Group 33 / Final Assignment

Course: 5SMB0 System Identification 2024/2025

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# Understanding saturation and Butterworth filter

Bode diagram of F(q) can be represented in figure 1:

A graph of a normalized frequency

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Figure 1 Bode Plot of F(q)

The cut off frequency at -3dB (or in other words bandwidth) is 0.69969π

In order to determine M we need to design a signal r(t) to drive the system into saturation. This signal should be predictable and trigger the saturation of filter (-M and +M).

A ramp signal (linspace) from -100 to 100 is useful here to gradually increase the intensity over time to observe when the saturation filter comes into effect.

The saturation limit M is the maximum absolute value allowed by the actuator. Any signal beyond this magnitude is clipped, potentially distorting the input u(t). Determining M ensures that we do not unintentionally drive the actuator into saturation during experiments, which would bias identification results

The maximum amount output (u) that the function retuns is be 1.80

# Nonparametric identification

## Defining r(t) as a multi sine wave

A multi sine wave covers a wide range of frequencies whitin the system passband. It can also offer a linear signal, in the operating range of the system, without going beyond the given domain.

where:

* AkA\_kAk​ is the amplitude of the kkk-th sine component.
* fkf\_kfk​ are 100 logarithmically spaced frequencies within the passband of the Butterworth filter.
* ϕk\phi\_kϕk​ are random phase shifts to ensure good time-domain properties.
* N=1500N = 1500N=1500 samples.

Compared to white noise or PRBS, multisine signals allow direct spectral interpretation and reduce variance in frequency response estimation.

The FRF could be displayed as below:

A graph of a function

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Figure 2 Frequency Response Function for G0(q) based on a multi-sine input

The gain drops significantly at higher frequencies, suggesting that G0(q) has low-pass characteristics. Sharp peaks in the FRF indicate resonant modes in the system, useful for selecting model structures in parametric identification.

To estimate the noise spectrum we could benefit from the following equasion:

Where:

* is Power spectral density (PSD) of the output y(t)
* is Cross-power spectral density (CPSD) of input u(t) and output y(t)
* is Power spectral density (PSD) of the input u(t)

Using the cpsd matlab function, we can get an estimation of noise spectrum and plot this figure:

A graph showing a number of noise spectrum

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Figure 3 Nonparametric Estimated Noise Spectrum of G0(q)

This figure gives an estimate of the contribution of noise after removing the effect of input u(t). The Bode plot of Phi\_v helps reveal frequency bands where the noise has more power. As shown, this is at high frequencies or outside the system bandwidth.

# Experiment design

There are a number of signal types we can exlpore

For parametric identification (limited to 𝑁 = 3000), we used a PRBS (pseudo-random binary sequence) signal, as PRBS is persistently exciting, well-suited for identifying linear models. It also balances good frequency content with repeatability and discrete structure, making it ideal for system excitation without excessive high-frequency noise.

# Parametric identification and validationsing

Based on Lecture 4, we use 3 different types of model introucded and use AIC to compare which model is better. The Akaike Information Criterion (AIC) is a model selection metric that balances: goodness of fit (how well the model explains the data) and model complexity (how many parameters it uses). In matlab we can simply measure AIC by aic(model) command

1. Comparing different model structures with best order for open loop

| Model Structure type | Best Orders | Minimum AICa |
| --- | --- | --- |
| BJ | [4,5,5,4,1] | -1.451 |
| OE | [4,5,1] | 2.0643 |
| ARMAX | [5,5,5,1] | -0.8719 |

1. Akaike Information Criterion

Since Box-Jenkins (BJ) has the lowest AIC we will use this model structure which has the following structure:

Where:

To idenitify the orders of this model ([nb, nc, nd, nf, nk]) we specify a range for each order, and use a for loop to identified that the least AIC occure when

* nb = 4
* nc = 5
* nd = 5
* nf = 4
* nk = 1

The chosen model and structure is giving consistent results which can be proved through different validation tests.

Two validation tests:

* Pole-Zero Map via pzmap() & isstable()
* Residual analysis via resid()

A graph of a polar axis

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Figure 4 Validation Test 1: All poles are inside the unit circle (model is stable)

A graph of a model

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Figure 5 Validation Test 2: most lags are within the confidence band (95%)

All validations are passed and isstable() command returns “true” because all poles are inside the unit circle (as can be seen in figure 4). As shown in the figure 5, both autocorrelation and x correlation graphs shows lags within the confidence band (95%), which means the BJ model is capturing the dynamics of G pretty well.

Comparing the estimated model with the results obtain in part 2 can be seen in figure 6. The model captures the spa() output with reasonleble accuracey for the passband

A comparison of a comparison of a model

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Figure 6 Comparing the BJ model with FRF of G0(q)

To get an estimate of the esitimated model parameters we can use getcov() command. In our case, the BJ model had the lowest AIC, is stable and residual analysis showed low input-residual and auto correlation. Therefore we can conclude this approach also to a minimum variance estimate.

# Experimental verification of variance estimates

**5.1** In our Monte Carlo simulation, we repeated the identification of the BJ model 100 times using independently generated PRBS input signals. Each run yielded a slightly different set of parameter estimates for the polynomials B(q), F(q), C(q) and D(q).

The results were not exactly the same in every run. The parameter estimates varied slightly from one simulation to the next. The reason for such variance is due to noise and random nature of input signal (PRBS).

**5.2** For theoritical vairnace we use MatLab built-in command getcov(). The diagonal elements of this matrix represent the estimated variances of each individual parameter based on a single experiment.

The theoritcal veriance is reliable if the model structure matches the true system (i.e., correct orders), and the input signal is persistently exciting (which our PRBS is). Moreover, the residuals are white and uncorrelated with the input. The only limitation here is number of data points which may introcude some error in our estimation.

**5.3** The Monte Carlo and theoretical variances were reasonably close, suggesting the model is well-identified and close to minimum variance, but the match is not perfect due to practical data (data point limited to 3000) and noise limitations

As can be observed in figure 7, the difference between month carlo and theoritical variance is slim to none (max difference is arround 0.045)

A graph with blue and red bars

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Figure 7 Bar plot for computed variance of BJ model’s B(q) and F(q) paramters and theoritical variance

# Closed-loop identification

Similar to part 4, we compare AIC value of three different model structure with best orders to see which structure provide the least AIC. For closed loop mode, ARMAX model is more fitting to G0(q)

Table 1 Comparing different model structures with besst order for close loop

| Model Structure type | Best Orders | Minimum AICa |
| --- | --- | --- |
| BJ | [4,4,3,4,1] | -2.8474 |
| OE | [5,4,0] | -0.32207 |
| ARMAX | [5,5,30,1] | -4.1366 |

We use the same PRBS input sigal data to excite this system and compare it to open loop estimate. Figure 8 shows how paramteric model compare against non parametric.

A graph of a comparison of a frr and a frequency model

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Figure 8 Comparing ARMAX (closed Loop) model with FRG of G0(q)

A graph of a polar axis

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Figure 9 Validation Test 1: All poles are whitin the unit circle, except a zero

According to validation test 1 (figure 9) all poles and zeros are within the unit circle and the model is stable.

A graph of a model

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Figure 10 Risidual Analaysis of ARMAX (closed loop) model

Figure 10 shows the result of second validation test, which indicate the model is fully capturing the noise model, where are lags are whiting the confidence band.

I tried different models (IV4, ARX, BJ) and niether was able to fully cpature the noise model. Therefore the nc was increase to 30 (which is high) to be able to capture the noide model (also lowest AIC).

The identified ARMAX model [5 5 30 1] achieved a 94.43% fit on an independent validation dataset, confirming that it generalizes well beyond the training set. Despite the high noise model order, the residuals were sufficiently white and uncorrelated, suggesting that the model captures both the dynamics and the noise characteristics effectively under closed-loop operation.

A blue line graph with white text

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Figure 11 ARMAX Model Performance on New validation data

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Figure 12 BJ Model Performance on new validation data

The preferred spelling of the word “acknowledgment” in America is without an “e” after the “g”. Avoid the stilted expression “one of us (R. B. G.) thanks ...”. Instead, try “R. B. G. thanks...”. Put sponsor acknowledgments in the unnumbered footnote on the first page.

Comparing closed vs open loop model, we can observe that the AIC value is much lower in closed loop, goodness of fit for unseen validation data also is higher compared to open loop (arround 76.27% percent for open loop figure 11 and 94.43% percent for closed loop according to figure 12)

##### Acknowledgment

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* Brainstorming and refining modeling strategies
* Generating MATLAB code templates for model comparison and validation
* Clarifying theoretical concepts such as residual analysis, model structures, and stability
* Summarizing results and drafting explanatory text for report sections

The use of AI was limited to general functionalities and did not replace core academic work such as experimental design, model estimation, or interpretation of results. All generated content was critically reviewed, adapted, and validated by the author to ensure compliance with academic standards and scientific integrity, as outlined in the TU/e Code of Scientific Conduct.