

Friction Force

When surface of two bodies are in contact, they develop a force at the plane of their contact which opposes their relative motion. This opposition force is known as friction force.

Characteristics:

- ① It opposes the direction of motion.
- ② It depends on the nature of the surfaces in contact.
- ③ It depends on the weight of the obj.
- ④ It is independent of the area of contact.

Dependency:

- ① Nature of surface.
- ② Degree of smoothness & roughness.
- ③ Temperature.
- ④ Increase/ Decrease in the presence of foreign materials.

- Two main types of friction force.

① static friction: This type of friction prevents the initiation of motion between two surfaces in contact. It acts when an external force tries to move an obj but hasn't yet overcome the frictional force.

② kinetic friction: kinetic friction occurs when two surfaces are in relative motion, between two surfaces in contact. It opposes the movement between the surfaces and acts as the resistance to sliding motion.

• Why don't object's move till the appropriate/optimum amount of force is applied?

⇒ Objects don't start moving instantly when force is applied due to presence of friction. Friction is a resistive force that opposes the motion of objects when they are in contact with each other. It arises from the interactions between the surfaces of two objects and is influenced by factors.

When an external force is applied to an obj; it needs to overcome the force of static friction before the obj starts moving. Static friction is the frictional force that prevents the initial motion of an object at rest.

Until the applied force exceeds the force of static friction the object remains stationary.

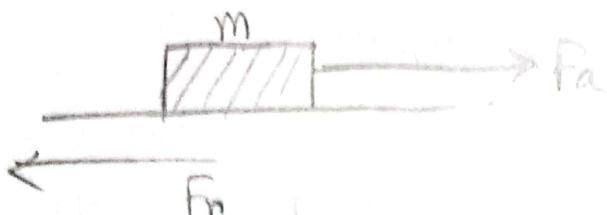
$$W = mg$$

$$D \times F = 81.2 \leftarrow$$

$$W \times g = 81.2 \leftarrow$$

$$(3.8 \times 10) \times 10 = 38 \leftarrow$$

$$W \times g =$$



$$N = mg$$

$$F_{\text{net}} = Fa - F_f$$

$$F_k \propto N$$

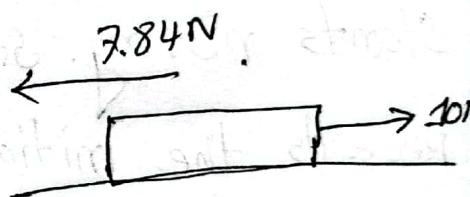
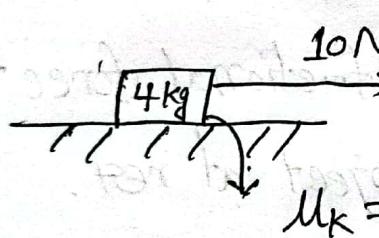
$$F_s \propto N$$

$$F_k = \mu_k N$$

↳ Coefficient of
kinetic friction

$$F_s = \mu_s N$$

↳ Coefficient
of static friction.



① Find friction

$$\Rightarrow F_k = \mu_k \times (4 \times 9.8)$$

$$= 7.84 N$$

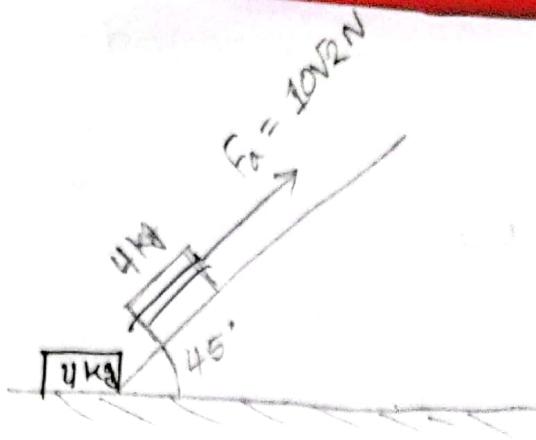
② a of the box?

$$F_{\text{net}} = ma$$

$$\Rightarrow 2.16 = 4 \times a$$

$$\Rightarrow a = \frac{2.16}{4} = 0.54 \text{ m s}^{-2}$$

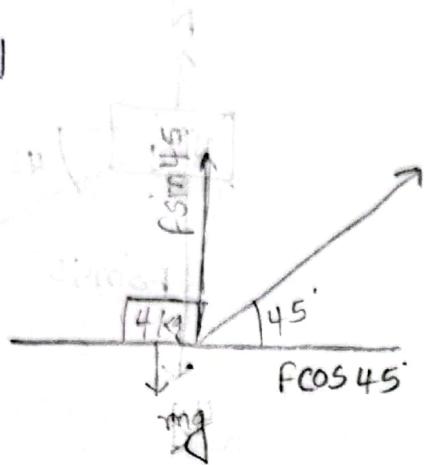
(A)



$$\mu_k = 0.3$$

(force applied with 45°)

① Find N



$$N + F \sin 45^\circ = mg = 4g$$

$$\Rightarrow N = 4g - 70$$

$$\approx 30N$$

$$N \alpha =$$

② find \bar{a} ?

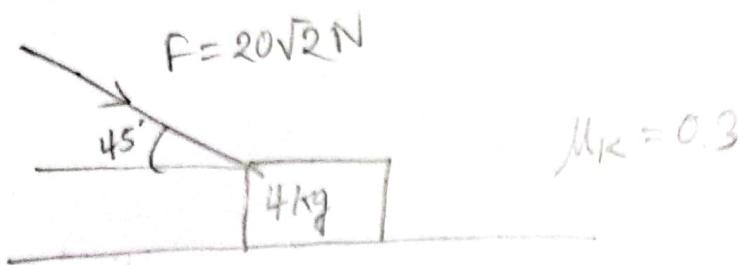
$$9 - F \cos 45^\circ = 4a$$

$$\Rightarrow a = \frac{1}{4} = 0.25 \text{ m s}^{-2}$$

① find f_k ?

$$\Rightarrow f_k = \mu_k N = 0.3 \times 30$$

$$\Rightarrow f_k = 0.3 \times 30 \\ = 9N$$

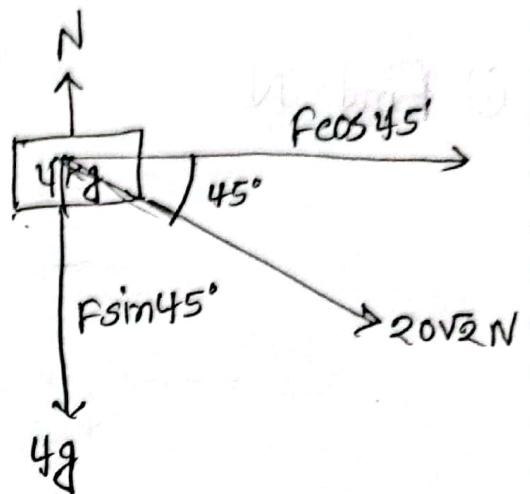


① Find N ?

$$\Rightarrow N + f \sin 45^\circ = 4g$$

$$\Rightarrow N =$$

$$\begin{aligned}\Rightarrow N &= 4g + f \sin 45^\circ \\ &= 40 + 20 \text{ N} \\ &= 60 \text{ N}\end{aligned}$$



$$② \text{ if } f_k = \mu_k \times N$$

$$= 0.3 \times 60 = 18 \text{ N}$$

$$③ F \cos 45^\circ - 18 = ma$$

$$\Rightarrow a = 0.5 \text{ m s}^{-2}$$

Static friction

① Tendency of relative motion.

② Variable \rightarrow self Adjusting.

③ $0 \leq f_s \leq F_{\text{limiting}}$

$$[f_s \propto N]$$

$$F_{\text{limiting}} = \mu_s N$$

*

\ddot{x} ?



[Body moves only if $F_{\text{applied}} > \text{Max static friction}$]

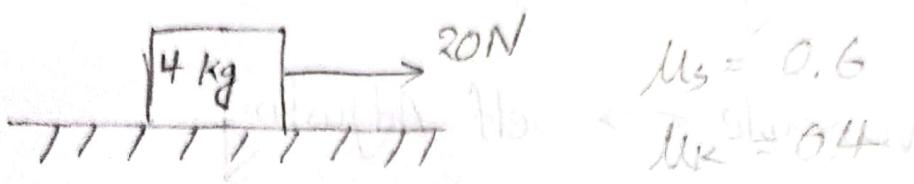
$$\Rightarrow F_{\text{limiting}} = \mu_s N$$

$$= 0.2 \times 4 \times 10$$

$$= 8N$$

So the object will ~~statistic~~ motion.

- find friction force & a.



$$\Rightarrow f_{\text{limiting}} = \mu_s \times 40 = 24 N$$

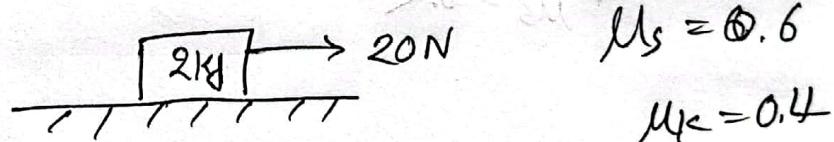
so the obj will ^{not} have a motion.

$$F_n = \mu_k \times 40 = 16 N$$

body at rest.

$$\text{So, } 20 - 16$$

- find friction force & a,



$$\Rightarrow f_{\text{limiting}} = \mu_s \times 20 = 12 N$$

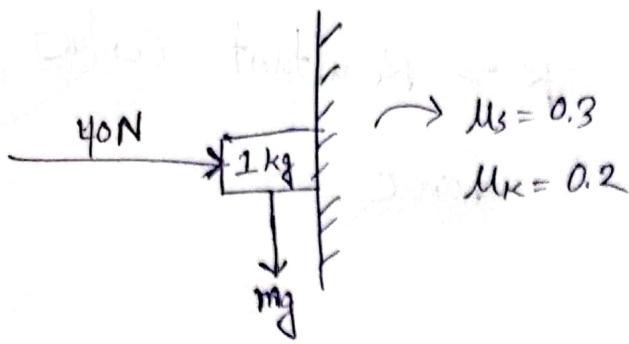
so the body will have a motion.

$$f_n = \mu_k \times 20 = 8 N$$

$$\text{So, } 20 - 8 = ma$$

$$\therefore a = \frac{12}{2} = 6 \text{ m/s}^2$$

• Find f_r and a .



$$\Rightarrow N = 40\text{ N}$$

$$F_{\text{limiting}} = 0.3 \times 40 = 12\text{ N}$$

~~$$f_r = 0.2 \times 40 = 20\text{ N}$$~~

applied forcee = 10N

(18)

$F_{\text{limiting}} > F_{\text{applied}}$
[So, body will ^{not} have
a certain motion]

50.

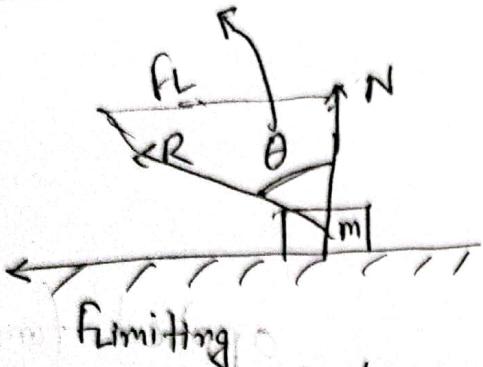
$$20 - 12 = ma$$

$$\Rightarrow a = 8\text{ ms}^{-2}$$

$$20 - \frac{m}{r} \cdot \frac{v^2}{r} = \frac{m}{r} \cdot a = 8\text{ ms}^{-2}$$

$$20 - \frac{m}{r} v^2 = 8\text{ ms}^{-2}$$

• Angle of friction:



$R \rightarrow$ Resultant Contact
Force

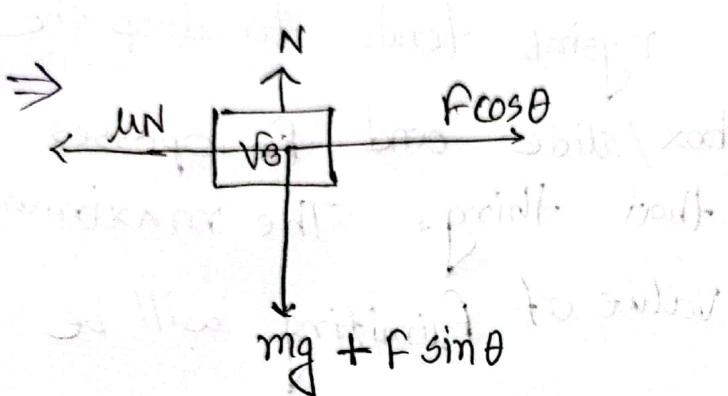
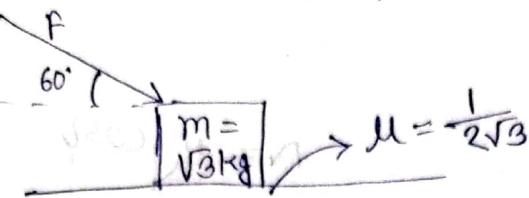
It is the angle between Resultant contact force and Normal force.

→ Tangential Component of Resultant contact force is known as frictional force.

$$\tan \theta = \frac{f_r}{N} = \frac{\mu_s N}{N} = \mu_s$$

$$\therefore \boxed{\theta = \tan^{-1} \mu_s}$$

o find Max value of F such that block doesn't move.

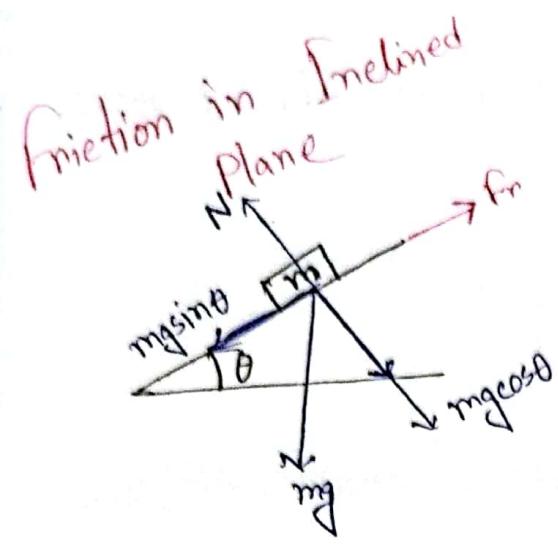


$$\text{so, } N = \sqrt{3}g + \frac{F\sqrt{3}}{2}$$

$$\Rightarrow F \cos 60^\circ = \mu N$$

$$\Rightarrow \frac{F}{2} = \frac{1}{2\sqrt{3}} \left(\sqrt{3}g + \frac{F\sqrt{3}}{2} \right)$$

$$\Rightarrow F_{\max} = 20 \text{ N}$$



$$N = mg \cos \theta$$

$mgsin\theta$ tends to drop the box/slide and f_r opposes that thing. The maximum value of f_r limiting will be

$$F_L = \mu_s N$$

$$F_L = \mu_s mg \cos \theta$$

Angle of Repose θ_R so if the value of $mgsin\theta > \mu_s mgcos\theta$ then the box will drop.

Just about to slip

$$mgsin\theta_R = \mu_s mgcos\theta_R$$

Possible Questions:

- ① Max θ at which Block stays at Rest.
- ② θ at which block is just about to slide.

Angle of Repose $mg \sin \theta_R = \mu_s mg \cos \theta_R$

$$\Rightarrow \theta_R \boxed{\mu_s = \tan \theta_R}$$

Given $\theta < \theta_R$

No sliding

Rest

Static friction = Applied force

$$\theta = \theta_R$$



Just about to
slide

Limiting friction

$$F_L = \mu_s N$$

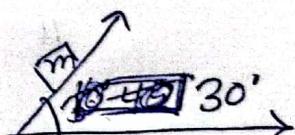
$$\theta > \theta_R$$

Sliding starts
Motion.

Kinetic friction

$$F_K = \mu_k N$$

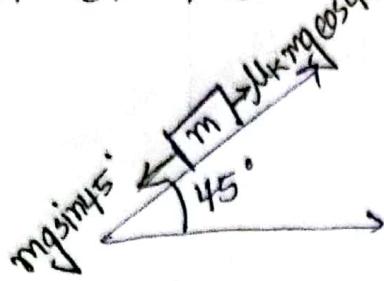
Q5: If angle of repose is 45° . Find if body is at rest or motion. Calculate friction force and \ddot{a} .



→ No sliding

Rest

Q2: If angle of repose is 37° . Find if body is at rest or motion. Calculate friction force and α



$$\theta_R = 37^\circ$$

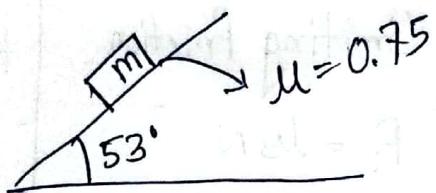
$$\theta > \theta_R$$

Body will slide.

$$\rightarrow mg \sin 45^\circ - \mu_k mg \cos 45^\circ = ma$$

$$\Rightarrow a = g \sin 45^\circ - g \cos 45^\circ \cdot \mu_k$$

Q3:



① Find Rest/motion

② Friction force?

③ a ?

Hence, $N = mg \cos 53^\circ$

and the force applied downward is $mg \sin 45^\circ = 7.99m$

∴ friction force is $\mu mg \cos 45^\circ = 6.14$

So, the obj will have a motion.

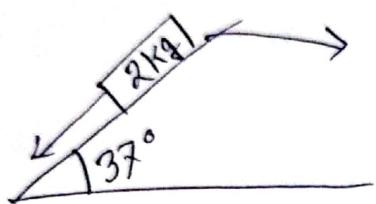
$$\rightarrow f_r = \mu_k mg \cos 53^\circ$$

$$= 6.14 m$$

$$\rightarrow mg \sin 53^\circ - \mu_k mg \cos 53^\circ = ma$$

$$\begin{aligned}\rightarrow a &= g \sin 53^\circ - 0.75 \times g \times \cos 53^\circ \\ &= 9.81 - 6.14 \\ &= 1.85 \text{ m s}^{-2}\end{aligned}$$

(*)



$$\begin{array}{l} \mu_s = 0.4 \\ \mu_k = 0.3 \end{array}$$

- ① Rest/Motion
- ② Friction force.

$$\Rightarrow N = mg \cos 37^\circ = 15.97$$

$$f_s = \mu_s \times N = 0.4 \times 15.97 = 6.39$$

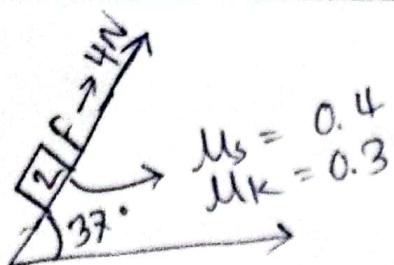
$$f_a = mg \sin \theta = 12.03$$

So it will have a motion.

$$\begin{aligned}\rightarrow f_k &= \mu_k N \\ &= 0.3 \times 15.97 \\ &= 4.791\end{aligned}$$

$$\rightarrow a = \frac{12.03 - 4.791}{12} = 3.6 \text{ m s}^{-2}$$

Q)



- find frictional force
- find \bar{a} .

$$\Rightarrow N = 2 \times 10 \times \cos 37^\circ$$

$$\text{and } f_s = \mu_s \times N$$

$$= 0.4 \times 2 \times 10 \times \frac{4}{5}$$

$$= 6.4 \text{ N}$$

$$f_a = 2g \sin 37^\circ = 12 \text{ N}$$

$$F_{\text{net}} = 12 - 4 = 8 \text{ N}$$

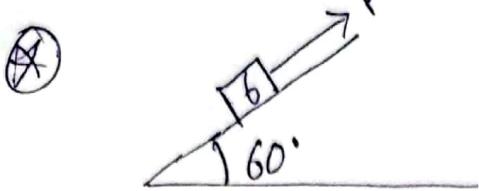
$f_k < F_{\text{net}}$ so, it will have motion,

$$f_k = 0.3 \times 2 \times 10 \times \cos 37^\circ$$

$$= 4.79 \text{ N}$$

$$a \rightarrow 8 - 4.79 = 2a$$

$$\Rightarrow a = \frac{8 - 4.8}{2} = 1.6 \text{ m s}^{-2}$$



$$\mu_s = 0.6$$

$$\mu_k = 0.4$$

$$F = ?$$

① Block stationary:

- ① move downward with constant velocity.
- ② move upward with $a = 4 \text{ ms}^{-2}$.

③ For,

$$F_L = \mu_s N = 0.6 \times 6 \times 10 \times \cos 60^\circ = 18 \text{ N}$$

$$F_a = 6g \sin 60^\circ = 52 \text{ N}$$

so, we should apply $(52 - 18) \text{ N}$ for stationary phase.

(ii) $f_k = \mu_k N = 0.4 \times 69 \cos 60^\circ$
 $= 12 \text{ N}$

$\boxed{a=0} \rightarrow \text{constant velocity.}$

$F = (52 - 12) = 40 \text{ N}$

(iii) $F - 52 - 12 = 6a$

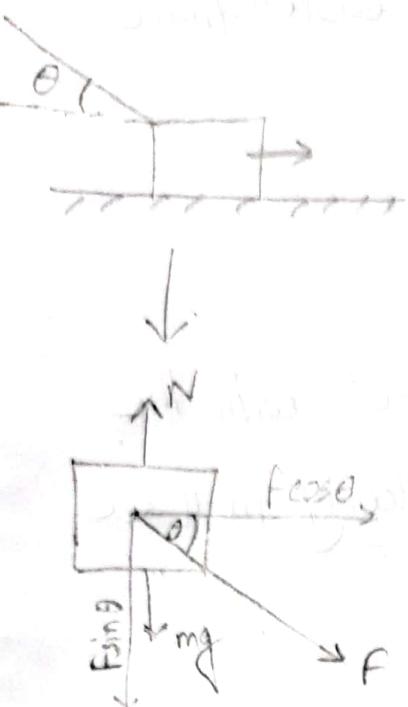
$\Rightarrow F = 24 + 64 = 88 \text{ N}$

$A = 0.2 \times 0.1 \times 0.8 \times 0.8 = 0.0128 \text{ m}^2$

$N = 88 \times 0.0128 = 1.12 \text{ kN}$

or $N = (88 - 52)$
= 36 N

Pull - Push



$$N = mg + f_{\text{parallel}} \sin \theta$$

and for horizontal

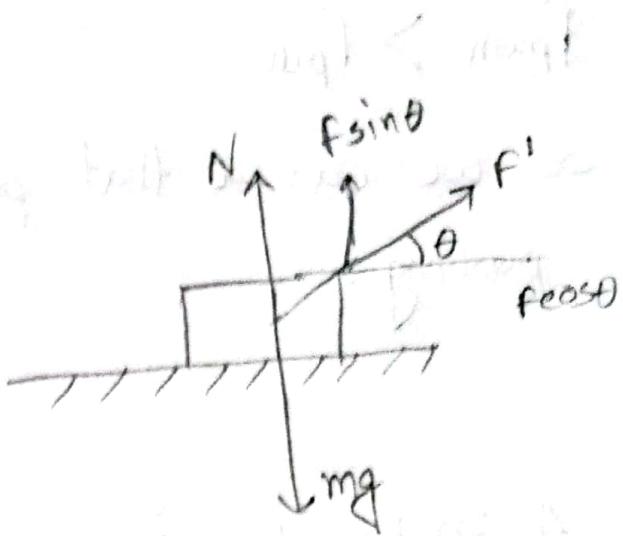
$$f_{\text{parallel}} = N \times \mu_s \text{ at } \text{POT} = \cos \theta + \sin \theta$$

$$\Rightarrow f_{\text{parallel}} = (mg + f_{\text{parallel}} \sin \theta) \times \mu_s$$

$$\Rightarrow f_{\text{parallel}} = \mu_s mg + \mu_s f_{\text{parallel}} \sin \theta$$

$$\Rightarrow f(\cos \theta - \mu_s \sin \theta) = \mu_s mg$$

$$\Rightarrow F = \frac{\mu_s mg}{\cos \theta - \mu_s \sin \theta}$$



$$\text{Hence, } N + f_{\text{parallel}} \sin \theta = mg$$

$$\therefore N = mg - f_{\text{parallel}} \sin \theta$$

and for horizontal

$$f_{\text{parallel}} = \mu_s N$$

$$\Rightarrow f_{\text{parallel}} = \mu_s \times (mg - f_{\text{parallel}} \sin \theta)$$

$$\Rightarrow f(\cos \theta + \mu_s \sin \theta) = \mu_s mg$$

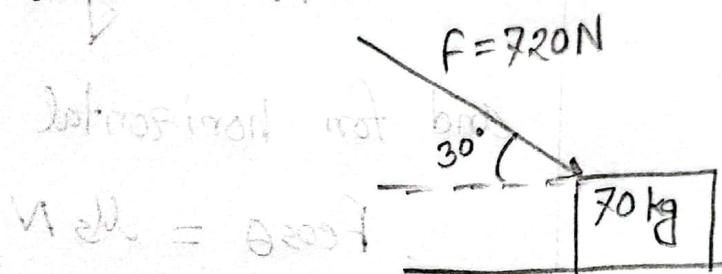
$$\Rightarrow F = \frac{\mu_s mg}{\cos \theta + \mu_s \sin \theta}$$

$$F_{\text{push}} > F_{\text{pull}}$$

so we can see that pulling an object is easier than pushing.

- Q) A 70 kg box is pushed by 720N at 30° with the ground. If the box has gained 16 m s^{-1} velocity in 4 sec

find μ .



$$(m(g - \mu g)) \times a = 686 \text{ N}$$

$$\Rightarrow N = mg + f \sin \theta = 686 + 360 = 1046 \text{ N}$$

$$V = u + at \quad \cancel{a = \frac{V-U}{t}}$$
$$\Rightarrow a = \frac{V-U}{t} = 4 \text{ m s}^{-2}$$

And now,

$$F_k = \mu \times N$$
$$= \mu \times 1046$$

We can write

$$f \cos 30 - \mu 1046 = 70 \times 4$$

$$\Rightarrow 623.54 - \mu 1046 = 280$$

$$\Rightarrow \mu 1046 = 343.54$$

$$\Rightarrow \mu = \frac{343.54}{1046} = 0.33$$

Soln:

$$\begin{aligned} f = ma &= 70 \times \left(\frac{v-u}{t}\right) \\ &= 70 \times 4 \\ &= 280 N \end{aligned}$$

$$f_r = (720 - 280) N$$

$$= 440 N$$

Now,

$$f_r = \mu (mg + f \sin \theta)$$

$$\Rightarrow \mu = \frac{f_r}{mg + f \sin \theta} = \frac{440}{70 \times 9.8 + 720 \sin 30} = 0.42$$

(Ans)