Motion

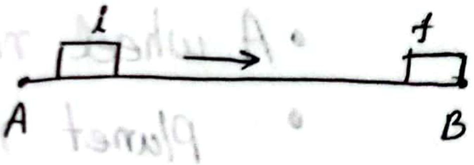
Motion is the change of position of an objeet respeet of surrounding overtime is called motion.

Types of Motion:

Translational Motion: If all the particles of an objeet travell in the "same distance" at the same time in a particular dirreetion then it's motion is called Translational Motion.

Translational Motion

→ **Reetilinear Motion:** The motion of an obj in which its center of mass moves in straight line.



→ **Curvilinear Motion:** The motion of an obj in which its center of mass moves in a carvalinear path.

→ **Circular Translational Motion:** sattelite's movement surround the plannet.

Mechanics: Mechanics is a branch of physics concern with the static body and it's motion.

- **Static Body:** Objects are at rest or moving with constant motion.
- **Dynamic Body:** Objects are moving with acelaration and deacceleration.

Dynamic

- **Kinetic:** Deals with motion as well as its cause of motion. It consider mass and speed. Uses mathematical expression to describe the relationship between forces and motion. More complex.
- **Kinematic:** Deals with only geomatricall aspects of motion. Don't consider mass, speed or cause of motion. Uses geometric concept to describe the motion of objects. Less complex.

Oscillatory Motion: This is a type of motion in which an object moves back and forth or up & down about a fixed point. The motion repeats itself over and over again and the object passes through its equilibrium position twice in each cycle.

- Beating of Human Heart.
- The swinging of a pendulum.
- Vibration of a spring.

Rotational Motion: This is the motion of an object around an axis.

- A wheel rotating on an axis.
- Planet rotating around the sun.

Random Motion: This is a type of motion in which the direction and speed of an obj change continiously and unpredictably. It is also called Brownian motion or zig zag motion.

- Pollen grains in water.
- Smoke
- Dust in air.

Circlular TM	Rotational Motion.
Object moves along a circular path without rotating around its own axis.	Object rotates around its own axis, while remaining relatively fixed in space.
No rotation around a specific Axis.	Rotates around a fixed axis.
Car moving along a Curved road, ~~changing~~ person running in a circular track.	Spinning top, rotating fan, Earth rotating on it's own axis.

Frame of Referenee: A frame of Referenee is a system of coordinates that is used to describe the position and motion of objects.

- There are two main types of frame of referenee.

Inertial frame of reference: This frame of referenee ~~is a~~ indicates those object which have no aceelaration it means the objects may be in constant velocity or static.

Non Inertail frame of referenee: This frame of referenee indicates those objects whose are in aceelaration or deaceilaration mode

Examples of IF.:

1. ~~Dr~~ A spaceship that is travelling in space at a constant velocity.
2. A point far away from any massive objects, such as star or galaxy.

Examples of NIF:

(i) A car that is accelarating.

(ii) A person sitting on a chair that is rotating.

Quantities Related Motion:

(i) Distance

(ii) Displacement

(iii) Speed

(iv) Velocity
- → Uniform Velocity.
- → Non-uniform velocity.
- → Relative velocity.
- → Instantenous velocity.
- → Non-Instantenous velocity.

(v) Accelaration.

Uniform Velocity:

(i) The velocity of the object is constant. This means that the object is travelling at the same speed in the same direction through its motion.

(ii) The accelaration of the object is zero. which means that the object is not speeding up, down, or changing it's direction.

(iii) The obj covers equal distance in equal intervals of time. It means if the obj travells 10 m in the first second. ~~the~~ in the second sec. it should travell 10 m.

(iv) The velocity - time graph of the obj is horizontal line.

Example:

(i) A car travelling at a constant speed on a straight road.

(ii) A satellite orbiting the earth at a constant speed.

Instantaneous Velocity: (তাৎক্ষণিক)

- This is the velocity of an object at an exact moment in time. In simple terms, it is the velocity at a particular point in time during the object's motion.

Example: When you throw a ball upward vertically then after travelling a certain distance at the peak point it will stop for a moment (when the velocity is 0) then it will travells downward from the peak location.

Non-Intantaneous Velocity: Non-instantaneous velocity refers to the avarage velocity of an object over a specific duration of time. It considers the total displacement of object during that time interval.

Example: A car travells 150 miles in 3 hours.

the non-Intantaneous velocity would be 50 mile/Hr.

Instantaneous velocity $v = \lim_{\Delta t \to 0} \frac{(\Delta r)}{(\Delta t)}$

Here Δr represents the chang in position
and Δt " " " in time.

This limit defination allows us to calculate the exact velocity of the object at any specific time.

Unifom Accelanation: It occurs when an object's velocity changes at a constant rate.

- Consider a car that accelarates from a standstill at a rate of $2m/s^2$.

Non-unifor Accelanation: Refer's to an object's velocity changing at an uneven rate.

- Rocket launching into space.

Relative Accelaration: This is the accelaration of one obj with respect to another moving obj.

- Consider two cyclist at the constant velocity of 10 km/h if one of them increase it's velocity to 15 km/h then the relative Accelanation will be 5 Km/hr unit time.

Instantaneous Accelanation

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}.$$

where "Δv" represents the change in velocity of the object over a small time interval "Δt".

Newton's Law of Inertia

First Law: (Law of Inertia): An object at rest will remain at rest, and an object in motion will continue moving with a constant velocity, unless external force applied on the object.

Or,

Objects tend to keep doing what they are already doing unless a force acts upon them.

* Why Newtown's 1st law called Law of Inertia?

→ The first Law of motion is also known as Law of inertia. Inertia is the tendency of an object to resist change in its motion. So, ~~m~~ the more inertia of an object has, the more force it takes to change its motion. For example, it takes more force to stop a bowling ball than it does to stop a ping pong ball.

Second Law [Law of Acceleration]: The rate of change of momentum of an object is directly propotional to the force applied and take's place in the direction in which the force is applied.

Third Law: For every action there is an equal and opposite reaction.

*** Derive 1st Law from the 2nd Law of motion.

→ As we know from 2nd Law of motion

$$\frac{dp}{dt} \propto f$$

$$\frac{dmv}{dt} \propto f$$

Const ← $m\left(\frac{dv}{dt}\right) = kf$ [$\because k=1$]

$$\frac{dv}{dt} = 0 \quad [\text{External force } 0]$$

so, $dv = 0$ [$dt \neq 0$]

$\therefore$ so ~~velocity~~ the change of velocity is 0. So we can say the obj in move with fixed ~~a~~ velocity or this is in ~~non mobi~~ rest.

* Derive Newton's third law of motion from his 2nd law of motion.

Newton's 2nd law, $\vec{F} = \frac{d\vec{p}}{dt}$

Newton's third Law, $\vec{F}_{BA} = -\vec{F}_{AB}$

2nd law → $\bar{F}_{BA} = \frac{\Delta P_1}{t}$ —(i) & $-\bar{F}_{AB} = \frac{\Delta P_2}{t}$ —(ii)

$$\bar{F}_{BA} + (-\bar{F}_{AB}) = \frac{\Delta P_1}{t} + \frac{\Delta P_2}{t}$$

$$\rightarrow \bar{F}_{BA} + \bar{F}_{AB} = \frac{1}{t}(\Delta P_1 + \Delta P_2)$$

$$\rightarrow \bar{F}_{BA} + \bar{F}_{AB} = 0$$

$$\rightarrow \bar{F}_{BA} = \bar{F}_{AB}$$

[We know that no external force is applied. Thus, momentum chang will be 0 because no change in velocity occurs]

[Proved]

Problem 1: From rest, a train starts moving with constant interaction $10 ms^{-2}$ at the same time, a car starts running parallel to the train with uniform velocity $100 ms^{-1}$. When and where the train will cross the car?

→ For train,

$u = 0$

$a = 10 ms^{-2}$

Now,

$S = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \cdot 10 \cdot t^2$

$\rightarrow S = 5t^2$ —— (1)

For car,

$V = 100 ms^{-1}$

Now,

$S = Vt = 100t$ —— (II)

Now combin (1) & 2

$5t^2 = 100t$

$\Rightarrow t^2 - 20t = 0$

$\Rightarrow t(t - 20) = 0$

$\Rightarrow t = 20$

∴ S for meet the car and train is (100×20)

$= 2000 \text{ m}$

Problem 2: A car traverse 30 m in first two sec. and 150 m in next 4 sec. If the accelaration of the car remains unchanged, then calculate the distance it will travell in the next one second.

→ For the first 2 seconds,

$$30 = 2u + \frac{1}{2} a \cdot 2^2$$

$$\Rightarrow 30 = 2u + 2a$$

$$\Rightarrow u + a = 15 \quad \text{—— (i)}$$

For the next 4 sec,

$$180 = 6u + \frac{1}{2} \cdot a \cdot 36$$

$$\Rightarrow 30 = u + 3a$$

$$\Rightarrow u + 3a = 30 \quad \text{—— (ii)}$$

Substract i from ii

$$\begin{aligned} u + 3a &= 30 \\ u + a &= 15 \\ \hline 2a &= 15 \end{aligned}$$

$$\therefore a = 7.5 \quad \& \quad u = 7.5$$

S in next 1 sec, means s in 7th second,

$$S_{7th} = u + a(n - 1/2)$$

$$= 7.5 + 7.5(7 - 1/2)$$

$$= 7.5 + 7.5 \times 6.5$$

$$= 56.25$$

(Ans)

Problem 3: A car traverse 30m in first two second and 150 m in next 4 sec. If the accelaration of the car remains unchanged, then calculate the distance it will travell in the nex ~~one~~ two second.

→ From problem 2, $a = 7.5$ & $u = 7.5$

so after 2 sec it will 8 sec. so $t = 8$

Now, $s = \frac{1}{2}at^2 + ut$

$= \frac{1}{2} \times 7.5 \times (8)^2 + 7.5 \times 8$

$= 300\ m$

The car travells $(30 + 150m) = 180m$ in the first 6 second. So, for the next 2 second it will travell $(300 - 180)m = 120\ m$

Principle of Conservation of Linear Momentum

- The vector sum of the linear momentum of all the particles in an system remain constant in the absence of any external forces

- A 1500 kg car moving with $100\ ms^{-1}$ collides with another car of 2500 kg at rest they couple upon collision. At what veloeity they move away?

$\rightarrow$

$$m_1 u_1 + m_2 u_2 = m_1 v + m_2 v$$

$$\Rightarrow v = \frac{m_1 u_1 + m_2 u_2}{(m_1 + m_2)}$$

$$= \frac{1500 \times 100 + 2500 \times 0}{(2500 + 1500)}$$

$$= \frac{150000}{4000}$$

$$= 37.5\ ms^{-1}$$

For Car 1

$m_1 = 1500\,kg$:

$u_1 = 100 ms^{-1}$

For Car 2

$m_2 = 2500\,kg$

$u_2 = 0$

Question: A 100g mass of bullet is shot from a gun of mass 2.5 kg at a velocity of 350 ms^{-1}. Find the recoil velocity of the gun.

~~→ We know that,~~

$\cancel{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$

$\cancel{\Rightarrow (0.1 \times 350) + (2.5 \times 0) = (0.1 \times 0) + (2.5 \times v_2)}$

$\cancel{\Rightarrow 35 = 2.5 v_2}$

$\cancel{\Rightarrow v_2 = 35/2.5 = 14\ ms^{-1}}$

~~(Ans)~~

~~for, bullet,~~

$\cancel{m_1 = 0.1\ kg}$

$\cancel{u_1 = 350\ ms^{-1}}$

$\cancel{u_1 = 0}$

~~for gun~~

$\cancel{m_2 = 2.5\ kg}$

$\cancel{u_2 = 0}$

$\cancel{v_2 = ?}$

⇒ We know that,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow (0.1 \times 0) + (2.5 \times 0) = 35 + 2.5 v_2$

$\Rightarrow 2.5 v_2 = 35$

$\Rightarrow v_2 = 35/2.5 = 14\ ms^{-1}$

for bullet,

$m_1 = 0.1$ kg

$u_1 = 0\ ms^{-1}$

$v_1 = 350$ m/s

for gun,

$m_2 = 2.5$ kg

$u_2 = 0$ m/s

$v_2 = ?$

Projectile Motion

Q1: what is projectile?

Q2: What is projetile motion?

Q3: What is the trajectory of a projectile? Draw necessary figure.

Q4: What are the assumption of projectile motion.

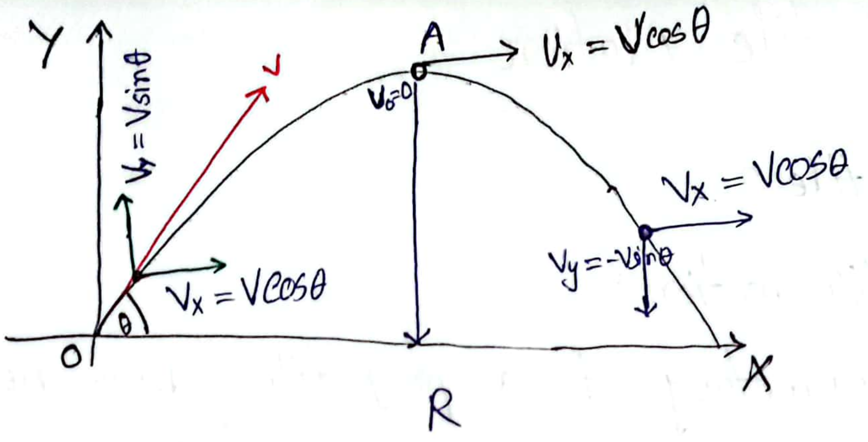
Answer of Q1:& 02

A body is projected with an initial at an angle with the horizontal in the vertical plane of the earth is called projectile. And the motion of it's known as projectile motion.

Answer3: The path traversed by a projectile is called it's trajectory.

Answer4: There are 3 assumption of projectile motion.

1. Friction by air.
2. Flow of air.
3. different values of g.



Q1: Find the position of a projectile at any time t.

→ As we know that projectile is 2D so the value of its position will be,

$$\bar{r} = x\hat{i} + y\hat{j}$$
$$|\bar{r}| = \sqrt{x^2 + y^2}$$

x = Horizontal Displacement
y = Vertical Displacement

Here, $V_x = V\cos\theta$

$V_y = V\sin\theta$

So the displacement on X axis, $X = V_0\cos\theta t + \boxed{\frac{1}{2}at^2}$

" " " Y " , $Y = V_0\sin\theta t - \frac{1}{2}gt^2$

So the position a t second is

$$|r| = \sqrt{(V_0\cos\theta t)^2 + (V_0\sin\theta t - \frac{1}{2}gt^2)^2}$$

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Q2: A ball is thrown with an initial velocity $30ms^{-1}$ making an $45°$ angle with the ground. Find the position of the ball at $t = 1.5$ second.

$\rightarrow$

$$r = \sqrt{(V_0 \cos\theta t)^2 + (V_0 \sin\theta t) + \frac{1}{2}gt^2)^2}$$

$$= \sqrt{30\cos\theta . 1.5 + (30\sin\theta . 1.5 - \frac{1}{2} 9.8 (1.5)^2)^2}$$

$$= \sqrt{(45\cos\theta)^2 + (45\sin\theta - 22.05)^2}$$

$$= \sqrt{(45.\cos 45)^2 + (45 \sin 45 - 22.05)^2}$$

$$= \cancel{37}\ 35.71 \ ms^{-1}$$

Q3: Find the verticle position / Height of the projectile at $t = 2sec$.

So, $y = V_0 \sin\theta t - \frac{1}{2} g t^2$

$= 30 \sin 45 \times 2 - \frac{1}{2} \times 9.8 \times (2)^2$

$= 51.05 - 19.6$

$= 31.45$ m.

$V_0 = 30 ms^{-1}$

$\theta = 45^\circ$

$t = 2$

Q54: Find the velocity of the projectile at a any time t.

→ Let's differentiate the value of Y respective of time

$V_{oy} = \frac{dy}{dt} \left(V_0 \sin\theta t - \frac{1}{2} g t^2 \right)$

$= V_0 \sin\theta - g t$

$V_{ox} = \frac{dx}{dt} \left(V_0 \cos\theta t \right)$

$= V_0 \cos\theta.$

#QS5: Show that the varticle veloeity component of a projectile varies with time but horizontal component remains constant.

→ As we get from QS 4

The formula for define velocity at x axis or horizontally

$$V_x = V_0 \cos\theta$$

for y axis

$$V_y = V_0 \sin\theta - gt$$

If we consider this two statement as the velocity of a object at vertically and horizontaly then we can see there is no variable for the velocity of x axis.

On the other hand there's a variable in 2nd statement "t". so the result ~~of~~ of V_x will always returns an Constant value when the V_y will returns different-different value with the change of time.

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Define maximum Height H.

⇒ Equation for vertical distance

$$y = V_0 \sin\theta\, t - \frac{1}{2} g t^2$$

At 't' = $T/2$ (In the Heighest position time will be $T/2$)

Now Replace y with H

$$\boxed{\cancel{H = V_0 \sin 90^\circ (T/2) - \frac{1}{2} g (T/2)^2}}$$

$$H = V_0 \sin\theta . V_0 \sin\theta / g - \frac{1}{2} g \left(\frac{V_0 \sin\theta}{g}\right)^2$$

$$= \frac{V_0^2 \sin^2\theta}{g} - \frac{V_0 \sin^2\theta}{2g}$$

$$\boxed{H = \frac{V_0^2 \sin^2\theta}{2g}}$$

As we know,

$$V_y = V_0 \sin\theta - g t$$

$$\Rightarrow 0 = V_0 \sin\theta - g t \quad [\text{At heighest position } V_y = 0]$$

$$\Rightarrow t = \frac{V_0 \sin\theta}{g}$$

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Define the Range of R.

$$R = V_{0x}T \quad (S = Vt)$$

$$= V_0 \cos\theta \cdot 2V_0 \sin\theta / g$$

$$\boxed{R = \frac{V_0^2 \sin 2\theta}{g}}$$

When the value of R will be max? Hint: 45°

Time of Flight T.

$V_y = V_0 \sin\theta - gt$ at, $t = \frac{T}{2}$, $V_y = 0$

$$\Rightarrow 0 = V_0 \sin\theta - gt$$

$$\Rightarrow 0 = V_0 \sin\theta - \frac{gT}{2}$$

$$\Rightarrow \boxed{T = \frac{2V_0 \sin\theta}{g}}$$

All Formula for Projectile

① Components of Velocity at time t

$$V_0 \cos\theta = V_x$$
$$V_y = V_0 \sin\theta - gt$$

(ii) Position at time t

$$x = V_0 \cos\theta\, t$$
$$y = V_0 \sin\theta\, t - \frac{1}{2} g t^2$$

(iii) Time of maximum Heighet $\rightarrow$ $t_{max} = \dfrac{V_0 \sin\theta_0}{g}$

(iv) Time of Flight $\Rightarrow$ $T_{total} = \dfrac{2 V_0 \sin\theta}{g}$

(v) Maximum Height $H_{max} = \dfrac{(V_0 \sin\theta)^2}{2g}$

(vi) Horizontal range of projectile $\Rightarrow$ $R = \dfrac{V_0^2 \sin 2\theta}{g}$

(vii) Maximum Horizontal Range $\Rightarrow$ $R_{max} = \dfrac{V_0^2}{g}$

(viii) Equation of path of projetile motion

$$y = (\tan\theta)x - gx^2 / 2(V_0 \cos\theta_0)^2$$

Qs: A soccer player kicks a ball at an angle 37° from the horizontal with an initial speed of 20 m/s. Assume that the ball moves in a vertical plane.

ⓐ Find the time t_1 at which the ball reaches the highest point of its trajectory?

ⓑ What is the horizontal range of the ball and how long is it in the air?

ⓑ How high the ball will go?

ⓓ What is the velocity of the ball as it strikes ground

ⓐ $\longrightarrow$

$$T_{max} = \frac{2V_0 \sin\theta}{g} = \frac{2 \times 20 \sin 37}{9.8} = 2.47 \text{ seconds.}$$

∴ On the heighest point 1.23 second

ⓑ $$H_{max} = \frac{(V_0 \sin\theta)^2}{2g} = \frac{(20 \sin 37)^2}{19.6} = 7.39\,m.$$

ⓒ $$R_{max} = \frac{V_0^2 \sin 2\theta}{g} = \frac{400 \sin 74}{9.8} = 39.24\,m.$$

and T = 2.47.

ⓓ The ball will hit the ground at $T = 2.47$ second.

$$V = \frac{R}{T} = \frac{39.24}{2.47} = 15.89$$

QS2: ~~A soccer~~ plav In a contest to drop a package on a target, one contest's plane is flying at a constant horizontal velocity of 155 km/hr at an elevation of 225 m toward a point directly above the target. At what angle of sight ~~to~~ should the package be released to strike the target?

→

Here

~~R = 225 m~~

$V_x = 155$ km/hr

$= 43.06$ m/s

$h = 225$ m

$R = V_x \times t$ —— ①

And we know that

$h = \frac{1}{2} g t^2 \quad (u = 0)$

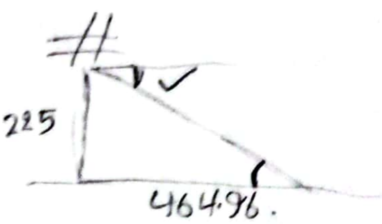
$\Rightarrow 225 = \frac{1}{2} \times 9.8 \cdot t^2$

$\therefore t = 10.81$ sec.

Now, Evaluate equation no. 1.

$R = 43.06 \times 10.81$

$= 464.96$ m.



And $\tan\theta = \frac{225}{464.96}$ m

$\therefore \theta = \tan^{-1}\left(\frac{225}{464.96}\right)$ m

$= 25.16$ degrees.

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Q53: A bomber is flying at a constant horizontal velocity of 820 miles/hr at an elevation of 52000 ft toward a point directly above its target. At what angle of sight should a ~~bob~~ bomb be released to strike the target? ($g = 32\ ft/sec^2$)

$\Rightarrow$

$h = \cancel{0}\ \frac{1}{2} g t^2$

$\Rightarrow \cancel{366}\ 15849.6 = \frac{1}{2}\ 9.76 \times t^2$

$\Rightarrow t^2 = \cancel{56}\ 57\ sec.$

$V_B = 366.6\ m/s$

$g = 32\ ft/s$
$= 9.76\ m/s$

$h = 52000\ ft$
$= 15849.6\ m$

So, $R = V_B \times t = 366.6 \times 57 = 20{,}892\ m$

and the angle will be,

$$\tan\theta = \frac{15849.6}{20892} = 37.16 \text{ degrees}.$$

20

Q54: A rescue plane files at a 123 miles/hr and constant height $h = 1640$ ft. toward a point directly over a victim, where a rescue capsule is to land. What should be the angle of sight the pilot's line of sight to the victim when the capsule release is made. ($g = 32\, ft/s^2$)

$\Rightarrow$

$H = 1640\, ft$

$180.4\, ft/s = V_x = 123\ miles/hr$

$g = 32\, ft\, s^{-2}$

$V_y = 0$