Tutorial 08, 16th of June 2025

Optimal Abatement in a static macroeconomy

The Economics of Climate Change

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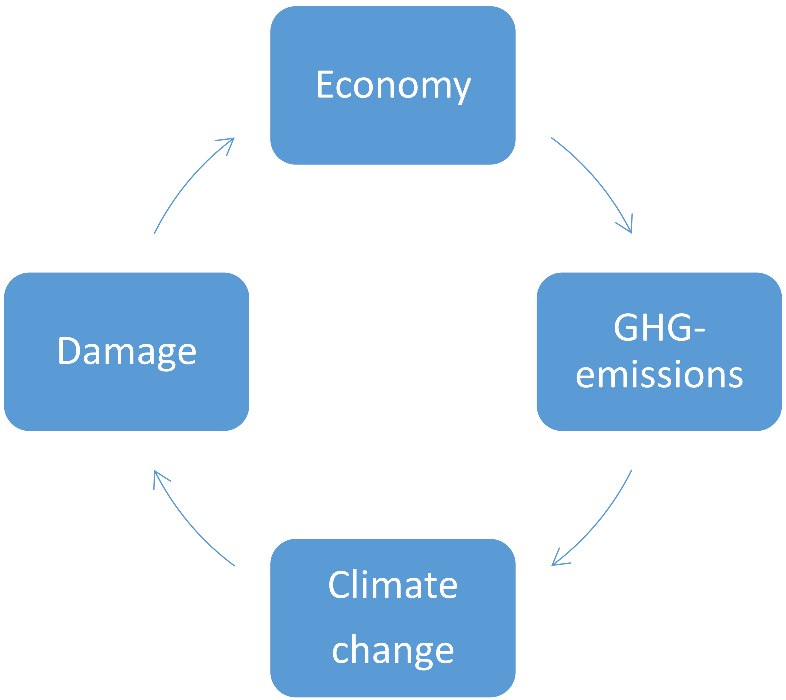
Agenda

Note: Assignment 05 online today, due next week before class

1. Ingredients of Integrated Assessment Model (like DICE)
2. Simple Environmental Macro Model
   - Social Cost of Carbon
   - Marginal Abatement Costs
3. Adding utility function to E-Macro model
4. Labor supply in E-Macro model

Integrated Assessment Models: Interaction Economy, GHGs, and Climate



- What does each component look like in an Integrated Assessment Model? (For example DICE)

"Economy" can include anything. Also aspects with a non-market valuation (ecosystems, health, et cetera).

Simple Environmental Macro Model

- Gross output is given: Y^{gr}
- Emissions E (flow) cause damages $D(E)$ *(strictly convex):*

$$Y^{net} = \big(1 - D(E)\big)Y^{gr}$$

- Abatement a reduces emissions (γ is emission intensity):

$$E = \gamma(1 - a)Y^{gr}$$

- Total Abatement costs *(strictly convex):*

$$\mathrm{TAC} = \Gamma(a)\big(1 - D(E)\big)Y^{gr}$$

- Consumption is net output – abatement costs:

$$C = \big(1 - \Gamma(a)\big)\big(1 - D(E)\big)Y^{gr}$$

Social Planner

Social Planner

- Social Planner can directly set all quantities in the economy

➢ Not constrained by “selfish behaviour” of agents

Meaning: there are NO implementability constraints

Decentralized Economy (next week)

- Agents optimize their own objective
  - ➢ Implementability constraints

- Government can only indirectly influence behaviour (with policies)

Example Redistribution of income: SP

- Social planner has utilitarian objective: $\max \sum_i \; U(c_i, l_i)$

Where i are the individuals, c is consumption (utility), l is labor supply (disutility!)

- Total budget: $\sum_i \; c_i = \sum_i \; w_i l_i$ (total cons. = total labor income + transfer)
- Social Planner can set c_i and l_i directly of each agent, Lagrangian:

$$L = \sum_i U(c_i, l_i) + \Lambda\left[\sum_i w_i l_i - \sum_i c_i\right]$$

- FOCs

$$U_{c,i} = \Lambda$$

(Marginal utility of consumption = Shadow price of the budget)

$$-U_{l,i} = \Lambda w_i$$

*(-Marginal utility of labor = Shadow price of the budget*wage)*

Interpretation? Is this solution “fair”?
Who has to work more, those with high or low wage?

Redistribution in Decentralized Economy with Government: Labor tax + Lump Sum transfer

- Households maximize utility of consumption and labour supply l: $\max U(c_i, l_i)$

- Budget constraint:

$$c_i = (1-\tau)w_i l_i + T$$

(consumption = net labor income + transfer)

- Lagrangian:

$$L = U(c_i, l_i) + \lambda_i[(1-\tau)w_i l_i + T - c_i]$$

- FOCs

$$U_{c,i} = \lambda_i$$
$$-U_{l,i} = \lambda_i(1-\tau)w_i$$

Note: shadow price of budget is marg. utility of cons.

- Substitute out λ:

$$-U_{l,i} = (1-\tau)U_{c,i}w_i$$

(-marg. ut. labor = marg. ut. cons. * net wage)

Labor tax drives a wedge between marg. rate of substitution & gross wage: **labor supply is inefficient (distorted)**

Simple Environmental Macro Model (static)

***Social Planner perspective**: all choice variables are freely chosen, but subject to technological and climate constraints*

- **Total emissions**: $E = \gamma(1-a)Y^{gr}$

Where a is abatement (reduction of emissions), γ emission intensity, Y^{gr} gross output (given)

- **Total Abatement Costs** (TAC) (strictly convex): $\Gamma(a)\big(1 - D(E)\big)Y^{gr}$

- Objective: **maximize consumption** (output net of damage & abatement costs)

$$C = \big(1 - \Gamma(a)\big)\big(1 - D(E)\big)Y^{gr}$$

- Lagrangian (after subst. out C):

$$L = \big(1 - \Gamma(a)\big)\big(1 - D(E)\big)Y^{gr} + \mu[E - \gamma(1-a)Y^{gr}]$$

(I prefer to NOT substitute out E: μ has a useful interpretation. What is μ?)

Simple Environmental Macro Model (static)

- Lagrangian:

$$L = \big(1 - \Gamma(a)\big)\big(1 - D(E)\big)Y^{gr} + \mu[E - \gamma(1 - a)Y^{gr}]$$

- *Choice variable a determines the solution, BUT also need to consider effect of a on damages D through emissions E*

- FOCs w.r.t. E and a:

$$\mu = D'(E)\big(1 - \Gamma(a)\big)Y^{gr}$$

Interpretation: Shadow price of E $=$ Marg. damage of E (in terms of goods)

$$\mu\gamma Y^{gr} = \Gamma'(a)\big(1 - D(E)\big)Y^{gr}$$

Interpretation: Shadow price E x marg. change in emissions (γY^{gr}) (due to a) = Marginal costs of abatement

Solution after simplifying

$$\mu = D'(E)\big(1-\Gamma(a)\big)Y^{gr}$$
$$\mu\gamma Y^{gr} = \Gamma'(a)\big(1-D(E)\big)Y^{gr}$$

- Simplifying solution (substitute μ out of second equation):

$$\mu\gamma\ = \Gamma'(a)\big(1-D(E)\big)$$

$$\gamma D'(E)\big(1-\Gamma(a)\big)Y^{gr} = \Gamma'(a)\big(1-D(E)\big)$$

$$\boldsymbol{D'(E)\big(1-\Gamma(a)\big)Y^{gr} = \frac{\Gamma'(a)\big(1-D(E)\big)}{\gamma}}$$

Marginal damage from emissions = Marginal Abatement Costs (MAC)

(MAC = Marginal costs of reducing emissions)

Social Cost of Carbon (here: flow damage only)

- SCC = Marginal damage from emissions (usually $ per ton/CO2 equivalent)

- In this simple model:

$$\text{SCC}= -\frac{\partial C}{\partial E} = D'(E)\big(1-\Gamma(a)\big)Y^{gr}$$

- *Note 1: the objective, consumption, is already measured in dollars.*

- *Note 2: SCC is only about **marginal effects!***
  - *Very different from Compensating Variation or Equivalent Variation, which compare welfare at two different points*

Marginal Abatement Costs

In general:

- Defintion of Marginal costs: change in costs $C(Q)$ resulting from changing quantity (Q) with a marginal unit
  - $C(Q)$ is cost function =>$MC = \frac{dC}{dQ}$

- Definition of Marginal Abatement Costs:

Costs to reduce emissions with one unit:

$$MAC = -\frac{\partial Total\ Abatement\ Costs}{\partial E}$$

Note the minus: costs of abatement = costs of reducing emissions

Marginal Abatement Costs

$$MAC = -\frac{\partial Total\ Abatement\ Costs}{\partial E}$$

- Total abatment costs: $TAC = \Gamma(a)\big(1 - D(E)\big)Y^{gr}$
- Emissions: $\boldsymbol{E = \gamma(1 - a)Y^{gr}}$

Two components in our model:

$$\frac{\partial TAC}{\partial E} = \frac{\partial TAC/\partial a}{\partial E/\partial a}$$

$\frac{\partial TAC}{\partial a}$: *marginal costs of abatement effort* $\boldsymbol{a}$

$\frac{\partial E}{\partial a}$: *Marg. change in emissions due to abatement effort* $\boldsymbol{a}$

Marginal Abatement Costs

$$MAC = -\frac{\partial Total\ Abatement\ Costs}{\partial E} = -\frac{\partial TAC/\partial a}{\partial E/\partial a}$$

- Total abatement costs: $TAC = \Gamma(a)\big(1 - D(E)\big)Y^{gr}$:

$$\frac{\partial TC}{\partial a} = \Gamma'(a)\big(1 - D(E)\big)Y^{gr}$$

- Emissions: $\boldsymbol{E = \gamma(1 - a)Y^{gr}}$:

$$\frac{\partial E}{\partial a} = -\gamma Y^{gr}$$

Meaning:

$$MAC = -\frac{\partial TC}{\partial E} = -\frac{\partial TC/\partial a}{\partial E/\partial a} = \frac{\Gamma'(a)\big(1 - D(E)\big)Y^{gr}}{\gamma Y^{gr}}$$

$$\boldsymbol{MAC = \frac{\Gamma'(\eta)\big(1 - D(E)\big)}{\gamma}}$$

Recap of Solution of Simple E-Macro

$$D'(E)\big(1-\Gamma(a)\big)Y^{gr} = \frac{\Gamma'(a)\big(1-D(E)\big)}{\gamma}$$

Social Cost of Carbon = Marginal Abatement Costs (MAC)

Environmental Macro Model with utility function(static)

Previous example:

- Objective: maximize consumption (output net of damage & abatement costs)

$$C = \big(1 - \Gamma(a)\big)\big(1 - D(E)\big)Y^{gr}$$

- Constraint:

$$E = \gamma(1 - a)Y^{gr}$$

Assume now that we want to maximize utility over consumption:

- Objective is to maximize utility: $U(C)$

❖ Will optimum be different (compared to the previous model which maximized consumption)?

Static Climate Economic Model with utility of consumption: Social Planner solution

- Objective: maximize utility (strictly concave & monotonically increasing) $U(C)$
- Gross production: Y^{gr}
- Net production after damage: $\big(1 - D(E)\big)Y^{gr}$
- Abatement a reduces emissions (γ is emission intensity): $E = \gamma(1 - a)Y^{gr}$
- Abatement costs (total): $\Gamma(a)\big(1 - D(E)\big)Y^{gr}$
- Budget constraint: $C = \big(1 - \Gamma(a)\big)\big(1 - D(E)\big)Y^{gr}$

Lagrangian:

$$L = U(C) + \lambda\Big[\big(1 - \Gamma(a)\big)\big(1 - D(E)\big)Y^{gr} - C\Big] + \mu[E - \gamma(1 - a)Y^{gr}]$$

FOCs Social Planner

Lagrangian:

$$L = U(C) + \lambda\big[\big(1-\Gamma(a)\big)\big(1-D(E)\big)Y^{gr} - C\big] + \mu[E - \gamma(1-a)Y^{gr}]$$

FOCs w.r.t. C, a, and E:

$$U'(C) = \lambda$$

$$\lambda\Gamma'(a)\big(1-D(E)\big)Y^{gr} = \mu\gamma Y^{gr}$$

$$\lambda\big(1-\Gamma(a)\big)D'(E)Y^{gr} = \mu$$

FOCs are different: shadow prices λ and μ are measured in utility units!

- Solution:

$$\mu\gamma = \lambda\Gamma'(a)\big(1-D(E)\big)$$

$$\gamma\lambda\big(1-\Gamma(a)\big)D'(E)Y^{gr} = \lambda\Gamma'(a)\big(1-D(E)\big)$$

$$\boldsymbol{\big(1-\Gamma(a)\big)D'(E)Y^{gr} = \frac{\Gamma'(a)\big(1-D(E)\big)}{\gamma}}$$

$$(SCC \;=\; MAC)$$

Exactly the same as before!

Social Cost of Carbon (static)

- *Without utility function: SCC=* $-\frac{\partial C}{\partial E}$*: marginal damage of emission is change in consumption (in \$/ton CO2))*

- More general in static environment:

$$SCC = -\frac{\partial W / \partial E}{\partial W / \partial C}$$

where W is (total) welfare

- $\partial W / \partial E$ (numerator): effect on welfare of a marginal unit of emissions
- $\partial W / \partial C$ (denominator): is the marginal change in welfare due to a change in C
  - This converts welfare units to monetary units (\$)
- Marginal damage of emissions in terms of (discounted) welfare, expressed in monetary unit (\$/ton CO2)

Social Cost of Carbon

In our model: $\boldsymbol{W} = \boldsymbol{U}(\boldsymbol{C})$

$$SCC = -\frac{\partial U/\partial E}{\partial U/\partial C}$$

- $\frac{\partial U}{\partial E} = \frac{\partial U}{\partial C}\frac{\partial C}{\partial E} = U'(C) \cdot -\big(1 - \Gamma(\eta)\big)D'(E)Y^{gr} = -\mu$
- $\frac{\partial U}{\partial C} = U'(C) = \lambda$

$$\text{SCC} = -\frac{\partial U/\partial E}{\partial U/\partial C} = \frac{\mu}{\lambda}$$

With $L = U(C) + \lambda\big[\big(1 - \Gamma(a)\big)\big(1 - D(E)\big)Y^{gr} - C\big] + \mu[E - \gamma(1 - a)Y^{gr}]$

Note: here $\frac{\partial U}{\partial C}$ *appears in the numerator and denominator, but in dynamic setting this will change, because* $\partial W/\partial E$ *will include future damage*

Comparison with definition in lecture

- In our static model:

$$SCC = -\frac{\frac{\partial W}{\partial E}}{\frac{\partial W}{\partial C}} = -\frac{\frac{\partial U}{\partial E}}{\frac{\partial U}{\partial C}} = -\frac{\frac{\partial U}{\partial C}\frac{\partial C}{\partial E}}{\frac{\partial U}{\partial C}}$$

- In the lecture discussed SCC in dynamic setting:

$$SCC = \frac{\sum_{t=1}^{T} \Delta C_t \frac{\partial W}{\partial C_t}}{\frac{\partial W}{\partial C_1}}$$

- ΔC_t = monetary loss of *global* consumption at time t, caused by emitting one ton of CO_2 today
- W = social welfare, aggregated over time horizon $t = 1, \dots, T$
- $\frac{\partial W}{\partial C_t}$ = change in social welfare caused by one additional Euro of consumption at time t

Labor supply in E-Macro

- Assume: utility over consumption & labour
  - *See Example Redistribution*

- Production results in emissions => emissions result in damage (negative externality)

- No abatement technology => can only reduce emissions by producing less

- Use Social Planner

Labor supply in E-Macro

- Additive separate utility from consumption (+) and labour (-):
$$U(C, L)$$
- Production (concave): $Y^{gr} = F(L)$
- Emissions: $E = \gamma Y^{gr}$
- Net output: $Y^{net} = \big(1 - D(E)\big)Y^{gr}$
- Consumption: $C = Y^{net}$

Lagrangian:

$$\mathrm{G} = U(C, L) + \lambda\Big[\big(1 - D(E)\big)Y^{gr} - C\Big] + \mu[E - \gamma Y^{gr}] + \varphi[F(L) - Y^{gr}]$$

Note: choosing L determines everything

❖ What do you expect to happen with optimal labour supply? (compared to no damage from emissions)

Optimization

$$G = U(C,L) + \lambda\left[\left(1 - D(E)\right)Y^{gr} - C\right] + \mu[E - \gamma Y^{gr}] + \varphi[F(L) - Y^{gr}]$$

FOCs w.r.t. C, L, E, Y^{gr} :

$$U_C = \lambda$$

Marginal utility cons. = shadow price of consumption (in utility units)

$$-U_L = \varphi F'(L)$$

*-Marg. ut. labor = shadow price of gross output (in utility units) * marg. prod. labor*

$$\mu = \lambda D'(E)Y^{gr}$$

Shadow price of emissions = marg. damage (in utility units)

$$\varphi = \lambda\left(1 - D(E)\right) - \gamma\mu$$

Shadow price of gross output (in utility units) = λ marg. change in cons. – marg. change in emissions* shadow price of emissions*

Simplifying

$$G = U(C,L) + \lambda\left[\left(1-D(E)\right)Y^{gr} - C\right] + \mu\left[E - \gamma Y^{gr}\right] + \varphi\left[F(L) - Y^{gr}\right]$$

$$-U_L = \varphi F'(L)$$

$$-U_L = \left[\lambda\left(1-D(E)\right) - \gamma\mu\right]F'(L)$$

$$-U_L = \left[\lambda\left(1-D(E)\right) - \gamma\lambda D'(E)Y^{gr}\right]F'(L)$$

$$\boldsymbol{-U_L = \lambda\left[\left(1-D(E)\right) - \gamma D'(E)Y^{gr}\right]F'(L)}$$

*-Marg. ut. labour = marg. ut. of cons. * effective marg. prod. labour (net of damage (level), and taking account of marg. damage)*

$\Rightarrow$Effective marg. productivity of labour goes down when damages are included

$$\left(1-D(E)\right) - \gamma D'(E)Y^{gr} < 1$$

- But does not necessarily mean labour supply goes down (see Extra slides)

Extra: Income & Substitution effect with additive separable utility function

- Common macroeconomic utility:

$$U(c,l) = \frac{c^{1-\nu} - 1}{1-\nu} - \chi \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

$$U_c = c^{-\nu}$$

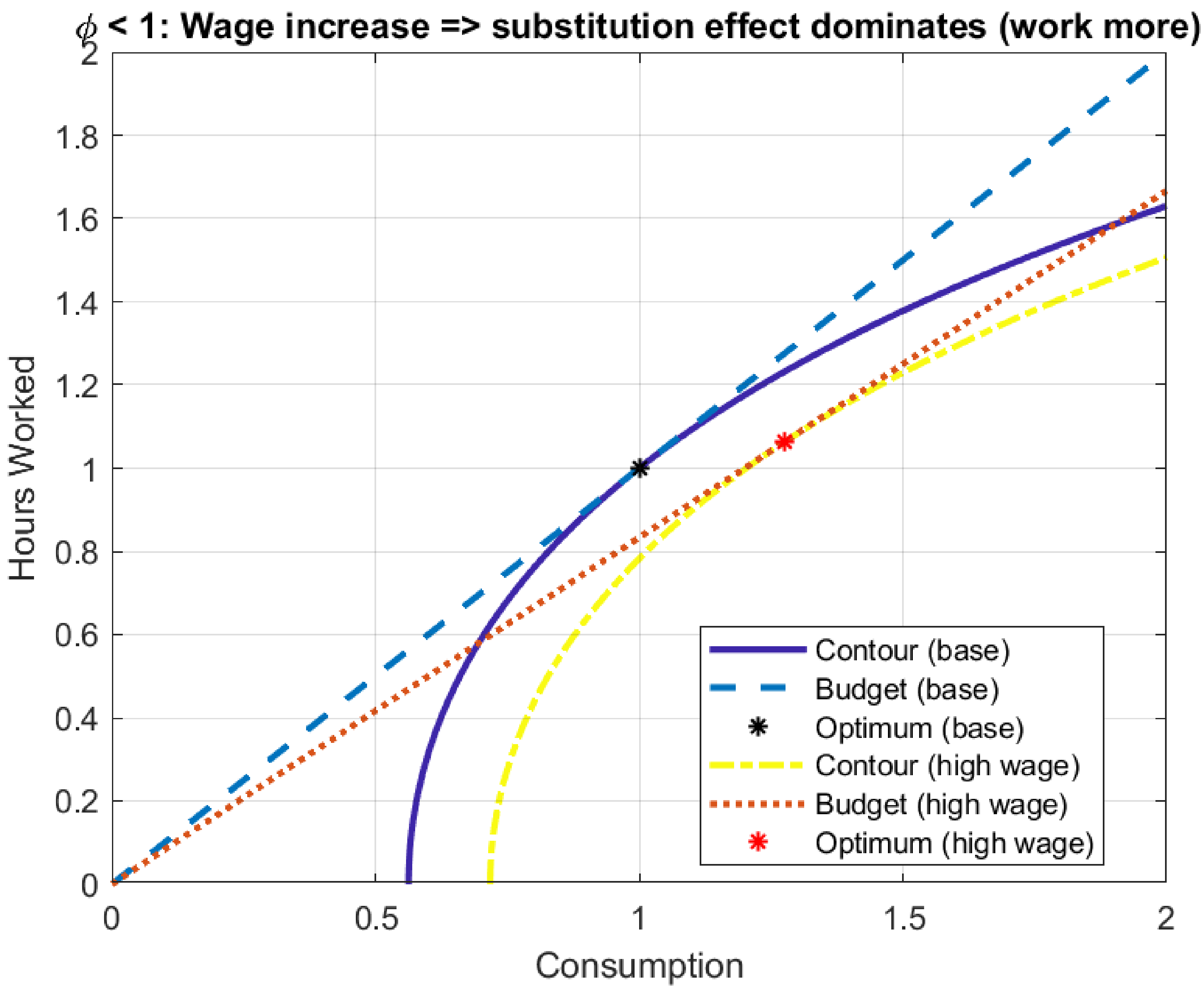
$$U_l = -\chi l^{\frac{1}{\varphi}}$$

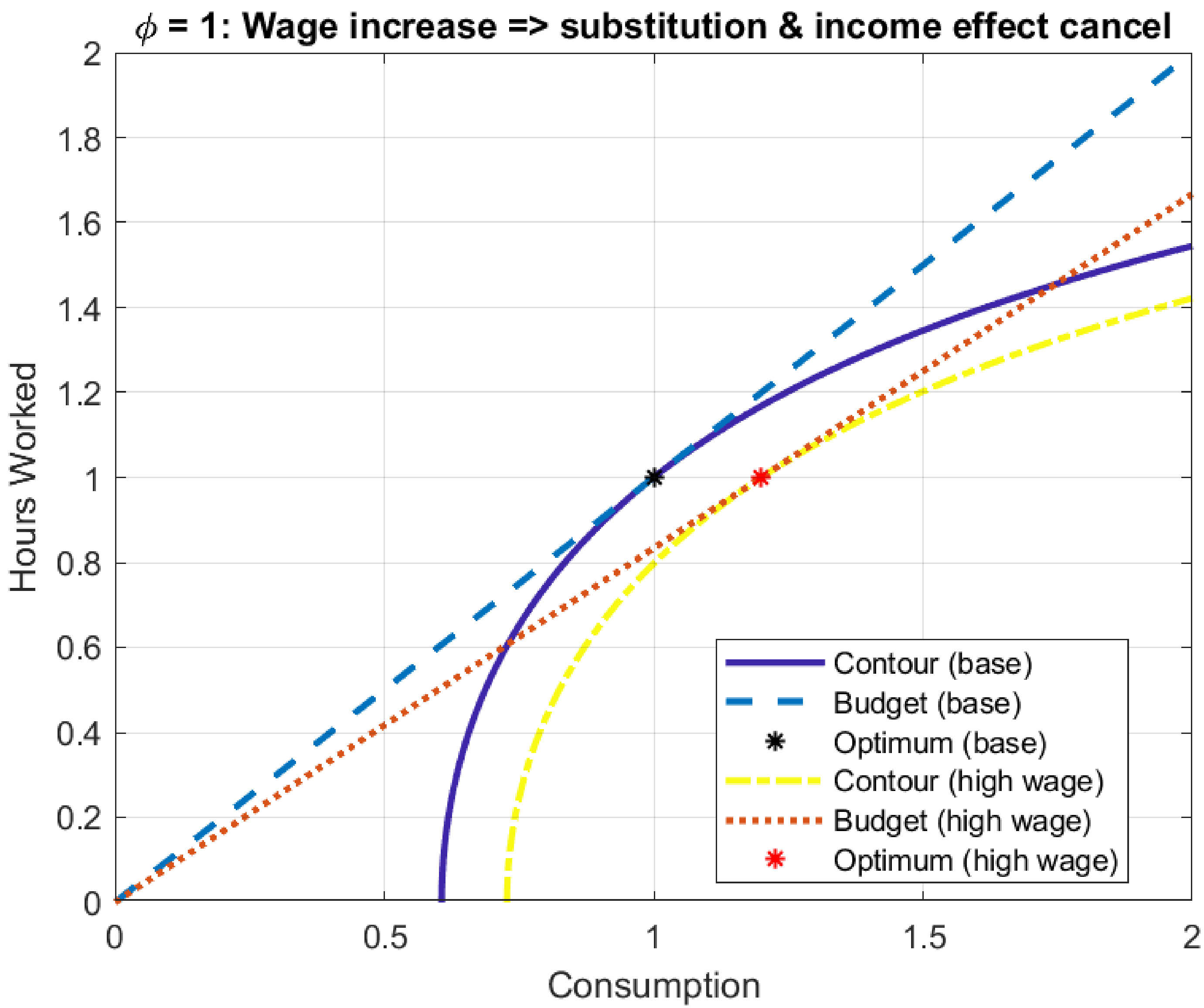
- Assume $c = wl$: $-\frac{U_l}{U_c} = w$ => $\frac{\chi l^{\frac{1}{\varphi}}}{w^{-\nu} l^{-\nu}} = w$ =>

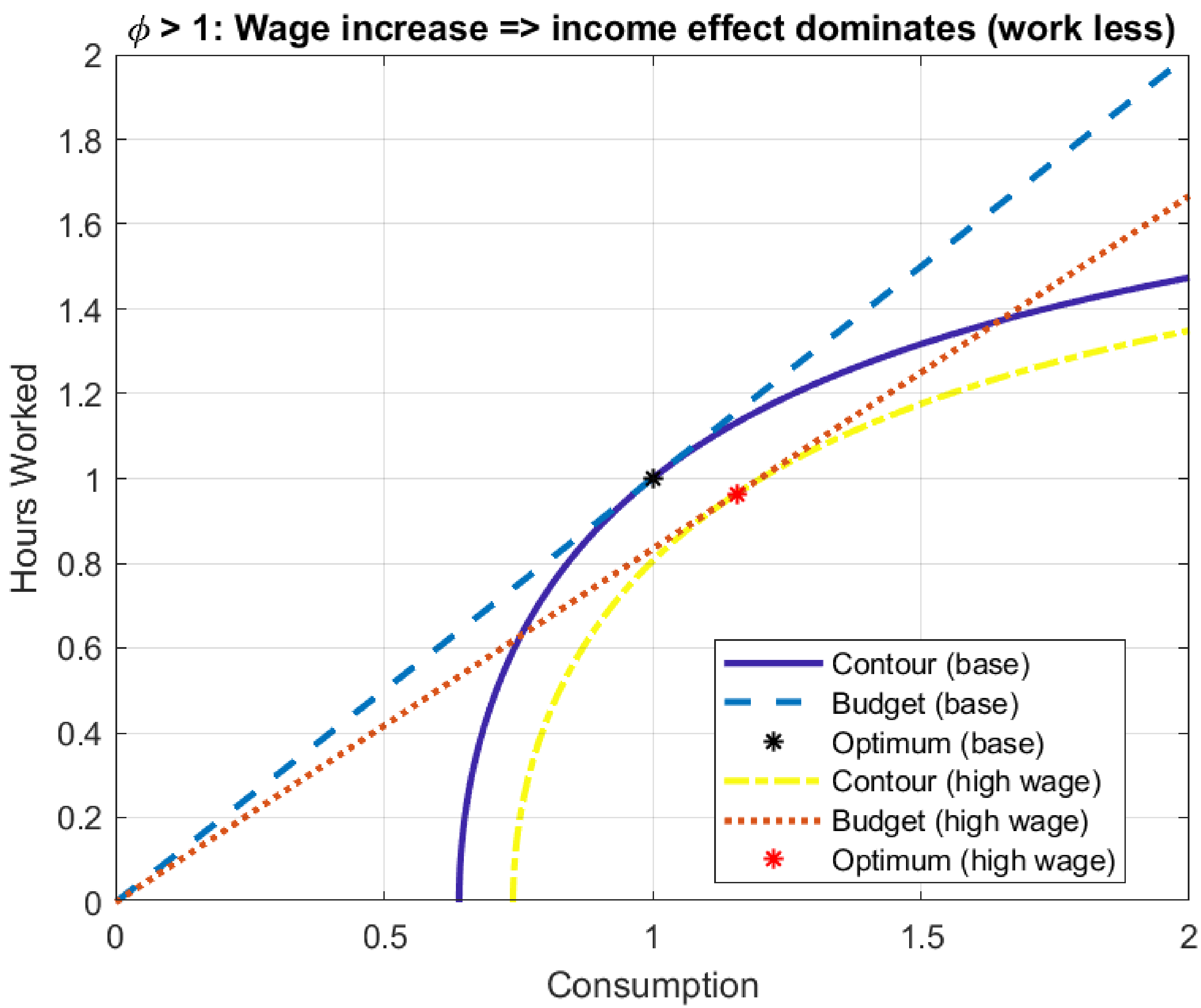
$$\chi l^{\frac{\nu\varphi+1}{\varphi}} = w^{1-\nu}$$

Note: $\frac{\nu\varphi + 1}{\varphi} > 0$

- $\nu < 1$ => $\frac{dl}{dw} > 0$: substitution effect dominates
- $\nu > 1$ => $\frac{dl}{dw} < 0$: income effect dominates
- $\nu = 1$ (*log utility*) => $\frac{dl}{dw} = 0$: income effect and substitution effect cancel


ϕ < 1: Wage increase => substitution effect dominates (work more)
Hours Worked
Consumption
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
0.5
1
1.5
2
Contour (base)
Budget (base)
Optimum (base)
Contour (high wage)
Budget (high wage)
Optimum (high wage)


ϕ = 1: Wage increase => substitution & income effect cancel
Hours Worked
Consumption
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
0.5
1
1.5
2
Contour (base)
Budget (base)
Optimum (base)
Contour (high wage)
Budget (high wage)
Optimum (high wage)



ϕ > 1: Wage increase => income effect dominates (work less)
Hours Worked
Consumption
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
0.5
1
1.5
2
Contour (base)
Budget (base)
Optimum (base)
Contour (high wage)
Budget (high wage)
Optimum (high wage)