Course: 5SMB0 System Identification 2024/2025

Final Assignment

Version: 24 February 2025.

Instructions and general recommendations

- Submit one copy of your group report through Canvas. The report consists of a typewritten pdf file, that is saved as **group__xx__names.pdf**, where xx is your group number. Handwritten reports shall not be considered.
- Give concise and focused answers that get straight to the point, do not get lost into lengthy explanations. Make sure to discuss and interpret the obtained results. Also make sure that your results are reproducible based on the report alone. So someone should be able to implement a version of your code based on your report.
- Try to fit your report into 8 pages with the standard IEEE double-column template. Figures should have labeled axis, including units (e.g. dB, Hz or rad/s) as well as a caption explaining the figure content. Every figure should be referred to in the main body of the text.
- While using Matlab, you are free to either use the System Identification Toolbox GUI, or the inline commands (e.g. `arx()`), or both. You should hand in a separate .zip folder of your Matlab code (named as **group__xx__names.zip**), containing a `main.m` file (with a fair level of code documentation) and all files required to run the `main.m` file, such as a `data.mat` file and/or the `assignment_sys_xx.p` file that we can use to reproduce all your results.
- Most of the algorithms to generate inputs, identify systems, etc., are already implemented in Matlab! Check the exercise sets or type `help ident` in Matlab for a complete list.
- The questions can be answered in multiple ways. You have the flexibility to determine how to achieve the goals, e.g. intermediate signal processing steps can be added. These steps should be described and motivated in detail in the report such that the goals achieved are reproducible.

General setup

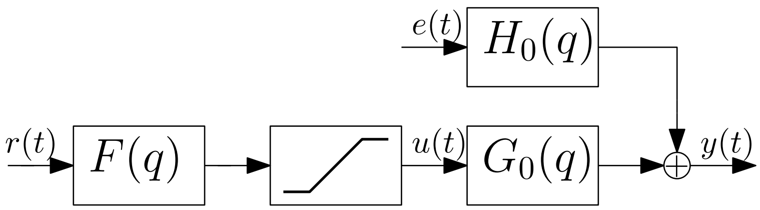

Figure 1: Block scheme of the system to identify

We are given a linear time-invariant system $G_0(q)$ which we are asked to identify as accurately as possible. The output $y(t)$ obeys the equations:

$$y(t) = G_0(q)u(t) + H_0(q)e(t)\,,$$

where $H_0(q)$ is a stable and monic filter and $e(t)$ is white noise.

The system is represented through an encrypted Matlab function (extension `.p`) named `assignment_sys_xx.p`, where `xx` is the Group number. The function implements the block scheme of Fig. 1: it accepts as input a vector r representing the samples of the signal $r(t)$ and sends out two vectors u and y representing the samples of the input and output signals of $G_0(q)$.

For example, the function can be called as follows:

```
[u,y] = assignment_sys_xx(r,mode)
```

where `mode` needs to be set as `'open loop'` for Parts 1-5 and `'closed loop'` for Part 6. Therefore, you have the freedom to feed the system with a prescribed signal $r(t)$, which will be converted inside the function into the actual input $u(t)$. The conversion is made through the following two blocks of Fig. 1. The generator dynamcis, given by $F(q)$, is a linear time-invariant filter; more precisely, it is a Butterworth filter given by

$$F(q) = \frac{0.505 + 1.01q^{-1} + 0.505q^{-2}}{1 + 0.7478q^{-1} + 0.2722q^{-2}}.$$

These dynamics prevent the system from being excited at high frequencies. The filter damps all the frequency contents that are above a certain cut-off frequency. Furthermore, the actuator contains a hard saturation $S(\cdot)$ on its output, namely a static function that transforms every sample of x as follows:

$$S(x) = \begin{cases} M & \text{if } x \geq M \\ x & \text{if } -M < x < M \\ -M & \text{if } x \leq -M \end{cases}$$

In the above equation, M is a real number that is unknown.

Part 1: Understanding saturation and Butterworth filter

1.1 It is important to understand the characteristics of $F(q)$. Plot its frequency response (Bode diagram). What is the cut-off frequency at -3dB (expressed as $x\,\pi$, where $x \in (0, 1)$)?
Hint: A linear scale in the horizontal axis may be preferable in this case.

1.2 It is required to determine M. To this end, design a signal $r(t)$ that can be used to accomplish this task. Explain why it is important to determine M.

Part 2: Nonparametric identification

2.1 We first want to get a rough idea of the frequency behavior of $G_0(q)$. To this end, we want to perform a preliminary experiment and apply nonparametric system identification using the collected data. The task is then to design an input with length $N = 1500$ that at least excites the system at 100 frequencies within the passband of the Butterworth filter. What type of input shall be used? Give an expression of $r(t)$ and motivate your choice.

2.2 Display the Bode plot of the identified FRF of the system. What can we conclude about:

- High-frequency behavior of $G_0(q)$;
- Resonance peaks.

2.3 Use the data generated in Part 2.1 , or modify the signal generated in Part 2.1, to obtain an estimation of the nonparametric noise model, i.e., derive an expression for estimating the noise spectrum Φ_v and plot the magnitude of the estimated noise spectrum in a Bode diagram. What can you conclude from the estimated spectrum? Remember: the input should have length $N = 1500$ and should at least excite 100 frequencies within the Butterworth filter passband.
Remark: In case you choose to use the `cpsd` matlab function, be aware that it implements Welch's averaging method.

Part 3: Experiment design

Based on the findings of the nonparametric identification, we want to get a parametric model of $G_0(q)$. However, the experiment is limited to only $N = 3000$ data points. Choose a signal type of $r(t)$ and motivate your choice.

Part 4: Parametric identification and validation

4.1 Using the designed input from Part 3, we want to perform a parametric identification of the system that yields *a consistent* estimate of G_0. For this purpose use a data set of length $N = 3000$. Motivate the choice(s) that you make for the

- model structure
- model orders

and provide at least two validation tests. Moreover, comment on the relationship between the obtained results and the results for Part 2.

4.2 Does your chosen approach also lead to a minimum variance estimate? Why or why not?

Part 5: Experimental verification of variance estimates

5.1 We would like to get an understanding of the mechanism behind the estimation of the variance of identified parameters. To this end, we are going to introduce experiment repetition as a way to estimate the variance of an estimated model. We do this by performing Monte Carlo simulations, that is, repeating Question 4 for 100 times. Inspect the parameters obtained from the Monte Carlo simulations. Do we always get similar results from the different simulation runs? If not, could you explain why?

5.2 Compute the *theoretical variance*, which is the estimated covariance matrix based on one single experiment. This estimated covariance matrix can be obtained directly from Matlab using the command `getcov(M_i)`, which implements the covariance matrix estimate (4.121)-(4.122)[1] for all model structures with a parametrized noise model, and (4.138)-(4.140) for model structures with a non-parametrized noise model. The estimated parameter variances occur as the diagonal elements of the estimated covariance matrix. Under which conditions is this estimated (theoretical) covariance an accurate estimate of the real covariance?

5.3 Using the collected data, we would like to compare the simulated statistics (via Monte Carlo) with the *theoretical variance.* The simulated statistics are obtained by computing the mean and variance of the identified coefficients of $B(q)$ and $F(q)$ (over the 100 Monte Carlo runs)— commands are `mean` and `var`.
Do the Monte Carlo variances match the theoretical ones? Why or why not?

[1] Equation numbers are taken from the Lecture Notes.

Part 6: Closed-loop identification

Next, we will consider a closed-loop system as shown in Fig. 2, where the output y is fed back through the controller K. Change the mode into 'closed loop', and recall:

```
[u,y] = assignment_sys_xx(r,mode)
```

Your task is to identify the system G_0 starting from your designed reference signal, the system input u, and the system output y. Describe and motivate the chosen approach. Compare the obtained system estimate, with the estimate obtained in Part 4.

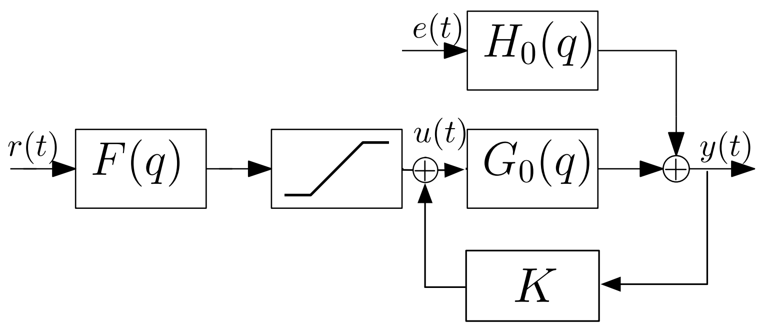

Figure 2: Block scheme of the system to identify