Control System Lab

Experiment No-1

Control System Toolbox and Symbolic Math Toolbox

Objectives:

- Using 'MATLAB base program + Control System toolbox', for control systems analysis and design.
- Brief introduction to Symbolic Math toolbox.

MATLAB is presented as an alternate method of solving control system problems. You are encouraged to solve problems first by hand and then by MATLAB so that insight is not lost through mechanized use of computer programs.

In Module 1, we have presented a tutorial on MATLAB window environment and some of its basic commands. In the present module, our objective is to take a primary look at MATLAB Control System Toolbox, which expands MATLAB base program to include control-system specific commands. In addition, presented is a MATLAB enhancement - Symbolic Math Toolbox, that gives added functionality to MATLAB base program and Control System Toolbox.

Control System Toolbox

Control System Toolbox is a collection of commands to be used for control systems' analysis and design. We will be using only some of these commands, because of the limited nature of course profile. Description of these commands will be distributed in different modules.

In this module, we will present commands related to transfer functions and system responses. To see all the commands in the Control System Toolbox and their functionalities, type **help control** in the MATLAB command window. MATLAB will respond with

>> help control

Find all the commands appeared in the screen

MODELDEV/CONTROL

In this module, we will learn how to represent transfer functions in the MATLAB, partial fraction expansion of rational expressions, representation of transfer functions as LTI objects, and to obtain time domain responses of LTI systems. Important commands for this module are:

roots – Find polynomial roots
poly – Convert roots to polynomial
polyval – Evaluate polynomial value
conv – Convolution and polynomial multiplication
deconv – Deconvolution and polynomial division
residue – Partial-fraction expansion (residues)
tf – Creation of transfer functions or conversion to transfer functions
pole – Compute the poles of LTI models
zero – Transmission zeros of LTI systems
tfdada – Quick access to transfer function data
zpkdata – Quick access to zero-pole-gain data
pzmap – Pole-zero map of LTI models
zpk – Create zero-pole-gain models or convert to zero-pole-gain format
step – Step response of LTI models
impulse – Impulse response of LTI models
lsim – Simulate time response of LTI models to arbitrary inputs
gensig – Periodic signal generator for time response simulations with lsim

Polynomials

Consider a polynomial '$s^3 + 3s^2 + 4$', to which we attach the variable name **p**. MATLAB can interpret a vector of length $n+1$ as the coefficients of an n^{th}-order polynomial. Coefficients of the polynomial are interpreted in descending powers. Thus, if the polynomial is missing any coefficient, we must enter zeros in the appropriate places in the vector. For example, polynomial **p** can be represented by the vector **[1 3 0 4]** in MATLAB. For example:

```
>> p=[1 3 0 4]
```

p =

1 3 0 4

Roots of the polynomial can be obtained by **roots** command.

>> r=roots(p) ↵

r =

-3.3553

0.1777 + 1.0773i

0.1777 - 1.0773i

Given roots of a polynomial in a vector, a vector of polynomial coefficients can be obtained by the command **poly**.

>> p=poly(r) ↵

p =

1.0000 3.0000 0.0000 4.0000

Use the command **polyval(p, s)** to evaluate the polynomial represented by vector **p** at arbitrary value of **s**. For example, to evaluate the polynomial '$s^3 + 3s^2 + 4$' at $s = \sqrt{2}$, type

>> polyval(p, sqrt(2)) ↵

ans =

12.8284

The product of two polynomials is found by taking the convolution of their coefficients. The function **conv** will do this f

or us. Consider an example of

multiplying polynomial '$s^3 + 3s^2 + 4$' with '$s + 2$':

```
>> p1=[1 3 4]; ↵

>> p2=[1 2]; ↵

>> p3=conv(p1,p2) ↵

p3 =

    1     5    10     8
```

The function **deconv** divides two polynomials and returns quotient as well as the remainder polynomial.

```
>> [q,r]=deconv(p1,p2) ↵

q =

    1     1

r =

    0     0     2
```

where **q** is the quotient and **r** is the remainder polynomial.

Exercise M2.1

Verify the deconvolution result given in vectors **q** and **r**.

Hint: Check whether **p1 = conv(q,p2) + r** or not?

Exercise M2.2

Let $G(s) = \dfrac{1}{500s^2}$ and $H(s) = \dfrac{s+1}{s+2}$. Obtain $M(s) = \dfrac{G(s)}{1-G(s)H(s)}$.

Consider the rational fractions of the form:

$$G(s) = \frac{b_0 s^m + b_1 s^{m-1} + \ldots + b_{m-1} s + b_m}{s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n} = \frac{N(s)}{D(s)}; m \le n$$

where the coefficients a_i and b_i are real constants, and m and n are integers.A fraction of the form *G*(*s*) can be expanded into partial fractions. To do this, first of all we factorize the denominator polynomial *D*(*s*) into n first-order factors. The roots of *D*(*s*) can be real or complex; distinct or repeated.

Let, vectors **N** and **D** specify the coefficients of numerator and denominator polynomials $N(s)$ and $D(s)$ respectively. The command **[A,p,K]=residue(N,D)**returns residues in column vector **A**, the roots of the denominator in column vector **p**, and the direct term in scalar **K**. If there are no multiple roots, the fraction $\frac{N(s)}{D(s)}$ can be represented as:

$$\frac{N(s)}{D(s)} = \frac{A(1)}{s - p(1)} + \frac{A(2)}{s - p(2)} + \ldots + \frac{A(n)}{s - p(n)} + K$$

If there are roots of multiplicity m_r , *i.e.* , $p(j) = \ldots = p(j + m_r - 1)$, then the expansion includes terms of the form:

$$\frac{A(j)}{(s - p(j))} + \frac{A(j+1)}{(s - p(j))^2} + \ldots + \frac{A(j + m_r - 1)}{(s - p(j))^{m_r}}$$

If $m < n$, *K* is empty (zero).

Supplying 3 arguments **A, p,** and **K** to **residue** converts the partial fraction expansion back to the polynomial with coefficients in **N** and **D.**

Consider the rational fraction:

$$\frac{N(s)}{D(s)} = \frac{10s + 40}{s^3 + 4s^2 + 3s}$$

MATLAB solution to partial fraction problem can be given by:

```
>> N=[10 40]; ↵
```

```
>> D=[1 4 3 0]; ↵
>> [A,p,K]=residue(N,D) ↵
A =
   1.6667
 -15.0000
  13.3333
p =
   -3
   -1
    0
K =
    [ ]
```

Exercise M2.4

Consider the rational fraction:

$$G(s)=\frac{K}{s(s+1)(s^2+2s+5)};K\geq 0$$

Obtain partial fraction form in terms of *K*. Solve using MATLAB for $K=5$ and countercheck your answer.

Exercise M2.5

Consider the rational fraction: $Y(s)=\dfrac{2s^2+3s+1}{s^2+4s+3}$.

Identify the points where:

1. $Y(s) = 0$ and

2. $Y(s) = \infty$

Note: The roots of the numerator polynomial, *i.e.* , $Y(s) = 0$, are known as the *zeros* of $Y(s)$ and the roots of the denominator polynomial, *i.e.* , $Y(s) = \infty$, are known as the poles of $Y(s)$.

Transfer Functions

Transfer functions can be represented in MATLAB as LTI (Linear Time Invariant) objects using numerator and denominator polynomials. Consider the transfer function given by $G(s) = \frac{s+1}{s^2+3s+1}$. It can be represented in MATLAB as:

```
>> num = [1 1];

>> den = [1 3 1];

>> G = tf(num,den)


Transfer function:

     s + 1
-----------------
s^2 + 3 s + 1
```

Example M2.2

The function **conv** has been used to multiply polynomials in the following MATLAB session for the transfer function

$$GH(s) = \frac{100(5s+1)(15s+1)}{s(3s+1)(10s+1)}$$

```
>> n1 = [5 1];

>> n2 = [15 1];

>> d1 = [1 0];

>> d2 = [3 1];

>> d3 = [10 1];

>> num = 100*conv(n1,n2);

>> den = conv(d1,conv(d2,d3));

>> GH = tf(num,den)


Transfer function:

7500 s^2 + 2000 s + 100
----------------------------------
   30 s^3 + 13 s^2 + s
```

To learn more about LTI objects given by **tf**, type **ltimodels tf** in MATLAB command window. Type **ltiprops tf** on MATLAB prompt to learn the properties associated with an LTI object represented by **tf**.

Transfer functions can also be entered directly in polynomial form as we enter them in the notebook using LTI objects. For example, observe the following MATLAB session.

>> s=tf('s') %Define 's' as an LTI object in polynomial form

Transfer function:

s

>> G1=150*(s^2+2*s+7)/[s*(s^2+5*s+4)] % Form G1(s) as an LTI transfer function

% in polynomial form.

Transfer function:

```
150 s^2 + 300 s + 1050
--------------------------------
     s^3 + 5 s^2 + 4s
```

>> G2=20*(s+2)*(s+4)/[(s+7)*(s+8)*(s+9)] % Form G2(s) as an LTI transfer

% function in polynomial form.

Transfer function:

```
    20 s^2 +120 s + 160
-----------------------------------
s^3 + 24 s^2 + 191s + 504
```

The commands **pole** and **zero** calculate the poles and zeros of LTI models.

>> pole(G1)

ans =

0

-4

-1

>> zero(G1)

ans =

-1.0000 + 2.4495i

-1.0000 - 2.4495i

To extract numerator and denominator polynomials, use the function **tfdata** .

>> [num,den]=tfdata(G,'v')

num =

0 1 1

den =

1 3 1

To extract zeros and poles of transfer function simultaneously, use the function **zpkdata** .

>> [z,p]=zpkdata(G,'v')

z =

-1

p =

-2.6180

-0.3820

If we know zeros and poles of the system with gain constant, the transfer function of LTI system can be constructed by **zpk** command. For example, to create a unity gain transfer function *G*3(*s*) with zero at -1 and two poles at -2.618 and -0.382, follow the MATLAB session given below.

>> G3=zpk(-1,[-2.618 -0.382],1)

Zero/pole/gain:

```
        (s+1)
--------------------------
(s+2.618)(s+0.382)
```

The polynomial transfer function created with **tf** can be converted to zero-pole-gain model by the command **zpk** and *vice versa* . The following MATLAB session gives the zero-pole-gain format of LTI system represented by *G*(*s*).

>> zpk(G)

Zero/pole/gain:

```
        (s+1)
--------------------------
(s+2.618)(s+0.382)
```

To observe the polynomial form of the transfer function *G*3(*s*), enter

>> tf(G3)

Transfer function:

```
        s+1
---------------------
   s^2 + 3s +1
```

To learn more about LTI objects given by **zpk**, type **ltimodels zpk** in MATLAB command window. Type **ltiprops zpk** on MATLAB prompt to learn the properties associated with an LTI object represented by **zpk**.

The function **pzmap(G)** plots the poles and zeros of the transfer function *G(s)* on complex plane. When used with two left hand side arguments, **[p,z] = pzmap(G)**, the function returns the poles and zeros of the system in two column vectors **p** and **z**. For example:

>> [p,z]=pzmap(G)

p =

-2.6180

-0.3820

z =

-1

System Response

Step and impulse responses of LTI objects can be obtained by the commands **step** and **impulse** . For example, to obtain the step response of the system represented in LTI object *G*, enter

>> step(G)

The MATLAB response to this command is shown in Fig. M2.1.

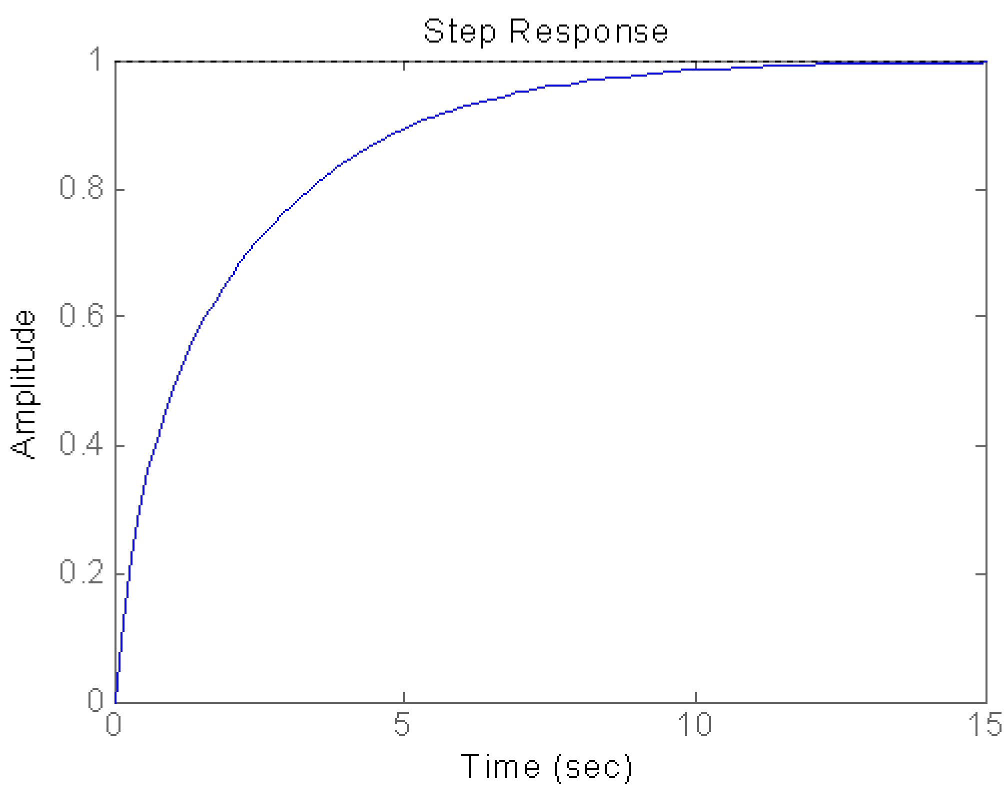


Fig. M2.1

To obtain the impulse response, enter

>> impulse(G)

The MATLAB response to this command is shown in Fig. M2.2.

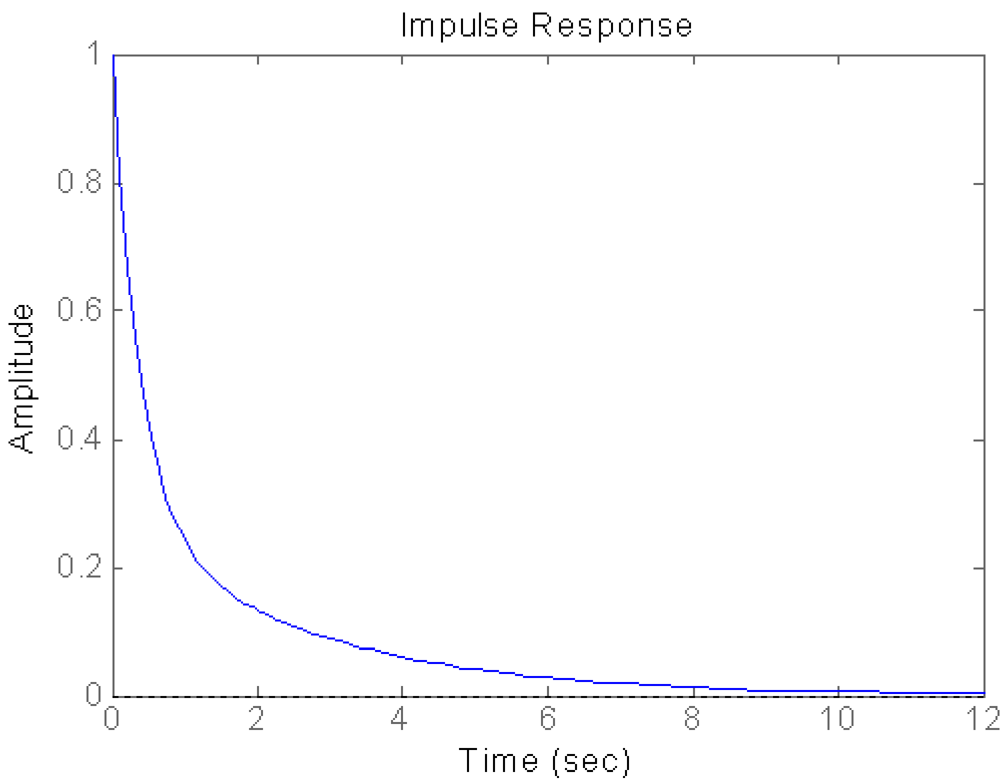


Fig. M2.2

Step and impulse response data can be collected into MATLAB variables by using two left hand arguments. For example, the following commands will collect step and impulse response amplitudes in **yt** and time samples in **t**.

[yt, t] = step(G)

[yt, t] = impulse(G)

Response of LTI systems to arbitrary inputs can be obtained by the command **lsim**. The command **lsim(G,u,t)** plots the time response of the LTI model **G** to the input signal described by **u** and **t**. The time vector **t** consists of regularly spaced time samples and **u** is a matrix with as many columns as inputs and whose i^{th} -row specifies the input value at time **t(i)**. Observe the following MATLAB session to obtain the time response of LTI system **G** to sinusoidal input of unity magnitude.

```
>> t=0:0.01:7;

>> u=sin(t);

>> lsim(G,u,t)
```

The response is shown in Fig. M2.3.

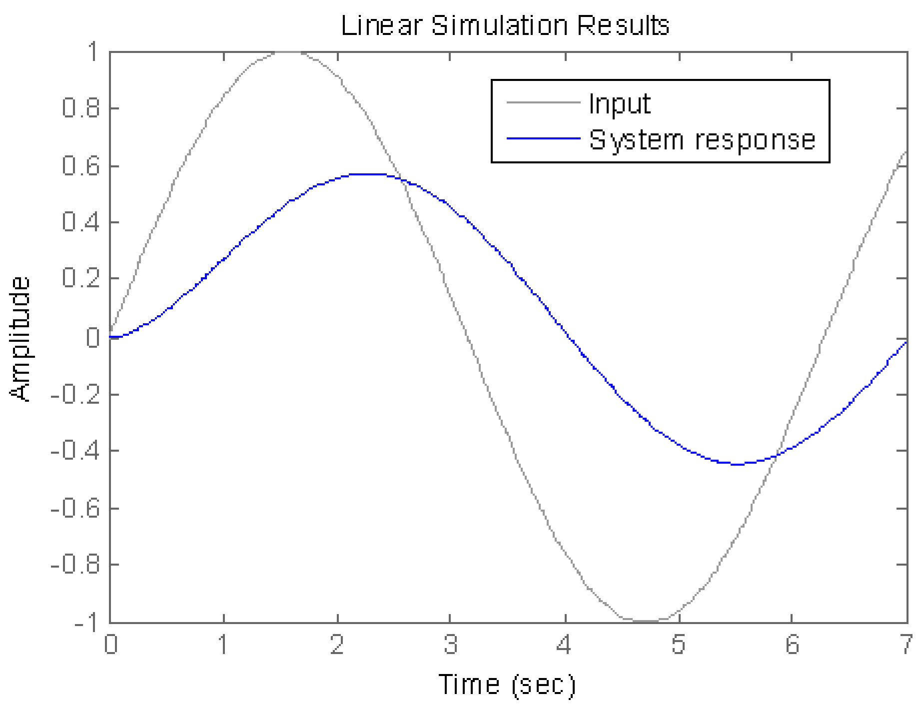


Fig. M2.3

Exercise M2.6

i. Obtain the response of $G(s) = \frac{s+1}{s^2+3s+1}$ to ramp and parabolic inputs using **lsim** command.

ii. Obtain the response of $G(s) = \frac{s+1}{s^2+3s+1}$ to ramp and parabolic inputs using **step** command.

The function **gensig** generates periodic signals for time response simulation with **lsim** function. It can generate sine, square, and periodic pulses. All generated signals have unit amplitude. Observe the following MATLAB session to simulate $G(s)$ for 20 seconds with a sine wave of period 5 seconds.

```
>> [u,t]=gensig( 'sin' ,5,20); %Sine wave with period 5 sec and duration 20 sec
```

```
>> lsim(G,u,t) %Simulate G(s) with u and t.
```

The response is shown in Fig. M2.4.

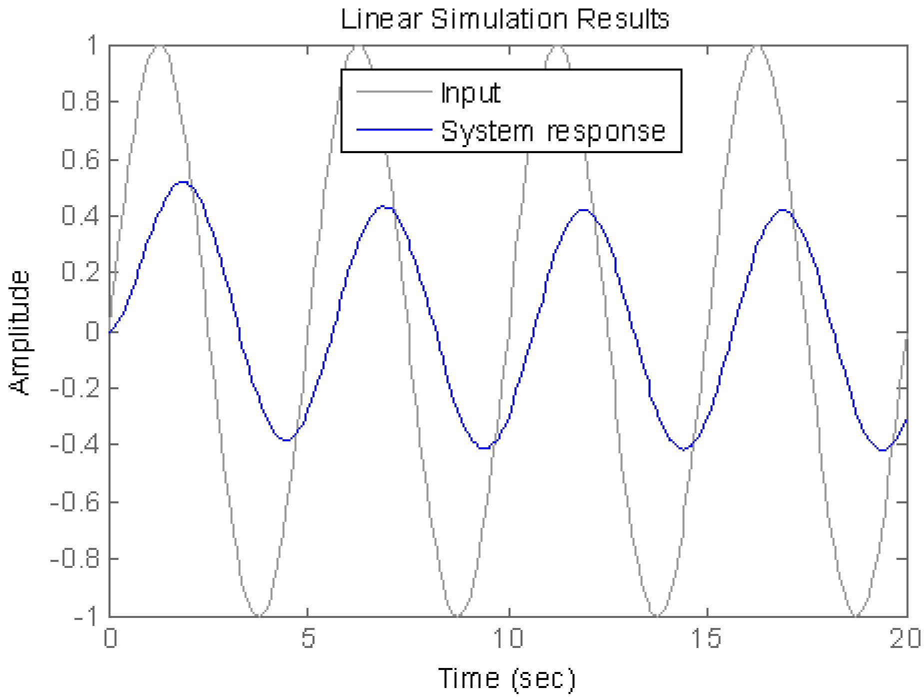


Fig. M2.4

Exercise M2.7

Generate square and pulse signals with the period of 4 seconds and obtain time response of $G(s) = \dfrac{s+1}{s^2+3s+1}$ for a duration of 30 seconds.

Example M2.3

The following MATLAB script calculates the step response of second-order system

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

with $\omega_n = 1$ and various values of ζ.

```
t=[0:0.1:12]; num=[1];
zeta1=0.1; den1=[1 2*zeta1   1];
zeta2=0.2; den2=[1 2*zeta2   1];
zeta3=0.4; den3=[1 2*zeta3   1];
zeta4=0.7; den4=[1 2*zeta4   1];
zeta5=1.0; den5=[1 2*zeta5   1];
zeta6=2.0; den6=[1 2*zeta6   1];
[y1,x]=step(num,den1,t); [y2,x]=step(num,den2,t);
[y3,x]=step(num,den3,t); [y4,x]=step(num,den4,t);
[y5,x]=step(num,den5,t); [y6,x]=step(num,den6,t);
plot(t,y1,t,y2,t,y3,t,y4,t,y5,t,y6)
xlabel( 't' ), ylabel( 'y(t)' )
grid
```

Response through the above MATLAB script is shown in Fig M2.5.

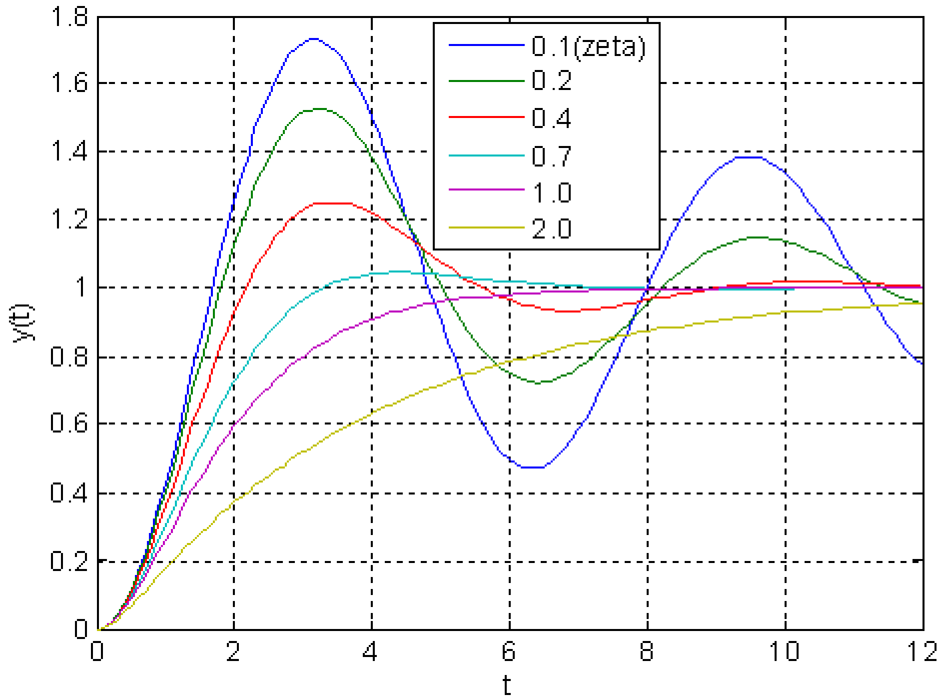


Fig. M2.5

The following MATLAB script calculates the impulse response of second-order system

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

with $\omega_n = 1$ and various values of ζ.

```
t=[0:0.1:10]; num=[1];

zeta1=0.1; den1=[1 2*zeta1   1];

zeta2=0.25; den2=[1 2*zeta2   1];

zeta3=0.5; den3=[1 2*zeta3   1];

zeta4=1.0; den4=[1 2*zeta4   1];
```

```
[y1,x,t]=impulse(num,den1,t);

[y2,x,t]=impulse(num,den2,t);

[y3,x,t]=impulse(num,den3,t);

[y4,x,t]=impulse(num,den4,t);

plot(t,y1,t,y2,t,y3,t,y4)

xlabel( 't' ), ylabel( 'y(t)' )

grid
```

Response through the above MATLAB script is shown in Fig M2.6.

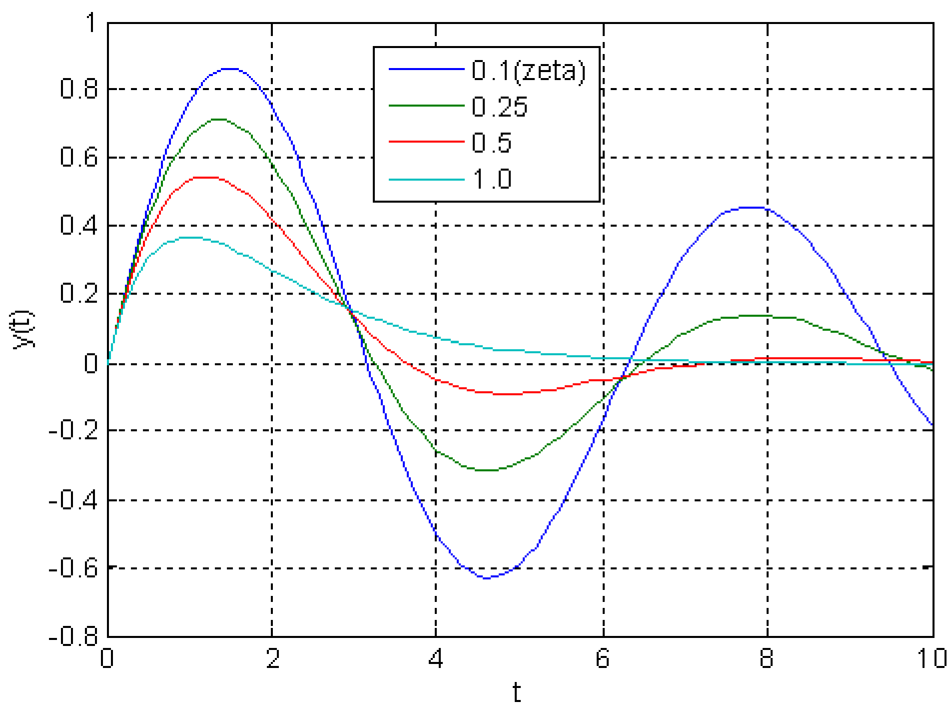


Fig. M2.6

Right-clicking away from the curves obtained by **step**, **impulse,** and **lsim** commands brings up a menu. From this menu, various time-response characteristics can be obtained and plotted on the graph

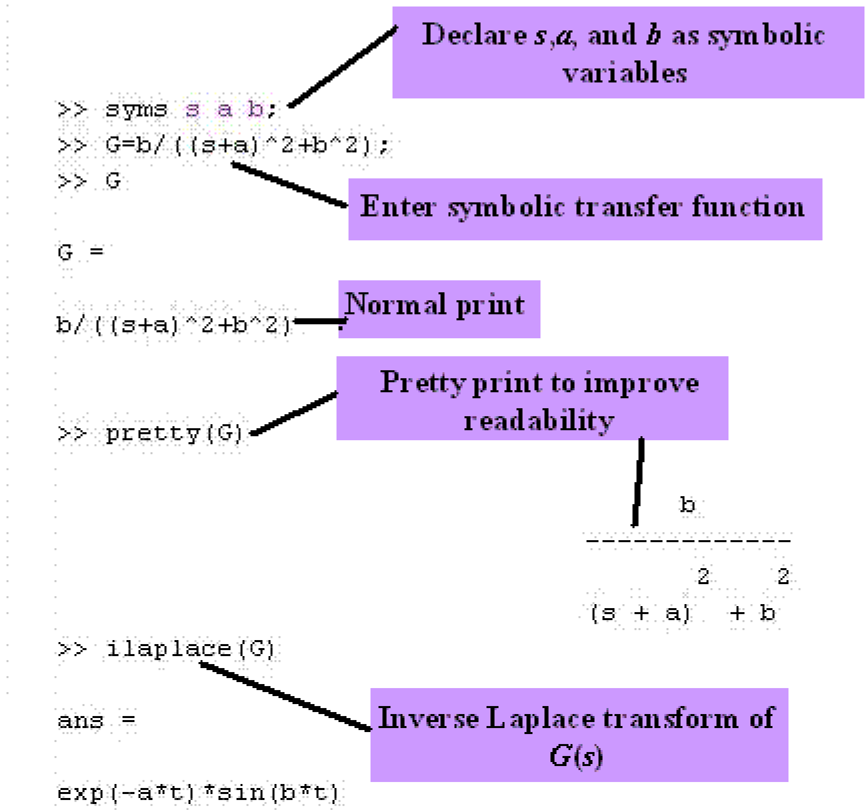
Symbolic Math Toolbox

MATLAB's Symbolic Math (**Symbolic Math** ematics) toolbox allows users to perform symbolic mathematical computations using MATLAB. The only basic requirement is to declare symbolic variables before they are used. For control systems analysis and design, symbolic math toolbox is particularly important because of the following:

1. Transfer functions and other expressions can be entered in symbolic form as we write them in the notebook.
2. Symbolic expressions can be manipulated algebraically and simplified.
3. It is straightforward to convert symbolic polynomials to the vectors of corresponding power-term coefficients.
4. Expressions can be ‘pretty printed' for clarity in the MATLAB command window.

Type **help symbolic** on the MATLAB command prompt to see all the functionalities available with Symbolic Math toolbox. In this section, we will learn commands of Symbolic Math toolbox particularly useful for control engineering.

First we demonstrate the power of Symbolic Math toolbox by calculating inverse Laplace transform. The following MATLAB session gives the steps performed during the calculation of inverse Laplace transform of $G(s) = \dfrac{b}{(s+a)^2 + b^2}$ using Symbolic Math.



Exercise M2.8

Initialize **s** and **T** as symbols. Using Symbolic Math tools, find the response of the first- order system $G(s) = \frac{K}{Ts+1}$,with the step excitation of strength A .

Hint: Find the inverse Laplace transform of $G(s) = \frac{A}{s} \times \frac{K}{Ts+1}$.

Exercise M2.9

Find manually the Laplace transform of $g(t) = 1 + \frac{1}{3}e^{-4t} - \frac{4}{3}e^{-t}$. Using Symbolic Math tools, declare **t** as symbolic variable and countercheck your result. Use the function **laplace(g)** to calculate the Laplace transform.

The function **simple(G)** finds simplest form of *G(s*) with the least number of terms. **simplify(G)** can be used to combine partial fractions. Symbolic Math toolbox also contains other commands that can change the look of the displayed result for readability and form. Some of these commands are:**collect(G)** - collects common-coefficient terms of *G*(*s*); **expand(G) -** expands product of factors of *G*(*s*); **factor(G)**-factors *G*(*s*); **vpa(G, places)** - stands for variable precision arithmetic (this command converts fractional symbolic terms into decimal terms with a specified number of decimal places).

Consider the function $G(s)=\frac{1}{s}+\frac{1}{3}\times\frac{1}{s+4}-\frac{4}{3}\times\frac{1}{s+1}$. MATLAB response to the simplification of $G(s)$ using the function **simplify** is shown below.

```
>> syms s

>> G = (1/s)+(1/3)*(1/(s+4))-(4/3)*(1/(s+1))

G =

1/s+1/3/(s+4)-4/3/(s+1)

>> pretty(G)


               1               1
1/s + 1/3 -------   - 4/3   -------
             s + 4           s + 1

>> pretty(simplify(G))


          4
-------------------
s (s + 4) (s + 1)
```

In the above example, the symbolic fractions 1/3 and 4/3 will be converted to 0.333 and 1.33 if the argument **places** in **vap(G, places)** is set to 3.

```
>> pretty(vpa(G,3))

         0.333     1.33
1/s + ---------- - ----------
         s + 4.     s + 1.
```

Exercise M2.10

Consider the function $g(t) = 1 + \frac{b}{a-b} \times e^{-at} - \frac{a}{a-b} \times e^{-bt}$. Show using Symbolic Math tools that the simplified form of its Laplace transform is $G(s) = \frac{ab}{s(s+a)(s+b)}$.

Basic mathematical operators +, $-$, $\times$, and $/$ are applicable to symbolic objects also. For example, simplification of the closed-loop system with forward gain K, forward-path transfer function $G(s) = \frac{2.5}{s(s+5)}$, and feedback-path transfer function $H(s) = \frac{1}{0.1s+1}$, is given by $M(s) = \frac{KG(s)}{1+G(s)H(s)}$. This can be done using Symbolic Math toolbox as follows

```
>> syms s K;

>> G = 2.5/(s*(s+5));

>> H = 1/(0.1*s+1);

>> M = K*G/(1+G*H)

M =

5/2*K/s/(s+5)/(1+5/2/s/(s+5)/(1/10*s+1))

>> pretty(M)

                                           K
5/2 ---------------------------------------------
              /                           1               \
   s  (s + 5) |  1 + 5/2      ----------------------      |
              \                s  (s + 5)  (1/10 s + 1)   /

>> pretty(simplify(M))

          (s + 10)  K
5/2  -------------------------
       3       2
      s + 15 s + 50 s + 25
```

With Symbolic Math toolbox, symbolic transfer functions can easily be converted to the LTI (Linear Time-Invariant) transfer function objects. This conversion is done in two steps. The first step uses the command **numden** to extract the symbolic numerator and denominator of *G* (*s*). The second step converts, separately, the numerator and denominator to vectors using the command **sym2poly** . The last step consists of forming the LTI transfer function object by using the vector representation of the transfer function's numerator and denominator. The command **sym2poly** doesn't work on symbolic expressions with more than one variable.

As an example, we form the LTI object from $M(s)$ obtained in the above example with $K = 1$.

```
>> K = 1;
>> M = K*G/(1+G*H);
>> [nums, dens] = numden(M)
```

Extract numerator and denominator polynomials in symbolic form

```
nums =

5*s*(s+5)*(s+10)


dens =

2*s*(s+5)*(s^3+15*s^2+50*s+25)


>> num = sym2poly(nums)

num =

     5    75   250     0

>> den = sym2poly(dens)

den =

     2    40   250   550   250     0
```

Convert numerator and denominator symbolic polynomials to the vectors of coefficients of power terms

```
>> M_LTI = tf(num,den)

Transfer function:
                5 s^3 + 75 s^2 + 250 s
------------------------------------------
2 s^5 + 40 s^4 + 250 s^3 + 550 s^2 + 250 s

>> M_LTI = minreal(M_LTI)

Transfer function:
       2.5 s + 25
------------------------
s^3 + 15 s^2 + 50 s + 25
```

Form LTI transfer function from numerator and denominator coefficient vectors

The command **poly2sym** converts polynomial coefficient vectors to symbolic polynomial. L