Computer aided mathematics and visualization
Practice

1. Which one(s) of the followings is/are correct?
   A. If the cross product of two vectors is null vector, then then the two vectors are perpendicular to each other.
   B. If the dot product of two vectors is zero, then then the two vectors are orthogonal to each other.
   C. If the dot product of two vectors is zero, then the two vectors have opposite directions.

2. True or false? Justify your answer!
   The graph of function $f(x) = 2x^4 + 3x - 3$ intersects the y axis at point $(0, 3)$.

3. True or false? Justify your answer!
   The graph of function $f(x) = 5x - 6$ goes through the point $A = (2, 4)$.

4. Consider the following function:

$$f : \mathbb{R} \to \mathbb{R},\ f(x) = 3x^6 - 2.9x^5 - 4.9x^4 + 2.7x^3 + 0.9x^2 + 1.1x + 0.5$$

   (a) Draw the graph of function f!
   (b) Draw the points corresponding to the zeros of function f!
   (c) Justify that $x = 0$ is not a zero of the function f!
   (d) Draw the points corresponding to the extrema of f!
   (e) Determine all extremum points of f, and categorize them as local or global, minimum or maximum.
   (f) Draw the point on the graph where $x = 1.23$! Do not use the *Point* command.
   (g) Draw the tangent line of f where $x = 1.23$, using the help of the derivative! Do not use the *Tangent* command.
   (h) Draw the following function with the given domain:

$$g : [0.2, 1.5] \to \mathbb{R},\ g(x) = 3x^6 - 2.9x^5 - 4.9x^4 + 2.7x^3 + 0.9x^2 + 1.1x + 0.5$$

5. Given the points $A = (2, 4)$ and $B = (3, -2)$, what is the slope of the line that goes through A and B?

6. Let us have point $P = (1, 6)$ and vector $\mathbf{v} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$. What is the tip point of vector $\mathbf{v}$ if we suppose that its tail point is P?

7. Let us consider the following curve:

$$x^3 + y^3 - 5xy^2 - x + 1 = 0$$

   (a) Draw the curve with blue color!
   (b) Can this shape be written in the form of an explicit, real-valued function?
   (c) Given the point $P = (1, 4)$, which of the following is true? Justify your answer using the mathematical background of implicit curves!
      A. P is on the curve.

B. P is not on the curve.

8. True or false? The zeros of the polynomial function f gives the zeros of f'.

9. True or false? A line segment cannot be written in a parametric form.

10. If F is a relation such that: $F = \{(2,4),(3,8),(4,5),(6,5)\}$, $F \subseteq A \times B$, where $A = \{1,2,3,4,5,6\}$ and $B = \{4,5,6,7,8\}$ Which of the followings are true?
    A. F is a function
    B. F is not a function
    C. the inverse of F is a function
    D. the inverse of F is not a function
    E. the domain is A
    F. the domain is B

11. Let us consider the the following curve:

$$x(t) = (a-b)\cos(t) + b\ \cos\left(\left(\frac{a}{b} - 1\right)t\right)$$
$$y(t) = (a-b)\sin(t) - b\ \sin\left(\left(\frac{a}{b} - 1\right)t\right)$$
$$t \in [0, 12\pi], a = 8.5, b = 3.9$$

    (a) Draw the curve! Hide the the graphs of the coordinate functions.
    (b) Draw the point P corresponding to the parameter value $t = 9\pi$! Do not use the *Point* command.
    (c) Draw the tangent vector of the curve defined at P with dashed style. Assure that the tail point of the vector is P.

12. Let us consider the unit circle centered at origin as a parametric curve.
    (a) Draw the curve!
    (b) Define the point P corresponding to parameter value $t_0 = \frac{\pi}{4}$!
    (c) Create a slider that changes the value of t_0 between 0 and π!
    (d) Create a parametric circle whose center is P with radius 0.5!

13. Let us have points $P_0 = (140, 80)$, $P_1 = (60, 80)$, $P_2 = (90, 50)$, and $P_3 = (120, 100)$. Create a third degree polynomial curve that goes through the points P_0, P_1, P_2, P_3 at parameter values 0, 1, 2, 3 respectively.
    (a) Draw the given curve! The start point of the curve should be P_0, and the end point should be P_3. Hide any other object.
    (b) Determine the tangent vector at P_3, and draw it! Use P_3 as its tail point. The vector should be in red colour and with a dashed style.