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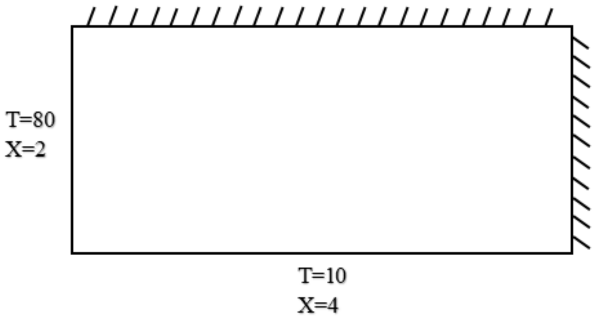
Calculation of temperature distribution by discretizing an advection-diffusion equation with the CDS method in the following two-dimensional figure below:



Discretization:

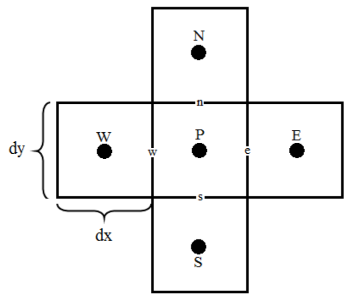
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad \Longrightarrow \quad \frac{\partial}{\partial \mathrm{x}}\left(\frac{\partial \mathrm{T}}{\partial \mathrm{x}}\right) + \frac{\partial}{\partial \mathrm{y}}\left(\frac{\partial \mathrm{T}}{\partial \mathrm{y}}\right) = 0$$

$$\int_W^E \frac{\partial}{\partial \mathrm{x}}\left(\frac{\partial \mathrm{T}}{\partial \mathrm{x}}\right) \mathrm{dx} + \int_S^N \frac{\partial}{\partial \mathrm{y}}\left(\frac{\partial \mathrm{T}}{\partial \mathrm{y}}\right) \mathrm{dy} = 0$$

$$\Longrightarrow \quad \left(\frac{\partial \mathrm{T}}{\partial \mathrm{x}}\right)\Big|_w^e + \left(\frac{\partial \mathrm{T}}{\partial \mathrm{y}}\right)\Big|_s^n = 0$$

$$\Longrightarrow \quad \left(\frac{\partial \mathrm{T}}{\partial \mathrm{x}}\right)\Big|_e - \left(\frac{\partial \mathrm{T}}{\partial \mathrm{x}}\right)\Big|_w + \left(\frac{\partial \mathrm{T}}{\partial \mathrm{y}}\right)\Big|_n - \left(\frac{\partial \mathrm{T}}{\partial \mathrm{y}}\right)\Big|_s = 0$$

$$\Longrightarrow \quad \frac{\mathrm{T_E} - \mathrm{T_P}}{\mathrm{dx}} - \frac{\mathrm{T_p} - \mathrm{T_W}}{\mathrm{dx}} + \frac{\mathrm{T_N} - \mathrm{T_P}}{\mathrm{dy}} - \frac{\mathrm{T_p} - \mathrm{T_s}}{\mathrm{dy}} = 0$$



First row coefficients

$$A_1 = \begin{bmatrix} \frac{3}{\Delta x}+\frac{1}{\Delta y} & \frac{-1}{\Delta x} & 0 & 0 & 0 \\ \frac{-1}{\Delta x} & \frac{2}{\Delta x}+\frac{1}{\Delta y} & \frac{-1}{\Delta x} & 0 & 0 \\ 0 & \frac{-1}{\Delta x} & \frac{2}{\Delta x}+\frac{1}{\Delta y} & \frac{-1}{\Delta x} & 0 \\ 0 & 0 & \frac{-1}{\Delta x} & \frac{2}{\Delta x}+\frac{1}{\Delta y} & \frac{-1}{\Delta x} \\ 0 & 0 & 0 & \frac{-1}{\Delta x} & \frac{3}{\Delta x}+\frac{1}{\Delta y} \end{bmatrix}$$

Coefficients of the middle rows

$$A_1 = \begin{bmatrix} \frac{3}{\Delta x}+\frac{2}{\Delta y} & \frac{-1}{\Delta x} & 0 & 0 & 0 \\ \frac{-1}{\Delta x} & \frac{2}{\Delta x}+\frac{2}{\Delta y} & \frac{-1}{\Delta x} & 0 & 0 \\ 0 & \frac{-1}{\Delta x} & \frac{2}{\Delta x}+\frac{2}{\Delta y} & \frac{-1}{\Delta x} & 0 \\ 0 & 0 & \frac{-1}{\Delta x} & \frac{2}{\Delta x}+\frac{2}{\Delta y} & \frac{-1}{\Delta x} \\ 0 & 0 & 0 & \frac{-1}{\Delta x} & \frac{3}{\Delta x}+\frac{2}{\Delta y} \end{bmatrix}$$

Last row coefficients

$$A_1 = \begin{bmatrix} \frac{3}{\Delta x}+\frac{3}{\Delta y} & \frac{-1}{\Delta x} & 0 & 0 & 0 \\ \frac{-1}{\Delta x} & \frac{2}{\Delta x}+\frac{3}{\Delta y} & \frac{-1}{\Delta x} & 0 & 0 \\ 0 & \frac{-1}{\Delta x} & \frac{2}{\Delta x}+\frac{3}{\Delta y} & \frac{-1}{\Delta x} & 0 \\ 0 & 0 & \frac{-1}{\Delta x} & \frac{2}{\Delta x}+\frac{3}{\Delta y} & \frac{-1}{\Delta x} \\ 0 & 0 & 0 & \frac{-1}{\Delta x} & \frac{3}{\Delta x}+\frac{3}{\Delta y} \end{bmatrix}$$

After determining the matrices Ax = B, the equations were solved and the repetition process was calculated until convergence of 10^{-6}. Tecplot 360 software was used to display the results and matrixToTecplotBinaryFile was used to match it with MATLAB. Finally, the MATLAB code for 100 cells is as follows:

```
clear all
clc

n=100;                               %    number of nodes in x axis
disp('Number of nodes in x axis = ')
disp(n)
m=round(n/2);
x=4;
y=x/2;
dx=x/n;
dy=y/m;

T_old=50*ones(m,n);

for i=1 : n-1                         %creat 3 diagonal matrix A1
   A1(i+1,i)=-1/dx;
   A1(i,i+1)=-1/dx;
   A1(i,i)=2/dx+1/dy;
end
A1(1,1)=3/dx+1/dy;
A1(n,n)=1/dx+1/dy;
for i=1 : n-1                         %creat 3 diagonal matrix A2
   A2(i+1,i)=-1/dx;
   A2(i,i+1)=-1/dx;
   A2(i,i)=2/dx+2/dy;
end
A2(1,1)=3/dx+2/dy;
A2(n,n)=1/dx+2/dy;
for i=1 : n-1                          %creat 3 diagonal matrix A3
   A3(i+1,i)=-1/dx;
   A3(i,i+1)=-1/dx;
   A3(i,i)=2/dx+3/dy;
end
A3(1,1)=3/dx+3/dy;
A3(n,n)=1/dx+3/dy;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for ITERATION=1:1:2000
   T=T_old;
   for i=1:n
      B1(i,1)=T_old(2,i)/dy;
   end
   B1(1,1)=B1(1,1)+160/dx;
   T_old(1,:)=(A1\B1);                 %first row solve

   for i=2:m-1
      for j=1:n
         B2(j,1)=(T_old(i-1,j)+T_old(i+1,j))/dy;
      end
      B2(1,1)=B2(1,1)+160/dx;
      T_old(i,:)=(A2\B2);              %Middle row solve
   end

   for i=1:n
```

```
        B3(i,1)=T_old(m-1,i)/dy+20/dy;
    end
    B3(1,1)=B3(1,1)+160/dx;
    T_old(m,:)=(A3\B3);                  %last row solve
    error = max(max(abs(T_old-T)));      %Convergence
    if error <1e-6
        break
    end
end
disp('Iteration = ')                     %num of iteration
disp(ITERATION)

disp('Mean of temperature = ')
disp(mean(mean(T)))

% imagesc(T_old),colorbar,colormap(gray)
T_old=flipud(T_old);
output_file_name='D:\test.plt';
[a,b]=meshgrid(1:n,1:m);
D=ones(n,m);
matrixToTecplotBinaryFile(a,b,T_old,output_file_name);
system('tec360 D:\test.plt');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

The results of Tecplot are shown as temperature distribution and contour with its meshing in the following figures: