STAT387 HOMEWORK#3

INTRODUCTION TO STATISTICAL LEARNING

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Due February 22, 11:59 PM

Conceptual

1. 3(a, b; page 220)

We now review k-fold cross-validation.
(a) Explain how k-fold cross-validation is implemented.

The *k-fold cross-validation* is implemented by randomly dividing the set of observations into k groups, or folds, of approximately equal size. The first fold is treated in a validation set (test set), and the method is fit on the remaining $k - 1$ folds. The mean squared error, MSE_1 is computed on the observations in the held-out fold. The procedure is repeated k times. Each time, a different group of observations is treated as a validation set. This process results in k estimates of the test error, $MSE_1, MSE_2, \cdots, MSE_k$. And we compute the average of these errors.

(b) What are the advantages and disadvantages of k-fold cross-validation relative to:

i. The validation set approach?

Advantages of k-fold cross validation: The validation set approach's estimate of the **test error rate can be highly variable**, depending on precisely which observations are included in the training set and which observations are included in the validation set. Also, validation set error rate may **tend to overestimate** the test error rate for the model fit on the entire data set, since it only uses half of the sample to fit the model (in general, a larger sample size leads to lower test error).

Disadvantages of k-fold cross validation: Compare to the k-fold cross validation, the validation set approach is conceptually **simple and easy to implement**. Also, it takes **less computation** since it only fits the model once.

ii. LOOCV?

Advantages of k-fold cross validation: LOOCV requires fitting the statistical learning method n times. This means, it is possible that this method **could be computationally expensive** than k-fold cross validation. Also, k-fold cross validation often gives more accurate estimates of the test error rate than does LOOCV.

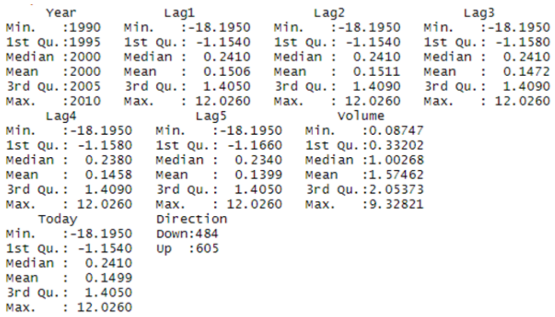
Disadvantages of k-fold cross validation: LOOCV has **less bias** than k-fold cross validation since it uses almost all of the points of the data set, nearly unbiased.

Applied

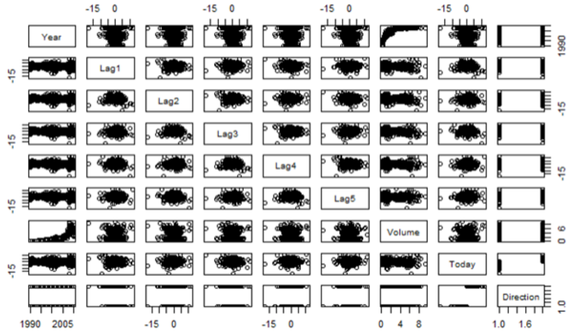
2. 13 (a, b, c, d, e, f, g, h, i; page 193)

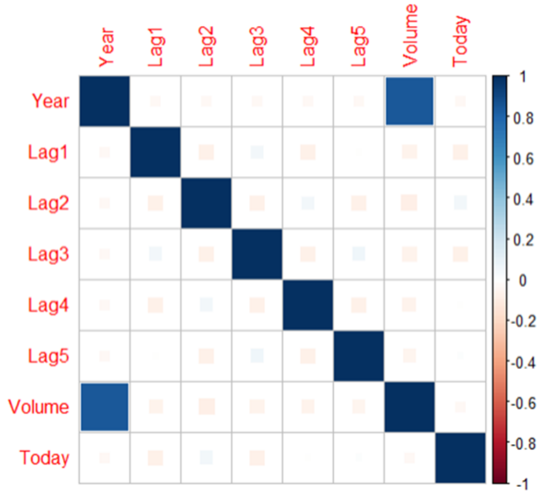
This question should be answered using the Weekly data set, which is part of the ISLR2 package. This data is similar in nature to the Smarket data from this chapter's lab, except that it contains 1,089 weekly returns for 21 years, from the beginning of 1990 to the end of 2010.

(a) Produce some numerical and graphical summaries of the Weekly data. Do there appear to be any patterns?



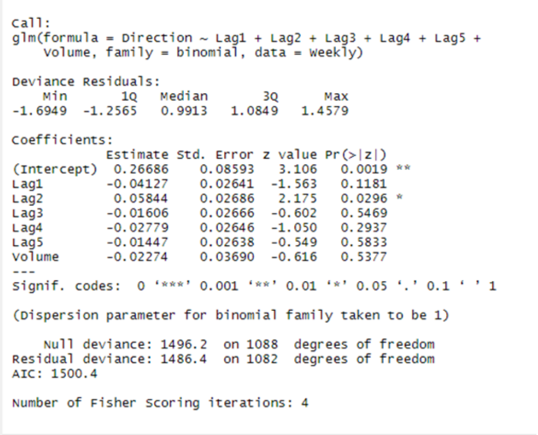
```
      Year           Lag1               Lag2               Lag3
 Min.   :1990   Min.   :-18.1950   Min.   :-18.1950   Min.   :-18.1950
 1st Qu.:1995   1st Qu.: -1.1540   1st Qu.: -1.1540   1st Qu.: -1.1580
 Median :2000   Median :  0.2410   Median :  0.2410   Median :  0.2410
 Mean   :2000   Mean   :  0.1506   Mean   :  0.1511   Mean   :  0.1472
 3rd Qu.:2005   3rd Qu.:  1.4050   3rd Qu.:  1.4090   3rd Qu.:  1.4090
 Max.   :2010   Max.   : 12.0260   Max.   : 12.0260   Max.   : 12.0260
      Lag4               Lag5              Volume
 Min.   :-18.1950   Min.   :-18.1950   Min.   :0.08747
 1st Qu.: -1.1580   1st Qu.: -1.1660   1st Qu.:0.33202
 Median :  0.2380   Median :  0.2340   Median :1.00268
 Mean   :  0.1458   Mean   :  0.1399   Mean   :1.57462
 3rd Qu.:  1.4090   3rd Qu.:  1.4050   3rd Qu.:2.05373
 Max.   : 12.0260   Max.   : 12.0260   Max.   :9.32821
     Today          Direction
 Min.   :-18.1950   Down:484
 1st Qu.: -1.1540   Up  :605
 Median :  0.2410
 Mean   :  0.1499
 3rd Qu.:  1.4050
 Max.   : 12.0260
```





Through the third summary of data, the variables that appear to have any significant linear relation are Year and Volume. The correlation plot does not illustrate that any other variables are linearly related.

(b) Use the full data set to perform a logistic regression with Direction as the response and the five lag variables plus Volume as predictors. Use the summary function to print the results. Do any of the predictors appear to be statistically significant? If so, which ones?

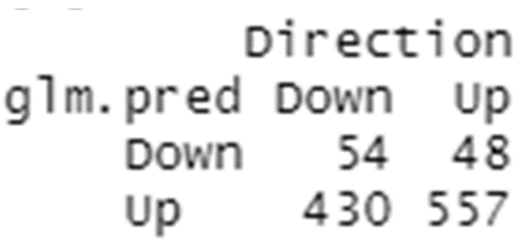


```
Call:
glm(formula = Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 +
    Volume, family = binomial, data = Weekly)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.6949  -1.2565   0.9913   1.0849   1.4579

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  0.26686    0.08593   3.106   0.0019 **
Lag1        -0.04127    0.02641  -1.563   0.1181
Lag2         0.05844    0.02686   2.175   0.0296 *
Lag3        -0.01606    0.02666  -0.602   0.5469
Lag4        -0.02779    0.02646  -1.050   0.2937
Lag5        -0.01447    0.02638  -0.549   0.5833
Volume      -0.02274    0.03690  -0.616   0.5377
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 1496.2  on 1088  degrees of freedom
Residual deviance: 1486.4  on 1082  degrees of freedom
AIC: 1500.4

Number of Fisher Scoring iterations: 4
```

*Lag*2 is the only variable that was statistically significant at the level of significance $\alpha = 0.05$. The other variables' p-values are all greater than 0.05.

(c) Compute the confusion matrix and overall fraction of correct predictions. Explain what the confusion matrix is telling you about the types of mistakes

made by logistic regression.
The confusion matrix is:



```
         Direction
glm.pred Down  Up
    Down   54  48
    Up    430 557
```

The overall fraction of correct predictions is:

$$\frac{TN+TP}{TN+FP+FN+TP} = \frac{54+557}{54+48+430+557} = \frac{611}{1089} = 0.5611 \approx 56.11\%$$

This illustrates that the model predicted the weekly market trend correctly 56.11%.
To talk about the types of mistakes made by logistic regression, there are false positive rate and false negative rate. The false positives are the cases that have been predicted as positive but they do not have that disease. And the false positive rate can be calculated as:

$$\frac{the\ number\ of\ negative\ events\ wrongly\ categorized\ as\ positive\ (false\ positives)}{the\ total\ number\ of actual\ negative\ events\ (regardless\ of\ classification)}.$$

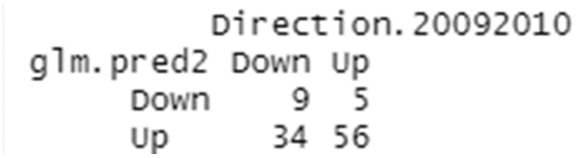
$$falsepositive = \frac{430}{54+430} = 0.8884 \approx 88.84\%$$

Now, false negatives are the cases that have been predicted as negative but they actually have that disease.

$$\frac{the\ number\ of\ positive\ events\ wrongly\ categorized\ as\ negative\ (false\ negatives)}{the\ total\ number\ of actual\ positive\ events\ (regardless\ of\ classification)}.$$

$$falsepositive = \frac{48}{48+557} = 0.9207 \approx 92.07\%$$

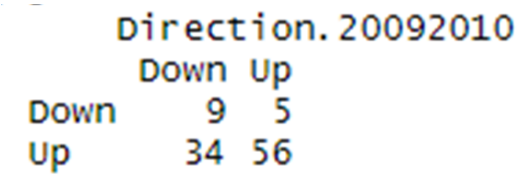
(d) Now fit the logistic regression model using a training data period from 1990 to 2008, with $Lag2$ as the only predictor. Compute the confusion matrix and the overall fraction of correct predictions for the held out data (that is, the data from 2009 and 2010).
The confusion matrix is:



```
          Direction.20092010
glm.pred2 Down Up
     Down    9  5
     Up     34 56
```

The overall fraction of correct predictions for the held out data is: $(9+56)/(9+5+34+56) = 65/104 = 0.6250$. This means, when we split up the whole Weekly dataset into a training and test dataset, the model correctly predicted weekly trends at rate of 62.5%, which is an improvement from the model that used the whole dataset.

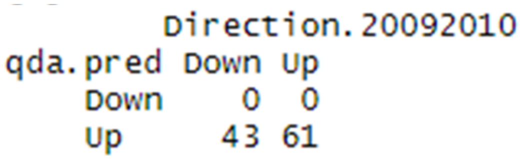
(e) Repeat (d) using LDA.
The confusion matrix is:



```
         Direction.20092010
          Down Up
  Down       9  5
  Up        34 56
```

The overall fraction of correct predictions for the held out data is: $(9+56)/(9+5+34+56) = 65/104 = 0.6250$.

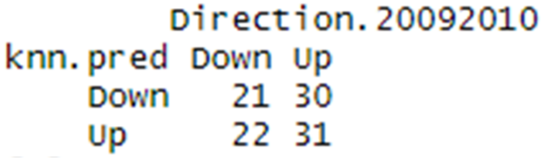
(f) Repeat (d) using QDA.
The confusion matrix is:



```
        Direction.20092010
qda.pred Down Up
    Down    0  0
    Up     43 61
```

The overall fraction of correct predictions for the held out data is: $(0+61)/(0+0+43+61) = 61/104 = 0.5865$.

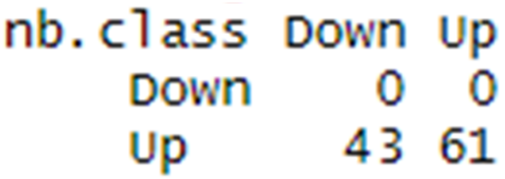
(g) Repeat (d) using KNN with $K = 1$.
The confusion matrix is:



```
        Direction.20092010
knn.pred Down Up
    Down   21 30
    Up     22 31
```

The overall fraction of correct predictions for the held out data is: $(21+31)/(21+30+22+31) = 52/104 = 0.5$.

(h) Repeat (d) using naive Bayes.
The confusion matrix is:



```
nb.class Down Up
    Down    0  0
    Up     43 61
```

The overall fraction of correct predictions for the held out data is: $(0+61)/(0+0+43+61) = 61/104 = 0.5865$.

(i) Which of these methods appears to provide the best results on this data?

The methods that appear to provide the best results on this data are the Logistic Regression and Linear Discriminant Analysis. These both are having rates of $0.6250 = 62.5\%$.

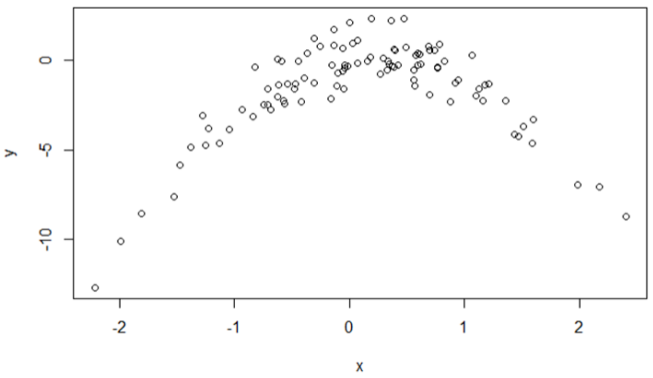
3. 8 (a, b, c; pages 222-223)

We will now perform cross-validation on a simulated data set.
(a) Generate a simulated data set as follows:

```
> set.seed (1)
> x <- rnorm (100)
> y <- x - 2 * x ^2 + rnorm (100)
```

In this data set, what is n and what is p? Write out the model used to generate the data in equation form.

n is 100 and p is 2 (x and x^2) since the model equation is $Y = X - 2X^2 + \epsilon$.
(b) Create a scatterplot of X against Y. Comment on what you find.



I could see that the data shows us a curved relationship.

(c) Set a random seed, and then compute the $LOOCV$ errors that result from fitting the following four models using least squares:
i. $Y = \beta_0 + \beta_1 X + \epsilon$

The computed $LOOCV$ error for i is: 7.288162

ii. $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \epsilon$

The computed $LOOCV$ error for ii is: 0.9374236

iii. $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + \epsilon$

The computed $LOOCV$ error for iii is: 0.9566218

iv. $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + \beta_4 X^4 + \epsilon$
The computed *LOOCV* error for *iv* is: 0.9539049

Note you may find it helpful to use the data.frame() function to create a single data set containing both X and Y.

R Code

```r
# Applied

####################################
## 2.13 (a, b, c, d, e, f, g, h, i)
####################################
### Problem (a)
library(corrplot)
attach(Weekly)
summary(Weekly)
plot(Weekly)
corrplot(cor(Weekly[, -9]), method = "square")



### Problem (b)
glm.fit = glm(Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 + Volume, data = Weekly, family = binomial)
summary(glm.fit)



### Problem (c)
glm.probs <- predict(glm.fit, type = "response")
glm.pred <- rep("Down", 1089)
glm.pred[glm.probs > .5] <- "Up"

# create a confusion matrix
table(glm.pred, Direction)



### Problem (d)
train = (Year < 2009)
test = Weekly[!train,]
glm.fit2 = glm(Direction ~ Lag2, data = Weekly, family = binomial, subset = train)

# compute the confusion matrix
glm.probs2 <- predict(glm.fit2, test, type = "response")
glm.pred2 <- rep("Down", 104) # 104 is the length of glm.probs2
glm.pred2[glm.probs2 > .5] <- "Up"
Direction.20092010 = Direction[!train]
table(glm.pred2, Direction.20092010)

mean(glm.pred2 == Direction.20092010)

```

```r


### Problem (e)
library(MASS)
lda.fit <- lda(Direction ~ Lag2, data = Weekly, family = binomial, subset = train)
lda.pred <- predict(lda.fit, test)
table(lda.pred$class, Direction.20092010)

mean(lda.pred == Direction.20092010)



### Problem (f)
qda.fit <- qda(Direction ~ Lag2, data = Weekly, subset = train)
qda.pred <- predict(qda.fit, test)$class
table(qda.pred, Direction.20092010)

mean(qda.pred == Direction.20092010)



### Problem (g)
library(class)
Weekly.train = as.matrix(Lag2[train])
Weekly.test = as.matrix(Lag2[!train])
train.Direction = Direction[train]
set.seed(1)
knn.pred = knn(Weekly.train, Weekly.test, train.Direction, k = 1)
table(knn.pred, Direction.20092010)

mean(knn.pred == Direction.20092010)



### Problem (h)
library(e1071)
nb.fit = naiveBayes(Direction ~ Lag2, data = Weekly, subset = train)
nb.class = predict(nb.fit, Direction.20092010)
table(nb.class, Direction.20092010)

mean(nb.class == Direction.20092010)









####################################
## 3.8 (a, b, c)
####################################
### Problem (a)
set.seed(1)
```

```r
x <- rnorm(100) # vector
y <- x - 2 * x^2 + rnorm(100)



### Problem (b)
plot(x, y)



### Problem (c)
# i
library(boot) # need this library for LOOCV
Data = data.frame(x,y)
glm.fit1 = glm(y ~ x)
cv.err = cv.glm(Data, glm.fit1)
cv.err$delta

# ii
glm.fit2 = glm(y~poly(x,2))
cv.err = cv.glm(Data, glm.fit2)$delta[1]

# iii
glm.fit3 = glm(y~poly(x,3))
cv.err = cv.glm(Data, glm.fit3)$delta[1]

# iv
glm.fit4 = glm(y~poly(x,4))
cv.err = cv.glm(Data, glm.fit4)$delta[1]

cv.error <- rep(0,4)
for (i in 1:4) {
  glm.fit <- glm(y ~ poly(x, i), data = Data)
  cv.error[i] <- cv.glm(Data, glm.fit)$delta[1]
}
cv.error
```