Project Parameter Estimation
Prof. Dr. Andreas Wieser

PPE Kriging	**Fall Semester 2020**	**Subject NR. 103-0787-00 L**

HOMEWORK 1: THE ORIGINS OF KRIGING

1 Contents and Aim of Homework

- **Understanding historical context and first applications of Kriging**
- **Get to know definitions, prerequisites, properties and formula of the Kriging estimator**
- **Grasping concept of support and the role of hypothesis concerning the underlying structure of a random field**
- **Relearn, how to calculate variances and expected values**

Note: This homework is to be handed in at noon, 12th of November. E-mail: jemil.butt@geod.baug.ethz.ch

Since we are going to have a detailed discussion on some aspects mentioned in the paper, please read it carefully. The amount of questions makes it look like a lot of work but most of the questions have actually a pretty straightforward and crisp answer.

2 Tasks

2.1 Understanding the text

a) Please read the text carefully, make notes and mark important sections.

b) Write a short summary of the text, capturing what you think is potentially essential information to us (Maximum: 10 sentences).

c) Make a mindmap of the paper detailing the topics and their connections.

2.2 Detailed Questions

a) *P 244: „Clearly, the sample mean of the assays multiplied by the estimated ore-body volume, is not a very good estimate of total recoverable ore."*

Why is this not a good estimate? Please be precise in your explanation.

b) *P 241 "… notice that this is not a filtering or state-estimation problem where the goal is to predict…"*

What is the difference between state-estimation and prediction that is hinted at by this sentence?

c) *P 241 " …zero-mean stochastic process with known covariance function, … "*

How would you tackle the problem that in real-life situations you actually don't know the covariance function?

2.3 Understanding the math

a) *P 241 „Should some of the data be defined on a larger support (e.g* $\int_{B_i} Z(s)ds / \int_{B_i} ds$ *instead of* $Z(s_i)$ *), …"*

Explain what this formula means and what the whole idea of "support" is about. Can you imagine a geodetic application, where you would have to consider this?

b) *P 245 "He also commented that the equal weights he was proposing would not be far from the optimal (kriging) weights, in the circumstances for which he was using them."*

Please imagine a situation, such that the optimal weights are all equal. For that, it might be helpful to look to Equation 11 for inspiration and discuss what the properties of a random field best estimated by equal weights would need to be.

c) *Bonus points:* Please try to relate adjustment procedures (e.g. Gauss-Markov model $Ax = y + e$) to what is presented in the paper by linking equation 12 on page 242 with the well-known estimators $\hat{x} = (A^T PA)^{-1} A^T Py$ and $\hat{e} = [I - A(A^T PA)^{-1} A^T P]y$ for parameters and residuals respectively.

2.4 Expectation and covariance

The following few tasks should (re-)familiarize you with the properties of expected values and covariances. They are not related to the paper discussed above and can be solved without any involved mathematics. You will however need the linearity and bilinearity properties of expectation and covariance operator discussed briefly during class (Slide 31) and basic knowledge about random variables.

a) Let X be the random variable corresponding to a dice throw. Calculate $E[X]$ and $\sigma(X,X)$ by hand.

b) Let $X_k, k = 1,...,n$ be independent measurements, $X_k \sim N(0,1)$, and $\overline{X}$ be the random variable corresponding to the arithmetic mean: $\overline{X} = \frac{1}{n}\sum_{k=1}^{n} X_k$. Calculate $E[\overline{X}]$ and $\sigma(\overline{X},\overline{X})$.

c) Let $Y_k, k = 1,...,n$ be correlated measurements, $Y_k \sim N(0,1)$. Their covariance shall be given by: $\sigma(Y_k, Y_k) = 1,\ \sigma(Y_k, Y_{k+1}) = 0.5,\ \sigma(Y_k, Y_{k+d}) = 0\ \forall d \geq 2$. Can this be? Or are e.g. Y_1 and Y_3 necessarily correlated, if Y_1, Y_2 and Y_2, Y_3 are correlated?

d) With Y_k as given above, calculate again expected value und variance of the mean $\overline{Y} = \frac{1}{n}\sum_{k=1}^{n} Y_k$.

Good Luck!