarrow \left| \begin{array}{l} x \rightarrow \frac{dv}{dx} = v \\ \frac{dv}{dx} = \frac{d^2x}{dx^2} = kx \end{array} \right.$$

$\frac{d^2x}{dx^2} = kx$

of which depends on the value of k .

P.T.O

$$0 = \frac{d^2x}{dx^2} + kx$$

differentiable enough for

[not enough]

solution of a diff equation:-

General solution (GS) *

Particular solution (PS)

etc ...

* Any solution that includes an arbitrary constant is known as C.S.

$y = e^{ax+b}$ - non



Particular solution

$y = x^2 + 2$ [P.S] equivalent to

Linear & non-Linear differential equation:-

$$(x+1) \frac{dy}{dx} + y = 5 \quad [\text{linear}]$$

$$(x+1) \frac{d^2y}{dx^2} + y = x^2 + 1 \quad [\text{non}]$$

$$\sqrt{y} \frac{dy}{dx} + (x+1) \frac{dy}{dx} = 10 \quad [\text{non}]$$

P.T.O

- * Simplifying by $\frac{dy}{dx} + a_1(x) \frac{d^{n-1}y}{dx^{n-1}} = 0$
- * $a_0(x) \frac{dy}{dx^n}$ is not linear $\frac{dy}{dx^n} \neq 0$
- (2) $\frac{dy}{dx} + a_{(n-1)}(x) \frac{d^{n-1}y}{dx^{n-1}} + a_n = 0$

here $a_1, \dots, a_n \rightarrow$ are the coefficients and function of x
 \rightarrow [linear] \rightarrow non-linear

- * otherwise all non-linear.
- ## # Techniques of solving differential equation:-

(1) $\frac{dy}{dx} = f(x)$

[General]

$$y = \int f(x) dx + C$$

[For]

$$\frac{dy}{dx} = 2x+5 \quad [y(1)=3]$$

[Now]

$$y = \int 2x+5 dx + C$$

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$x = 2, y = 3$ (dangos solving #)

$$y = 1 + 5 + c(t)B + (c)T = \frac{66}{x^6}$$

$$\Rightarrow c = -3 \quad \text{so } c(t)B = -6(c)T \quad \text{so}$$

$$\therefore y = x^5 + 5x^{-3}$$

Given that $y = f(x)$ and the gradient of y is $-3x^6 + 4x + 1$ and the curve passes through the point $(-5, 7)$. Find y in terms of x .

$$y = -3x^6 + 4x + 1 + C$$

$$= -\frac{3x^3}{3} + \frac{4x^2}{2} + x + C$$

$$y = -\frac{x^6}{3} + 2x^5 + x + C$$

$$\Rightarrow 7 = -\frac{3+(-5)^3}{3} + 2+(-5)+(-5)+C$$

$$\Rightarrow 7 = 170 + C \quad \therefore y = -\frac{3x^3}{3} + 2x^5 + x - 163$$

$$\Rightarrow C = -163$$

$$\therefore y = -x^3 + 2x^5 + x - 163$$

Variable separable :-

$$\frac{dy}{dx} = f(x) \times g(y) \quad \text{or } f(y) dy = f(x) dx$$

$$\text{or, } f(y) dy = f(x) dx$$

* $\frac{dy}{dx} = f(x) f(y)$

then we have $f(y) dy = f(x) dx$ \Rightarrow $\int f(y) dy = \int f(x) dx$

but $\int f(y) dy = F(y) + C$ \Rightarrow $F(y) = \int f(x) dx - C$

$$\Rightarrow \int f(y) dy = \int f(x) dx \quad \text{using } x \text{ to matrix}$$

$$\therefore \frac{dy}{dx} = (x^2 + 1) \sqrt{y} \quad \Rightarrow \quad y^{-\frac{1}{2}} dy = (x^2 + 1)^{\frac{1}{2}} dx$$

$$\frac{dy}{\sqrt{y}} = (x^2 + 1)^{\frac{1}{2}} dx$$

$$\Rightarrow \int y^{-\frac{1}{2}} dy = \int (x^2 + 1)^{\frac{1}{2}} dx$$

$$\Rightarrow \frac{y^{\frac{1}{2}}}{\frac{1}{2}} = \frac{1}{3} x^3 + x + C = f(x)$$

$$\Rightarrow 2\sqrt{y} = \frac{1}{3} x^3 + x + C$$

$$\Rightarrow 2\sqrt{y} = \frac{1}{3} x^3 + x + C = f(x)$$

$$*(1+j) dx \rightarrow x dj$$

$$\Rightarrow \frac{(1+j)}{dj} A = \frac{x}{dx} \rightarrow \frac{Ab}{rb}$$

$$\Rightarrow \frac{dj}{(1+j)} = \frac{dx}{x}$$

~~$$= \frac{(1+j)^{-1} dj}{x} = \frac{rb}{rb} \times \frac{Ab}{rb}$$~~

~~$$= \frac{(1+j)^0 dj}{x} = \frac{rb}{rb} \times \frac{Ab}{rb}$$~~

~~$$= \int \frac{1}{(1+j)} dj \rightarrow \int \frac{1}{x} dx$$~~

~~$$= \ln(1+j) + C_1 + \frac{Ab}{rb} C$$~~

~~$$= \ln(1+j) + C_2 + \frac{Ab}{rb} C$$~~

~~$$\Rightarrow \ln(1+j) = \frac{Ab}{rb} C$$~~

~~$$\Rightarrow 1+j = e^{\frac{Ab}{rb} C}$$~~

~~$$\Rightarrow j = Cx - 2 + \frac{N}{2}$$~~

~~$$\Rightarrow \frac{N}{2} = N$$~~

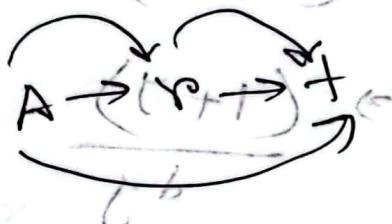
ω

C_2

~~Diff Eq~~

$$dt^b \propto = \kappa b(t+1)$$

$$\frac{dA}{dt} \propto \sqrt{\frac{\kappa b}{t+1}}$$



$$\Rightarrow \frac{dA}{dt} = k \times \frac{4}{3} \pi r^3$$

$$\Rightarrow \frac{dA}{dr} \times \frac{dr}{dt} = \frac{4k}{3} \pi r^3$$

$$\Rightarrow 8\pi r \times \frac{dr}{dt} = \frac{4k}{3} \pi r^3$$

$$\Rightarrow \frac{dr}{dt} + 3r = \frac{4k \pi r^3}{6(8\pi)} \text{ (cancel 8\pi)}$$

$$\Rightarrow \frac{dr}{dt} = \frac{V - r(t+1)}{6} \text{ (cancel r)}$$

$$\Rightarrow 2 = \frac{k}{6} \times 5 = \frac{5k}{6} \Rightarrow k = 12$$

$$\Rightarrow k = \frac{12}{25}$$

$$\therefore \frac{dr}{dt} = \frac{\frac{12}{25}r^2}{25 \times 6} = \frac{1}{25r}$$

$$5.0 - 480.0 = \frac{1}{25r} - \text{(i)}$$

$$480.0 - 5.0 = 0.08 r$$

$$480.0 - 5.0 = \frac{1}{25r}$$

$$\frac{dr}{dt} = \frac{0.08r}{480.0 - 5.0}$$

~~$$0.08r = 480.0 - 5.0$$~~ (ii)

$$\int \frac{dr}{r} = \int 0.08 dt + 80.0$$

$$\frac{1}{r} = 0.08t + \frac{80.0}{r}$$

$$\int \frac{1}{r^2} dr = 0.08t + C$$

$$\Rightarrow \int r^{-2} dr = 0.08t$$

$$\Rightarrow -r^{-1} = 0.08t + C$$

$$\Rightarrow -\frac{1}{r} = 0.08t + C$$

$$\Rightarrow -\frac{1}{5} \geq C \Leftrightarrow C \leq -0.2$$

$$\Rightarrow -\frac{1}{r} \geq 0.08 + -0.2$$

$$\Rightarrow \frac{1}{r} = 0.2 - 0.08 +$$

$$\Rightarrow r = \frac{0.2 - 0.08 +}{0.2 - 0.08 +}$$

(iii)

$$0.2 - 0.08 + > 0$$

$$\Rightarrow 0.08 + < 0.2$$

$$\Rightarrow + < \frac{0.2}{0.08}$$

$$\Rightarrow 0.2 < 2.5$$

$$+ 80.0$$

$$+ 80.0$$

$$+ 80.0$$

$$\frac{1}{r} = e$$

~~Q10~~

(a) $\frac{dv}{dt} = -kv$

$$\frac{dv}{dt} = -kv$$

(b) $t=0, v=10000$

$$\frac{dv}{dt} = -kv$$

$$\int \frac{dv}{v} = \int -k dt$$

$$kvt \Rightarrow v = e^{-kt + C}$$

$$v = e^{-kt + C}$$

$$v = e^{-kt + C} \cdot 10000$$

$$\Rightarrow 10,000 = e^c \cdot e^{(v-u)t} \quad (a)$$

$$\Rightarrow e^c = 10000$$

$$V = e^{(v-u)t} = \frac{v}{u}$$

$$\therefore V = 10000 e^{-ut}$$

$$10000 e^{-36u} = 5000 \quad (b)$$

$$t = 36, \quad V = 5000$$

$$\Rightarrow 5000 = 10000 \times e^{-\frac{36u}{6}}$$

$$\Rightarrow e^{-\frac{36u}{6}} = \frac{1}{2} \quad ?$$

$$\Rightarrow -36u = \ln(\frac{1}{2}) \quad ?$$

$$\Rightarrow u = -\frac{0.693}{36} \quad ?$$

$$\Rightarrow 0.01925 \quad ?$$

$$-0.01925 \quad ?$$

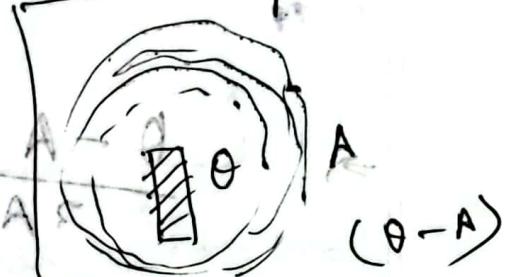
$$\therefore V = 10000 e^{-0.01925 t}$$

$$A \varepsilon = 10000 e^{-0.01925 \times 24} = (A - \theta) \quad | t = 24 \text{ m}$$

$$= 6.3608 \cdot 22 \left(\frac{A - \theta}{A \varepsilon} \right) \text{ rad.}$$

~~θ~~

t_N



$$(1) \quad \frac{d\theta}{dt} = A(\theta - A) \quad | t_N$$

$$\Rightarrow \frac{d\theta}{dt} = -k(A - \theta) \quad | t_N$$

$$\int \frac{d\theta}{\theta - A} = -k \int dt \quad | t_N$$

~~$\ln(\theta - A) = -kt + C$~~

$$\Rightarrow \ln(\theta - A) = -kt + C$$

$$\Rightarrow \ln(4A - A) = 0 + C$$

$$\Rightarrow \ln 3A = C$$

$$\ln \left(\frac{\theta - A}{3A} \right) = -kt + \ln 3A$$

$$\ln \left(\frac{\theta - A}{3A} \right) = s - kt \quad ?$$

$$\therefore \frac{\theta - A}{3A} = e^{-ut}$$

$$\Rightarrow \theta - A \underset{(\theta = A)}{=} 3A e^{-ut} \quad (i)$$

$$\Rightarrow \theta = A + 3A e^{-ut}$$

$$(ii) 3A = A + 3A e^{-ut}$$

$$\Rightarrow 2A = 3A e^{-ut} = \frac{A}{e^{ut}}$$

~~$$\Rightarrow \frac{2}{3} = e^{-ut} \cdot (\theta - A) \text{ wl}$$~~

~~$$\Rightarrow -ut = \frac{2}{3} \ln (\theta - A)$$~~

~~$$\Rightarrow u_2 + \ln = \frac{3}{2} \ln (\theta - A) \text{ wl}$$~~

~~$$J = A e^{-ut}$$~~

Linear & First order differential
equation :-

$$\boxed{\frac{dy}{dx} + P(x)y = Q(x)}$$

Integration Factor = $e^{\int P(x) dx}$

$$x \cdot \frac{dy}{dx} + (2x+1)y = e^{\int (2x+1) dx} = e^{x^2+x}$$

$$x \cdot \frac{dy}{dx} - 4y = -x^2 - x$$

$$\frac{dy}{dx} - \frac{4}{x}y = x^2 + x \quad (\text{standard form})$$

$$y = A(6(1+3e^{-xt}) - \ln \frac{3}{2} \times 2)$$

$$= A(1 + 3e^{-xt} + 3 - \ln \frac{3}{2} \times 2)$$

$$= \frac{7}{3}A(1 + 3e^{-xt} + 3 - \ln \frac{3}{2} \times 2)$$

$$= \frac{7}{3}A(1 + 3e^{-xt} + 3 - \ln \frac{3}{2} \times 2)$$

homogenes mit $\frac{4}{x^2}$ und $\frac{6}{x^3}$ \rightarrow I.F. = $e^{-\frac{4}{x^2}dx}$

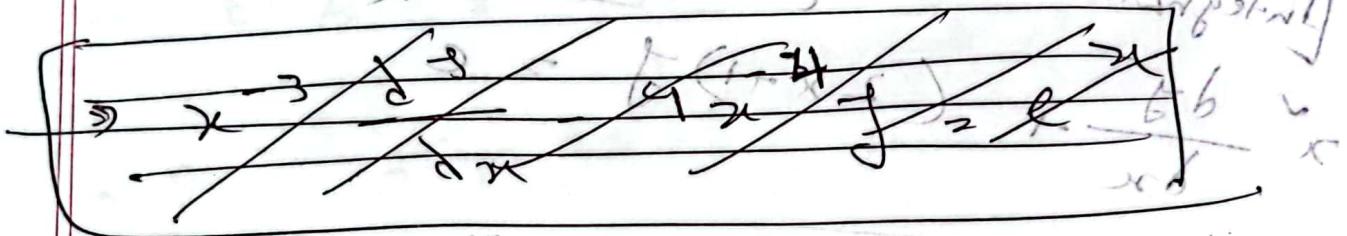
\therefore weiter

- und

$$\left(\frac{d}{dx} - \frac{4}{x^2} \right) y + \frac{6}{x^3} = 0$$

separabel nach y

$\Rightarrow y = \frac{1}{x^6} \cdot C_1 e^{-\frac{4}{x^2}}$ = reell mit Punkt



$$x^{-4} \frac{dy}{dx} - 4x^{-6} y = x^{-6} e^x$$

$$\Rightarrow \frac{d}{dx} \left[x^{-4} y \right] = x^{-6} e^x$$

$$x^{-4} y = x^{-6} e^x + C_1 \quad A = 0$$

$$x^{-4} y = x^{-6} e^x + C_1 x^4 e^{-4}$$

$$\Rightarrow y = x^4 e^{-x} + C_1 x^4 e^{-4}$$

~~$\frac{dy}{dx}$~~ $y = 5 + \frac{tb}{x^b} \rightarrow A$

~~* $(x^v+9) \frac{dy}{dx} + x^v y' = 0 - \frac{tb}{x^b}$~~

$\Rightarrow \frac{dy}{dx} + \left(\frac{x}{x^v+9} \right) y = 0$

I.F. = $e^{\int \frac{x}{x^v+9} dx}$

 $\Rightarrow e^{\frac{1}{2} \int \frac{2x}{x^v+9} dx}$

~~$\int \frac{f'(x)}{f(x)} dx = \ln f(x)$~~

$\Rightarrow e^{\frac{1}{2} \ln(x^v+9)}$

$\Rightarrow e^{\frac{1}{2} \ln(x^v+9)} = 5 + \frac{tb}{x^b} \rightarrow$

$\Rightarrow e^{\frac{1}{2} \ln(x^v+9)} = (5x) \frac{tb}{x^b}$

$\Rightarrow 5x = \sqrt{x^v+9} + \frac{tb}{x^b}$

$\sqrt{x^v+9} \frac{dy}{dx} + \left(\frac{x}{\sqrt{x^v+9}} \right) y = 0$

$\Rightarrow \frac{d}{dx} (\sqrt{x^v+9} \cdot y) = 0$

$\Rightarrow \sqrt{x^v+9} \cdot y = C$

$\therefore y = \frac{C}{\sqrt{x^v+9}}$

$$\# x \frac{dy}{dx} + y = 2x, \quad y(1) = 0$$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x}y = \frac{2}{x^2} (e+x)$$

$$O \cdot \delta \left(\frac{1}{x} \frac{dy}{dx} \right) + \frac{dy}{x^2}$$

$$\Rightarrow T.F = e$$

$$(e+x)^2$$

$$c$$

$$c^2$$

$$x \frac{dy}{dx} + y = 2x$$

$$\Rightarrow \frac{dy}{dx} (x+y) = 2x$$

$$x \frac{dy}{dx} = x^2 - 1$$

$$\Rightarrow x \frac{dy}{dx} = x^2 + c$$

$$O \cdot \delta \left(\frac{1}{x} \frac{dy}{dx} \right)$$

$$O = y + c$$

$$\frac{dy}{dx} = x - \frac{1}{x}$$

$$\Rightarrow x \frac{dy}{dx} = x^2 - 1$$

$$\Rightarrow C = -1$$

$$* \frac{dy}{dx} = y + e^x \quad \text{.....(ii)}$$

$$* \frac{dx}{dy} = x + y^2$$

$$* y' = 2y + x(e^{3x} + e^{2x})$$

$$\frac{dy}{dx} = y + \frac{x^6}{e^6} \quad y(0) = 2$$

$$* \frac{dy}{dx} = y + e^{2x} \quad (x \cdot e^{6-x}) \frac{d}{dx}$$

$$I.F = e^{\int \frac{1}{e^{-2}} dx} = e^{6-2x}$$

$$e^{-x} \frac{dy}{dx} - e^{-x} y = 1$$

$$\Rightarrow \frac{d}{dx} (e^{-x} \cdot y) = 1$$

$$\Rightarrow e^{-x} y = x + C$$

$$\Rightarrow y = x e^x + C e^x$$

$$(ii) \frac{dx}{dt} - x = \int_0^t \frac{ab}{x^6} dt \quad (*)$$

$$\text{Int F} = e^{\int -\frac{ab}{x^6} dx} = \frac{x^6}{b^6}$$

$$\Rightarrow e^{-\int \frac{ab}{x^6} dx} = e^{\frac{x^6}{b^6}}$$

$$\Rightarrow e^{-\int \frac{ab}{x^6} dx} - xe^{-\int \frac{ab}{x^6} dx} = \int e^{-\int \frac{ab}{x^6} dx} dt$$

$$\Rightarrow \frac{d}{dt} (e^{-\int \frac{ab}{x^6} dx} \cdot x) = \int e^{-\int \frac{ab}{x^6} dx} dt$$

$$\Rightarrow x e^{-\int \frac{ab}{x^6} dx} = \int \int e^{-\int \frac{ab}{x^6} dx} dt = T \cdot T$$

$$T = b^{\frac{1}{5}} t - \frac{ab}{x^6} \dots$$

$$T = (t - \frac{ab}{x^6}) \frac{b}{abt} \dots$$

$$t - \frac{ab}{x^6} = \frac{b}{abt} \dots$$

$$t^2 - \frac{ab^2}{x^{12}} = \frac{b^2}{a^2b^2t^2} \dots$$

Assignment 2

~~Q*~~ ~~Find partial derivatives to eval. at point A~~

$$w = x^2y - 3x^2z^2 + yz^2$$

$$x = p \sqrt{a}$$

$$y = p^2 \sqrt{b}$$

$$z = \sqrt[3]{p}$$

$$\frac{\partial w}{\partial p} = \frac{\partial w}{\partial x} \times \frac{\partial x}{\partial p} + \frac{\partial w}{\partial y} \times \frac{\partial y}{\partial p} + \frac{\partial w}{\partial z} \times \frac{\partial z}{\partial p}$$

[using] $\frac{\partial x}{\partial p} = \frac{\partial y}{\partial p} = \frac{\partial z}{\partial p} = 1$

[using] $\frac{\partial x}{\partial p} = \frac{\partial y}{\partial p} = \frac{\partial z}{\partial p} = 1$

$\frac{\partial w}{\partial p} = 2p\sqrt{a} - 6p^2\sqrt{b} + p^2\sqrt{b}$

~~GW~~

~~Stagnation point~~

* Newton's law of cooling

* Analysis of function: $y = x^{\frac{1}{n}}$

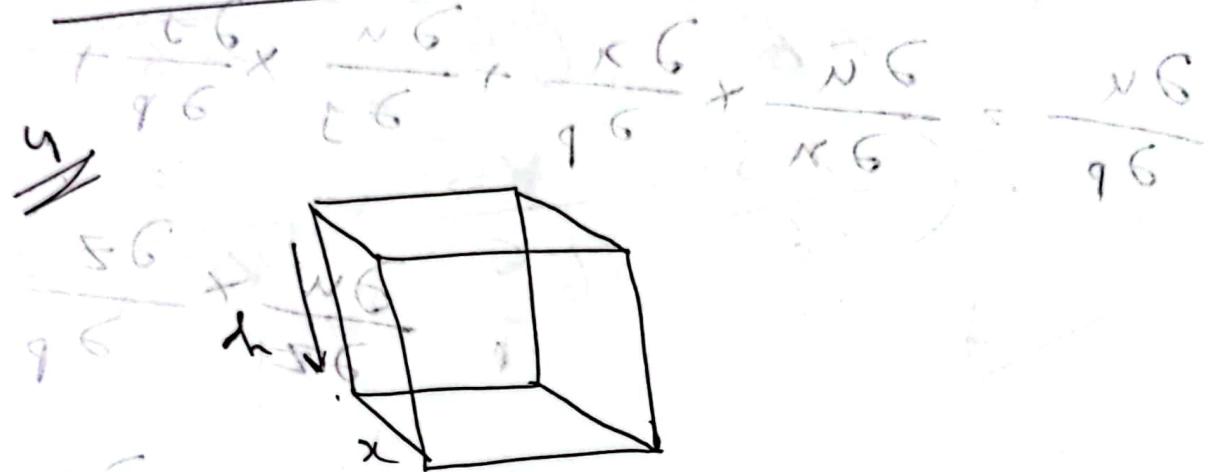
* Turning point / stationary point

$$f(x) = x^{\frac{1}{n}}$$
$$f'(x) = \frac{1}{n} x^{\frac{1}{n}-1}$$
$$\text{Max, } f'(x) = 0 \Rightarrow x^{\frac{1}{n}-1} = 0 \Rightarrow x = 1$$
$$\text{Min, } f'(x) > 0 \Rightarrow \frac{1}{n} x^{\frac{1}{n}-1} > 0 \Rightarrow x^{\frac{1}{n}-1} > 0 \Rightarrow x > 0$$

$f'(x) > 0$ [increasing]

$f'(x) < 0$ [decreasing]

* optimization:



$$+ \frac{56}{96}x + \frac{x}{96} + \frac{2x}{96} + \frac{2x}{96} + \frac{2x \times h}{96} = \frac{120}{96}$$

$$A = 2 \left(x \times 2x + x \times h + \frac{2x \times h}{96} \right) = 120$$

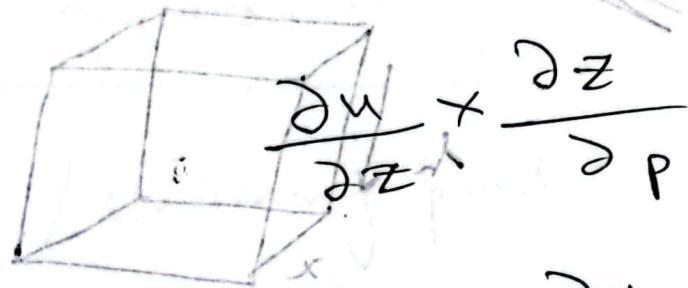
$$\Rightarrow 4x^2 + 6xh = 120$$

$$\Rightarrow h = \frac{60 - 2x}{3x}$$

$$V = \cancel{\frac{56}{96}x \times \frac{x}{96} \times \frac{2x}{96} \times h} = \frac{w b}{96}$$
$$\therefore \left(\frac{56}{96}x \times \frac{x}{96} \times \frac{60 - 2x}{3x} \right) = \frac{40x - \frac{4x^3}{3}}{96}$$

~~partial derivatives~~ ~~independence~~

$$\frac{\partial u}{\partial p} = \frac{\partial u}{\partial x} \times \frac{\partial x}{\partial p} + \frac{\partial u}{\partial y} \times \frac{\partial y}{\partial p} +$$



$$\frac{\partial u}{\partial Q} = \frac{\partial u}{\partial x} \times \frac{\partial x}{\partial Q} + \frac{\partial u}{\partial y} \times \frac{\partial y}{\partial Q} +$$

$$\text{or } = (\cancel{\frac{\partial u}{\partial x}} \times \cancel{\frac{\partial x}{\partial Q}}) + \frac{\partial u}{\partial z} \times \frac{\partial z}{\partial Q}$$

$$\text{or } = \cancel{\frac{\partial u}{\partial x}} + \cancel{\frac{\partial u}{\partial y}}$$

~~$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \times \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \times \frac{\partial y}{\partial t} + \frac{\partial u}{\partial z} \times \frac{\partial z}{\partial t}$$~~

~~$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \times \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \times \frac{\partial y}{\partial t} + \frac{\partial u}{\partial z} \times \frac{\partial z}{\partial t}$$~~

~~E lastik struk~~

$$\left(\frac{\partial w}{\partial u} \times \frac{\partial u}{\partial y} + \frac{\partial w}{\partial t} \right) +$$

$$\left(\frac{\partial w}{\partial v} \times \frac{\partial v}{\partial y} + \frac{\partial w}{\partial s} \right) (v)^{1/2}$$

$$+ \frac{\partial w}{\partial e} - \frac{\partial w}{\partial s} = \cancel{(v)^{1/2}}$$

$$+ \frac{\partial w}{\partial e} - \frac{\partial w}{\partial s} = \cancel{(v)^{1/2}}$$

$$+ 0 = 2$$

$$P = C$$

$$2 + \frac{x}{2} = \frac{m}{x} = 5$$

0.79

Work sheet 3

(2)

+ ~~Aim to solve question no: 1 to 6~~

$$f'(x) = \cancel{2x} - \frac{1}{3} + \cancel{\frac{1}{3}}$$

$$f(x) = \frac{2x^{\frac{2}{3}}}{\frac{2}{3}} + \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + C$$

$$\Rightarrow 5 = \frac{2 \cdot (8)^{\frac{2}{3}}}{\frac{2}{3}} - \frac{(8)^{\frac{4}{3}}}{\frac{4}{3}} + C$$

$$\Rightarrow 5 = 0 + C$$

$$\Rightarrow C = 5$$

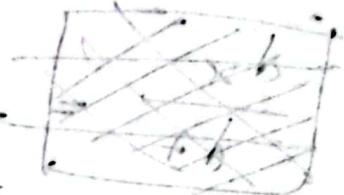
$$y = \frac{2x^{\frac{2}{3}}}{\frac{2}{3}} - \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + 5$$

A

P.T.O.

A m to the ques no: 2 (d)

$$\frac{dy}{dx} = 6x^5 - \frac{4}{x^3}$$



$$\begin{aligned}\frac{dy}{dx} &= \frac{6x^3}{3} - 4 \cdot \frac{x^{-2}}{x^5} \\ &= 2x^3 + \frac{2}{x^5}\end{aligned}$$

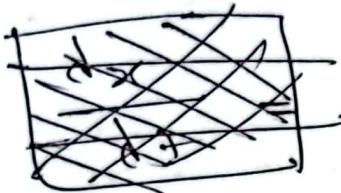
$$\begin{aligned}y &= \frac{2x^4}{4} + 2 \cdot x^{-2} \\ &= \frac{x^4}{2} + 2 \cdot \frac{1}{x^2}\end{aligned}$$

$$(1) \quad x = -1, \quad \frac{dy}{dx} = 6(-1)^3 - \frac{4}{(-1)^3} = 10 > 0$$

The function has local minimum.

(b) ~~s : on exp left of A~~

$$\frac{ds}{dt} = 5 \text{ unit/s}$$

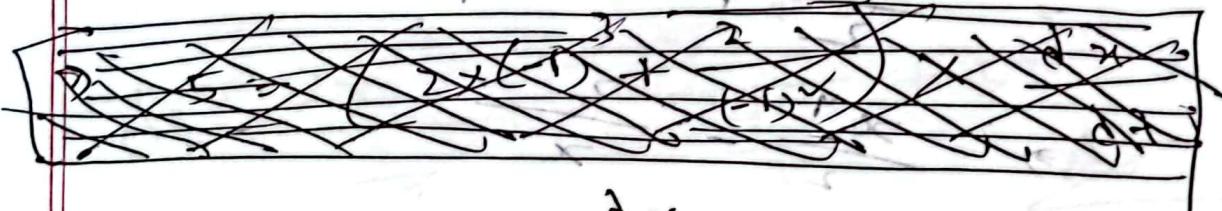


$$\frac{ds}{dt} = \frac{\partial s}{\partial x} \frac{\partial x}{\partial t} = \frac{6b}{x^2}$$

$$\frac{ds}{dt} = \frac{\partial s}{\partial x} \times \frac{\partial x}{\partial t} = \frac{6b}{x^2}$$

$$\rightarrow (2x^3 + 2) \times \frac{\partial x}{\partial t}$$

$$\Rightarrow 5 = (2x^3 + 2) \times \frac{\partial x}{\partial t} = 6$$



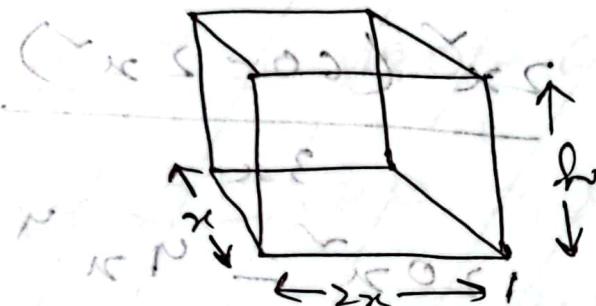
$$\Rightarrow 5 = 4 \times \frac{\partial x}{\partial t}$$

$$\Rightarrow \frac{\partial x}{\partial t} = \frac{6b}{4} = 1.5 \text{ unit/s}$$

$$\Rightarrow \frac{\partial x}{\partial t} = 1.25 \text{ unit/s}$$

Ans
answering book and worksheet 3, 17

Ans to the que no 4



$$\begin{aligned}a &= x \\b &= 2x \\c &= h\end{aligned}$$

(i)

$$2(ab + bc + ca) = 120$$

$$\Rightarrow 2(2x \cdot 2x \cdot h + 2x \cdot h + x \cdot h) = 120$$

~~$\Rightarrow 2(4x^2h + 2xh + xh) = 120$~~

$$\Rightarrow 2x^2h + 2xh + xh = 60 \quad (iii)$$

$$\Rightarrow 2xh + xh = 60 - 2x^2h$$

$$\Rightarrow 3xh =$$

$$\Rightarrow h = \frac{60 - 2x^2}{3x}$$

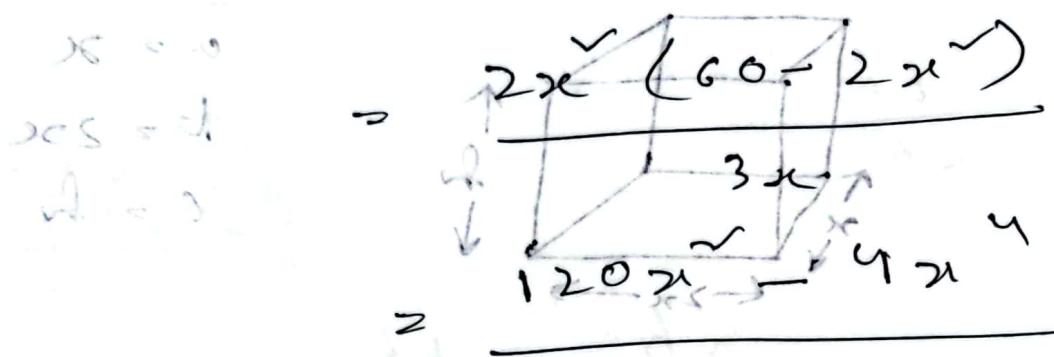
~~$\Rightarrow \frac{60 - 2x^2}{3x} = x$~~

• forms cubic eqn.

• write part

times solve x
Grapic method

$$(ii) \frac{dV}{dx} = 2x^3 h - 2x \left(\frac{60 + 4x^2}{3x} \right)$$



$$dV/dx = (4x^3 + 120x) - \frac{8x^3}{3}$$

$$dV/dx = (2x^3 + 40x) - \frac{4x^3}{3}$$

(iii)

$$\frac{dV}{dx} = 40 - \frac{12x^2}{3} + \frac{4x^3}{3}$$

~~$$x = 3.16$$~~

~~$$40 - 4x^2 = 0$$~~

~~$$\Rightarrow x = (3.16 - 3.16)$$~~

{x value can't
be negative}

~~{x value can't be
negative}~~

$$\frac{4x}{3} = \frac{120 - 12x}{3}$$

$$\Rightarrow 12x = 180 - 6x$$

$$\Rightarrow 18x = 180$$

$$\Rightarrow x = 10$$

~~some str.~~

$h = \frac{4x}{3}$

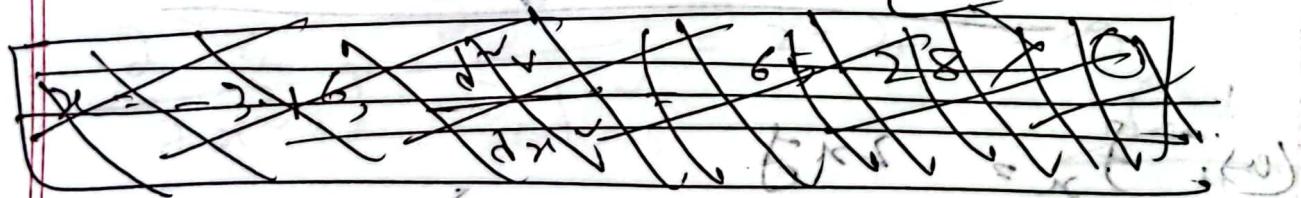
$h = \frac{40}{3} \approx 13.33$

$\therefore V = 40 \times 10 \times 13.33 = 5333.33 \text{ cu m}$

$$\frac{dV}{dx} = 8x$$

$$125.28 \text{ cu m}$$

$$x = 10, \frac{dV}{dx} = 80 \text{ cu m} \leftarrow \text{maximum width of local minima}$$



$$(s-1) + (14 \times (3.16)^3)$$

$$V = 40 \times 3.16 - \frac{N-3}{84.33 \text{ cu m}}$$

$$= 84.33 \text{ cu m}$$

$$\therefore V = (s-1) \times 21 \text{ cu m} = 67 \text{ cu m}$$

Aim to the question: 5

$$f_x(1, 3) = 6x^2 - y^2 + 4$$

$$= 6 \times 1 \times (3)^2 + 4$$

$$\rightarrow 58$$

$$f_y(1, 3) = 4x^3 - y + 2$$

$$= 4(1)^3 - 3 + 2$$

Aim to the question no: 6 = 15

(a) $f_x = 2x^2 \dots \checkmark$

$$(2 \times 2) + (1) \times (-2)$$

$$= 21 \cdot 2 + 0 \cdot 1 = 15$$

(b) $f_y = x^2 + 15 \checkmark$

$$= 1^2 + 15 \times (-2) = 60$$

Ans → to the ques no. 7 (i)

(a) $Z = 6x^v - j^v + 2xj$

$$\frac{\partial Z}{\partial x} = 2x + 2j \quad \frac{56}{56}$$
$$\frac{\partial Z}{\partial x^v} = 2 \quad \frac{56}{56}$$
$$\frac{\partial Z}{\partial j} = -2(j + 2x) \quad \frac{56}{56}$$
$$\frac{\partial^v Z}{\partial j^v} = -2 \quad \frac{56}{56}$$
$$\frac{\partial^v Z}{\partial x^v} + \frac{\partial Z}{\partial j^v} = 2 - 2 \quad \frac{56}{56} = 0$$

$$(b) \frac{\partial z}{\partial x} = e^x \sin y + e^y (-\sin x)$$

$$\frac{\partial^2 z}{\partial x^2} = e^x \sin y - e^y \cos x \quad (1)$$

$$e^x \sin y - e^y \cos x = \frac{56}{\sqrt{56}}$$

$$\frac{\partial z}{\partial y} = e^x \cos y + e^y \cos x \quad \frac{56}{\sqrt{56}}$$

$$\frac{\partial^2 z}{\partial y^2} = e^x (-\sin y) + e^y \cos x \quad \frac{56}{\sqrt{56}}$$

$$= -e^x \sin y + e^y \cos x \quad \frac{56}{\sqrt{56}}$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^x \sin y - e^y \cos x -$$

$$e^x \sin y + e^y \cos x \quad \frac{56}{\sqrt{56}}$$

$$= 0$$

Ans

Answer to the question no: 8

$$Z = x^2 y + t^2 + 6x + 6 - (t, x) \text{ & } (x)$$

$\frac{\partial Z}{\partial x} = 2xy + 6$

$$\begin{aligned}\frac{\partial Z}{\partial t} &= \frac{\partial Z}{\partial x} \times \frac{\partial x}{\partial t} + \frac{\partial Z}{\partial y} \times \frac{\partial y}{\partial t} \\&= 2xy \times 2t + x^2 \times 3t - (t, x) \\&= 4xyt + 3x^2 t\end{aligned}$$

$$\begin{aligned}Z &= x^2 y = \cancel{x^2} \cancel{y} = \cancel{6x^2 y} \\&\Rightarrow \cancel{(x^2 \cdot y)} = \cancel{6x^2 y} \\&\Rightarrow \cancel{3x^2 + 1} \cancel{y} = \cancel{6x^2 y} = 0 \\&= x^2 \cancel{\frac{\partial}{\partial t}(y)} + \cancel{y} \cancel{\frac{\partial}{\partial t}(x^2)} \\&\Rightarrow x^2 3t + y 4t \\&\Rightarrow x^2 3t + 4t + y \\&\Rightarrow x^2 3t + 4xy + [t = x]\end{aligned}$$

8. \checkmark Answer to the question: 9

(a) $f(x, y) = y^2 + xy + 3y + 2x + 3$

$f_x = y + 2 = 0$

$f_y = 2y + x + 3 = \frac{1}{6} \times \frac{56}{6} = \frac{56}{6}$

$(x, y) = \underline{\underline{(2, -2)}}$

$f_{xx} = 0$

$f_{yy} = 2$

$f_{xy} = 1$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$= 0 - (-1)^2$

$= -1 < 0$

Local maximum available.

$(x=2) + 6x^2 + 2x^2 + 3x^2 = 0$

$$(b) f(x, y) = x^2 + xy - 2y - 2x + 1$$

$$f_x = 2x + y - 2 \rightarrow (0, x^2 + x + 1)$$

$$f_y = x - 2 = 0$$

$$(x, y) = \left(\frac{2}{1+x^2 + x} \right) = \frac{2}{1+x^2}$$

$$f_{xx} = \frac{1}{(1+x^2)^2} > 0 \text{ (local min)}$$

$$f_{yy} = 0$$

$$f_{xy} = 1 - \frac{1}{(1+x^2)^2}$$

$$D = 0 - 1 = -1 < 0$$

This function has local maximum

value when $x = 0$.

(Local max) $\Rightarrow (1+x^2) \text{ when } x = 0$

(Local min) $\Rightarrow (1+x^2) \text{ when } x = \pm 1$

$\therefore f(x, y) = x^2 + xy - 2y - 2x + 1$

~~3x^2 + x^2 - 5x - f(x) = g(x)~~ (b, c) & (d)

$$(3x^2 + x^2 + 3x + 1) \frac{dy}{dx} - 6y = x^2$$

$$\int \frac{dy}{y} = \int \frac{1}{(3x^2 + x^2 + 3x + 1)^2} dx \quad (b, c)$$

$$\rightarrow \ln y = \int \frac{1}{(x+1)(3x+1)} dx = x^2$$

$$\rightarrow \int \frac{3}{2(3x+1)} - \frac{1}{2(x+1)} dx$$

$$= \frac{3}{2} \int \frac{1}{(3x+1)} dx - \frac{1}{2} \int \frac{1}{x+1} dx$$

$$= \frac{3}{2} \times \frac{1}{3} \int \frac{1}{u} du - \frac{1}{2} \ln(x+1)$$

$$= \frac{3}{2} \times \frac{1}{3} \ln(3x+1) - \frac{1}{2} \ln(x+1)$$

$$= \underline{\underline{\frac{\ln(3x+1) - \ln(x+1)}{2}}}$$

$$u = 3x+1$$

~~using direct~~

~~using first diff~~

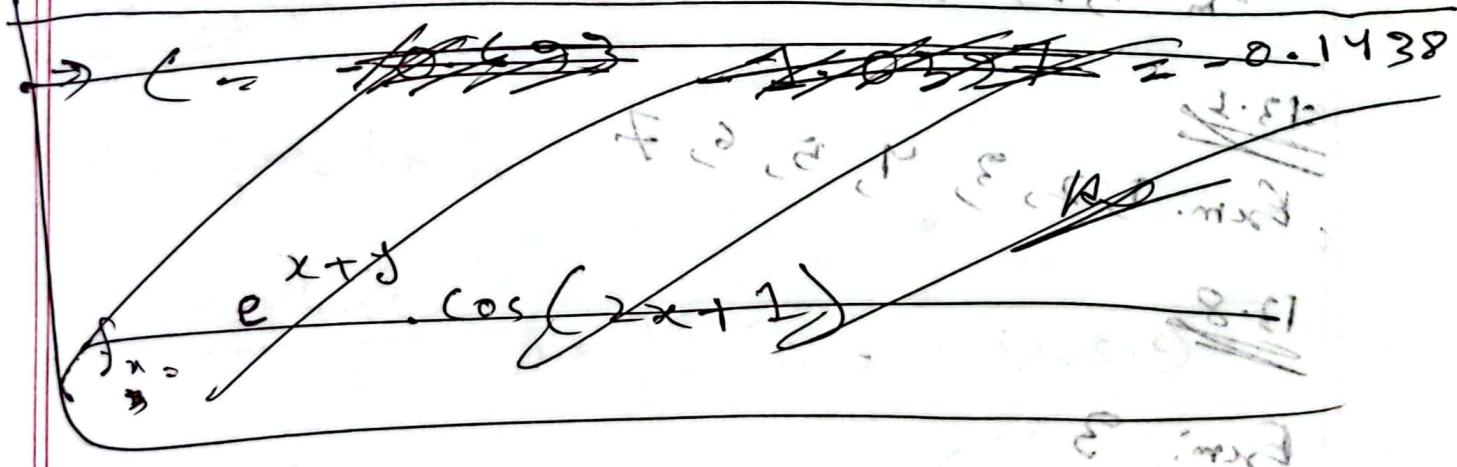
$$du$$

$$\left\{ \begin{array}{l} x=1 \\ y=2 \end{array} \right.$$

~~8.5 P.E.C. int~~

$$\ln y = \frac{(\ln(3x+1) - \ln(x+1))}{2} + C$$

$$\Rightarrow \ln y - (\ln(3x+1) - \ln(x+1)) = C$$



$$\Rightarrow \ln(2) - \frac{(\ln(6) - \ln(3))}{2} = C$$

$$\Rightarrow 0 - 0.144 = C$$

$$\Rightarrow -0.144 \quad y = 1.37$$

$$x=3$$

Some basic Rule

Circle:

Area, $A = \pi r^2$ length, $L = 2\pi r$

Volume, $V = \frac{4}{3}\pi r^3$ area, πr^2

Rectangle:

Length, $L = 2(l+w)$

Area, $A = l \times w$ $l = \text{length}$

Volume, $V = l \times w \times h$ $w = \text{width}$ $h = \text{height}$

Triangle:

$A = \frac{1}{2} \times b \times h$

$s = \frac{a+b+c}{2}$ $A = \sqrt{s(s-a)(s-b)(s-c)}$

triangular prism: $V = A \times h$ $[A = \text{Area}]$

n pyramid: $V = \frac{1}{3} A \times h$

~~short word was~~

cylinder:

~~is short~~

Lateral area $L = 2\pi r h$ $\Rightarrow A_{\text{lateral}}$

Total surface area $A = 2\pi r^2 + 2\pi r h$

Volume $V = \pi r^2 h$

~~(at + 1) \times 5 = 5 \times 1000~~ ~~is short~~

whilst \Rightarrow

$$3x^2 + 2x \frac{dy}{dx} - 2y \frac{dy}{dx} + 3 + 2 \cdot \frac{dy}{dx} = 0$$

~~whilst~~

$$\Rightarrow 2 \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} + 2 \cdot \frac{dy}{dx} = -3x$$

~~whilst~~

$$\Rightarrow \frac{dy}{dx} = (2 - 2y + 2) \cdot \cancel{x} \cdot \cancel{\frac{1}{x}} = A$$

$$\Rightarrow \frac{(2 - 2y + 2)}{x} (-2y) \Big|_2 = -3x \frac{\cancel{x} \cdot \cancel{-2y}}{\cancel{x}} = ?$$

$$\Rightarrow \frac{2(2y - 1)}{x} = \frac{w \times 3x^2}{x} \text{ using only operations}$$

~~writing only operations~~

$$\Rightarrow 2(2y - 1) = 3w^2$$

$$3x^{\frac{1}{2}} - \frac{3\epsilon + \kappa\epsilon}{5+2x^{\frac{1}{2}}} = 0 \quad \frac{\kappa\epsilon}{5+2x^{\frac{1}{2}}} =$$

$$\Rightarrow \frac{3x^{\frac{1}{2}} - 3}{5+2x^{\frac{1}{2}}} = 0$$

$$\Rightarrow 3x^{\frac{1}{2}} = 3 \quad x^{\frac{1}{2}} = \frac{1}{\kappa\epsilon} = \frac{1}{\kappa}\epsilon$$

$$\Rightarrow x = \frac{3}{3} = \frac{1}{\kappa\epsilon} = \frac{1}{\kappa}\epsilon$$

$$(4) \quad \frac{dy}{dx} = \frac{3}{\cancel{x}} x^{-\frac{1}{2}} + \frac{3}{2} x^{-\frac{3}{2}}$$

$$x=1 \quad \frac{dy}{dx} = 3 > 0$$

~~$$0 = \kappa\epsilon + \kappa\epsilon$$~~

~~$$0 = (\kappa + \kappa)r\epsilon$$~~

~~$$0 < \kappa + r\epsilon$$~~

~~$$0 < \sqrt{\kappa^2 + 2\kappa r}$$~~

$$\frac{dx}{dt} = k(100-x)^{0.81}$$

$$\Rightarrow \int \frac{1}{100-x} dx = \int k(100-x)^{0.81} dt$$

$$\Rightarrow -\ln(100-x) = kt + C$$

$$\Rightarrow \ln(100-x) = -kt - C$$

$$\Rightarrow 100-x = e^{-kt-C}$$

$$\Rightarrow x = -e^{-kt-C+100}$$

$$\Rightarrow x = -e^{-kt-C+100}$$

$$100-x = e^{-kt} \cdot e^{-C}$$

$$\Rightarrow e^{-C} = \frac{100-25}{e^0} \cdot F^8$$

$$= 75$$

$$\Rightarrow -C = \ln(75) = 4.32$$

$$\Rightarrow C = -4.32$$

$$t = 180 \quad (\kappa \approx 0.1) \quad 8.5 \quad \frac{x_b}{t_b}$$

$$\Rightarrow -\ln(100 - 8.5) = k \times 180 + 4.32$$

$$\Rightarrow -2.72 + 4.32 = \frac{1}{k \times 180} \quad \text{or}$$

$$\Rightarrow \frac{1.61}{180} = k \quad (\kappa \approx 0.1) \quad \text{wh.}$$

$$\Rightarrow k = 8.94 \times 10^{-3} \quad (\kappa \approx 0.1) \quad \text{wh.}$$

$$u = e^{\frac{-k(t-s)}{0.1} + 100} + 4.32$$
$$= e^{-\frac{-(8.94 \times 10^{-3}) \times 200}{0.1} + 100} + 4.32$$

$$\Rightarrow \frac{87.42}{0.9} \quad \text{or}$$

28

$$-\Delta E.P = (28) \cdot 1 \quad \text{or}$$

$$\Delta E.P = -28 \quad \text{or}$$

$$x^3 + 2xy - y^2 + 3x + 2y + 7 = 0 \quad \frac{b}{\sqrt{b}}$$

$$fx = 3x^2 + 2y + 3 \quad \frac{\sqrt{b}}{\sqrt{b}}$$
$$fy = \cancel{2x} - 2y + 2 \quad \frac{\sqrt{b}}{\sqrt{b}}$$

$$\frac{\partial z}{\partial x} = \frac{-fx + \cancel{3x^2} - \cancel{3x}}{fy} = \frac{\sqrt{b}}{\sqrt{b}}$$
$$= \frac{-(3x^2 + 2y + 3)}{2x - 2y + 2}$$

TR or DTR

TR 0. DTR 3

DTRD TR TR

new 50

$$\frac{dV}{dr} = \frac{d}{dr}(56\pi r^5 - \pi r^3 - 6r^2 + 5)$$

$$\frac{d^2V}{dr^2} = 5 + 15r^2 - 12r - 12 = 6r^2 - 12r + 6 = 6(r-1)^2$$

$$\frac{d^3V}{dr^3} = -60\pi r^2$$

$$(6r^2 - 12r + 6) = \frac{16}{x^5}$$

[প্রযোগ কর না]

আমরা এখন

এই প্রাপ্তি - 0 মেল

কো মাত্র আমরা,

কো মেল.

~~2009, 2010~~

considering leftmost fib as
 old mag & others as
 fib right most as

middle leftmost fib axis

$\tau_{\text{fib}}(r, x)$ will act on $\frac{6}{2}$ middle leftmost fib A

new leftmost fib box $\frac{6}{2}$ will be $\rho = \rho_b(r, x) M$

$$\frac{3}{2} \times \frac{3}{2} \times \frac{2\sqrt{2}}{x^2} = \frac{M2}{\gamma^2} \quad \text{to}$$

$$(r, x) M = [(r, x)^2] \frac{3}{x^2} \cdot \text{area}$$

$$(r, x) M = [(r, x)^2] \frac{3}{\gamma^2}$$

$\rho(r, x)$ will built of sequence into new box

$$[(r, x)^2] \frac{2\sqrt{2}}{x^2 \times 2} = [(r, x)^2] \frac{3}{x^2} \frac{2}{\gamma^2} = \frac{M2}{\gamma^2}$$

$$[(r, x)^2] \frac{2\sqrt{2}}{\gamma^2 \times 2} = [(r, x)^2] \frac{3}{\gamma^2} \frac{2}{x^2} = \frac{M2}{x^2}$$

* Differential Equations

* Variable Separable

* Linear first order diff

* Exact differential equation:

A differential equation of the form $M(x,y) \frac{dx}{dy} + N(x,y) = 0$ will be exact differential equation.

$$\text{of } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\text{where, } \frac{\partial}{\partial x} [f(x,y)] = M(x,y)$$

$$\frac{\partial}{\partial y} [f(x,y)] = N(x,y)$$

and we are supposed to find the $f(x,y) = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y} \left[\frac{\partial}{\partial x} (f(x,y)) \right] = \frac{\partial^2}{\partial y \partial x} (f(x,y))$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x} \left[\frac{\partial}{\partial y} (f(x,y)) \right] = \frac{\partial^2}{\partial x \partial y} (f(x,y))$$

$$\nabla \cdot x^2y^3 dx + x^3y^2 dy = 0 \quad (\text{check } \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x})$$

$$M(x, y) = x^2y^3 \quad \text{and} \quad \frac{\partial M}{\partial y} = 3x^2y^2$$

$$N(x, y) = x^3y^2 \quad \therefore \quad \frac{\partial N}{\partial x} = 3x^2y^2$$

\therefore This is an exact differential equation.

$$\therefore \frac{\partial}{\partial x} [f(x, y)] = M = x^2y^3$$

$$\therefore f(x, y) = \int x^2y^3 dx + g(y)$$

$$\therefore f(x, y) = \frac{1}{3}x^3y^3 + g(y)$$

$$\frac{\partial}{\partial y} [f(x, y)] = x^3y^2 + g'(y) = N = x^3y^2$$

$$\therefore g'(y) = 0 \quad \therefore g(y) = C$$

$$\therefore f(x, y) = \frac{1}{3}x^3y^3 + C$$

$$\nabla \cdot 2xy^2 dx + (x^2 - 1) dy = 0 \quad \text{exponent of } y^2$$

$$M(x, y) = 2xy^2 \quad \text{and} \quad \frac{\partial M}{\partial y} = 4xy \quad (\text{check } \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x})$$

$$N(x, y) = x^2 - 1 \quad \text{and} \quad \frac{\partial N}{\partial x} = 2x$$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$; This is an exact diff. equation

$$0 = xb(4x - 2x) + xb(2x + 2x)$$

\therefore solution will look like this

$$\frac{\partial}{\partial x} f(x,y) = 2xy^0 = eb^2 e^{bx} + xb^2 e^{bx} \quad \text{and}$$

$$\therefore f(x,y) \stackrel{M_2}{=} \int 2xy \, dx + g(y) \quad e^{bx} = (e^x)^M$$

$$\therefore f(x,y) \stackrel{x^2y}{=} \frac{e^{bx^2} y}{x^2} + g(y) \quad \therefore e^{bx} = (e^x)^n$$

$$\therefore \frac{\partial f}{\partial y} = x^2 \text{ is a diff. term in } f \text{ w.r.t. } y \text{ which is constant.}$$

$$x^2 + g'(y) = x^2 - 1 \quad e^{bx} = M = [(e^x)^2] \frac{2}{x^2} \quad \therefore$$

$$\Rightarrow g'(y) = -1$$

$$\therefore g(y) = -y + C \quad \left(e^{bx} + xb^2 e^{bx} \right) = (e^x)^2 \quad \therefore$$

$$\therefore f(x,y) = x^2y - y + C \quad (e^{bx} + e^{bx}) \frac{1}{e^b} = (e^x)^2 \quad \therefore$$

$$e^{bx} = h = (e^x)^2 + e^{bx} = [(e^x)^2] \frac{2}{e^b}$$

$$\text{Or, } \frac{\partial}{\partial y} f(x,y) = x^2 - 1 \quad 0 = (e^x)^2 \quad \therefore$$

$$\therefore f(x,y) = x^2y - y + g(x) \quad e^{bx} \frac{1}{e^b} = (e^x)^2 \quad \therefore$$

* Solving Homogeneous differential equations *

If a function $f(x,y)$ possesses the following property than it will be called homogeneous differential equation.

i.e. $f(tx, ty) = t^m f(x, y)$; m is a constant

$$(x^2 + y^2) dx + (x^2 - xy) dy = 0$$

This is not exact diff. equation.

$$(x^2+y^2)dx + (x^2-xy)dy = 0 \rightarrow \text{Homogeneous} \quad [\text{if } \frac{dy}{dx} = \frac{y}{x} \text{ same}]$$

x^2+y^2	$+^2x^2+tx\cdot ty$
$+^2x^2+y^2$	$+^2x^2+^2xy = vb \cdot \frac{2+1+v}{1+v}$
$\Rightarrow +^2(x^2+y^2)$	$= +^2(x^2-xy)$
$+^2x^2 = vb \left(\frac{2}{1+v} - 1 \right)$	

* A substitution $y = vx$ can convert a homogeneous equation into a differential equation of variable separable.

$$\begin{aligned}
 & (x^2+y^2)dx + (x^2-xy)dy = 0 \\
 \Rightarrow \frac{dy}{dx} &= -\frac{x^2+y^2}{x^2-xy} \quad \left| \begin{array}{l} y = vx \\ \frac{dy}{dx} = v + x \frac{dv}{dx} \end{array} \right. \\
 \therefore v+x \frac{dv}{dx} &= -\frac{x^2+v^2x^2}{x^2-vx^2} \\
 \Rightarrow v+x \frac{dv}{dx} &= -\frac{x^2(1+v^2)}{x^2(1-v)} \\
 \Rightarrow x \frac{dv}{dx} &= \frac{v^2+1-v}{v-1} - v \quad \frac{dv}{\frac{v^2+1-v}{v-1}-v} = \frac{dx}{x} \\
 \Rightarrow x \frac{dv}{dx} &= \frac{1+v^2-\sqrt{1+v^2}}{v-1} \quad \frac{v^2x}{x^2v+1} = \frac{vb}{x} \\
 \Rightarrow x \frac{dv}{dx} &= \frac{v+1}{v-1} \quad \frac{v^2x}{(v^2+1)x} = \frac{vb}{x} \\
 \therefore \frac{v-1}{v+1} dv &= \frac{1}{x} dx
 \end{aligned}$$

$$\therefore \int \frac{v-1}{\sqrt{v+1}} dv = \int \frac{1}{x} dx$$

$$\Rightarrow \int \frac{v+1-2}{v+1} dv = \ln x + C$$

$$\Rightarrow \int \left(1 - \frac{2}{v+1}\right) dv = \ln x + C$$

$$\Rightarrow v - 2\ln(v+1) = \ln x + C$$

$$\Rightarrow \frac{y}{x} - 2\ln(\frac{y+x}{x} + 1) = \ln x + C$$

$$\therefore \frac{y}{x} - 2\ln(\frac{y+x}{x} + 1) = \ln x + C$$

$$\Rightarrow \frac{y}{x} = \ln \left(\frac{(y+x)^2}{x^2} + \ln x + C \right)$$

$$\Rightarrow \frac{y}{x} = \ln \left\{ x \left(\frac{y+x}{x} \right)^2 \right\} + C$$

$$\frac{\frac{dy}{dx} + \frac{y}{x}}{\frac{dy}{dx} - \frac{y}{x}} = \frac{vb}{xb} x + v \quad \therefore$$

$$\therefore \frac{dy}{dx} = \frac{xy}{x^2+y^2} \text{ or, } xy \frac{dx}{dy} - (x^2+y^2) \frac{dy}{dx} = 0$$

$$\therefore v+x \frac{dv}{dx} = \frac{x \cdot vx}{x^2+(vx)^2}$$

$$\Rightarrow v+x \frac{dv}{dx} = \frac{x^2 v}{x^2+v^2 x^2}$$

$$\Rightarrow v+x \frac{dv}{dx} = \frac{x^2 v}{x^2(1+v^2)}$$

$$vb \cdot \frac{1}{x} = vb \frac{1-v}{1+v} \quad \therefore$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v}{1+v^2} \rightarrow v \text{ 77/6 robos bws 8}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v - v - v^3}{1+v^2} \text{ 77/6 waitings 6 vA}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{-v^3}{1+v^2} \text{ 77/6 waitings 6 vA}$$

$$\Rightarrow \int \frac{1+v^2}{-v^3} dv = \int \frac{dx}{x}$$

$$\Rightarrow \int -\frac{1}{v^3} + \frac{1}{v} dv = \ln x + C \text{ 77/6 robos}$$

$$\Rightarrow \frac{-v^{-2}}{-2} + \ln v = \ln x + C \text{ 77/6 waitings 6 vA}$$

$$\Rightarrow \frac{1}{2} \cdot \frac{1}{v^2} + \ln v = \ln x + C \text{ 77/6 robos bws 8}$$

$$\Rightarrow \frac{1}{2} \cdot \frac{x^2}{y^2} + \ln(\gamma/x) - \ln x = C \text{ 77/6 robos bws 8}$$

$$\Rightarrow \frac{1}{2} \cdot \frac{x^2}{y^2} + \ln(\gamma/x) = C + \frac{6b}{x^6} \text{ 77/6 waitings 6 vA}$$

* Second order linear homogeneous equation with constant coefficients:

$$3 \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + y = 0 \quad \leftarrow O = (x) \text{ 77/6}$$

77/6

* 2nd Order diff equation: $\frac{dy}{dx} = \frac{vb}{xb} \dots$

Any equation of the form $\frac{dy}{dx} = \frac{vb}{xb} \dots$

$$P_1 \frac{d^n y}{dx^n} + P_2 \frac{d^{n-1} y}{dx^{n-1}} + \dots + P_{n-1} \frac{dy}{dx} + P_n y = f(x)$$

where P_1, P_2, \dots, P_n are either constant or function of x , is known as n th order linear differential equation.

$$\frac{dy}{dx} = vb + \frac{1}{xb} \dots$$

* 2nd order line

$$\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = f(x) \text{ is a}$$

second order diff equation with constant coefficient.

* $f(x) = 0 \rightarrow$ Homogeneous.

$$0 = k + \frac{vb}{xb} \beta + \frac{k^2 b}{x^2 b} \dots$$

P.T.

* $\frac{d^2y}{dx^2}$ (to find O.D. for) ~~A~~

$$y = \sin x \quad \Rightarrow y'' + y = 0$$

$$\frac{dy}{dx} = \cos x \quad \text{O.D.} = 2$$

$$\frac{dy}{dx} = -\sin x \quad \text{O.D.} = m$$

* $c_1 \frac{d^2y}{dx^2} + c_2 \frac{dy}{dx} + c_3 y = 0$

Corresponding Auxiliary equation:-

$$c_1 m^2 + c_2 m + c_3 = 0$$

$$\text{Solving } m \rightarrow (m_1, m_2)$$

if $m_1 \neq m_2$ are real & distinct

$$(x^{m_1} + x^{m_2})$$

then $y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$$(x^{m_1} + x^{m_2}) + x^{m_1} + x^{m_2}$$

~~37~~

~~A.E~~

(real & distinct)

$$\frac{b^2 b}{m \times b}$$

8

$$m = 6 + 2i, 2, 3$$

$$i. g = C_1 e^{-x} + C_2 e^{-2x} + C_3 e^{-3x}$$

$$m = -2, -1, 4, 5$$

$$g = C_1 e^{-2x} + C_2 e^{-x} + C_3 e^{4x} + C_4 e^{5x}$$

~~If roots are repeated~~

$$m = 2, 2 \rightarrow (e^{2x} + m_2 x e^{2x})$$

$$g = (C_1 + C_2 x) e^{2x}$$

$$(m = -3, -3 + 3i)$$

$$g = (C_1 + C_2 + C_3 x) e^{-3x}, m = 2i$$

$$m \rightarrow 2, -2, -2 \rightarrow (e^{-2x} + 6)$$

$$g = C_1 e^{-x} + (C_2 + C_3 x) e^{-2x}$$

P72

$$(iv) \text{ Given } \lambda_1 = -\frac{1}{2}, \lambda_2 = -\frac{1}{2} + i\sqrt{\frac{3}{2}}, \lambda_3 = -\frac{1}{2} - i\sqrt{\frac{3}{2}}$$

$$y = (c_1 + c_2 x) e^{-\frac{1}{2}x} + (c_3 + c_4 x) e^{-\frac{1}{2}x} \cos \sqrt{\frac{3}{2}}x$$

* If roots are imaginary

$$\lambda \pm i\beta$$

$$y = e^{\lambda x} \left[c_1 \cos \beta x + c_2 \sin \beta x \right]$$

$$* \text{ Given } \lambda_1 = -\frac{1}{2} + i\sqrt{\frac{3}{2}}, \lambda_2 = -\frac{1}{2} - i\sqrt{\frac{3}{2}}$$

$$y = e^{-\frac{1}{2}x} \left[c_1 \cos \sqrt{\frac{3}{2}}x + c_2 \sin \sqrt{\frac{3}{2}}x \right]$$

$$0 = 2 + \lambda_1 + \lambda_2 + \lambda_3$$

$$-5 = -1 - 1 - 1 - 2$$

$$2\lambda_1 + 2\lambda_2 + 2\lambda_3 + 6 = 0$$

adunpoS

$$* \frac{d^3 y}{dx^3} - 13 \frac{dy}{dx} + 12y = 0 \quad (v)$$

AE

$$m^3 - 13m + 12 = 0 \text{ for } 2x$$

$$\Rightarrow m_1 = 1, m_2 = 2 \quad \text{for } 2x$$

$$\therefore \text{solution: } y = C_1 e^x + C_2 e^{2x} + C_3 e^{-2x}$$

$$* \frac{d^3 y}{dx^3} - 13 \frac{dy}{dx} + 11 \frac{d^2 y}{dx^2} + 6y = 0$$

$$y^3 + 6y^2 + 11y + 6 = 0$$

$$m = \cancel{-3}, -2, -1$$

$$\therefore \text{sol} \Rightarrow y = C_1 e^{-3x} + C_2 e^{-2x} + C_3 e^{-x}$$

Normal Solution

$$\frac{d^4x}{dt^4} - \frac{d^3x}{dt^3} - \frac{dx}{dt} + 2x = 0$$

~~AE~~

$$m^4 - m^3 - m + 1 = 0$$

$$\Rightarrow m^3(m-1) - 2(m-1) = 0$$

$$\Rightarrow \cancel{(m-1)}(m^3-2) = 0 \quad \text{Observe}$$

$$(1) \quad m^3 - 2 = 0$$

$$m-2=0$$

$$\Rightarrow m=2$$

$$(m+1)(m+m+1) = 0$$

$$m=2, (m+m+1)=0$$

$$m = \frac{2 \pm \sqrt{2-4}}{2 \cdot 2}$$

$$m = 2, 2, -\frac{1}{2} \pm \frac{\sqrt{3}i}{2}$$

~~IT:~~

~~solt~~

$$x = \left(c_1 + c_2 t \right) e^t + c_3 e^{-\frac{1}{2}t} - \frac{x^m b}{m+b}$$

$$\rightarrow \left[c_3 \cos \frac{\sqrt{3}}{2}t + c_4 \sin \frac{\sqrt{3}}{2}t \right] \quad \text{TA}$$

* ~~$y'' - 2y' + y = e^x$~~

~~homogeneous~~ $\rightarrow (c_1 e^x) (c_2 e^x)$

* ~~$y'' - 2y' + y = 0$~~ , $y(0) = 5$, $y'(0) = 10$

$$(t^m + m t^{m-1} + \dots + m!)(1 - t^m)^0$$

$$(m-2)! = 0$$

$$\Rightarrow m = (2, 1)$$

$$\therefore y = (c_1 + c_2 x) e^x$$

Ans

$$\text{mol} \left(\begin{matrix} 1 & 2 & 5 \\ 1 & 2 & 5 \end{matrix} \right) \text{ molar } \left(\begin{matrix} 3 & 1 & 7 \\ 1 & 2 & 5 \end{matrix} \right) e^x +$$

$$\rightarrow \begin{matrix} x & 5 & 0 \\ 1 & 2 & 5 \end{matrix} \stackrel{\text{d}}{=} \begin{matrix} 5 \\ 1 & 2 & 5 \end{matrix} \rightarrow \begin{matrix} e^x \cdot c_2 \\ c_2 \cdot c_1, c_2 = 10 \end{matrix}$$

$$5 \rightarrow (c_1 + 0) \cdot 1 \rightarrow 0 = 5 \rightarrow 5 + 0 \stackrel{!}{=} 10, \text{ i.e. } c_2 = 10$$

$$\rightarrow \begin{matrix} c_1 & 5 & 0 \\ 1 & 2 & 5 \end{matrix} \rightarrow \begin{matrix} c_1 & 5 & 0 \\ 1 & 0 & 5 \end{matrix} \stackrel{!}{=} 5 + 5 \rightarrow c_1 = 5$$

fibro rocks \rightarrow $c_1 = 5$

water rich rocks \rightarrow $c_2 = 10$

water rich rocks \rightarrow $c_1 = 5$

fibro rocks \rightarrow $c_2 = 10$

water rich rocks \rightarrow $c_1 = 5$

fibro rocks \rightarrow $c_2 = 10$

water rich rocks \rightarrow $c_1 = 5$

~~177~~

C.W
29.3.23

Diff eq

+ Friday, (7th-8) pm (Extra) Class

* $y'' + C_1 y' + C_2 y = f(x)$

* $y'' + C_1 y' + C_2 y = 0$

* If $f(x)$ & $g(x)$ are the solution of the above diff eq.

then the linear combination of the solutions is also the solution of the above ordinary diff eq.

* $\sin x$ & $\cos x$ are the solution.
∴ $C_1 \sin x + C_2 \cos x$ is also the solution.

P.T.B

Ans $y_1 = e^{rx}$, $y_2 = e^{-rx}$ are the solution
 $\therefore C_1 e^x + C_2 e^{-x}$ is also the
 solution $\Rightarrow r = \pm i$ $\therefore A = 5$ $\therefore I.9$

$\frac{dy}{dx} + \frac{dy}{dx} + jA = \sin x$
 \downarrow
 $\frac{dy}{dx} + jA = \sin x$

+ (General solution:-
 Complementary function + Particular
 function) \Rightarrow Complementary function + Particular
 function \Rightarrow $(A + j\alpha)$ $\sin x$

$\frac{dy}{dx} + \frac{dy}{dx} + j(A + \alpha - A) \sin x$

$\Rightarrow m^2 + m + j = 0 \quad A + \alpha - A$

$$m = \frac{-1 \pm \sqrt{1+4A^2}}{2}$$

$$= \frac{-1 \pm \sqrt{1+39}}{2}$$

$$= \frac{1}{2} \pm \frac{\sqrt{3}}{2} j$$

$$\therefore \alpha = 0$$

$$\therefore \beta = 0$$

$$\text{vector form of } f \text{ is } e^{-\frac{1}{2}x} \left[C_1 \sin \frac{\sqrt{3}}{2}x + C_2 \cos \frac{\sqrt{3}}{2}x \right]$$

$$P.I = y = A \sin x + B \cos x$$

$$y' = A \cos x - B \sin x$$

$$y'' = -A \sin x - B \cos x$$

$$\text{L.H.S.} = (\underbrace{A \cos x + B \sin x}_{\text{particular solution}}) + \underbrace{(A \cos x + B \sin x)}_{\text{homogeneous part}} + \underbrace{(A \cos x + B \sin x)}_{\text{particular solution}} =$$

$$\sin(-A - B + A) + \cos x (-B + A + B) =$$

$$0 + \frac{6}{\sin x} + \frac{6}{\cos x}$$

$$\Rightarrow -B \sin x + A \cos x = \frac{6}{\sin x} + m + n$$

$$-B = 2$$

$$\therefore B = -2$$

$$\boxed{A = 0} \quad \boxed{t = -m}$$

$$\boxed{i P.E.I = t - \frac{\cos x}{\sin x}}$$

$$i \frac{dt}{dx} + \frac{t}{\sin x} = 0$$

~~Equation~~ Equating the coefficient of like terms.

LW

~~Matrix~~

~~O = L + R~~ ~~linear~~

* Diagonal Matrix :-

Diagonals of which entries are 0.

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* Identity Matrix:-

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

~~W
1.4.23~~

linear

* solve the differential equation

$$\frac{dy}{dx} + y = e^{2x}$$

$$A.E \Rightarrow m^2 + 1 = 0$$

$$\Rightarrow m^2 = -1 \text{ without imaginary}$$

$$\Rightarrow m = \pm i \sqrt{-1}$$

$$= \pm i \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$G.S = C.F + P.I$$

$$C.F = C_1 \begin{bmatrix} \cos x \\ \sin x \end{bmatrix} + C_2 \begin{bmatrix} \sin x \\ \cos x \end{bmatrix}$$

$$y = A e^{2x}$$

$$y' = 2A e^{2x}$$

$$y'' = 4A e^{2x}$$

$$4Ae^{2x} + Ae^{2x} \rightarrow e^{2x} + 5e^{2x}$$

$$\Rightarrow 5Ae^{2x} \rightarrow e^{2x} \quad (\text{operator method})$$

$$5Ae^{2x} = 1$$

$$Ae^{2x} = \frac{1}{5}$$

$$3j'' + 2j' + 5j = f(x)$$

$$(3D^2 + 2D + 5) j = f(x)$$

$$j \cdot P.I = \frac{1}{5} e^{2x}$$

$$\therefore w.s = (C_1 \cos x + C_2 \sin x) + \frac{1}{5} e^{2x}$$

* Finding P.I using operator.

$$j = Ae^{2x}$$

$$j'' + j = e^{2x}$$

$$(D^2 + 1) j = e^{2x}$$

$$j = \frac{e^{2x}}{(D^2 + 1)}$$

$$\Rightarrow j = \frac{e^{2x}}{z^2 + 1} = \frac{e^{2x}}{5}$$

$$* \quad j''' + 3j'' + 2j' \underset{-2x}{\cancel{e}} + \underset{-2x}{\cancel{A}} N$$

~~$$\rightarrow (D^3 + 3D^2 + 2) j \underset{-2x}{\cancel{e}} \underset{-2x}{\cancel{A}} N$$~~

~~$$(D^3 + 3D^2 + 2) j \underset{-2x}{\cancel{e}} \underset{-2x}{\cancel{A}} N = (D^3 + 3D^2 + 2) \underset{\frac{1}{2}}{\cancel{e}} \underset{-8+12}{\cancel{A}}$$~~

$$j \underset{-2x}{\cancel{e}} + \underset{\frac{1}{2}}{\cancel{e}} = -8+12+1 = 5 \underset{6}{\cancel{6}}$$

~~$$3j'' + j' - 2j \underset{-2x}{\cancel{e}} = 2w$$~~

~~$$(D^2 + D - 2) j \underset{-2x}{\cancel{e}}$$~~

~~$$j \underset{-2x}{\cancel{e}} = 6 \underset{3D+D-2}{\cancel{e}}$$~~

~~$$j \underset{-2x}{\cancel{e}} = 6 \underset{3D+D-2}{\cancel{e}}$$~~

~~$$j \underset{-2x}{\cancel{e}} = \frac{6}{3D+D-2} = x e^{-x} \times \frac{1}{6r+1}$$~~

~~$$j \underset{-2x}{\cancel{e}} = 6 e^{-x}$$~~

Rules

* $\frac{e^{ax}}{f(b)} \text{ or } \frac{e^{ax}}{f'(b)} \cdot e^x ; f'(a) \neq 0$

* $\frac{e^{ax}}{f(b)} \cdot e^x \text{ where } f'(a) = 0$

$$= xe^{ax} \cdot \frac{1}{f'(a)} \quad [f'(a) \neq 0]$$

* $\frac{\sin ax}{f(b)} \text{ or } \frac{\cos ax}{f(b)} \quad [f(-a) \neq 0]$

$$= \frac{1}{f(-a)} \sin ax \quad \frac{1}{f(-a)} \cos ax$$

$$\Rightarrow xe^{ax} \cdot \frac{1}{f''(a)} \quad [f''(a) \neq 0]$$

$$P = 6x + \frac{6b}{x^b} e^{-\frac{x^b b}{x^b}} = \frac{6^b b}{x^b} e^{-b}$$

$$P = 6 \left(x + ae - q \right)^{-\frac{b}{a}}$$

$$\frac{9}{x^b} = 6 \cdot e^{-b}$$

$$*\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 5y = 3e^{3x}$$

$$(D^2 - 3D + 5) y \Rightarrow 3e^{3x} \quad *$$

$$y = \frac{0 + (A) e^{3x}}{(D - 3D + 5)} + C e^{-5x}$$

$$\frac{(0 + (A) e^{3x})}{(D - 3D + 5)} \xrightarrow{x \rightarrow \infty} \frac{(A) e^{3x}}{(D - 3D + 5)} \quad *$$

$$\frac{3^x - 9 + 5}{(D - 3D + 5)} \xrightarrow{x \rightarrow \infty} \frac{5}{(D - 3D + 5)}$$

$$\boxed{\frac{0 + (A) e^{3x} - 9 + 5}{3e^{3x} (D - 3D + 5)}} \xrightarrow{x \rightarrow \infty} \boxed{\frac{5}{(D - 3D + 5)}} \quad *$$

$$*\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = e^{2x}$$

$$\Rightarrow (D^2 - 3D + 2)y = e^{2x}$$

$$\Rightarrow y = \frac{e^{2x}}{D^2 - 3D + 2}$$

$$= x e^{2x} + \frac{1}{f'(0)} + \frac{5^6}{6!}$$

$$= x e^{2x} + \frac{1}{2^n - 3} (e + 4k^2 x^n)$$

$$= x e^{2x} + \frac{1}{2^n - 3} e^{2x} = x e^{2x} + \frac{1}{n-3}$$

$$\frac{dy}{dx} + u y = \frac{\cos 3x}{e^{2x}}$$

$$(D^2 + 4) y = \frac{\cos 3x}{e^{-2x}} \quad f(D) = D^2 + 4$$

$$y = \frac{\cos 3x}{D^2 + 4} \quad f(D) = -a^2 + 4$$

$$= -\frac{2}{5} e^{-2x} \cos 3x \quad f(-2) = -3^2 + 4 = -9 + 4 = -5$$

$$= -\frac{2}{5} e^{-2x} \cos \left(\frac{n+q}{f} \right) \quad f(D) = f(-2)$$

$$= \left(\cos \pi + \cos 2\pi \right) \frac{1}{25} =$$

$$* \frac{d^2 y}{dx^2} + (D^2 + 5) y = \sin 3x$$

$$\frac{1}{D+5} (D^2 + D + 5) y = \sin 3x$$

$$\therefore y = \frac{\sin 3x}{D^2 + D + 5} + C_1 x^{-1} + C_2 x^{-3}$$

$$\Rightarrow \frac{\sin 3x}{-9 + D + 5} = \frac{\sin 3x}{D - 4}$$

$$\frac{1}{D - 4} \cdot \sin 3x$$

$$u + v = (a)$$

$$u + v = (a) \quad \frac{D+4}{D-16} \cdot \sin 3x$$

$$u + v = \frac{D+4}{-9-16} \sin 3x \quad \left\{ D = \text{differentiation} \right.$$

$$(u+v)(a) = -\frac{1}{25} (D+4) \sin 3x$$

$$= -\frac{1}{25} (3 \cos 3x + 4 \sin 3x)$$

~~Ex 4/23~~

Linear

$$\star \frac{\sin(ax+q)}{f(D^v)} \text{ in } (a) \theta = \frac{(x)\theta}{(a)\theta}$$

$$\Rightarrow \frac{\sin(x(a\theta)-\alpha)}{f(-a\theta)} \text{ w } f(-a\theta) \neq 0$$

$$\therefore \frac{\sin((x-a)(a\theta)-\alpha)}{f(-a\theta)} \text{ w } f(-a\theta) = 0,$$

$$\Rightarrow x \cdot \frac{\sin((x-a)(a\theta)-\alpha)}{f'(D^v)(a\theta)} \left(f'(D^v) \neq 0 \right)$$

$$\star \boxed{\frac{1}{D} = \int dx}$$

$$\star (2D+1)(\cos^3 x)^{x=0} = 6(x+1)$$

$$\Rightarrow 2(-\sin 3x \cdot 3) + \cos^3 x = 6(x+1)$$

$$\Rightarrow -6 \sin^3 x + \cos^3 x = 6$$

$$(x+2x) \cdot \frac{x}{D+2}$$

$$(x+2x) \cdot \frac{x}{(x+\frac{2}{x})x} =$$

* $\frac{g(x)}{f(D)}$, $g(x)$ is polynomial

$$* (1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 +$$

$$\frac{n(n-1)(n-2)}{3!} x^3 + \dots$$

$$* (1+D)^{-2} = 1 + (-2) D + \frac{(-1)(-2)}{2!} D^2 +$$

$$\boxed{\frac{(-2)(-3)(-4)}{3!} D^3 + \dots}$$

$$* (x^2 + 2x)^{-1} = x^{-1} \frac{1}{(x^2 + 2x)} (x^2 + 2x)$$

$$\Rightarrow (D+2)^{-1} = \frac{x^{-1}}{(x^2 + 2x)}$$

$$\Rightarrow J = \frac{x^{-1}}{D+2} + x^2 -$$

$$\Rightarrow \frac{1}{D+2} (x^{-1} + 2)$$

$$= \frac{1}{2(\frac{D}{2} + 1)} (x^{-1} + 2)$$

$$\begin{aligned}
 & x + \frac{x}{2} + \left(3 + \frac{D}{2} \right)^{-2} + \left(\frac{x^2 + 2}{x^2 - 4} \right) \\
 & = \frac{1}{2} \left[1 + (-2)^{\frac{D}{2}} + \frac{(-2)(-2)}{6} \left(\frac{D}{4} \right) \right] \\
 & = \frac{1}{2} \left(1 - \frac{D}{2} + \frac{1}{6} D^2 + \frac{1}{2} D \right) \\
 & \quad \text{O} = \left\{ 5 + m^2 + 2m + 2 \right\} m \in \mathbb{N} \\
 & = \frac{1}{2} \left\{ x^{\frac{1}{2}} \cdot 2x + \frac{1}{4} \cdot 2^2 \right\} m \in \mathbb{N} \\
 & = \frac{1}{2} \left(x^{\frac{3}{2}} - x + \frac{3}{2} \right) m \in \mathbb{N} \\
 & \text{P.T.: } y = \frac{1}{2} \left(x^{\frac{3}{2}} - x + \frac{3}{2} \right) + 7, \quad x = 7.5
 \end{aligned}$$

$$x^{\frac{3}{2}} - x + \frac{3}{2} = 5^{\frac{3}{2}} + 7.5$$

$$\frac{x^{\frac{3}{2}} - x}{5^{\frac{3}{2}} + 7.5} + \frac{\cancel{P.T.} \quad x^{\frac{3}{2}}}{5^{\frac{3}{2}} + 7.5} = 5$$

$$* y''' + 2y'' + y' \overset{D = 1/2 + i\omega}{=} e^{2x} + x^2 + 2x$$

$$\left[\frac{1}{(s-1)} \cdot (s-1)(s-1) \right]$$

$$\Rightarrow \left(D^3 + 2D^2 + D \right) y \overset{D = 1/2 + i\omega}{=} e^{2x} + x^2 + 2x$$

$$(s+1)^3$$

$$m^3 + 2m^2 + m = 0$$

$$\Rightarrow m(m^2 + 2m + 1) = 0$$

$$m = 0,$$

$$m^2 + 2m + 2 = 0$$

$$\Rightarrow m(-1, -2)$$

$$C.F = C_1 e^{0x} + (C_2 + C_3 x) e^{-2x}$$

$$\rightarrow D(D+2)^2 y = e^{2x} + x^2 + 2x$$

$$\Rightarrow y = \frac{e^{2x}}{D(D+2)^2} + \frac{x^2 + 2x}{D(D+2)^2}$$

$$\frac{x^2 - \frac{5x}{2} + \frac{25}{4}}{D(D+2)} = \frac{5e^{\frac{2x}{D}} + 1}{2x^2}, D=6$$

$$\begin{aligned} & \frac{x^2 + x}{D(D+2)} = (0) 6 \\ & \Rightarrow (x^2 + x) \frac{1}{D} (D+2)^{-2} = 6 \\ & \Rightarrow \frac{1}{D} (1+D)^{-2} (x^2 + x)^6 = 6 \\ & = \frac{1}{D} (1 - 2D + 3D^2 - 4D^3 + \dots) (x^2 + x) \\ & = \left(\frac{1}{D} - 2 + 3D - 4D^2 + \dots \right) (x^2 + x) \end{aligned}$$

$$\begin{aligned} & = \frac{x^3}{3} - 2x^2 + 6x - 8 + \frac{x^2}{2} - 2x + 3 \\ & = \frac{x^3}{3} - 2x^2 + 4x + \frac{x^2}{2} - 5 \\ & = \frac{x^3}{3} - \frac{3x^2}{2} + 4x - 5 \end{aligned}$$

$$y = C_1 + (C_2 + (C_3 x) e^{-x}) e^{-x} + \frac{x^2}{3} - \frac{3x^3}{2}$$

$$+ 4x - 5 + \frac{1}{18} e^{2x}$$

$$x=0, y =$$

$$x=0, y' = (C_1 + C_2)$$

$$x=0, y'' = (C_3 + C_2)$$

$$y(0) = \frac{x+x}{x}$$

$$y'(0) = (C_1 + C_2)$$

$$y''(0) = (C_3 + C_2)$$

$$(C_1 + C_2) (C_3 + C_2) \frac{1}{1}$$

$$(C_1 + C_2) (C_3 + C_2) \frac{1}{1}$$

$$C_1 + C_2 = \frac{1}{a} + 8 - x^2 + x^3 - \frac{1}{a}$$

$$C_3 + C_2 = \frac{1}{a} + 8 - x^2 + x^3 - \frac{1}{a}$$

$$\frac{1}{a} - \frac{1}{a} + x^2 + x^3 - \frac{1}{a}$$

$$\frac{1}{a} - x^2 + \frac{1}{a} + x^3 - \frac{1}{a}$$

6/11/23

linear

- * $A^{-1} \rightarrow$ existing matrix
- * solving sequential form
↓ ↓
↓ ↓

* 90° check $= A^{-1} - A$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0+3 & -2+0 \\ -2+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$$

↓ -ve
↑ +ve

- * rotation

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

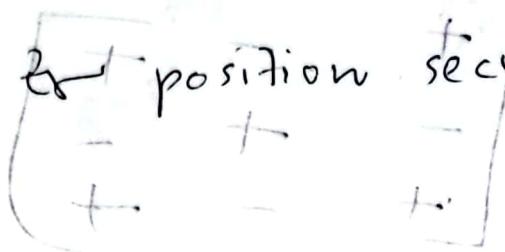
I.T.

* Inverse matrix of a matrix
is denoted by A^{-1}

$$A A^{-1} = A^{-1} A = I$$
$$T_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} T_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

~~4(2)~~

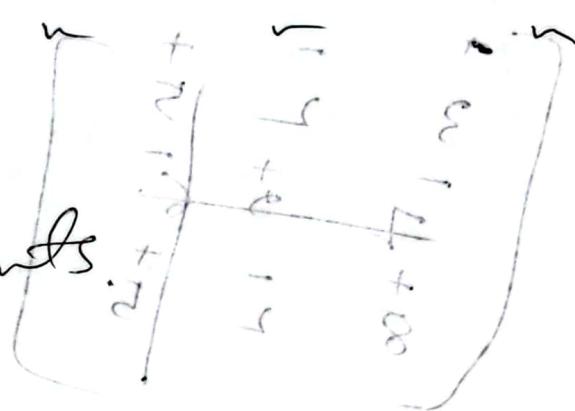
(i) School C 20 ~~at position secure~~



School D 20

- 1

(ii) (a) total points



(iii)

$$(21 - 3) - 3$$

(iv)

(v) total distance

~~linear~~

* System of linear equations.

A system of linear equation is a list of equations of same variable.

$$2x + y = 5 \quad \left[\begin{array}{l} \text{linear equation with 2} \\ \text{variables} \end{array} \right]$$

$$3x - 2y + z = 10 \quad \left[\begin{array}{l} \text{3 variables} \end{array} \right]$$

*

$$\left. \begin{array}{l} 3x + 2y - z = 10 \\ 4x - 2y + 7z = 15 \\ -x + 3y + 2z = 7 \end{array} \right\} \begin{array}{l} \text{is an example of} \\ \text{system of linear equation} \\ \text{with 3 equations} \\ \text{3 variables.} \end{array}$$

P.T.

~~Ex 1~~ $L_1: y = 4x + 5$ $L_2: y = 2x + 3$

Both equations will be solved \Rightarrow
 we will solve the equations simultaneously
 \rightarrow has unique solution

~~Ex 2~~ $L_1: y = 3x + 5$
 $L_2: y = 3x - 2$

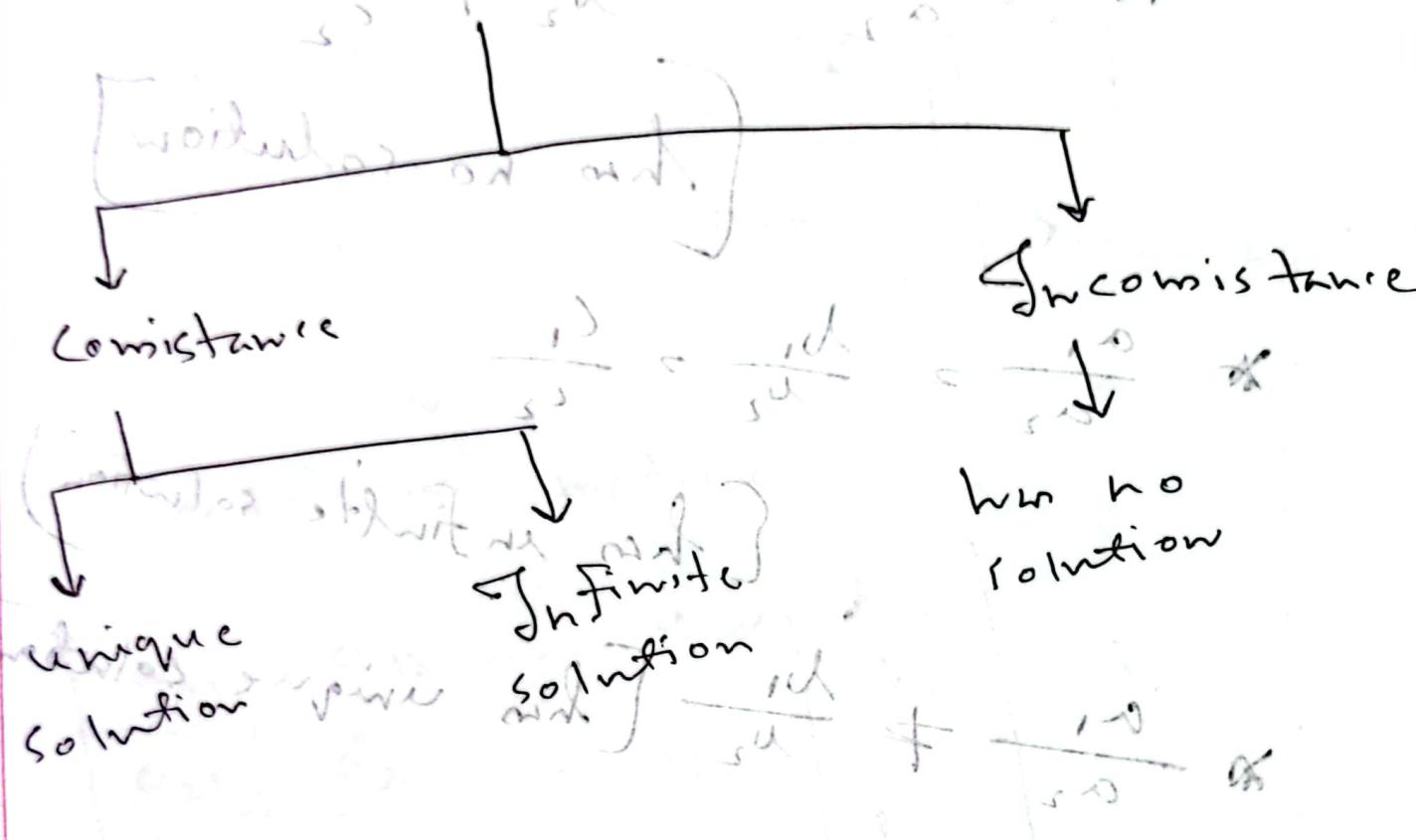
Both equations will be solved \Rightarrow
 has no solution

~~Ex 3~~ $L_1: 2x + 3y = 10$
 $L_2: 3x + 4y = 15$

Both equations will be solved \Rightarrow
 has infinite solutions

has infinite solutions

* System of linear equation



$$* a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}, B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\text{Condition } a_1b_2 - b_1a_2 = 0$$

$$\begin{aligned} 1) & A^{-1} \text{ exists} \Rightarrow a_1b_2 - b_1a_2 \neq 0 \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \\ & a_1b_2 - b_1a_2 = 0 \Rightarrow a_1b_2 = b_1a_2 \\ & \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \end{aligned}$$

* If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

[then no solution]

* $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

[then infinite solution]

* $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ [then unique solution]

$$a_{11}x_1 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + \dots + a_{2n}x_n = b_2$$

$$\vdots \quad \vdots \quad \vdots$$

$$a_{m1}x_1 + \dots + a_{mn}x_n = b_m$$

where, (1) x_1, x_2, \dots, x_n are variables

(2) $a_{ij}, b_i \rightarrow$ constants

$$\frac{1}{a_{11}} \quad \frac{1}{a_{21}} \quad \dots \quad \frac{1}{a_{m1}}$$

$$E = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

$$\begin{bmatrix}
 a_{11} & a_{12} & \cdots & a_{1n} \\
 a_{21} & a_{22} & \cdots & a_{2n} \\
 \vdots & & \ddots & \\
 a_{m1} & a_{m2} & \cdots & a_{mn}
 \end{bmatrix}
 \begin{bmatrix}
 x_1 \\
 x_2 \\
 \vdots \\
 x_n
 \end{bmatrix}
 = \begin{bmatrix}
 b_1 \\
 b_2 \\
 \vdots \\
 b_n
 \end{bmatrix}$$

↓

coefficients + free terms

$$\begin{bmatrix}
 a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\
 a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\
 \vdots & & \ddots & \\
 a_{m1} & a_{m2} & \cdots & a_{mn} & b_n
 \end{bmatrix}
 \xrightarrow{\text{Augmented Matrix}}$$

$$A = X^{-1}$$

$$X^{-1} A = I$$

P.T.O.

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* Solving system of linear equations
equation by Cramian
elimination method.

$$x + 2y + 3z = 1$$

$$x + 3y + 6z = 3$$

$$2x + 6y + 13z = 5$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 3 & 6 & 3 \\ 2 & 6 & 13 & 5 \end{array} \right] \xrightarrow{\text{R1-R2, R3-2R1}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 2 & 7 & 3 \end{array} \right] \xrightarrow{\text{R3-2R2}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

$$AX = B$$

$$X = BA^{-1}$$

$$\begin{matrix} 2.9 \\ \hline \text{PT:} \end{matrix}$$

$$x + 2y + 3z = 1$$

$$x + 2y + 3z = 2 \quad [P_2 = P_2 - P_1]$$

$$x + 2y + 3z = 2 \quad [P_3 = P_3 - P_1]$$

so bottom north wind
is now $x + 2y + 3z = 2$

$$x + 2y + 3z = 2 \quad [P_3 = P_3 - 2P_1]$$

$$\therefore z = -1$$

$$\therefore x = -6, y = 5 \quad n + 5d + 6e = 0$$

C.V
30 - 4.23

Gauss - Jordan elimination

method:-

The Gauss - Jordan elimination method is something like Gaussian elimination method except that here each "pivot" is used to place "0" (zero) both below and above it before working with the next pivot. Also one variation is that the pivot is to be unit.

$$x - 2y + 3z + u = 2$$

$$x + y + 4z - u = 3$$

$$2x + 5y + 9z - 2u = 8$$

P.I.O

Coefficient Matrix $A = \begin{bmatrix} 1 & -2 & 3 & 2 \\ 2 & 1 & 4 & -1 \\ 1 & 2 & 0 & 5 \\ 2 & 5 & 0 & -2 \end{bmatrix}$

and the corresponding Augment

Matrix =

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 2 \\ 2 & 1 & 4 & -1 & 3 \\ 1 & 2 & 0 & 5 & 8 \\ 2 & 5 & 0 & -2 & 6 \end{array} \right]$$

transforms \rightarrow pivots.

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 2 \\ 0 & 3 & 2 & -2 & 11 \\ 0 & 9 & 6 & 5 & 24 \end{array} \right]$$

$R_2 \leftarrow R_2 - R_1$

$R_3 \leftarrow R_3 - 3R_1$

transforms \rightarrow pivots.

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & 1 & 2 \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ 0 & 9 & 3 & -4 & 4 \end{array} \right]$$

$R_2' \leftarrow R_2 \div 3$

Treating as equation, not matrix

$$R = \begin{bmatrix} 2 & 0 & \frac{11}{3} \\ 0 & 2 & \frac{1}{3} \\ 0 & 0 & 2 \end{bmatrix} \quad R_1' = R_1 + 2R_2$$

$$R_3' = R_3 - 9 \times R_2$$

$\Rightarrow 2u = 1 \Rightarrow u = \frac{1}{2}$

$\therefore j + \frac{11}{3}z = \frac{2}{3}u = \frac{1}{3}$

$\Rightarrow j + \frac{11}{3}z = \text{constant}$

$x + \frac{11}{3}z = \text{constant}$

Put $z = 0$, find $x, j.$

$\Rightarrow x = \text{constant}$

Now write up
[constant]

$$\begin{bmatrix} 1 & 1 & 0 & \text{constant} \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

* Homogeneous Systems of linear Equations

A system of linear equations is said to

be homogeneous if all the constant terms are zero. Then a homogeneous

System has the form $AX = 0$.

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = 0$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = 0$$

$$a_{r1}x_1 + a_{r2}x_2 + \dots + a_{rn}x_n = 0$$

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = 0$$

Variable - n , Number of Eqns $\Rightarrow r$

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

P.S.

* If $\text{rank}(A) < n$, then the system has only zero solutions. All other cases will have $A \cdot \text{variables} = 0$.

For now with the \mathbb{Z} : work with mod 2 A

* If $\text{rank}(A) < n$, think it has non-zero solution. Work out one not

* $\text{rank}(A) \neq n$ with mod 2 A

$x + y - z = 0 \quad \text{Solve to find if it}$

$$2x - 3y + 2z = 0 \quad \text{has a zero or non-zero soln.}$$

$$x - 4y + 2z = 0 \quad \text{Zero soln.}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & -3 & 1 & 0 \\ 2 & -4 & 2 & 0 \end{array} \right] \quad \text{Row reduction}$$

except to reduce

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -5 & 3 & 0 \\ 0 & -3 & 2 & 0 \end{array} \right]$$

$$\sim \left[\begin{array}{ccc|c} x_1 & 1 & -x_2 & 0 \\ 0 & -5 & x_3 & 0 \\ 0 & -5 & 3 & 0 \end{array} \right] \quad \left[\begin{array}{ccc|c} x_1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \quad \begin{aligned} r_2' &= r_2 - r_1 \\ r_3' &= r_3 - 2r_2 \end{aligned}$$

$$\sim \left[\begin{array}{ccc|c} 2 & 1 & -3 & 0 \\ 0 & -5 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad (I_K - A) \times \dots$$

~~row 2 is non zero~~

$$\sim \left[\begin{array}{ccc|c} 2 & 1 & -3 & 0 \\ 0 & -5 & 3 & 0 \\ 0 & 1 & -3/5 & 0 \end{array} \right] \quad \begin{aligned} &\text{variable } -A \\ &\text{now - zero} \\ &\text{has solution.} \end{aligned}$$

* Eigen values & Eigen Vectors :-

$$\text{If, } A\vec{v} = \lambda\vec{v}$$

Then, \vec{v} = Eigen vector

λ = Eigen value

$$\begin{cases} x + y - z = 0 \\ x - 2y = 0 \\ y - \frac{3}{5}z = 0 \end{cases} \quad \begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases}$$

Eigen values & Eigen vectors

$$(1) \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix}$$

$Ax - \lambda x = 0$

$$\Rightarrow Ax - \lambda x = 0$$

$$\Rightarrow x(A - \lambda I) = 0$$

for non-zero soln

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 3-\lambda & 2 \\ 1 & -1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 3-\lambda & 2 \\ 1 & -1-\lambda \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 3-\lambda & 2 \\ 1 & -1-\lambda \end{bmatrix} = V^{-1} A V$$

$$\Rightarrow \begin{bmatrix} 3-\lambda & 2 \\ 1 & -1-\lambda \end{bmatrix} = V^{-1} A V$$

$$\sim (3-x)(-1-x) - 2 = 2x - 5 = 0$$

$$\Rightarrow \cancel{-3+x} + \cancel{2x} - 2 = 5x + x^2$$

$$\Rightarrow \cancel{x^2 - 2x - 5} = 6 - x$$

$$\Rightarrow x^2 - 3x + 3x + x^2 - 2x - 5 = 0$$

$$\Rightarrow x^2 - 2x - 5 = 0$$

$$\Rightarrow x = 1 + \sqrt{6}, \quad 1 - \sqrt{6}$$

$$x = 1 + \sqrt{6} \rightarrow \text{not an eigenvalue}$$

$$\begin{array}{lcl} 3x + 2j & = & (1 + \sqrt{6})x \\ x + -j & = & (1 + \sqrt{6})j \end{array}$$

$$\Rightarrow 3x + 2j - (1 + \sqrt{6})x = 0$$

$$\Rightarrow x - j - (1 + \sqrt{6})j$$

$$\Rightarrow (2 - \sqrt{6})x + 2j$$

$$\cancel{x + 2 - \sqrt{6}j} = 0$$

$$\begin{bmatrix} x \\ j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

eigen vector

$$\lambda = 2 - \sqrt{6} = 5 - (x+1-)(x-3)$$

$$3x + 2j = (2 - \sqrt{6})x$$

$$x - j = (2 - \sqrt{6})j$$

$$\Rightarrow (2 + \sqrt{6})x + 2j = 0$$

$$x + (2 + \sqrt{6})j = 0$$

$$\therefore x = 0, j = 0$$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ eigen vector $\sqrt{2}x + j = 0$

$$x(\sqrt{2} + 1)$$

$$j(\sqrt{2} + 1) = 0 + xj$$

$$0 - xj$$

$$x(\sqrt{2} + 1) = 0 + x\sqrt{2}$$

$$j(\sqrt{2} + 1) = 0 - x$$

$$0 = \sqrt{2}x + x(\sqrt{2} - 1)$$

$$0 = \sqrt{2}x + x \cancel{(\sqrt{2} - 1)}$$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ eigenvector