Standard RBC model

26th December 2024

1 Model

A standard Real Business Cycle (RBC) model with a representative agent including labour supply choice. The objective function of the agent is:

$$\max E_1 \sum_{t=1}^{\infty} \beta^{t-1} \frac{C_t^{1-\nu}}{1-\nu}$$

where C_t is period t consumption, and H_t is period t labor supply. The real budget constraint is:

$$C_t + K_{t+1} = Z_t K_t^{\alpha} + (1-\delta) K_t \tag{1}$$

where K_t is the capital stock at the beginning of period t, and δ is the depreciation rate of capital. Total Factor Productivity (TFP) Z_t evolves by an exogenous process:

$$z_t = \rho_z z_{t-1} + \sigma_z \epsilon_t \tag{2}$$

where $z_t = \log(Z_t)$, ρ_z is the autocorrelation coefficient, and σ_z is the standard deviation of the shocks. The shocks ϵ_t are standard normally distributed ($\epsilon_t \sim \mathcal{N}(0,1)$).

The optimization problem can be written with an infinite Lagrangian:
$\mathcal{L} = E \sum_{t=1}^{\infty} \beta^t \left\{ \frac{C_t^{1-\nu}}{1-\nu} + \lambda_t \left[Z_t K_t^{\alpha} + (1-\delta) K_t - C_t - K_{t+1} \right] \right\}$
where λ_t is the shadow price of the budget constraint, and K_1 is given. Maximization of this Lagrangian with respect consumption C_t and capital in next period K_{t+1} yields the following First Order Conditions:

$$\begin{aligned} C_t^{-\nu} &= \lambda_t \\ \lambda_t &= \beta \lambda_{t+1} \left[F_k(K_t) + 1 - \delta \right] \end{aligned}$$

Substituting out λ gives:

$$C_t^{-\nu} = \beta E_t \left\{ C_{t+1}^{-\nu} \left[Z_{t+1} \alpha K_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha} + 1 - \delta \right] \right\} \tag{3}$$

2 Steady state

To derive the analytical steady state we start with the Euler equation (3):

$$\begin{aligned} \overline{C}^{-\nu} &= \beta \left\{ \overline{C}^{-\nu} \left[\overline{Z} \alpha \overline{K}^{\alpha-1} + 1 - \delta \right] \right\} \\ \frac{1 - \beta\left(1 - \delta\right)}{\overline{Z}\alpha\beta} &= \overline{K}^{\alpha-1} \\ \overline{K} &= \Omega^{\frac{1}{\alpha-1}} \end{aligned}$$

with $\Omega = \frac{1-\beta(1-\delta)}{\alpha\beta\overline{Z}}$.

Substituting this into the resource constraint (1) yields:

$$\begin{aligned} \overline{C} + \overline{K} &= \overline{ZK}^{\alpha} + \left(1 - \delta\right) \overline{K} \\ \overline{C} &= \overline{ZK}^{\alpha} - \delta \overline{K} \end{aligned}$$