Further Improvement of State Feedback Controller Design for Networked Control Systems

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Abstract — *This paper proposes a design approach to synthesis static state-feedback controller for stabilization of networked control systems (NCSs). Variable delay is considered in the NCS model to represent both of the data packet latency and dropout. Utilizing an appropriate Lyapunov-Krasovskii functional, an improved sufficient condition is derived to determine the controller gain in terms of Linear Matrix Inequalities (LMIs). Illustrative examples demonstrate that the proposed method leads to remarkably less conservative results compared with the others' methods.*

Keywords: Networked Control Systems, Variable Delay, State Feedback Controller.

1. Introduction

A Networked Control System (NCS) is a feedback control structure wherein the control loop is closed through a communication network. The advantages of NCS such as low cost, simple installation and maintenance and high flexibility make it more and more popular in many applications including distributed industrial systems, intelligent traffic systems, cluster of unmanned air vehicles and multi-agent systems. However, the presence of communication network in control loop makes the analysis and design of the NCS more complicated compared with classical control systems. Main issues in this configuration are the packet dropout and latency in the communication channels between sensor, controller and actuator [1].

Many methods have been introduced in the recent years to design stabilizing controllers for networked control systems in which the packet dropout and delay occur in the sensor-to-controller and controller-to-actuator channels when sensor, actuator and controller exchange data. The problems induced by communication channel in networked control system were studied in [1]-[6]. In [2], networked control system with random transmission delays was modeled in the switched systems' framework. A predictive controller was designed to stabilize the system. In [3], the networked control system with data packet dropout was modeled as a jump linear system. A design method was introduced to obtain static stabilizing state-feedback controller by solving linear matrix inequalities (LMIs). In [4], Utilizing Lyapunov-Krasoskii theorem, a memoryless state-feedback controller was designed to stabilize a networked control system in which both the network-induced delay and the data packet dropout are taken into account. The feedback gain of the controller and the maximum allowable value of the network-induced delay were derived by solving a set of delay-dependent matrix inequalities. The result of [4] was improved in [5], wherein a state feedback was obtained in the similar manner to stabilize networked control system by solving a set of delay-dependent matrix inequalities. The Lyapunov-Razumikhin theorem was used in [6] to extract a delay-dependent condition for the stabilization of NCS in terms of LMIs.

In this note, inspired by the NCS design method developed in [4] and [5], a procedure is derived to design a memoryless state feedback stabilizing controller. Employing an appropriate Lyapunov-Krasovskii functional, a less conservative delay-dependent sufficient condition is obtained in terms of LMIs. Some numerical examples are presented to show that the resulting maximum allowable transfer delay to retain the stability of closed-loop system with the proposed controller is higher than the values obtained with the methods of [4] and [5].

The paper is organized as follows: In section II, networked control system model is described. Section III presents the main results of this paper wherein controller synthesis method is derived to determine the controller gain and allowable upper bound of the variable delay. In section IV, the suggested design method is illustrated by two examples and the results are compared with the existing design approaches. Finally, section V concludes this paper.

2. Problem Setup

The considered networked control system is shown in Fig 1, wherein the controller, sensor and the actuator are assumed to be separated and connected through a communication network. In the network, all the data are lumped together into one packet and transmitted at the same time (single packet transmission) and the sent packets are time stamped. The sensor is time-driven and controller and actuator are event-driven. The controller and actuator always use the new data packet and discard the old ones. When an old data packet arrives, it is dealt with as a packet loss. The controlled system is linear and time invariant and a zeroorder-hold is placed before the plant and the input is zero before the first controller packet arrives.

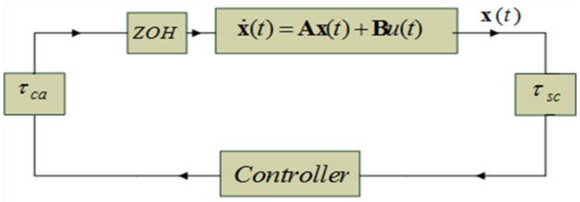

Fig. 1: Schematic Diagram of the NCS

Regarding the above assumptions on the NCS, the following dynamical equation can describe the closed-loop system behavior:

$$\dot{\mathbf{x}}(t)=\mathbf{A}\,\mathbf{x}(t)+\mathbf{B}\,\mathbf{u}(t),\qquad t\in[i_kh+\tau_{i_k},i_{k+1}h+\tau_{i_{k+1}}) \tag{1}$$

$$\mathbf{u}(t^+)=\mathbf{K}\,\mathbf{x}(t-\tau_{i_k}),\qquad t\in\left\{i_kh+\tau_{i_k},k=1,2,\cdots\right\} \tag{2}$$

where $\mathbf{x}(t)\in\mathbf{R}^n$ and $\mathrm{u(t)}\in\mathbf{R}^m$ are the state vector and the control input vector, respectively. $\mathbf{A}$ and $\mathbf{B}$ are two constant matrices with appropriate dimensions. h is the sampling period and i_k is an integer denoting the sampling instant of the state feedback for $\mathrm{k}=1,2,3,\ldots$. Transmission delays induced by the network is composed of two parts: sensor-to-controller delay τ_{sc} and controller-to-actuator delay τ_{ca}. If the feedback controller is static these two delays can be lumped together, so τ_{i_k} stands for the network-induced time delay from the instant i_kh ($\tau_{i_k}=\tau_{sc_{i_k}}+\tau_{ca_{i_k}}$). t^+ denotes the time interval ranging from $i_kh+\tau_{i_k}$ to $i_{k+1}h+\tau_{i_{k+1}}$. $\mathbf{K}$ is the feedback gain matrix which will be designed.

In the time interval $t\in[i_kh+\tau_{i_k},i_{k+1}h+\tau_{i_{k+1}})$, the closed-loop system model can be rewritten as follows:

$$\dot{\mathbf{x}}(t)=\mathbf{A}\,\mathbf{x}(t)+\mathbf{B}\mathbf{K}\,\mathbf{x}(i_kh),$$

By definition of $\tau(t)=t-i_kh$ one gets:

$$\dot{\mathbf{x}}(t)=\mathbf{A}\,\mathbf{x}(t)+\mathbf{B}\mathbf{K}\,\mathbf{x}(t-\tau(t)) \tag{3}$$

which is in the form of continuous-time system with delayed control input [7] with:

$$0\le\tau_{i_k}\le\tau(t)\le(i_{k+1}-i_k)h+\tau_{i_{k+1}}\le\eta \tag{4}$$

where η is supposed to be upper bound of tolerable delay to preserve the closed-loop stability and $\mathbf{x}(\theta)=\varphi(\theta)$ for $\theta\in[t_0-\eta,t_0]$ is the initial condition of the systems.

3. Controller Design

Since the networked control system was modeled as a linear time-delay system (3), the concepts of time-delay systems theory can be used for analysis and design of the NCS [7], [8]. The Lyapunov-Krasovskii functional candidate is considered as follows:

$$\begin{aligned}V(t)=\;&\zeta^T(t)\mathbf{P}\zeta(t)+\int_{t-\eta}^{t}\boldsymbol{\rho}^{\mathrm{T}}(s)\mathbf{Q}\boldsymbol{\rho}(s)ds+\\&+\int_{-\eta}^{0}\int_{t+\theta}^{t}\boldsymbol{\rho}^T(s)\mathbf{R}\boldsymbol{\rho}(s)dsd\theta\\&+\int_{-\eta}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{\mathbf{x}}^T(s)\mathbf{Z}\dot{\mathbf{x}}(s)dsd\lambda d\theta\end{aligned} \tag{5}$$

where $\zeta(t)=col\{\mathbf{x}(t),\,\mathbf{x}(t-\eta),\,\int_{t-\eta}^{t}\mathbf{x}^T(s)\,ds\}$, $\boldsymbol{\rho}(s)=col\,\{\mathbf{x}(s),\,\dot{\mathbf{x}}(s)\}$ and the matrices $\mathbf{P}=\mathbf{P}^T>0$, $\mathbf{Q}=\mathbf{Q}^T>0$, $\mathbf{R}=\mathbf{R}^T>0$ and $\mathbf{Z}=\mathbf{Z}^T>0$ will be determined. The sufficient condition to determine the mentioned matrices and feedback gain is given in the following theorem.

Theorem 1. For given scalars h and η if there exist following matrices

$$\mathbf{L}=\begin{bmatrix}\mathbf{L}_1\\\mathbf{L}_2\\\mathbf{L}_3\\\mathbf{L}_4\\\mathbf{L}_5\end{bmatrix},\;\mathbf{M}=\begin{bmatrix}\mathbf{M}_1\\\mathbf{M}_2\\\mathbf{M}_3\\\mathbf{M}_4\\\mathbf{M}_5\end{bmatrix},\;\mathbf{N}=\begin{bmatrix}\mathbf{N}_1\\\mathbf{N}_2\\\mathbf{N}_3\\\mathbf{N}_4\\\mathbf{N}_5\end{bmatrix},\;\mathbf{S}=\begin{bmatrix}\mathbf{S}_1\\\mathbf{S}_2\\\mathbf{S}_3\\\mathbf{S}_4\\\mathbf{S}_5\end{bmatrix},$$

$$\mathbf{P}=\begin{bmatrix}\mathbf{P}_{11}&\mathbf{P}_{12}&\mathbf{P}_{13}\\ *&\mathbf{P}_{22}&\mathbf{P}_{23}\\ *&*&\mathbf{P}_{33}\end{bmatrix}>0,$$

$$\mathbf{Q}=\begin{bmatrix}\mathbf{Q}_{11}&\mathbf{Q}_{12}\\ *&\mathbf{Q}_{22}\end{bmatrix}>0,$$

$$\mathbf{R}=\begin{bmatrix}\mathbf{R}_{11}&\mathbf{R}_{12}\\ *&\mathbf{R}_{22}\end{bmatrix}>0,$$

$$\mathbf{Z}>0$$

and $\mathbf{V}$ of appropriate dimensions such that the matrix inequality (6) –(8) holds, then the system (1) with the state feedback controller (2) and variable delay satisfying (4) is stable:

$$\boldsymbol{\Phi}<0 \tag{6}$$

$$\boldsymbol{\Pi}_1\ge0 \tag{7}$$

$$\boldsymbol{\Pi}_2\ge0 \tag{8}$$

$$\boldsymbol{\Phi}=\begin{bmatrix}\boldsymbol{\Omega}&\frac{1}{2}\eta^2\mathbf{L}\\ *&-\frac{1}{2}\eta^2\mathbf{Z}\end{bmatrix}$$

$$\boldsymbol{\Pi}_1=\begin{bmatrix}\mathbf{V}&\boldsymbol{\Gamma}_1\\ *&\mathbf{R}\end{bmatrix},\qquad\boldsymbol{\Gamma}_1=\begin{bmatrix}-\boldsymbol{\psi}+\mathbf{L}&\mathbf{S}\end{bmatrix}$$

$$\mathbf{\Pi}2=\begin{bmatrix}\mathbf{V} & \mathbf{\Gamma}_2\\ * & \mathbf{R}\end{bmatrix},\qquad \mathbf{\Gamma}_2=\begin{bmatrix}-\boldsymbol{\psi}+\mathbf{L} & \mathbf{N}\end{bmatrix}$$

$$\boldsymbol{\psi}=\begin{bmatrix}\mathbf{P}_{33} & 0 & -\mathbf{P}_{33} & \mathbf{P}_{13}^T & \mathbf{P}_{23}^T\end{bmatrix}^T$$

where $\mathbf{\Omega}=\mathbf{\Omega}_1+\mathbf{\Omega}_2+\mathbf{\Omega}_2^T+\mathbf{\Omega}_3+\mathbf{\Omega}_3^T+\eta\mathbf{V}$

$$\mathbf{\Omega}_1=\begin{bmatrix}(1,1) & 0 & -\mathbf{P}_{13}+\mathbf{P}_{23}^T & \mathbf{Q}_{12}+\eta\mathbf{R}_{12}+\mathbf{P}_{11} & \mathbf{P}_{12}\\ * & 0 & 0 & 0 & 0\\ * & * & (2,2) & \mathbf{P}_{12}^T & \mathbf{P}_{22}-\mathbf{Q}_{12}\\ * & * & * & (3,3) & 0\\ * & * & * & * & -\mathbf{Q}_{22}\end{bmatrix}$$

$(1,1)=\mathbf{Q}_{11}+\eta\mathbf{R}_{11}+\mathbf{P}_{13}+\mathbf{P}_{13}^T$,

$(2,2)=-\mathbf{P}_{23}-\mathbf{P}_{23}^T-\mathbf{Q}_{11}$

$(3,3)=\mathbf{Q}_{22}+\eta\mathbf{R}_{22}+\frac{1}{2}\eta^2\mathbf{Z}$

$\mathbf{\Omega}_2=\begin{bmatrix}\mathbf{N}+\eta\mathbf{L} & \mathbf{S}-\mathbf{N} & -\mathbf{S} & 0 & 0\end{bmatrix}$

$\mathbf{\Omega}_3=\begin{bmatrix}-\mathbf{MA} & -\mathbf{MBK} & 0 & \mathbf{M} & 0\end{bmatrix}$

The * denotes the symmetric entry in a symmetric matrix.

Proof: Calculating the time derivative of $V(t)$ in (5) with respect to t along the trajectories of the system (3) for $t\in[i_kh+\tau_{i_k},i_{k+1}h+\tau_{i_{k+1}})$ yields:

$$\begin{aligned}\dot V(t)&=2\zeta^T(t)\mathbf{P}\dot\zeta(t)+\boldsymbol{\rho}^T(t)\mathbf{Q}\boldsymbol{\rho}(t)-\boldsymbol{\rho}^T(t-\eta)\mathbf{Q}\boldsymbol{\rho}(t-\eta)\\ &+\eta\boldsymbol{\rho}^T(t)\mathbf{R}\boldsymbol{\rho}(t)-\int_{t-\eta}^{t}\boldsymbol{\rho}^{\mathrm T}(s)\mathbf{R}\boldsymbol{\rho}(s)ds \qquad (9)\\ &+\frac{\eta^2}{2}\dot{\mathbf{x}}^T(t)\mathbf{Z}\dot{\mathbf{x}}(t)-\int_{-\eta}^{0}\int_{t+\theta}^{t}\dot{\mathbf{x}}^T(s)\mathbf{Z}\dot{\mathbf{x}}(s)dsd\theta\end{aligned}$$

Inspired by [9], the term $\sum_{i=1}^{5}\alpha_i(t)$ is added to the $\dot V(t)$ in (9), where:

$$\alpha_1(t)=2\boldsymbol{\xi}^T(t)\mathbf{S}[\mathbf{x}(i_kh)-\mathbf{x}(t-\eta)-\int_{t-\eta}^{i_kh}\dot{\mathbf{x}}(s)ds]=0$$

$$\alpha_2(t)=2\boldsymbol{\xi}^T(t)\mathbf{M}[\dot{\mathbf{x}}(t)-\mathbf{Ax}(t)-\mathbf{BKx}(i_kh)]=0$$

$$\alpha_3(t)=2\boldsymbol{\xi}^T(t)\mathbf{N}[\mathbf{x}(t)-\mathbf{x}(i_kh)-\int_{i_kh}^{t}\dot{\mathbf{x}}(s)ds]=0$$

$$\begin{aligned}\alpha_4(t)&=2\boldsymbol{\xi}^T(t)\mathbf{L}[\eta\,\mathbf{x}(t)-\int_{t-\eta}^{i_kh}\mathbf{x}(s)ds\\ &-\int_{i_kh}^{t}\mathbf{x}(s)ds-\int_{-\eta}^{0}\int_{t+\beta}^{t}\dot{\mathbf{x}}(s)dsd\beta]=0\end{aligned}$$

$$\begin{aligned}\alpha_5(t)&=\eta\boldsymbol{\xi}^T(t)\mathbf{V}\boldsymbol{\xi}(t)-\int_{i_kh}^{t}\boldsymbol{\xi}^T(t)\mathbf{V}\boldsymbol{\xi}(t)ds\\ &-\int_{t-\eta}^{i_kh}\boldsymbol{\xi}^T(t)\mathbf{V}\boldsymbol{\xi}(t)ds=0\end{aligned}$$

with $\boldsymbol{\xi}(t)=col\{\mathbf{x}(t),\mathbf{x}(i_kh),\mathbf{x}(t-\eta),\dot{\mathbf{x}}(t),\dot{\mathbf{x}}(t-\eta)\}$.

With some manipulation, the obtained $\dot V(t)$ can be transformed to the following form:

$$\begin{aligned}\dot V(t)&=\boldsymbol{\xi}^T(t)\mathbf{\Omega}\boldsymbol{\xi}(t)-\int_{t-\eta}^{i_kh}\zeta^T(t,s)\mathbf{\Pi}_1\zeta(t,s)ds\\ &-\int_{i_kh}^{t}\zeta^T(t,s)\mathbf{\Pi}_2\zeta(t,s)ds\\ &-\int_{-\eta}^{0}\int_{t+\beta}^{t}[\boldsymbol{\xi}^T(t)\mathbf{L}+\dot{\mathbf{x}}^T(s)\mathbf{Z}]\mathbf{Z}^{-1}[\boldsymbol{\xi}(t)\mathbf{L}+\dot{\mathbf{x}}^T(s)\mathbf{Z}]^T dsd\beta\\ &+\boldsymbol{\xi}^T(t)[\frac{1}{2}\eta^2\mathbf{L}\mathbf{Z}^{-1}\mathbf{L}^T]\boldsymbol{\xi}(t)\end{aligned}$$

Where $\varsigma(t,s)=col\{\boldsymbol{\xi}(t),\boldsymbol{\rho}(s)\}$. So if $\mathbf{\Pi}_i\geq 0$, $(i=1,2)$, the following is obtained:

$$\dot V(t)\leq\boldsymbol{\xi}^T(t)[\mathbf{\Omega}+\frac{1}{2}\eta^2\mathbf{L}\mathbf{Z}^{-1}\mathbf{L}^T]\boldsymbol{\xi}(t)$$

Then, if $\mathbf{\Phi}<0$, by the Lyapunov-Krasovskii Theorem [7], the closed-loop system is stable. Note that the Schur Complement is used implicitly in the above reasoning. ■

Remark 1: Theorem 1 introduces a sufficient condition for the stability of the closed-loop control system (1)-(2). For a given controller gain $\mathbf{K}$, this Theorem can be employed to determine the maximum value of allowable delay η which retain the stability of system.

Utilizing free weighting matrices technique [10], the nonlinear matrix inequality condition in Theorem 1 is modified to obtain equivalent Linear Matrix Inequalities which is computationally more tractable using the existing efficient LMI feasibility methods.

Theorem 2: For given scalars h, η and p_i, $i=2,3,\cdots 5$, if there exist the following matrices:

$$\tilde{\mathbf{L}}=\begin{bmatrix}\tilde{\mathbf{L}}_1\\ \tilde{\mathbf{L}}_2\\ \tilde{\mathbf{L}}_3\\ \tilde{\mathbf{L}}_4\\ \tilde{\mathbf{L}}_5\end{bmatrix},\ \tilde{\mathbf{N}}=\begin{bmatrix}\tilde{\mathbf{N}}_1\\ \tilde{\mathbf{N}}_2\\ \tilde{\mathbf{N}}_3\\ \tilde{\mathbf{N}}_4\\ \tilde{\mathbf{N}}_5\end{bmatrix},\ \tilde{\mathbf{S}}=\begin{bmatrix}\tilde{\mathbf{S}}_1\\ \tilde{\mathbf{S}}_2\\ \tilde{\mathbf{S}}_3\\ \tilde{\mathbf{S}}_4\\ \tilde{\mathbf{S}}_5\end{bmatrix},$$

$$\tilde{\mathbf{V}}=\begin{bmatrix}\tilde{\mathbf{V}}_{11} & \tilde{\mathbf{V}}_{12} & \tilde{\mathbf{V}}_{13} & \tilde{\mathbf{V}}_{14} & \tilde{\mathbf{V}}_{15}\\ * & \tilde{\mathbf{V}}_{22} & \tilde{\mathbf{V}}_{23} & \tilde{\mathbf{V}}_{24} & \tilde{\mathbf{V}}_{25}\\ * & * & \tilde{\mathbf{V}}_{33} & \tilde{\mathbf{V}}_{34} & \tilde{\mathbf{V}}_{35}\\ * & * & * & \tilde{\mathbf{V}}_{44} & \tilde{\mathbf{V}}_{45}\\ * & * & * & * & \tilde{\mathbf{V}}_{55}\end{bmatrix}$$

$$\tilde{\mathbf{P}}=\begin{bmatrix}\tilde{\mathbf{P}}_{11} & \tilde{\mathbf{P}}_{12} & \tilde{\mathbf{P}}_{13}\\ * & \tilde{\mathbf{P}}_{22} & \tilde{\mathbf{P}}_{23}\\ * & * & \tilde{\mathbf{P}}_{33}\end{bmatrix}>0,$$

$$\tilde{\mathbf{Q}}=\begin{bmatrix}\tilde{\mathbf{Q}}_{11} & \tilde{\mathbf{Q}}_{12}\\ * & \tilde{\mathbf{Q}}_{22}\end{bmatrix}>0$$

$$\tilde{\mathbf{R}}=\begin{bmatrix}\tilde{\mathbf{R}}_{11} & \tilde{\mathbf{R}}_{12}\\ * & \tilde{\mathbf{R}}_{22}\end{bmatrix}>0,$$

$$\tilde{\mathbf{Z}}>0$$

and nonsingular matrices $\mathbf{X}$ and $\mathbf{Y}$ of appropriate dimensions such that the LMI (10)-(12) holds, then the system (1) with the networked static state feedback controller (2) is stable.

$$\tilde{\mathbf{\Phi}} < 0 \tag{10}$$
$$\tilde{\mathbf{\Pi}}_1 \geq 0 \tag{11}$$
$$\tilde{\mathbf{\Pi}}_2 \geq 0 \tag{12}$$

$$\tilde{\mathbf{\Phi}} = \begin{bmatrix} \tilde{\mathbf{\Omega}} & \frac{1}{2}\eta^2\tilde{\mathbf{L}} \\ * & -\frac{1}{2}\eta^2\tilde{\mathbf{Z}} \end{bmatrix}$$

$$\tilde{\mathbf{\Pi}}_1 = \begin{bmatrix} \tilde{\mathbf{V}} & \tilde{\mathbf{\Gamma}}_1 \\ * & \tilde{\mathbf{R}} \end{bmatrix}, \quad \tilde{\mathbf{\Gamma}}_1 = \begin{bmatrix} -\tilde{\boldsymbol{\psi}} + \tilde{\mathbf{L}} & \tilde{\mathbf{S}} \end{bmatrix},$$

$$\tilde{\mathbf{\Pi}}2 = \begin{bmatrix} \tilde{\mathbf{V}} & \tilde{\mathbf{\Gamma}}_2 \\ * & \tilde{\mathbf{R}} \end{bmatrix}, \quad \tilde{\mathbf{\Gamma}}_2 = \begin{bmatrix} -\tilde{\boldsymbol{\psi}} + \tilde{\mathbf{L}} & \tilde{\mathbf{N}} \end{bmatrix}$$

$$\tilde{\boldsymbol{\psi}} = \begin{bmatrix} \tilde{\mathbf{P}}_{33} & 0 & -\tilde{\mathbf{P}}_{33} & \tilde{\mathbf{P}}_{13}^T & \tilde{\mathbf{P}}_{23}^T \end{bmatrix}^T$$

where $\tilde{\mathbf{\Omega}} = \tilde{\mathbf{\Omega}}_1 + \tilde{\mathbf{\Omega}}_2 + \tilde{\mathbf{\Omega}}_2^T + \tilde{\mathbf{\Omega}}_3 + \tilde{\mathbf{\Omega}}_3^T + \eta\tilde{\mathbf{V}}$

$$\tilde{\mathbf{\Omega}}_1 = \begin{bmatrix} (1,1) & 0 & -\tilde{\mathbf{P}}_{13} + \tilde{\mathbf{P}}_{23}^T & \tilde{\mathbf{Q}}_{12} + \eta\tilde{\mathbf{R}}_{12} + \tilde{\mathbf{P}}_{11} & \tilde{\mathbf{P}}_{12} \\ * & 0 & 0 & 0 & 0 \\ * & * & (2,2) & \tilde{\mathbf{P}}_{12}^T & \tilde{\mathbf{P}}_{22} - \tilde{\mathbf{Q}}_{12} \\ * & * & * & (3,3) & 0 \\ * & * & * & * & -\tilde{\mathbf{Q}}_{22} \end{bmatrix}$$

$$(1,1) = \tilde{\mathbf{Q}}_{11} + \eta\tilde{\mathbf{R}}_{11} + \tilde{\mathbf{P}}_{13} + \tilde{\mathbf{P}}_{13}^T,$$
$$(2,2) = -\tilde{\mathbf{P}}_{23} - \tilde{\mathbf{P}}_{23}^T - \tilde{\mathbf{Q}}_{11},$$
$$(3,3) = \tilde{\mathbf{Q}}_{22} + \eta\tilde{\mathbf{R}}_{22} + \frac{1}{2}\eta^2\tilde{\mathbf{Z}}.$$
$$\tilde{\mathbf{\Omega}}_2 = \begin{bmatrix} \tilde{\mathbf{N}} + \eta\tilde{\mathbf{L}} & \tilde{\mathbf{S}} - \tilde{\mathbf{N}} & -\tilde{\mathbf{S}} & 0 & 0 \end{bmatrix}$$

$$\tilde{\mathbf{\Omega}}_3 = \begin{bmatrix} -\mathbf{AX}^T & -\mathbf{BY} & 0 & \mathbf{X}^T & 0 \\ -p_2\mathbf{A}X^T & p_2\mathbf{BY} & 0 & p_2\mathbf{X}^T & 0 \\ -p_3\mathbf{AX}^T & p_3\mathbf{BY} & 0 & p_3\mathbf{X}^T & 0 \\ -p_4\mathbf{AX}^T & p_4\mathbf{BY} & 0 & p_4\mathbf{X}^T & 0 \\ -p_5\mathbf{A}X^T & p_5\mathbf{BY} & 0 & p_5\mathbf{X}^T & 0 \end{bmatrix}$$

Furthermore, the state feedback control gain is obtained by $\mathbf{K} = \mathbf{Y}\mathbf{X}^{-T}$.

Proof : In the matrix $\mathbf{M}$ defined in the Theorem 1, replace $\mathbf{M}_1 = \mathbf{M}_0$, $\mathbf{M}_2 = p_2\mathbf{M}_0$, $\mathbf{M}_3 = p_3\mathbf{M}_0$, $\mathbf{M}_4 = p_4\mathbf{M}_0$ and $\mathbf{M}_5 = p_5\mathbf{M}_0$. Feasibility of inequality (10) implies that $\mathbf{M}_0$ is nonsingular. Let, $\mathbf{X} = \mathbf{M}_0^{-1}$ then pre and post multiply simultaneously the two sides of (6) with $diag(\mathbf{X}, \mathbf{X}, \mathbf{X}, \mathbf{X}, \mathbf{X}, \mathbf{X})$ and (7)-(8) with $diag(\mathbf{X}, \mathbf{X}, \mathbf{X}, \mathbf{X}, \mathbf{X}, \mathbf{X}, \mathbf{X})$ and its transpose, respectively. Therefore, inequality (6)-(8) leads to inequality (10)-(12) with $\mathbf{Y} = \mathbf{K}\,\mathbf{X}^T$. ■

4. Numerical Examples

Two illustrative numerical examples are presented to compare the proposed method with existing procedures in the literature. The YALMIP Toolbox is utilized to solve the LMI feasibility problems [11].

Example 1: Consider the following system [4]

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}\mathbf{u}(t) \tag{19}$$

Maximum allowable transfer interval (MATI) which also called MADB is a common criterion to measure the conservativeness of the NCS design methods [12]. By the suggested method, the controller gain is obtained as $\mathbf{K} = [-4, -12]$ which leads to the MATI equal to 1.0204. In TABLE I, the MATIs corresponding to the rival design methods are compared. It's obvious that presented method is notably lees conservative.

TABLE I. MATIs corresponding to the different design methods for Example 1

Ref.	[4]	[5]	Theorem 2
MATI	0.8334	0.9671	1.0204

Example 2: Consider the simplified model of the inverted system process [4]:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u}(t) \tag{18}$$

In [4], the maximum allowable value of $\eta_{\max}$ that guarantee the stability of the closed loop system is obtained equal to 0.97 whereas in [5], $\eta_{\max}$ is 0.994. Using Theorem 2, the maximum value of the delay is computed equal to 0.997 with $p_2 = 0.01$, $p_3 = 0.01$, $p_4 = 128$, $p_5 = 0.01$ and $\mathbf{K} = -[1.0021, 1.0030]$. The above discussion is summarized in TABLE II.

TABLE II. Maximum allowable delay corresponding to the different design methods for example 2

Ref.	[4]	[5]	Theorem 2
$\eta_{\max}$	0.97	0.994	0.997

5. CONCLUSION

This paper proposed a procedure to design stabilizing state feedback controller for the linear time invariant system which is controlled via communication network. Data packet loss and latency arising in NCS were incorporated in the system model as time-varying delay. In the suggested scheme stabilizing state feedback controller is constructed via the feasible solution of a set of linear matrix inequalities and the obtained result is less conservative than existing approaches. The advantages of the method were demonstrated by numerical examples.

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