Introduction to Algorithms

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The problem of sorting

Input: sequence $\langle a_1, a_2, \ldots, a_n \rangle$ of numbers.

Output: permutation $\langle a'_1, a'_2, \ldots, a'_n \rangle$ such that $a'_1 \leq a'_2 \leq \cdots \leq a'_n$.

Example:

Input: 8 2 4 9 3 6

Output: 2 3 4 6 8 9

Keys, satellite data, record

- The numbers to be sorted are also known as the keys. Although the problem is conceptually about sorting a sequence, the input comes in the form of an array with n elements.
- When we want to sort numbers, it's often because they are the keys associated with other data, which we call satellite data.
- Together, a key and satellite data form a record.

Example

Initial Array Representation

```
+-----+-------+-----------+
| ID  | Name  | Price     | Description
+-----+-------+-----------+
| 1   | Prod1 | P1 (500)  | Desc1
| 2   | Prod2 | P2 (300)  | Desc2
| 3   | Prod3 | P3 (450)  | Desc3
| 4   | Prod4 | P4 (150)  | Desc4
```

Example

cont'd

After Full Sort

```
+-----+-------+-----------+
| ID  | Name  | Price     | Description
+-----+-------+-----------+
| 4   | Prod4 | P4 (150)  | Desc4
| 2   | Prod2 | P2 (300)  | Desc2
| 3   | Prod3 | P3 (450)  | Desc3
| 1   | Prod1 | P1 (500)  | Desc1
```

Pseudocode

- we'll typically describe algorithms as procedures written in a pseudocode.
- In pseudocode, we employ whatever expressive method is most clear and concise to specify a given algorithm.
- Another difference between pseudocode and real code is that pseudocode often ignores aspects of software engineering.

Example of insertion sort

8 2 4 9 3 6

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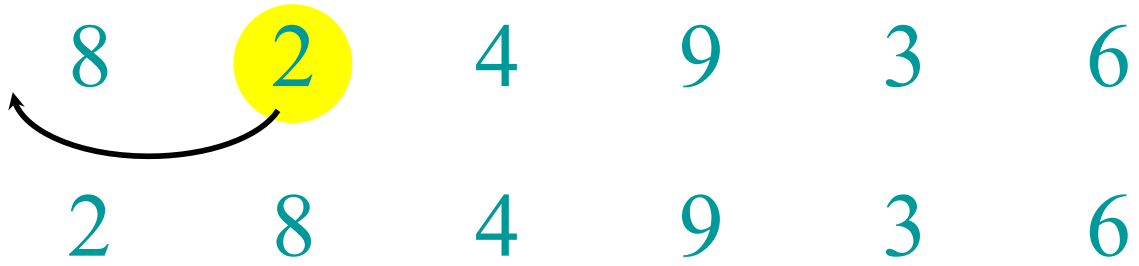
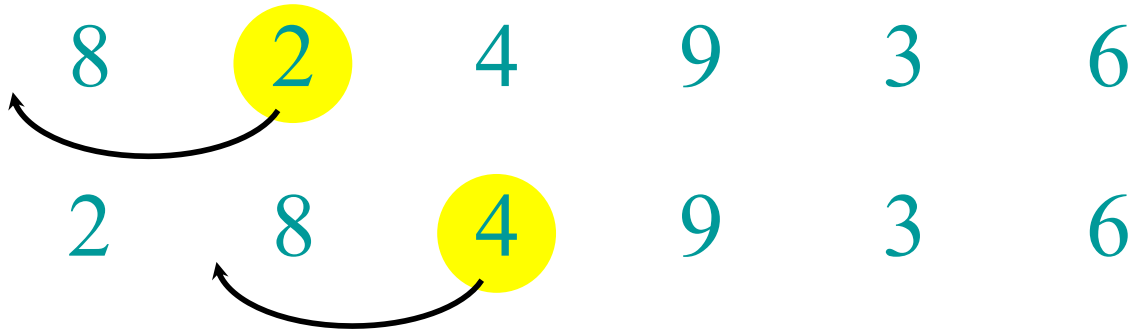


Example of insertion sort



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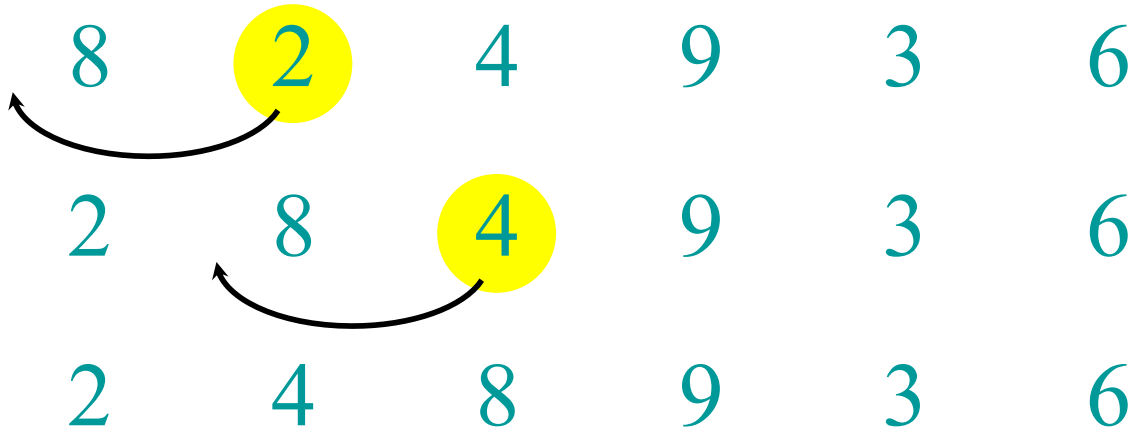
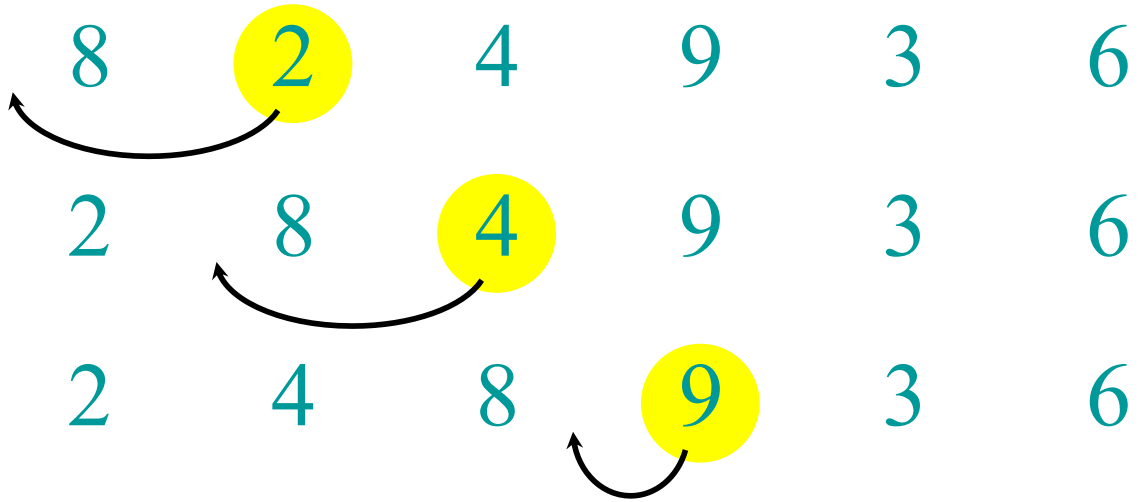
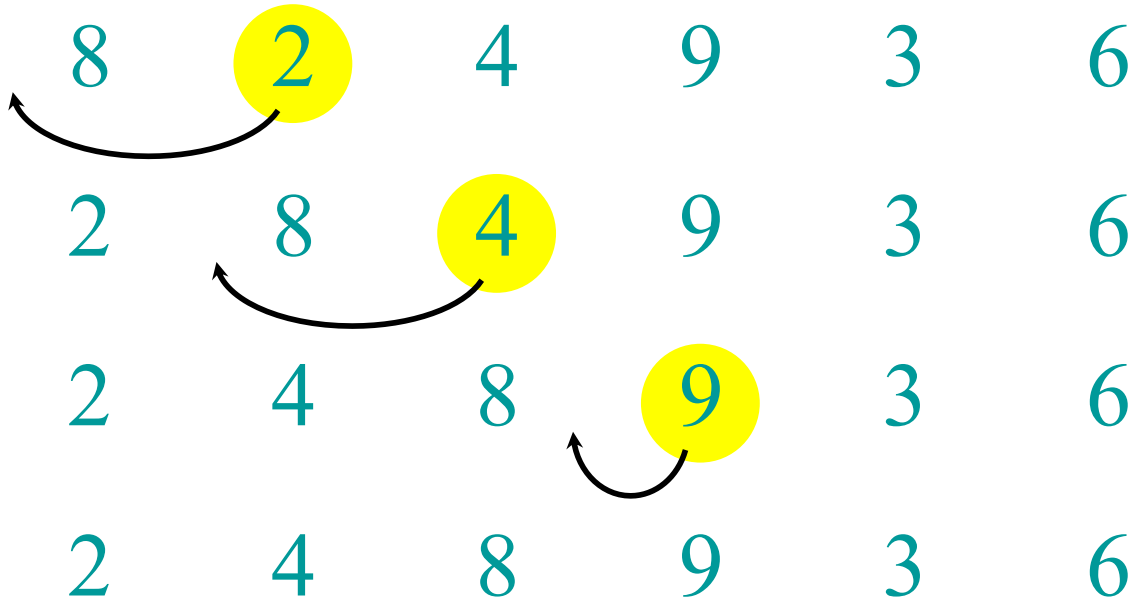


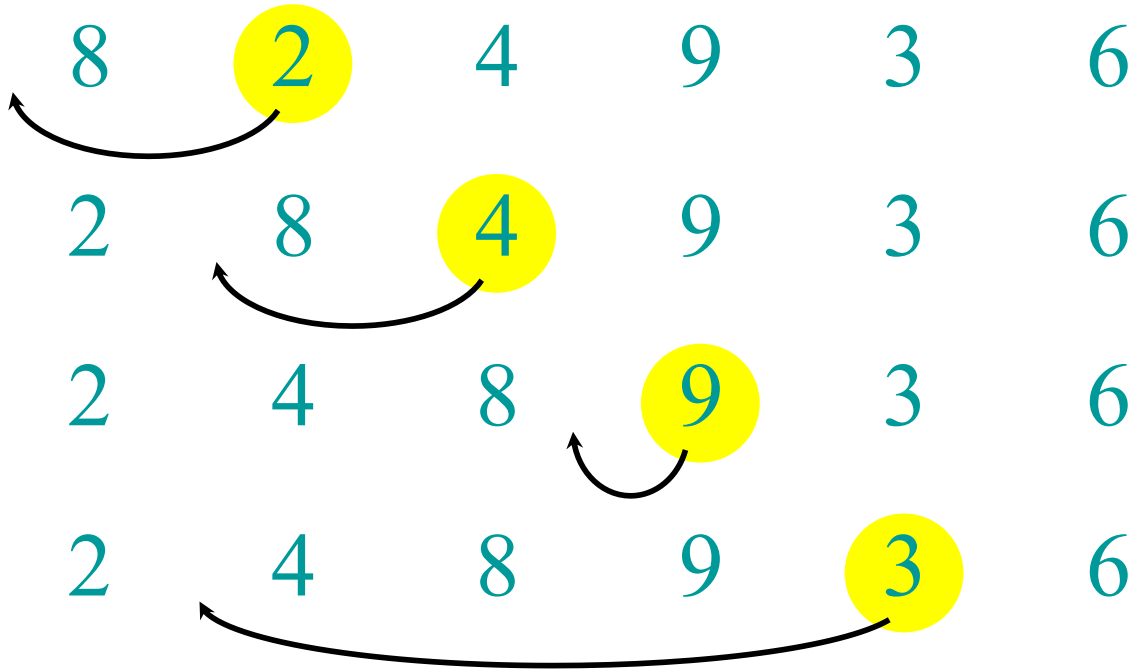
Example of insertion sort



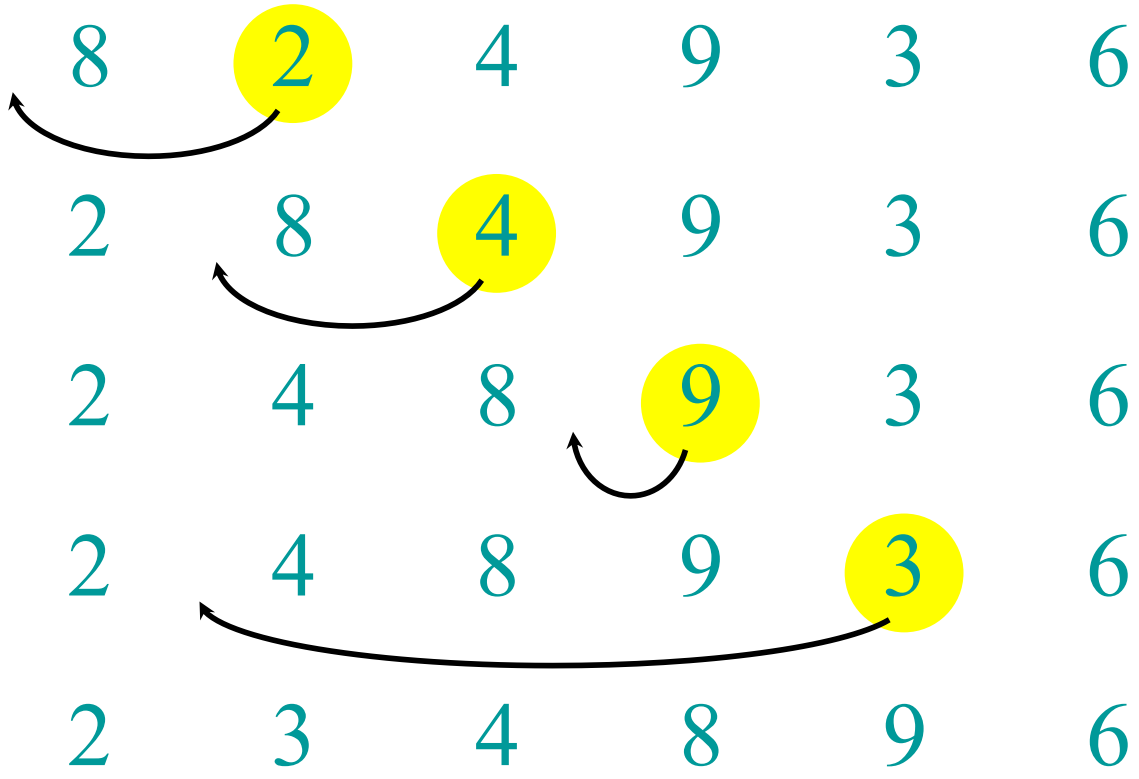
Example of insertion sort



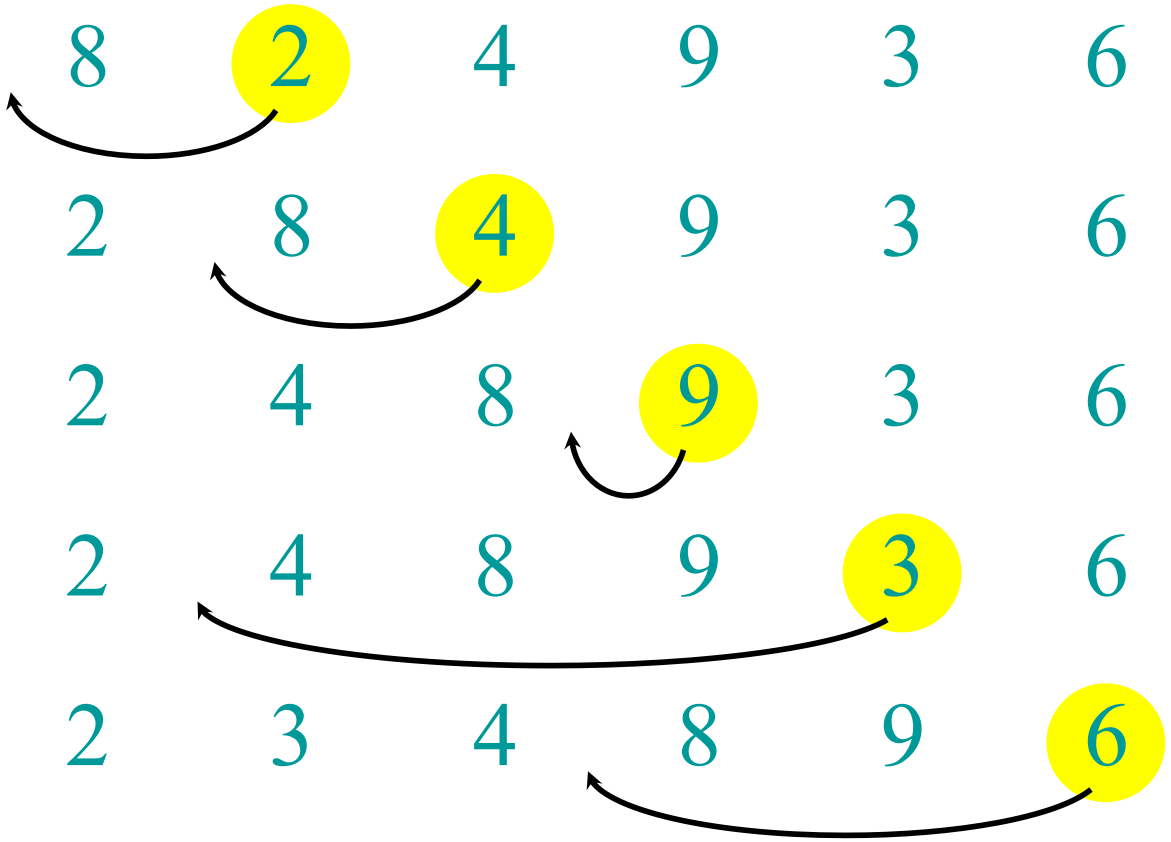
Example of insertion sort



Example of insertion sort



Example of insertion sort



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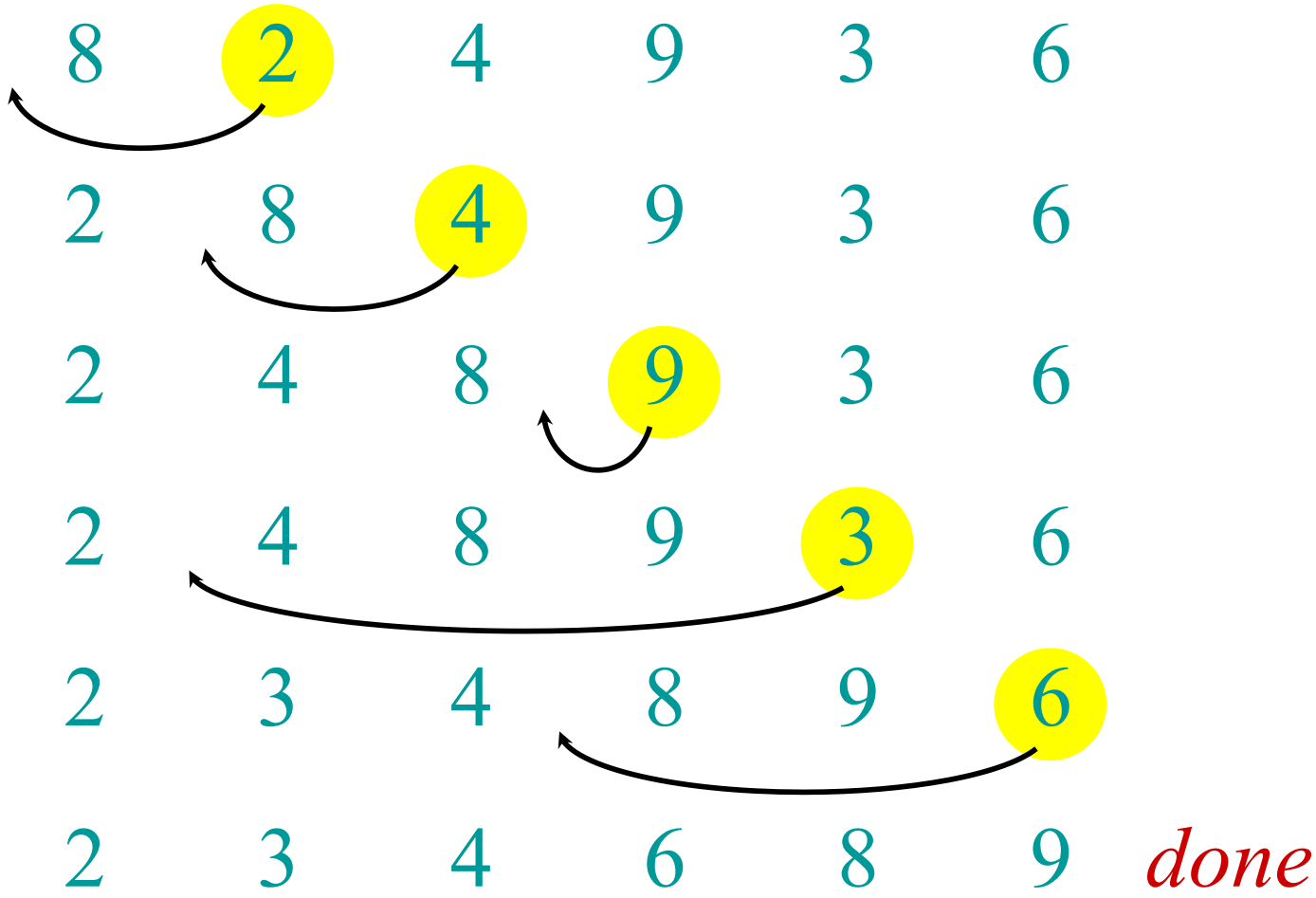
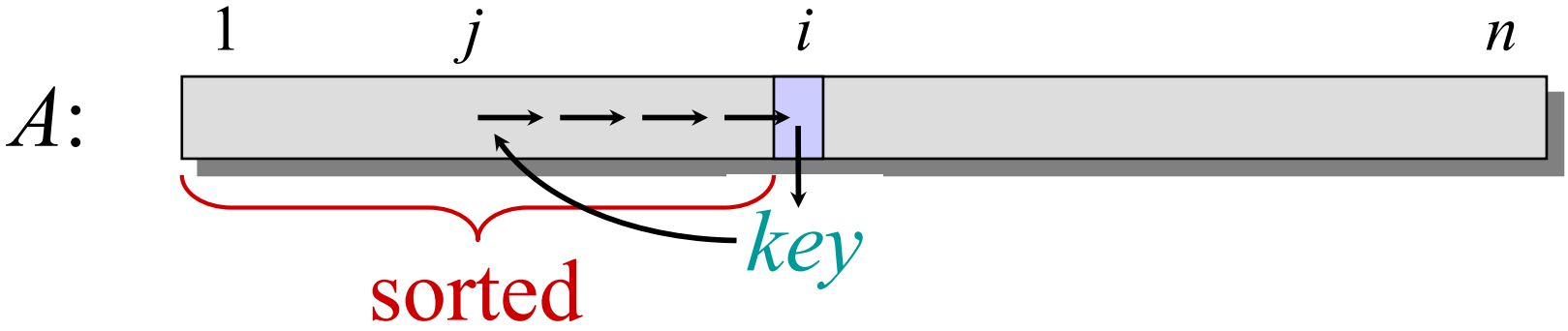


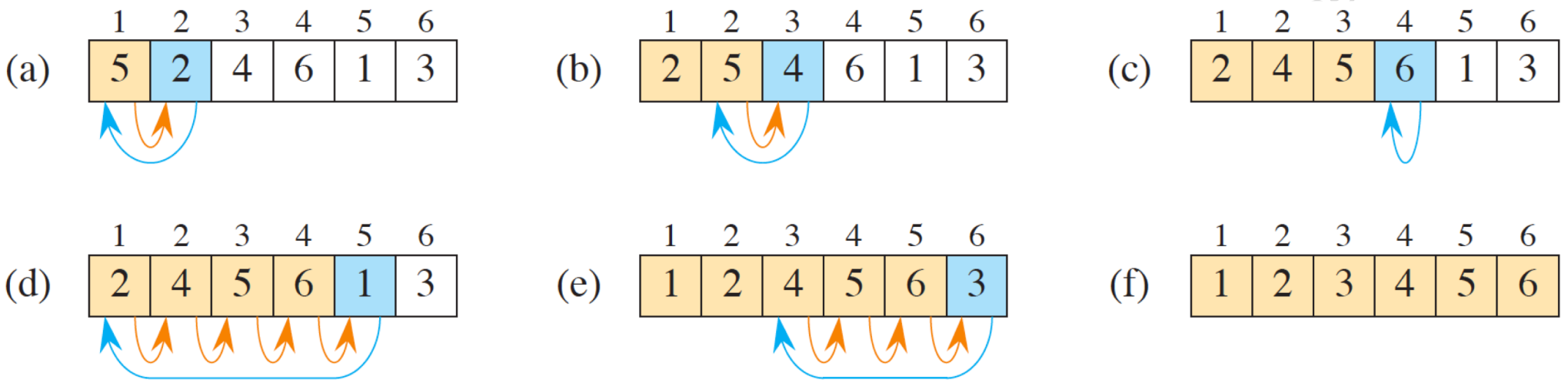
Insertion sort

```
INSERTION-SORT(A, n)
1   for i = 2 to n
2       key = A[i]
3       // Insert A[i] into the sorted subarray A[1 : i − 1].
4       j = i − 1
5       while j > 0 and A[j] > key
6           A[j + 1] = A[j]
7           j = j − 1
8       A[j + 1] = key
```



Example INSERTION-SORT(A,n)

sequence $\langle 5, 2, 4, 6, 1, 3 \rangle$



Pseudocode conventions

- Indentation indicates block structure.
- The looping constructs **while**, **for**, and **repeat-until** and the **if-else** conditional construct have interpretations similar to those in C, Python. we use the keyword **downto** when a for loop decrements its loop counter.
- The symbol // indicates that the remainder of the line is a comment.
- Variables (such as *i* , *j* , and *key*) are local to the given procedure.

Pseudocode conventions

- We access array elements by specifying the array name followed by the index in square brackets.
- Arrays in this book use 1-origin indexing.
- The notation ":" denotes a subarray. Thus, $A[i:j]$ indicates the subarray of A consisting of the elements $A[i], A[i+1], \ldots, A[j]$.

Loop invariants and the correctness of insertion sort

- **Initialization**: It is true prior to the first iteration of the loop.
- **Maintenance**: If it is true before an iteration of the loop, it remains true before the next iteration.
- **Termination**: The loop terminates, and when it terminates, the invariant-usually along with the reason that the loop terminated-gives us a useful property that helps show that the algorithm is correct.

Correctness of insertion sort

Initialization: We start by showing that the loop invariant holds before the first loop iteration, when $i = 2$.[2] The subarray $A[1 : i - 1]$ consists of just the single element $A[1]$, which is in fact the original element in $A[1]$. Moreover, this subarray is sorted (after all, how could a subarray with just one value not be sorted?), which shows that the loop invariant holds prior to the first iteration of the loop.

Correctness of insertion sort

Maintenance: Next, we tackle the second property: showing that each iteration maintains the loop invariant. Informally, the body of the **for** loop works by moving the values in $A[i-1]$, $A[i-2]$, $A[i-3]$, and so on by one position to the right until it finds the proper position for $A[i]$ (lines 4–7), at which point it inserts the value of $A[i]$ (line 8). The subarray $A[1:i]$ then consists of the elements originally in $A[1:i]$, but in sorted order. ***Incrementing*** i (increasing its value by 1) for the next iteration of the **for** loop then preserves the loop invariant.

Correctness of insertion sort

Termination: Finally, we examine loop termination. The loop variable i starts at 2 and increases by 1 in each iteration. Once i's value exceeds n in line 1, the loop terminates. That is, the loop terminates once i equals $n + 1$. Substituting $n + 1$ for i in the wording of the loop invariant yields that the subarray $A[1:n]$ consists of the elements originally in $A[1:n]$, but in sorted order. Hence, the algorithm is correct.