Bashundhara
Exercise Book
Write Your Future

Sadman
Statistics (c)
011221592

C.W 27.1.24

Stat

* statistical issue is for sample, not for individual. (collective).

C.W 27.1.23

Stat

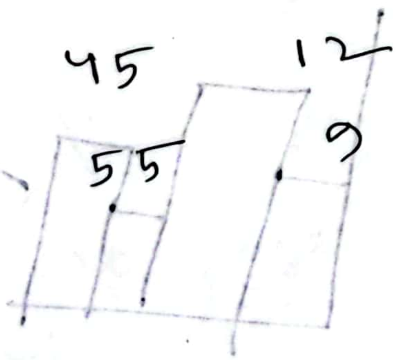
$$\boxed{\theta_i = \frac{f_i}{\Sigma f_i} \times 360^\circ}$$

graph-2

statistical basics

class	x → class marks	f → frequency	F → cumulative frequency
0-10	5	15	15
10-20	15	18	33
20-30	25	10	43
30-40	35	24	67
40-50	45	12	79
50-60	55	9	88

88 (sample size)



class size = 10 = C

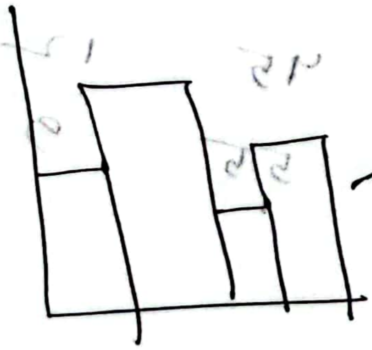
$x_i \pm 5$

graph-2

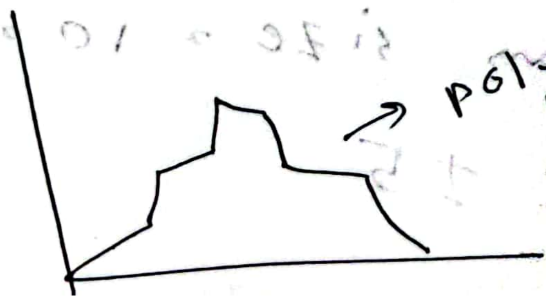
Cumulative frequency difference

Class	x	f	F
1 - 2	1.5	4	4
2 - 3	2.5	6	10
3 - 4	3.5	3	13
4 - 5	4.5	2	15
5 - 6	5.5	2	17
6 - 7	6.5	3	20
7 - 8	7.5	10	30
8 - 9	8.5	6	36
9 - 10	9.5	4	40

from class limit



Histogram



polygon

graph-3

class	x	f	F
10-15	12.5	20	20
15-20	17.5	25	45
20-25	22.5	30	75
25-30	27.5	15	90
30-35	32.5	10	100
35-40	37.5	5	105

modal class

$x - d/2 \to$ low

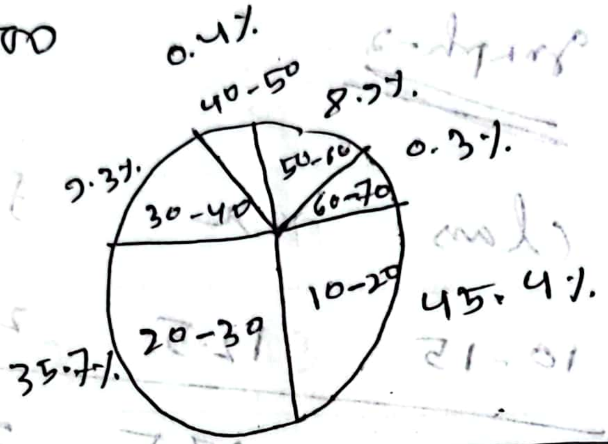
$x + d/2 \to$ up

class marks to class

P.T.O

graph-4 (pi chart)

Sample size = 500



$$\theta_i = \frac{f_i \times 360}{500}$$

$$f_i = \frac{\theta_i \times 500}{360}$$

$$\boxed{500 \times \%}$$

class	x	f	F
10-20		227	
20-30		179	
30-40		46	
40-50		2	
50-60		45	
60-70		1	
		500	

আগের টা কমাইলে, পরের টা বাড়াইতে হবে,

Am → Arithmetic Mean

Shifting origin

$$AM = \frac{\sum f_i x_i}{\sum f_i} \qquad u_i = \frac{x_i - a}{h}$$

h → class difference

scaled variable

$$\Rightarrow x_i = a + h u_i$$

$$\Rightarrow f_i x_i = a f_i + h f_i u_i$$

$$\Rightarrow \sum f_i x_i = a \sum f_i + h \sum f_i u_i$$

$$\Rightarrow \frac{\sum f_i x_i}{\sum f_i} = a + h \frac{\sum f_i u_i}{\sum f_i}$$

$$\boxed{\Rightarrow AM \to \bar{x} = a + \frac{h \sum f_i u_i}{\sum f_i}}$$

P.T.O.

$F_m \to (15)$

$F_{(m-1)}$

Geometric mean

$\frac{1}{\Sigma f_i}$

$G = \left(\Pi x_i^{f_i}\right)$

$\log G = \frac{1}{\Sigma f_i} \log\left(\Pi x_i^{f_i}\right)$

$$\boxed{\log G = \frac{\Sigma f_i \log x_i}{\Sigma f_i}}$$

$\therefore G = 10^{\frac{\Sigma f_i \log x_i}{\Sigma f_i}}$

Harmonic mean

HM → Inverse of avg of Inverse

$\to \left(\frac{\Sigma \frac{1}{x_i} \times f_i}{\Sigma f_i}\right)^{-1}$

$\to \left(\frac{\Sigma f_i/x_i}{\Sigma f_i}\right)^{-1}$

$$\boxed{H.M \to \frac{\Sigma f_i}{\Sigma (f_i/x_i)}}$$

$AM \leq GM \leq HM$ or

$AM \geq GM \geq HM$

$(GM)^2 = (AM)(HM)$

$$Mo = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times C$$

Δ_1 → diff of highest freq & pre highest freq

C → class diff

Δ_2 → diff of highest freq & post " "

l → lower class limit of highest freq class

$$\text{Median} = L + \frac{\frac{N}{2} - \boxed{F_{m-1}}}{f_m} \times C$$

L → lower limit of $\frac{\text{Cumulative freq}}{2}$ class

$\frac{N}{2}$ → $\frac{\text{Cumulative freq}}{2}$

f_m → $\frac{\text{Cumulative freq}}{2}$ class এর freq

F_{m-1} → $\frac{\text{Cumulative freq}}{2}$ এর আগের class এর Cumulative freq

C.W
30.1.23

~~Chapter 3~~

Stat

→ 1st decile → 10%

→ 2nd decile → upto 20%

→ 3rd " → upto 30%

→ quartile → 25%

Quartile, $Q_i = L + \dfrac{\dfrac{i \times N}{4} - F_{q-1}}{f_q} \times c$

L → lower limit of Quartile class

$\dfrac{i \times N}{4}$ → quartile class

N → total frequency

i = 1, 2, 3

from excel

$Q_1 = 57.5 + \dfrac{\dfrac{1 \times 50}{4} - 6}{9} \times 4$

$= 60.3889$ (up to)

decile class

$$D_i = L + \frac{\frac{i \times N}{10} - F_{d-1}}{f_d} \times C \quad i = 1 \to$$

$D_i \to$ Decile

$$D_6 = 65.5 + \frac{\frac{6 \times 50}{10} - 26}{16} \times 4$$

$= 66.5$ (upto)

percentile class

$$P_i = L + \frac{\frac{i \times N}{100} - F_{p-1}}{f_p} \times C$$

$P_i \to$ Percentile

$$P_{10} = 53.5 + \frac{\frac{10 \times 50}{100} - 0}{6} \times 4$$

$= 56.8333$ (upto)

Histogram → mod

polygon → cumulative frequency

→ graph → quantile, decile, percentile

* $MD_{(B)} = \dfrac{\sum f_i |x_i - B|}{\sum f_i}$

→ $CMD_{(B)} = \dfrac{MD}{B} \times 100(\%)$

* $SD = \sqrt{\dfrac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}}$ [Always positive]

$= \sqrt{\dfrac{\sum f_i \cdot x^2}{\sum f} - \left(\dfrac{\sum f_i x_i}{\sum f}\right)^2}$

↳ Always for Arithmitic Mean

* $CSD = \frac{SD}{AM} \times 100 \, (\%)$

$$\boxed{SD = h \sqrt{\frac{\sum f_i u_i^2}{\sum f_i} - \left(\frac{\sum f_i u_i}{\sum f_i}\right)^2}}$$

$$u_i = \frac{x_i - a}{h}$$

Stat

C.W 1.2-24

* Inter Quantile range = $Q_3 - Q_1$

* Coffficiend of " " = $\frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100\%$

↓

relative percentage

statistical indese

* $Z = \frac{x - \bar{x}}{\sigma}$ class mid values, $\bar{x} \rightarrow$ AM, $\sigma \rightarrow$ SD

$Z = (-3 - +3)$

organized

CamScanner

Moments, skewness & Kurtosis

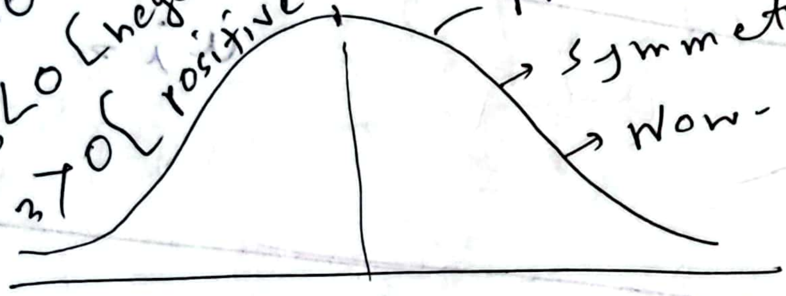
MA

$$m'_r = \frac{\sum f_i (x_i - A)^r}{\sum f_i} \quad r = 1, \cancel{2}, 3, 4$$

$\beta_3 = 0$ [Normal]

$\beta_3 < 0$ [negative skewed]

$\beta_3 > 0$ [positive]



Normal curve

→ symmetric

→ Non-skewed

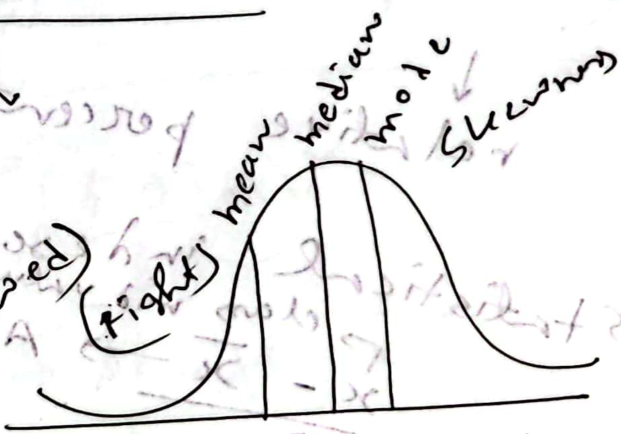
mode

Median

Mean

(left) skewness (+ skewed)

$M_o < Me < Mean$



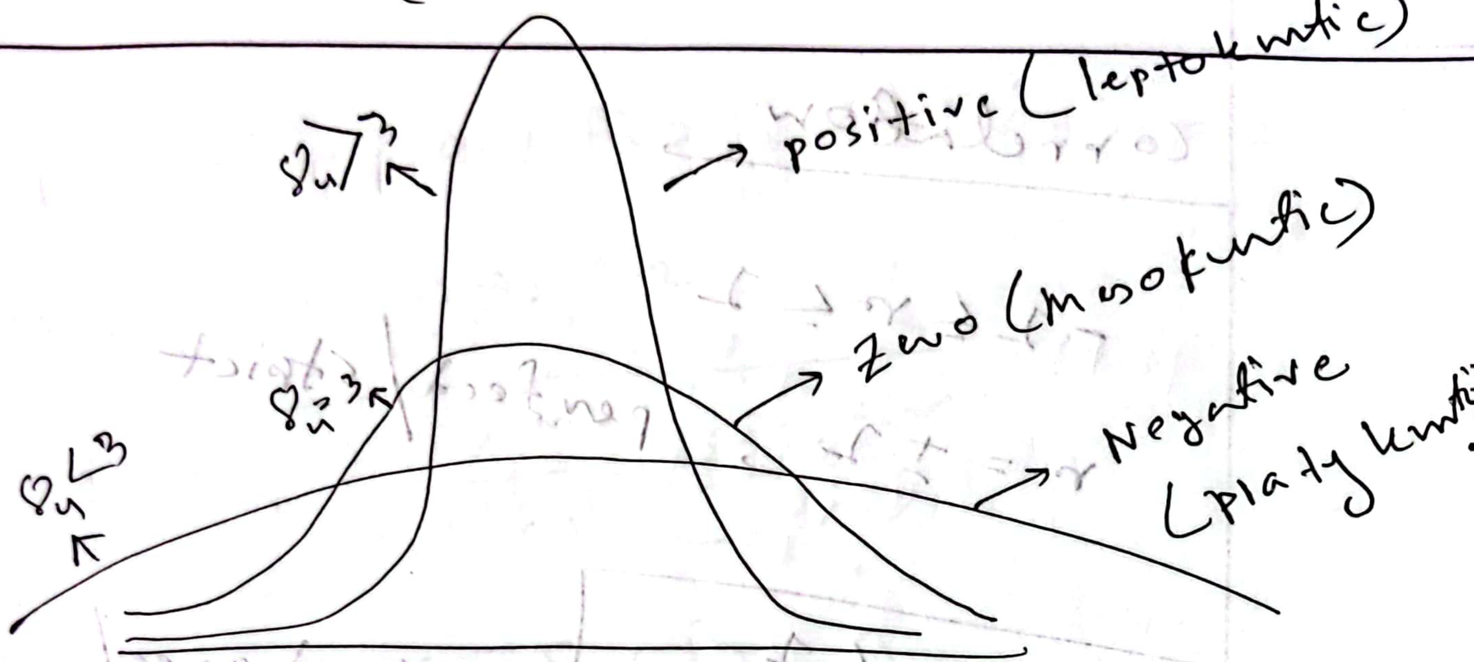
(right) skewness

(- skewed)

mean < Me < Mo

Standard

C.T →



$$m'_r = \frac{\sum f_i (x_i - \bar{x})^r}{\sum f_i}$$

$$u_i = \frac{x_i - A}{h}$$

$$\boxed{m'_r = h^r \frac{\sum f_i . u_i^r}{\sum f_i}}$$

$$\beta_3 = \frac{m_3}{\sqrt{m_2^3}}, \quad \beta_4 = \frac{m_4}{m_2^2}$$

$$m_2 = m'_2 - m'^2_1$$

$$m_3 = m'_3 - 3m'_2 m'_1 + 2m'^3_1$$

$$m_4 = m'_4 - 4m'_3 m'_1 + 6m'_2 m'^2_1 - 3m'^4_1$$

correlation

$-1 \le r \le 1$

$r = \pm 1$, perfect/strict

$\boxed{\begin{array}{l} 0.5 \le r < 1 \\ -1 < r \le -0.5 \end{array}}$ High degree/ strong

+ (positive correlation)
(-) (negative correlation)

$\boxed{r = 0}$ → no relation

$\boxed{\begin{array}{l} 0 < r < 0.5 \\ -0.5 < r < 0 \end{array}}$ low degree/ weak

C.W 6.2.24

Stat

$\Rightarrow \sum f_i x_i^2 - 2a \sum f_i x_i + a^2 \sum f_i = 2215$

$\Rightarrow \sum f_i x_i^2 = 1391.2$

$\Rightarrow SD = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2}$

class এর ক্ষেত্রে, $\sum f_i = 1$

$\sigma_w = \sqrt{\frac{\sum x^2}{N} - (47.2)^2}$

$\sum x_w^2 = 19,254.5$

$\sum x_m^2 = 29964.7$

$\sum x^2 = 49219.2$

$\sigma = \sqrt{\frac{49219.2}{23} - (44.08)^2}$

13)

$$H.M = \frac{4}{\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{2}}$$

$$= 3.12$$

* central tendency

↓

Median

* spread of the birth weight

↓

IQR $(Q_3 - Q_1)$

Correlation & Regression

$$\operatorname{cov}(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{N}$$

$\hookrightarrow$ Number of target objects.

$\downarrow$

$\sigma_{xy} = \sigma_{xy}$

$$\sigma_x^2 = \operatorname{var}(x) = \frac{\sum (x_i - \bar{x})^2}{N}$$

$$\sigma_y^2 = s_{yy} = \operatorname{var}(y) = \frac{\sum (y_i - \bar{y})^2}{N}$$

$$r_{xy} = r_{yx} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

$$= \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

P.T.O

$$\bar{x} = \frac{\sum x_i}{N}$$

$$\bar{y} = \frac{\sum y_i}{N}$$

$$\sigma_{xy} = \frac{N\sum xy - \sum x \sum y}{\sqrt{\{N\sum x^2 - (\sum x)^2\}\{N\sum y^2 - (\sum y)^2\}}}$$

$$\frac{x_i - a}{h} = u_i \qquad \frac{y_i - b}{k} = v_i$$

$$x_i - \bar{x} = h(u_i - \bar{u})$$

$$y_i - \bar{y} = k(v_i - \bar{v})$$

$$r_{uv} = \frac{\sum (u_i - \bar{u})(v_i - \bar{v})}{\sqrt{\sum (u_i - \bar{u})^2 \sum (v_i - \bar{v})^2}}$$

Stat

Formula

$$M_o = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times C$$

$$M_e = L + \frac{\frac{N}{2} - F_{m-1}}{f_m} \times C$$

$i_Q = 1, 2, 3$

$i_D = 1-9$

$i_P = 1-99$

$$(Q, D, P) = L + \frac{\frac{i \times N}{4, 10, 100} - F_{(Q, D, P)-1}}{f_{(Q, D, P)}} \times C$$

$$AM = \frac{\sum f_i x_i}{\sum f_i}, \quad a + \frac{h \sum f_i u_i}{\sum f_i}$$

$$GM = 10^{\frac{\sum f_i \log x_i}{\sum f_i}}$$

$$u_i = \frac{x_i - a}{h}$$

$$HM = \frac{\sum f_i}{\sum \left(\frac{f_i}{x_i}\right)}$$

$$MD = \frac{\sum f_i |x_i - B|}{\sum f_i}, \quad CMD = \frac{MD}{B} \times 100\%$$

$$SD = \sqrt{\frac{\sum f x^2}{\sum f} - (AM)^2}, \quad CSD = \frac{SD}{AM} \times 100\%$$

$$IQR = Q_3 - Q_1, \quad CIQR = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100\%$$

raw moments.

$$m'_r = \frac{h^r \sum f_i u_i^r}{\sum f_i} \quad r = 1, 2, 3, 4$$

$m_1 = 0$

$m_2 = m'_2 - m_1'^2$

$m_3 = m'_3 - 3 m'_2 m'_1 + 2 m_1'^3$

$m_4 = m'_4 - 4 m'_3 m'_1 + 6 m'_2 m_1'^2 - 3 m_1'^4$

skewness

$$\gamma_3 = \frac{m_3}{\sqrt{m_2^3}}$$

$\gamma_3 > 0$
positive skewed
mod < me < mean

$\gamma_3 < 0$
negetive skewed
mean < me < mode

$\gamma_3 = 0$ [nonskewed normal]

kurtosis

$$\gamma_4 = \frac{m_4}{m_2^2}$$

$\gamma_4 > 3$ [positive kurtosis] (leptokurtic)

$\gamma_4 < 3$ [negative kurtosis] (platykurtic)

$\gamma_4 = 3$ [balanced] (mesokurtic)

6.W
10.2.24

Stat

Regression

$$\hat{y} = b_{\frac{y}{x}}(x - \bar{x}) + \bar{y}$$

$$\hat{x} = b_{\frac{x}{y}}(y - \bar{y}) + \bar{x}$$

↳ regression cofficient

$$b_{\frac{y}{x}} = \frac{6_{xy}}{6_{x^2}} = \frac{N\Sigma xy - \Sigma x \Sigma y}{N\Sigma x^2 - (\Sigma x)^2}$$

$$b_{\frac{x}{y}} = \frac{N\Sigma xy - \Sigma x \Sigma y}{N\Sigma y^2 - (\Sigma y)^2}$$

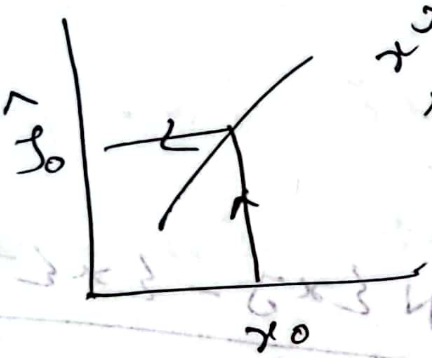
$$b_{\frac{y}{x}} = \frac{k}{h} \; b_{\frac{v}{u}}$$

$$b_{\frac{x}{y}} = \frac{h}{k} \; b_{\frac{u}{v}}$$

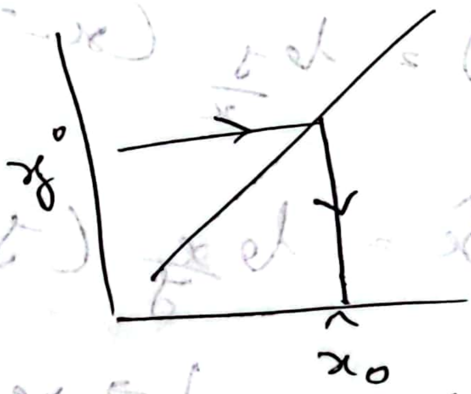
$\hat{y} = a_0 + a_1 x$

$a_0 = ?$

$a_1 = ?$



$\hat{x} = a_0 + a_1 y$



* $r_{xy} = \pm\sqrt{b_{\frac{y}{x}} \times b_{\frac{x}{y}}}$

* $b_{\frac{x}{y}} = \frac{\sigma_{xy}}{\sigma_y^2}$ $\qquad b_{\frac{y}{x}} = \frac{\sigma_{xy}}{\sigma_x^2}$

* $b_{\frac{x}{y}} =$

P.T.O

~~(six)~~

$$r_{rank} = 1 - \frac{6 \sum d_i^2}{N(N^2-1)}$$

check

$$\boxed{b_{yx} \times b_{xy} = r^2}$$

* value inside interpolition.

17.8585

C.W
12.2.24

Stat

$$SD = \sqrt{\frac{\Sigma fi \cdot x^2}{\Sigma fi} - (A.M)^2}$$

* $\frac{360^\circ}{\Sigma fi} = ?$ After that we it

*

C.W 13.2.24

stem → leaf plot

70, 135, 15, 10, 92, 103, 149, 42,
98, 116, 18, 62, 91, 14, 45

51, 21, 52, 80, 72, 8, 13, 25,
111, 35, 21, 97, 11, 22, 55,
16.

underline — leaf

stem → underline

raw

l_2		l_1
	5	
2	7	0
	13	5
6 1 3	11	5 0 8 4
7	9	2 8 1
	10	3
	14	9
1	4	2 5
7	11	6
1	6	2
5 9 1	5	
2 1 5 1	2	
0 8	8 0	

Organized by Ascending order

l_2	S	l_1
8	0	
6, 3, 1	1	0, 4 5 8 → O_1
5 2 1 1	2	
5	3	
	4	2 5
9 5 1	5	
	6	2
2	7	0 Me
0	8	
7	9	1 2 8
	10	3 → O_3
1	11	6
	~~12~~	
	13	5
	14	9

2|7|0

$l_1 = 72$

$l_2 = 70$

$[R_1 = 149 - 16 = 139]$ for l_1

$[R_2 = 111 - 8 = 103]$ for l_2

$Me = \frac{N+1}{2}$ th value [N is odd]

$me = \frac{\frac{N}{2} + (\frac{N}{2}+1)}{2}$ " " [N is even]

$Me_1 = 70$

$me_2 = \frac{25+35}{2} = 30$

median এর আগের Part এর
median => Q_1
পরের = Q_3

$IQR_1 = Q_3 - Q_1$

$= 103 - 18$

$= 85$

$IQR_2 = \frac{59+72}{2} - \frac{16+21}{2}$

$= 65.5 - 18.5$

$= 47$

cost minimization;

$\boxed{Me_2 < Me_1}$

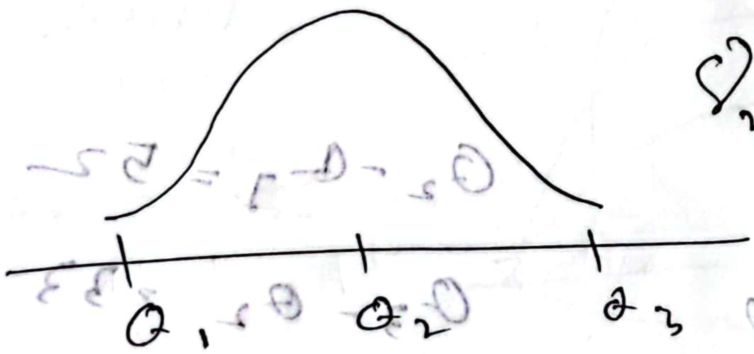
(Me) comparison for which data set is Better.

IQR যেইটার কম সেইটার Better.

$IQR_2 < IQR_1$

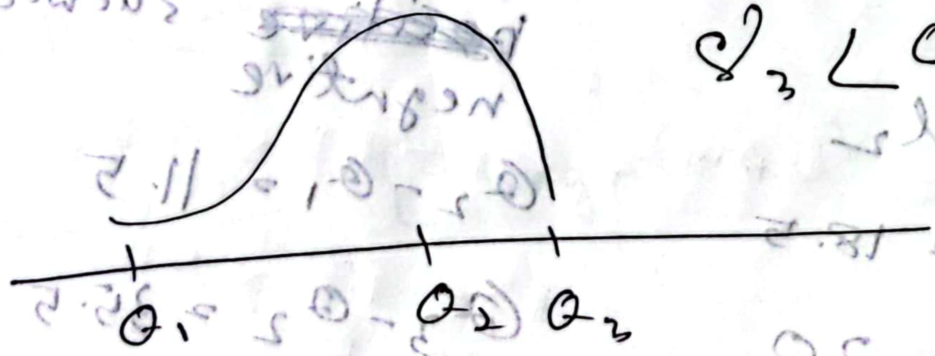
more consistent.

$Q_2 - Q_1 = Q_3 - Q_2$



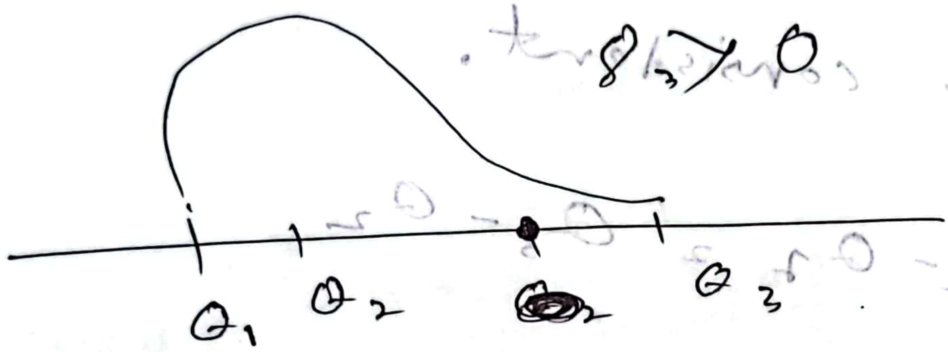
$\gamma_3 = 0$

$Q_2 - Q_1 > Q_3 - Q_2$



$\gamma_3 < 0$

$Q_2 - Q_1 < Q_3 - Q_2$



for l_1

$Q_1 = 18$

$Q_2 = 70$

$Q_3 = 103$

$Q_2 - Q_1 = 52$

$Q_3 - Q_2 = 33$

$52 > 33$

~~positive~~ negative skewed

for l_2

$Q_1 = 18.5$

$Q_2 = 30$

$Q_3 = 65.5$

$Q_2 - Q_1 = 11.5$

$Q_3 - Q_2 = 35.5$

$11.5 < 35.5$

positive skewed

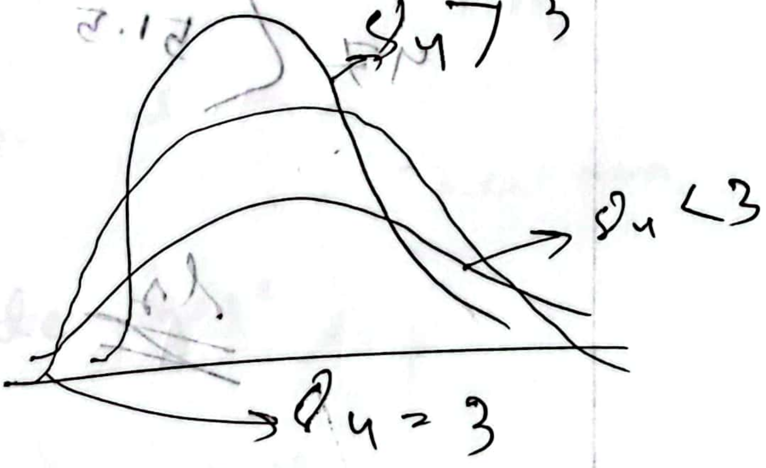
$IQR_1 = 85$ compare with $\frac{range}{2}$

$IQR_2 = 47$

$\frac{R_1}{2} = 69.5$

$\frac{R_2}{2} = 57.5$

$IQR = \frac{R}{2}$



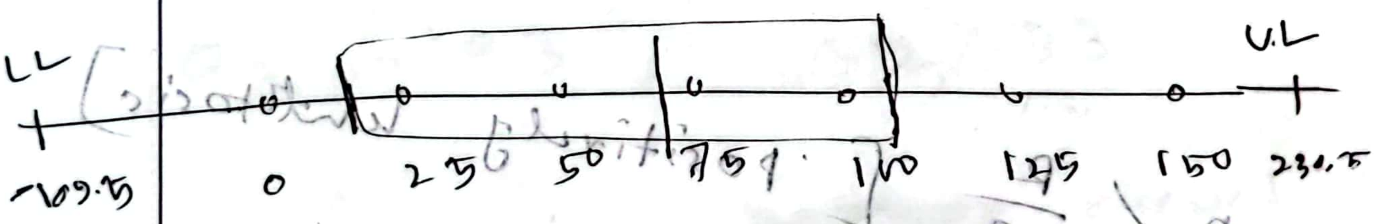
$IQR < \frac{R}{2}$ [positively kurtosis]

$IQR > \frac{R}{2}$ [negative kurtosis] for l_1

85 > 69.5 [negative kurtosis]

for l_2

47 < 51.5 [positive kurtosis]



(LL) lower limit = $Q_1 - 1.5 \times IQR$

$= 18 - 1.5 \times 85$

$= -109.5$

(UL) upper limit $= 103 + 1.5 \times 85$

$= 230.5$

outlier → value টেনে হবে কোনো না।

7-2-24

* key ta always likhte হবে।

* mode → max repentation

repetition না হলে, mode হবে না।

* How distributed nature
How the distribute shape } skewness

girls

Me = 1.6 (7th)

$Q_1 = \frac{0.4 + 0.4}{2} = 0.4$

$Q_3 = \frac{2.3 + 2.5}{2} = 2.4$

IQR = 2.0

Boys

Me = 1.9 (6th)

$Q_1 = 1.1$

$Q_3 = 2.5$

IQR = 2.5 − 1.1
= 1.4

Boys > girls (median)

Range Girls => 3.1 - 0.1

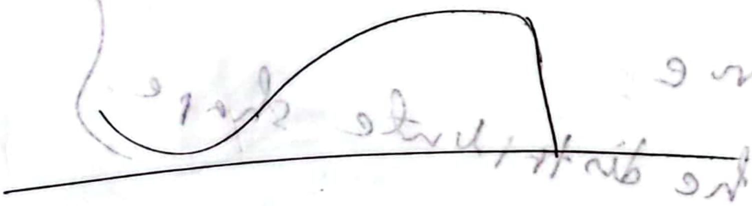
=> 3

" Boys = 3.2 - 0.2

=> 3

girls : $Q_2 - Q_1$ $Q_3 - Q_2$

=> 1.2 > 0.8

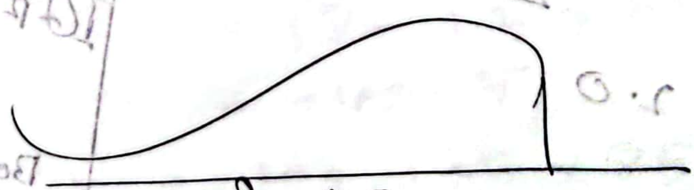


$\gamma_3 < 0$

[negative skewed]

boys : $Q_2 - Q_1$ $Q_3 - Q_2$

=> 0.8 > 0.6



$\gamma_3 < 0$

[negative skewed]

kurtosis

boys

IQR = 1.4

$\frac{\text{Range}}{2} = 1.5$ $\quad 1.4 < 1.5$ Positive kurtosis

girls

IQR = ~~ ~~ 2.0

$\frac{\text{Range}}{2} = 1.5$ $\quad 2.0 > 1.5$ negative kurtosis

Girls

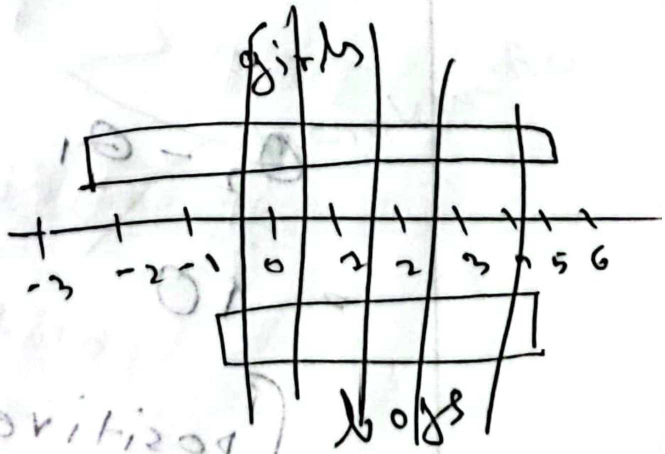
LL = $0.4 - 1.5 \times 2 = -2.6$

UL = $2.4 + 1.5 \times 2 = 5.4$

boys

LL = $1.1 - 1.5 \times 1.4 = -1$

UL = $2.5 + 1.5 \times 1.4 = 4.6$



LL = 20

UL = 90

Range = 90 - 20 = 70

$Q_1 = 40$

Me = 50 = Q_2

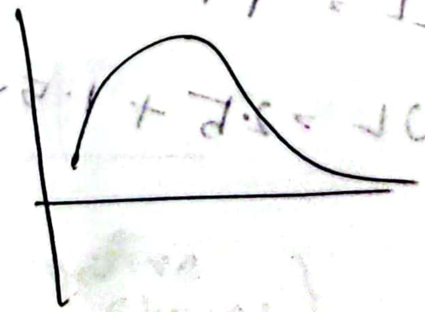
$Q_3 = 80$

IQR = 40

$\frac{\text{Range}}{2} = 35$

$Q_2 - Q_1 = 10 < 30$ $Q_3 - Q_2$

$\boxed{\text{positive skewed}}$



35 < 40 [negative kurtosis]

(platy kurtic)

* more consistance → small wishker box

* sydney

$\frac{\text{Range}}{2} = \frac{15-9}{2} = 3$

IQR = 14-11 = 3

Melbourne

mesokurtic

$\frac{\text{Range}}{2} = \frac{19-7}{2} = 6$

IQR = 16-10 = 6

* maximization
* more consistence

2) ~~A~~

$Q_1 = 30$

$Q_3 = 55$

Me = 48

IQR = 25 → more consistente

$\frac{\text{Range}}{2} = \frac{65-5}{2} = 30$

$\text{IQR} < \frac{\text{Range}}{2}$

~~positive skewed~~

{positive kurtosis}

P.T.O

$Q_2 - Q_1$ > $Q_3 - Q_2$

$= 18$ > 7

[negative skewed]

$Q_1 = 32$

$Q_3 = 59$

$me = 40$

$IQR = 27$ — $\frac{Range}{2} = \frac{66-7}{2}$

$= 29.5$

$27 < 29.5$ [positive kurtosis]

$Q_2 - Q_1$ < $Q_3 - Q_2$

$= 8$ < $= 19$

[positive skewed]

weight x	height y	x^2	y^2	xy
48	63	2304	3969	3024
43	63	1849	3969	2709
77	67	5929	4489	5159
87	68	7569	4624	5916
81	69	6561	~~4489~~ 4761	5589
$\Sigma = 336$	$\Sigma = 330$	$\Sigma = 24212$	$\Sigma = 21812$	$\Sigma = 22397$

$$r = \frac{(5 \times 22397) - (336 \times 330)}{\sqrt{\{(5 \times 24212) - (336)^2\}\{(5 \times 21812) - (330)^2\}}}$$

$$= 0.97$$

$$b_{y/x} = \frac{(5\times 22397) - (336\times 330)}{5\times 24212 - (336)^2}$$

$b = 0.135$

$$b_{x/y} = \frac{(5\times 22397) - (336\times 330)}{(5\times 21812) - (330)^2}$$

$= \boxed{6.906}$ [Large]

[dominent height]

$$\sqrt{b_{y/x}\times b_{x/y}} = 0.97$$

$$y = 0.135\left(x - \frac{336}{5}\right) + \frac{330}{5}$$

$= 0.135x - 9.072 + 66$

$= 0.135x + 56.928$

$$x = 6.906\left(y - \frac{330}{5}\right) + \frac{336}{5}$$

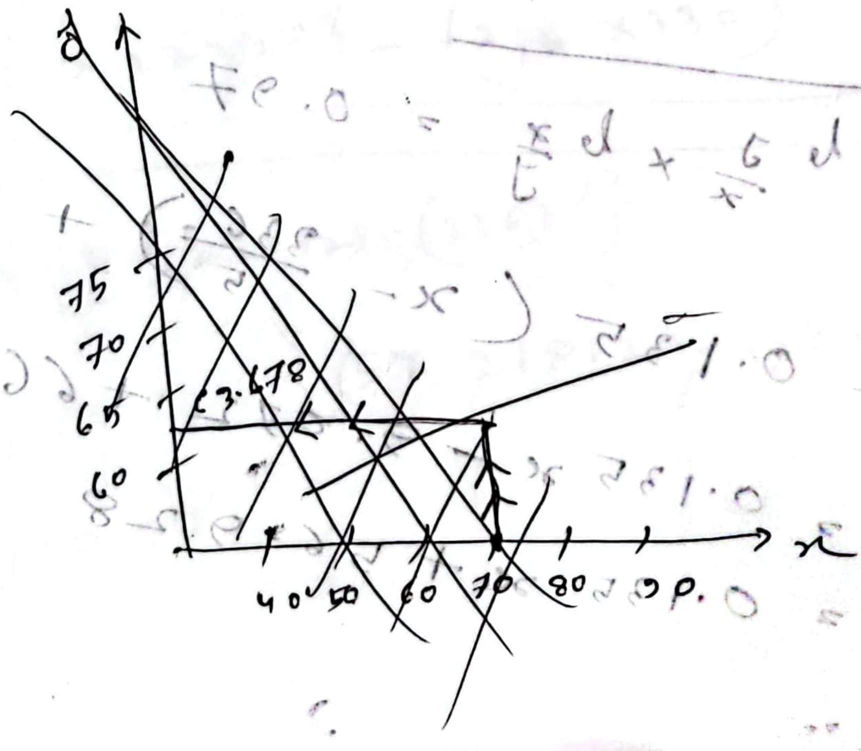
$$= 6.906y - 455.796 + 67.2$$

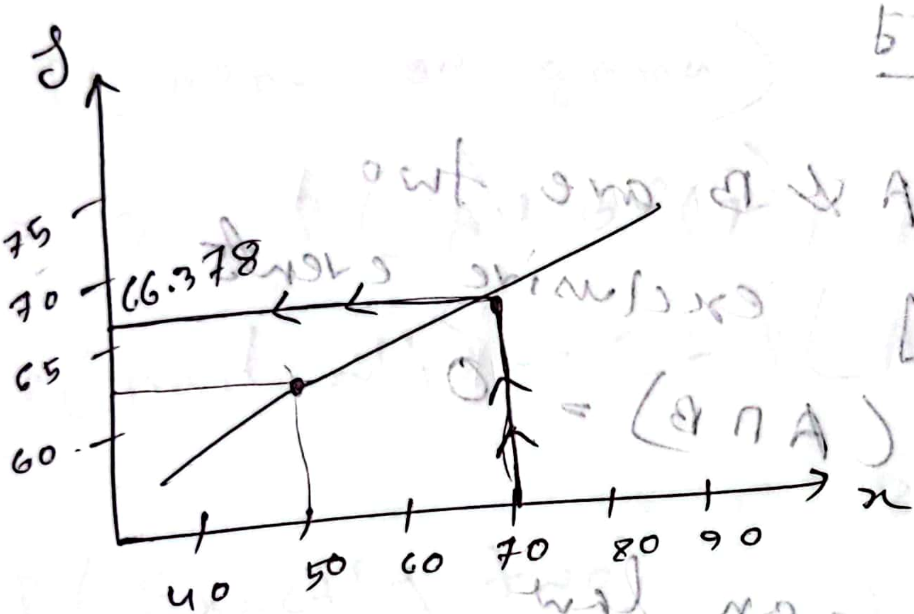
$$= 6.906y - 388.596$$

$$y = 0.135x + 56.928$$

$x = 50$, $y \to 63.678$

$x = 70$, $y \to 66.378$





N=5

x	y	d_i	d_i^2
1	2	-1	1
4	3	1	1
5	5	0	0
3	1	2	4
2	4	-2	4
			10

$$\therefore R_{rank} = 1 - \frac{6 \times 10}{5(25-1)} = 0.5$$

C.W 7.3.24

Stat

Probability

* If A & B are two mutually exclusive events then $P(A \cap B) = 0$

* De morgan law

* Probability positive

* " always 1

* Mathmetical statement যেকে maths করতে হয়। logical statement দিয়ে math করতে হবে।

* $P(A \cap B') = P(A) - P(A \cap B)$

* $P(A' \cap B) = P(B) - P(A \cap B)$

$P(\text{None of game})$

$= 1 - P(A \cup B \cup C)$

$P(\text{Not all}) = 1 - P(A \cap B \cap C)$

* $P(\text{Exactly two})$

$= P(A \cap B) + P(B \cap C) + P(A \cap C)$

* $P(\text{Getting Red but not Ace})$

$= P(A \cap B')$

$= \frac{26}{52} - \frac{2}{52}$

* $P(A) = P(A \cup B') - P[(A \cup B)']$

$= P(A \cup B') - \{1 - P(A \cup B)\}$

$P(C \cap D)$

$\bullet \; P(C) + P(D) - P(C \cap D) = P(C \cup D)$

$P(C) = 0.8$

$P(D) = 0.6$

$P(C \cap D) = ?$

$P(C \cup D) =$ not find

CW 5.3.24

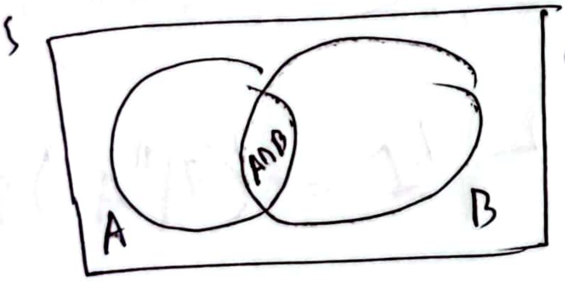
Stat

$\uparrow$ $x \geq a$; at least

$x > a$

$\downarrow$ $x \leq a$; at most

CW 19.3.24



requirement

condition

$n(S)$

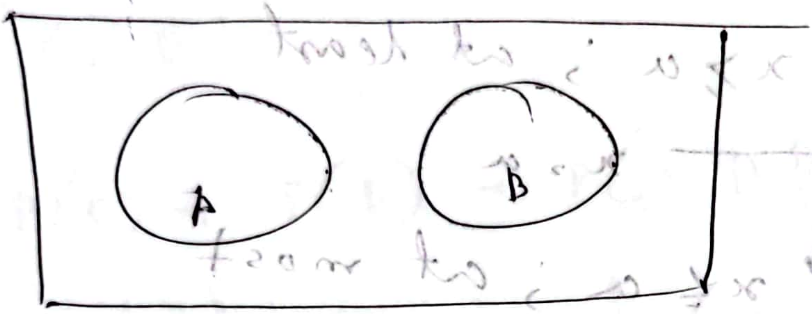
$n(A)$

$n(B)$

$n(A \cap B)$

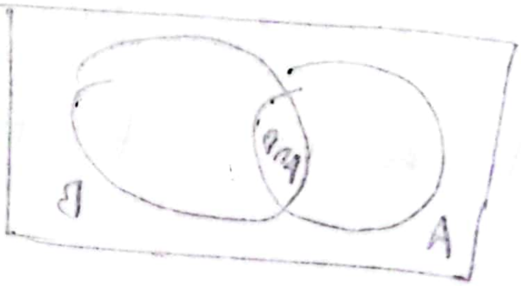
$$P(A|B) = \frac{n(A\cap B)/n(S)}{n(B)/n(S)} = \frac{P(A\cap B)}{P(B)}$$

$$P(B|A) = \frac{n(A\cap B)/n(S)}{n(A)/n(S)} = \frac{P(A\cap B)}{P(A)}$$



$n(A \cap B) = 0$

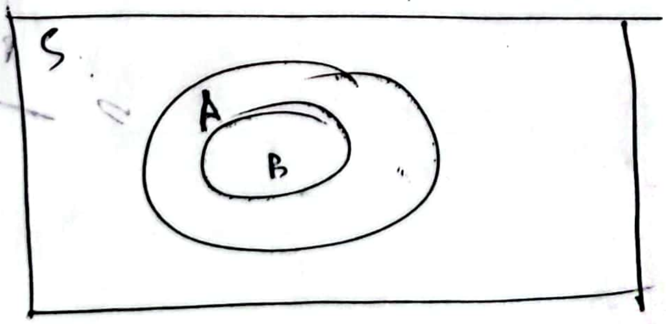
$P(A \cap B) = 0$



$P(A/B) = 0$

$P(B/A) = 0$

for disjoint set conditional probability zero.



$B \subset A$, $A \cap B = B$

$$P(A/B) = \frac{n(B)}{n(B)}$$

$$P(B/A) = \frac{n(B)}{n(A)} =$$

(compliment)

$$P(A'/B) = 1 - P(A/B)$$

* $P(A / B_1 \cup B_2 \cup B_3 \ldots)$

$$= \frac{P(A \cap B_1) + P(A \cap B_2) + \ldots}{P(B_1) + P(B_2) + \ldots}$$

$P'(B/B) = 1$

* $P(A_1' \cup A_2 \cup A_3 / B)$

$$= \frac{P(A_1 \cap B) + P(A_2 \cap B)}{P(B)}$$

[6, 9,] H.W

→ without replacement
indecate conditional
probability.

* P(w p)

1.3

Exercise (12, 13, 14) sum

14

	A_1	A_2	A_3	total
B_1	7	11	13	31
B_2	11	21	9	41
B_3	12	9	7	28
total	30	41	29	100

$$P(A_1) = \frac{30}{100}$$

$$P(A_3 \cap B_2) = \frac{9}{100}$$

$$P(A_2 \cup B_3) = \frac{12+9+7+21+11}{100}$$

$$P(A_1 / B_2) = \frac{22}{42}$$

$$P(B_1 / A_3) = \frac{13}{29}$$

4

$$\underbrace{\frac{12}{52}}_{\text{Heart without ace}} \times \overset{\text{All heart}}{\frac{4}{51}} + \underbrace{\frac{1}{52}}_{\text{Heart ace}} \times \overset{\text{(All ace - heart ace)}}{\frac{3}{51}}$$

6/

	PR	NPR
HD	111	110
N.H.D	223	338
	334	648

$$P(HD \mid NPR) = \frac{110}{648}$$

7/

$$\frac{2}{4}\times\frac{2}{3}+\frac{2}{4}\times\left(\frac{2}{3}+\frac{2}{4}\right)\times\frac{1}{3} = \frac{5}{6}$$

O_1 B_2 B_1 O_2 O_1 O_2

$$P(O_1/A+I) = \frac{2/6}{5/6} = \frac{2}{5}$$

8/

① WWL ② WLW

③ LWW ④ W^3

⑥

$$\frac{3}{20}\times\frac{2}{19}\times\frac{1}{18}$$

$3C_2 = 3$

15

A.W. (RB, BR)

If,

$P(A)\,P(B/A) \neq P(B)\cdot P(A/B)$

dependent

= Independent

$P(A \cap B) = P(A)\,P(B)$

Independent

(W.I) $P(A/B) = P(A)$,

$P(B/A) = P(B)$

Independent Event

~~Example~~ ~~1.4.1~~ Statistics

~~1.4.4~~,

[mutually independent]

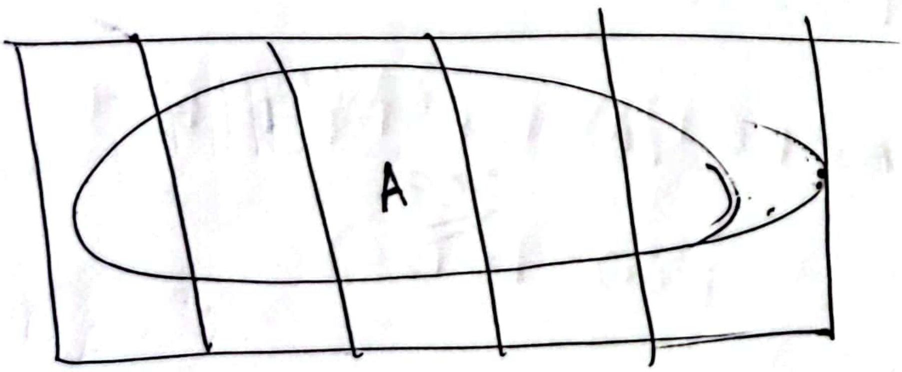
~~1.4.5~~, ~~1.4.6~~,

[complementary approach]

Bayes theorem

$P(A \cap B) = P(A)\,P(B/A) = P(B)\,P(A|B)$ [Independent]

$\Rightarrow P(B_k / A) = \dfrac{P(B_k)\,P(A/B_k)}{P(A)}$



$B_j \longrightarrow$

$$A = A \cap (B_1 \cup B_2 \cup \dots \cup B_n)$$

$$\Rightarrow A = (A \cap B_1) \cup (A \cap B_2) \dots \cup (A \cap B_n)$$

$$\Rightarrow P(A) = \sum_{i=1}^{n} P(A \cap B_i)$$

$$\Rightarrow P(A) = \sum_{i=1}^{n} P(B_i)\, P(A \mid B_i)$$

C.W 2.4.24

Probability

1.5.3

(11)

$P(T^-/C^+) = 0.16$

$P(T^+/C^+) = 0.84$

$P(C^+/T^+)$

$$= \frac{P(C^+ \cap T^+)}{P(T^+)}$$

$$= \frac{P(C^+)\,P(T^+/C^+)}{P(C^+)\,P(T^+/C^+) + P(C^-)\,P(T^+/C^-)}$$

P.T.O

(ii)

$A = \{\text{Heavy}\}$ $P(A) = 0.15$

$B = \{\text{light}\}$ $P(B) = 0.3$

$C = \{\text{non}\}$ $P(C) = 0.55$

$P(D/C) = x$

$P(D/B) = 3x$

$P(D/A) = 5x$

$$P(C/D) = \frac{P(C)\,P(D/C)}{P(A)\,P(D/A) + P(B)P(D/B) + P(C)\,P(D/C)}$$

$$= \frac{0.55x}{0.75x + 0.9x + 0.55x}$$

13.

$$P(A) = \frac{2}{5} = 0.4$$

$$P(B) = 0.6 = (1 - 0.4)$$

$$P(I^+/A) = 0.03$$

$$P(I^+/B) = 0.02$$

$$P(I^+) = \frac{2}{25} = 0.08$$

$$P(I^-) = (1 - 0.08) = 0.92$$

$$P(A|I^+)$$

$$= \frac{P(A)\,P(I^+/A) + P(B)\,P(I^+/B)}{\cancel{P(I^+)}}$$

$$= \frac{P(A \cap I^+)}{P(I^+)} = \frac{P(A)\,P(I^+/A)}{P(I^+)}$$

*

variable → X (declare)

X → f(x) = . . . x = -1, 0, 1

→ pmf → $F(x) = \sum_{1}^{0} f(x)$

$f(x) = \_\_\_$; $1 \leq x \leq 10$

→ pdf

$\boxed{\text{upto}}$ (~~[illegible]~~)

$$F(x) = \int_{2}^{5} f(x)\, dx$$

* random variable
* discrete random variable
* continuous random variable
* pmf, pdf

16/4.2.21

Statistics

2.1 $\boxed{f(x) = \frac{1}{m} \; ; \; x = 1, 2, 3, \ldots, m}$

$$F(x) = P(X \le x) = \begin{cases} 0 \; ; & x < 1 \\ \frac{k}{m} \; ; & 1 \le k < m \\ 1 \; ; & x \ge m \end{cases}$$

$$\frac{f(x) - P(X_1)}{P(X_2) - P(X_1)} = \frac{x - x_1}{X_2 - X_1}$$

$P(x=1) = \frac{1}{16}$

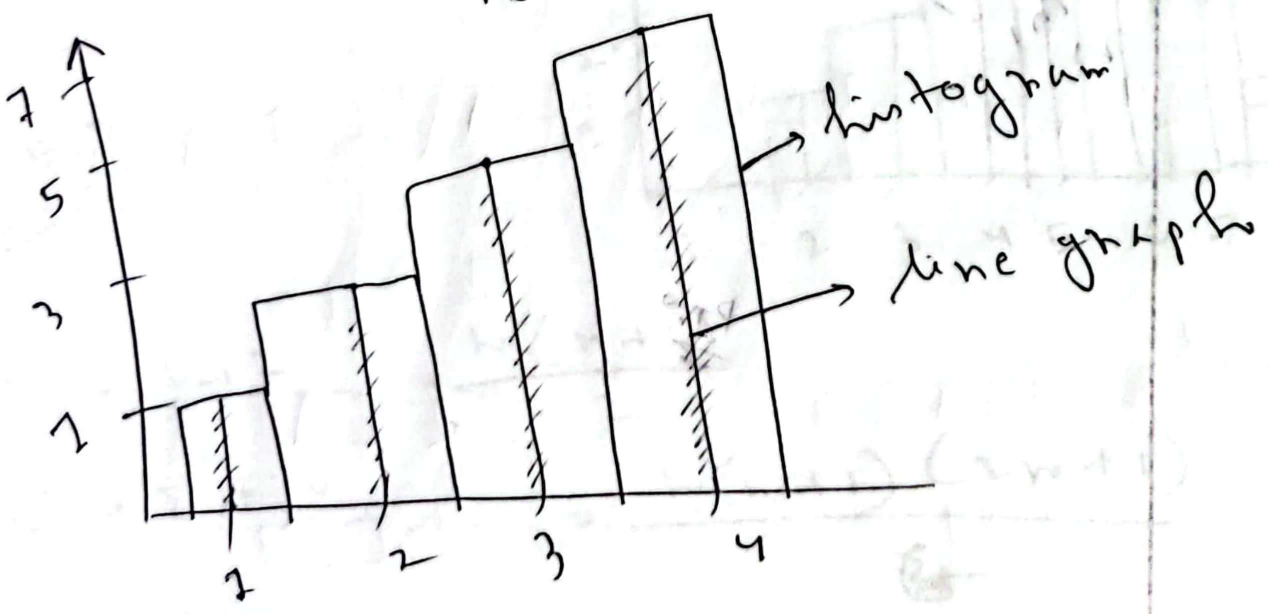
$P(x=2) = \frac{3}{16}$

$P(x=3) = \frac{5}{16}$

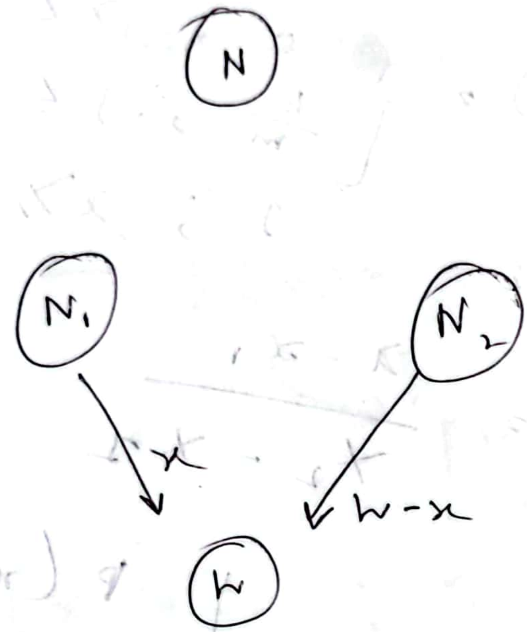
$P(x=4) = \frac{7}{16}$

$$\frac{f(x) - \frac{3}{16}}{\frac{5}{16} - \frac{3}{16}} = \frac{x-2}{3-2}$$

$$\Rightarrow f(x) = \frac{2x-1}{16}$$



Hypergeometric distribution



$n \le N$

$N_1 + N_2 = N$

$x \le n,\ N_1$

$n - x \le n,\ N_2$

$$P(X=x) = \frac{{}^{N_1}C_x \times {}^{N_2}C_{n-x}}{{}^{N}C_n}$$

Example:

2, 3, 4, 5, 6

P.T.O

7

$p(x=2) = \frac{1}{16}$

$p(x=3) = \frac{2}{16}$

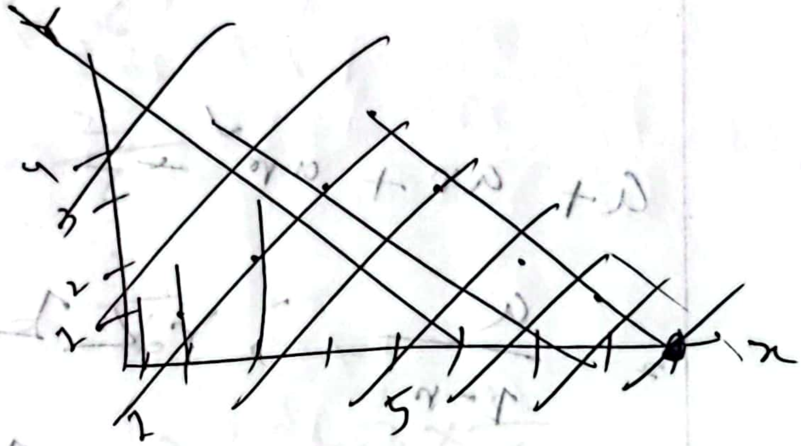
$p(x=4) = \frac{3}{16}$

$p(x=5) = \frac{4}{16}$

$p(x=6) = \frac{3}{16}$

$p(x=7) = \frac{2}{16}$

$p(x=8) = \frac{1}{16}$



$$f(x) = \frac{4-|x-5|}{16}$$

(1,5,9)

$$S_n = 1+2 \ldots = \frac{n(n+1)}{2}$$

$$S_n^2 = 1^2+2^2+\ldots = \frac{n(n+1)(2n+1)}{6}$$

$$S_{n^3} = 1^3 + 2^3 + \dots \left\{\frac{n(n+1)}{2}\right\}^2$$

$$a + ar + ar^2 + \dots$$

$$= \frac{a}{1-r} ; \; r < 1$$

$$= \frac{a}{r-1} ; \; r > 1$$

C.T = {2.5, (2.1 − 2.3)}

C.T = {3.1, 3.2, 3.3}

C.W 20.4.24

2.2.1

$P(A) = P(X=1) = \frac{3}{6}$

$P(B) = P(X=2) = \frac{2}{6}$

$P(C) = P(X=3) = \frac{1}{6}$

$E(X) = 1 \times \frac{3}{6} + 2 \times \frac{2}{6} + 3 \times \frac{1}{6}$

$E(X^2) = 1^2 \times \frac{3}{6} + 2^2 \times \frac{2}{6} + 3^2 \times \frac{1}{6}$

$\mu = E(X)$

$\sigma^2 = E(X^2) - \{E(X)\}^2$

$$\frac{f(x) - P(X = X_0)}{P(X = X_1) - P(X = X_0)} = \frac{x - x_0}{X_1 - X_0}$$

$$f(x) = \frac{4 - x}{6}$$

~~CW 22.4.24~~

Probability

2.2.7

$$E(x) = 2 \times \frac{75}{216} + 2 \times \frac{15}{216} \times (-1) \times \frac{125}{216}$$

$$= -\frac{17}{216}$$

$$\text{loss} = \frac{17}{216}$$

$$E(x^2) = 1^2 \times \frac{75}{216} + 2^2 \times \frac{15}{216} + 3^2 \times \frac{1}{216} + (-1)^2 \times \frac{125}{216}$$

$$SD = \sqrt{E(x^2) - \{E(x)\}^2}$$

$$f(x) = \frac{1}{m}$$

$$\mu = E(x) = \frac{m+1}{2}$$

$$\sigma^2 = E(x^2) - \{E(x)\}^2 = \frac{m^2-1}{12}$$

$$M(t) = \sum_{x=1}^{m} \frac{e^{tx}}{m}$$

$$\boxed{M(0) = 1}$$ for (mgf)

Exm: 2.3.1

$m = 6$

$$f(x) = \frac{1}{6} \; ; \quad x = 1, 2, 3, 4, 5, 6$$

$$\mu = \frac{6+1}{2} = \frac{7}{2}$$

$$\sigma^2 = \frac{6^2-1}{12} = \frac{35}{12}$$

2.3.2 (H.W)

$$M(t) = \sum_{x=1}^{6} \frac{e^{tx}}{6}$$

3.4

2.3.5

$P(X=1)$

$$M(t) = e^{t}\left(\frac{3}{6}\right) + e^{2t}\left(\frac{2}{6}\right) + e^{3t}\left(\frac{1}{6}\right)$$

$$= \sum e^{tx} f(x)$$

$$\frac{f(x) - P(X=1)}{P(X=3) - P(X=1)} = \frac{x-1}{3-1}$$

$$f(x) = \frac{4-x}{6} \quad ; \quad x = 1, 2, 3$$

$$M'(t) = \frac{3}{6} e^{t} + \frac{4}{6} e^{2t} + \frac{3}{6} e^{3t}$$

$$M''(t) = \frac{3}{6} e^{t} + \frac{8}{6} e^{2t} + \frac{9}{6} e^{3t}$$

$$\sigma^2 = M''(0) - \{M'(0)\}^2 = \frac{20}{6} - \left(\frac{10}{6}\right)^2$$

$$= \frac{20}{36}$$

* if $M(0) \neq 1$,
no corrosponding mean & variance

6/ $M(t) = \frac{e^t}{2}\left(1 - \frac{e^t}{2}\right)^{-1}$

$= \frac{e^t}{2}\left(1 + \frac{e^t}{2} + \frac{e^{2t}}{2^2} + \frac{e^{3t}}{2^3} + \dots\right)$

$= \frac{e^t}{2} + \frac{e^{2t}}{2^2} + \frac{e^{3t}}{2^3} + \dots$

$= \sum_{x=1}^{\infty} \frac{e^{tx}}{2^x} = \sum^{\infty} e^{tx} f(x)$

$f(x) = \frac{1}{2^x}$; $x = 1, 2, \dots$

Geometric

$f(x) = q^{x-1} p \; ; x = 1, 2, \ldots$

$M(t) = \frac{pe^t}{1 - qe^t}$

$\mu = \frac{1}{p} \;,\; \sigma^2 = \frac{q}{p^2}$

$f(x) = 1 \;;$

(b) $M(t) = \sum e^{tx} \;;\; x = 5$

(c) $M(t) = \sum_{x=1}^{3} e^{tx} \left(\frac{4-x}{6}\right)$

[Expected value = Avg]

* $^nC_x = \frac{n!}{x!\,(n-x)!}$

3

$E(x+y) = 10$

$\Rightarrow E(x) + y = 10$

$\Rightarrow E(x) = 6$

$E[(X+y)^2] = 116$

$\Rightarrow E\{X^2 + 8X + 16\} = 116$

$\Rightarrow E(x^2) + 8E(x) + 16 = 116$

$\Rightarrow E(x^2) = 116 - 16 - 8\times 6$

$= 52$

$\mu = E(x) = 6$

$\sigma^2 = V(x) = E(x^2) - \{E(x)\}^2$

$= 52 - 6^2$

$= 16$

$Z = \frac{x - \bar{x}}{SD} = \frac{X - \mu}{\sigma}$

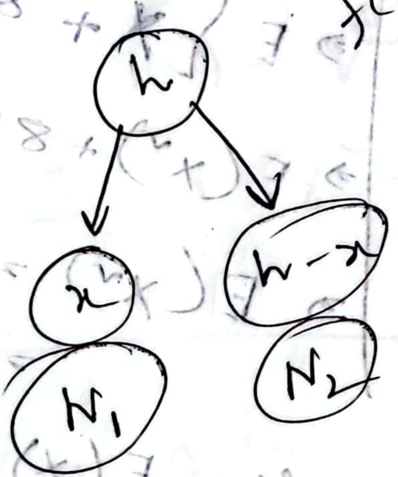
P.T.O

(binomial distribution)

Ex: 8 (H.W)

2.3.17

pmf: $f(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$



$(a+b)^n = {}^{n}C_{0}\, a^{0} b^{n} + {}^{n}C_{1}\, a^{1} b^{n-1} + \dots + {}^{n}C_{n}\, a^{n} b^{0}$

$= \sum_{x=0}^{n} {}^{n}C_{x}\, a^{x} b^{n-x}$

$$\sum_{x=0}^{n} {}^{n}C_x \, p^x q^{n-x}$$

$$= (p+q)^n = 1^n = 1$$

$$M(t) = \sum e^{tx} \, {}^{n}C_x \, p^x q^{n-x}$$

$$= \sum {}^{n}C_x (pe^t)^x q^{n-x}$$

$$= (q + pe^t)^n$$

$$M(0) = (q+p)^n = 1$$

$$\mu = M'(0) = np$$

$$\sigma^2 = M''(0) - \{M'(0)\}^2$$

$$= np(1-p)$$

$$= npq \qquad SD = \sqrt{npq}$$

$b(n, p)$

$b(12, \frac{1}{4})$ → rate of success

↳ trial

$b(n, p) \rightarrow b(n, q)$

↓

compliment

Exm

2.4.7

2.4.8

2.4.9

2.4.10

$n = 1$

$f(x) = {}^{1}C_{x}\, p^{x} q^{1-x}$

$x = 0, 1$

$\mu = p$

$\sigma^2 = pq$

Exercise

~~3~~, ~~4~~, ~~6~~, ~~7~~, ~~8~~, ~~9~~ (H.W), ~~10~~

~~13~~, ~~14~~ (H.W), ~~17~~,

~~19~~, ~~20~~

* name of the distribution
* pmf / pdf
* mean / vaience
* mgf

2.6

Poisson distribution

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, 3, \ldots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$= \sum_{r=0}^{\infty} \frac{x^r}{r!}$$

$$\cancel{\sum_{x \in S} f(x) = \sum_{x=0}^{\infty} \frac{\lambda^x e^{-\lambda}}{x!}}$$

$=$

Uniform (Discrete) distribution

$f(x) = \frac{1}{m}; x = 1,2,3,\ldots m$

$E(x) = \frac{m+2}{2}$

$E(x^2) = \frac{(m+1)(2m+1)}{6}$

$\mu = E(x) = \frac{m+2}{12}$

$\sigma^2 = E(x^2) - \{E(x)\}^2$

$= \frac{m^2-1}{12}$

$\sigma = \sqrt{\frac{m^2-1}{12}}$

$mgf = \sum_{x=1}^{m} \frac{e^{tx}}{m}$

Hypergeometric Distribution

$f(x) = \frac{{}^{N_1}C_x \, {}^{N_2}C_{n-x}}{{}^{N}C_n}; \quad x \le n$

$E(x) = n\left(\frac{N_1}{N}\right)$

$E(x^2) = \frac{n(n-1)N_1(N_1-1)}{N(N-1)} + n\left(\frac{N_1}{N}\right)$

$\mu = E(x)$

$\sigma^2 = n\left(\frac{N_1}{N}\right)\left(\frac{N_2}{N}\right)\left(\frac{N-n}{N-1}\right)$

$mgf = M(t) = \sum_{x \in S} e^{tx} f(x)$

Geometric Distribution $[p+q=1]$

$f(x) = q^{x-1} p \; ; \; x = 1,2,3,\ldots$

$\mu = E(x) = \frac{1}{p}$

Mgf $= M(t) = \sum_{x=1}^{\infty} e^{tx} f(x)$

$= \frac{pe^t}{1-qe^t}$

$M'(t) = \frac{pe^t}{(1-qe^t)^2}$

$M''(t) = \frac{pe^t(1+qe^t)}{(1-qe^t)^3}$

$\sigma^2 = M''(0) - \{M'(0)\}^2$

$= \frac{q}{p^2}$

Binomial distribution

$f(x) = {}^{n}C_x \, p^x q^{n-x} \quad x = 0,1,2\ldots n$

$\mu = E(x) = np$

mgf, $M(t) = (q+pe^t)^n$

$M'(t) = n(q+pe^t)^{n-1} pe^t$

$M''(t) = n(n-1)(q+pe^t)^{n-2}(pe^t)^2$

$+ n(q+pe^t)^{n-1} pe^t$

$\sigma^2 = npq$

C.W 27.4.24

Stat

Poisson distribution

resource, $n \to \infty$
probability of occurrence $p \to 0$
$\Bigg\}$ poisson mean, constant... $= \lambda$

$\lambda = np > 0$

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!} \; ; \; x = 0, 1, 2, 3 \ldots$$

$\mu = \lambda$, $\sigma^2 = \lambda$,

$M(t) = e^{\lambda(e^t - 1)}$

$\lambda = 5$, $f(x) = \dfrac{e^{-5} 5^x}{x!}$

$p(x \leq 6) = 1$ ~~$1 - P(\ldots)$~~

1, 2, 3, 4 (Exm)

$P(x>10) = 1 - P(x \leq 10)$

3.1

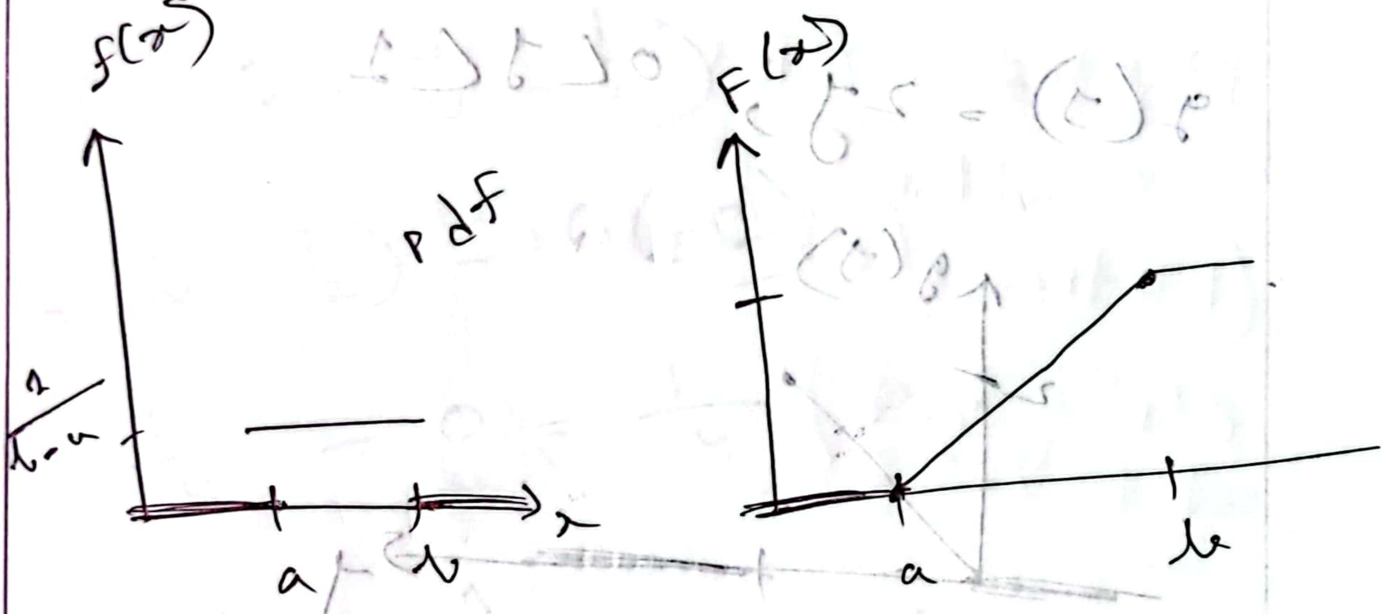
$$f(x) = \frac{1}{b-a} \; ; \; a \leq x \leq b$$

$$F(x) = \begin{cases} 0 \; ; & x < a \\ \int_a^x \frac{1}{b-a} dx = \frac{x-a}{b-a} \; ; & a \leq x \leq b \\ 1 \; ; & x > b \end{cases}$$

$$E(x) = \int_a^b x \frac{1}{b-a} dx = \frac{a+b}{2}$$

$$E(x^2) = \int_a^b x^2 \frac{1}{b-a} dx = \frac{a^2+ab+b^2}{3}$$

$\mu = E(x) \quad \sigma^2 = \frac{(b-a)^2}{12}$



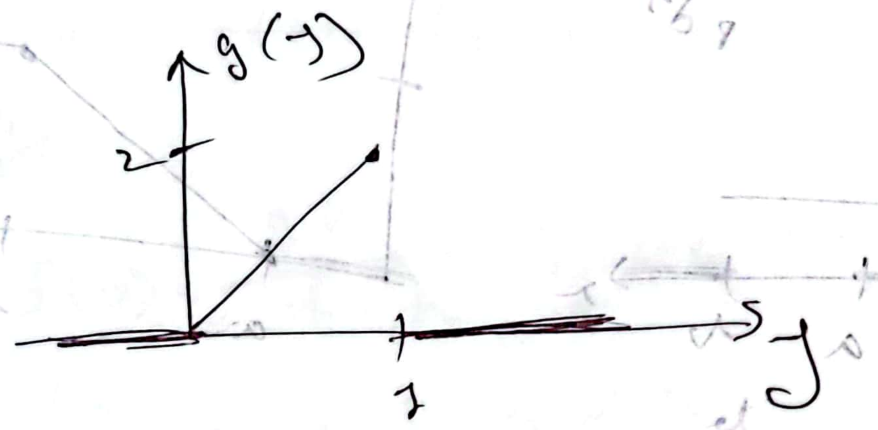
$$Mgf = \int_a^b e^{tx} \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a} \left. \frac{e^{tx}}{t} \right|_a^b$$

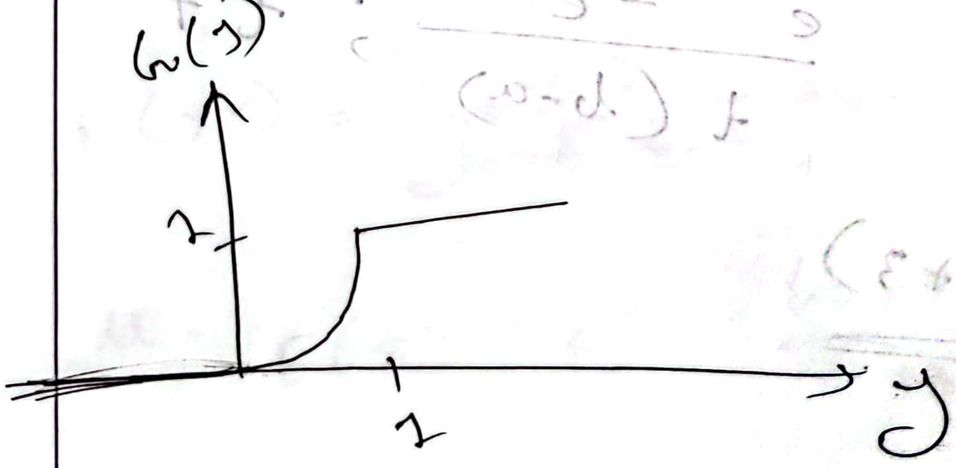
$$= \frac{e^{bt} - e^{at}}{t(b-a)} \; ; \; t \neq 0$$

Exm (1+3)

$g(y) = 2y \, ; \quad 0 < y < 1$



$$G(y) = \begin{cases} 0 \, ; & y < 0 \\ \int_0^y 2w\,dw = y^2 & 0 \le y \le 1 \\ 1 \, ; & y \ge 1 \end{cases}$$



$$P\left(-1 < y < \frac{1}{2}\right)$$

$$= \omega\left(\frac{1}{2}\right) - \omega(-1)$$

$$= \frac{1}{4} - 0 = \frac{1}{4}$$

$$E(Y) = \int_0^1 y\,(2y)\,dy$$

$$E(Y^2) = \int_0^1 y^2\,(2y)\,dy$$

$$M(t) = \int_0^1 e^{ty}\,2y\,dy$$

Monday

9.00 PM

(Extra class Online)

C.T {3.7}

* Continuos ~~F~~ probability from C.D.F

Exm 3.1.5

(16, 18) H.W

* Bigger time intersection of smaller time & subset of

" " .

Normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$N(\mu, \sigma^2)$

$$M(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

$\phi \rightarrow$ CDF

$M(0) = e^0 = 1$

$Z = \frac{x-\mu}{\sigma}$, $N_Z(0, 1)$ (from table)

$Z = a.bc$

$a.b \rightarrow 0.0c$

$\phi(-Z) = 1 - \phi(Z)$

2 →

3 →

$\cancel{3.3.2}$

$\cancel{3.3.2}$

$x = \mu + Z_1\sigma$

$\cancel{N(0,1)} \rightarrow$ (Z converted)

C.W

6.5.24

Exm 3.3.4 (H.W)

3.3.5 ,,

$$\boxed{\begin{array}{l} Z_\alpha \\ P(Z \le Z_\alpha) = \alpha \end{array}}$$

6.

$N(3, 16)$

$\mu = 3, \quad \sigma = 4$

$P(4 \le x \le 8)$

$= P\left(\frac{4-3}{4} \le \frac{x-3}{4} \le \frac{8-3}{4}\right)$

$= P(0.25 \le Z \le 1.25)$

$= \phi(1.25) - \phi(0.25)$

Exercise

CW

Probability

3/

a) $P(|Z| \le c) = 0.95$

$\Rightarrow 2\Phi(c) - 1 = 0.95$

$\Rightarrow \Phi(c) = 0.975$

$\Rightarrow c = 1.64$

Type I error

$P(E_1) \rightarrow \alpha \rightarrow$ risk factor

↳ significance

Type II error

$P(E_2) = \beta$ weakness

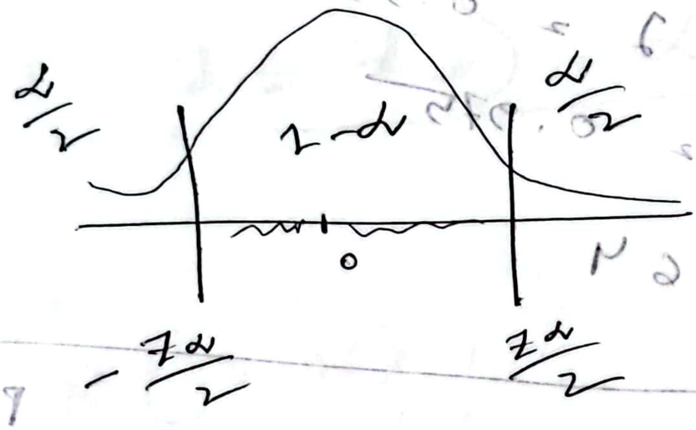
$\boxed{1-\beta \rightarrow \text{power of test}}$

P.T.

sustainable

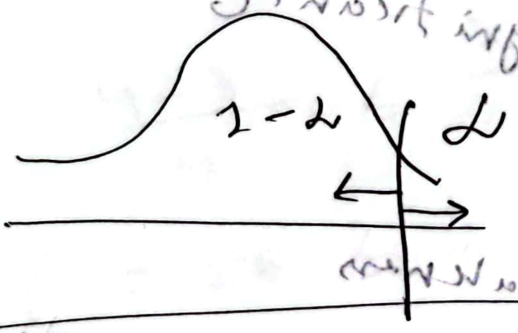
p-value

$\mu = \mu_0$



$$P\left(-Z_{\frac{\alpha}{2}} < Z < Z_{\frac{\alpha}{2}}\right) = 1-\alpha$$

$\mu > \mu_0$



$$\boxed{P(Z > Z_\alpha) = \alpha}$$

$$P(Z \le Z_\alpha) = 1-\alpha$$

$\mu < \mu_0$

$Z = \dfrac{x-\mu}{\sigma}$, $\dfrac{x-np}{\sqrt{npq}}$

(5)

Hc: $\boxed{P(Z > -Z_\alpha) = 1-\alpha}$ (C)

$P(Z < -Z_\alpha) = \alpha$