

$$a) T(n) = 4T(n/4) + 5n$$

$$a=4 \quad b=4 \quad c=1 \quad d=1$$

$$n^1 < 5n \rightarrow \boxed{5n}$$

$$n = n \rightarrow \boxed{n \log n}$$

$$b) T(n) = 4T(n/5) + 5n$$

$$a=4 \quad b=5 \quad c=\log_5 4 \quad d=1$$

$$n^{\log_5 4} < n \rightarrow n$$

$$c) T(n) = 5T(n/4) + 4n$$

$$a=5 \quad b=4 \quad c=\log_4 5 \quad d=1$$

$$n^{\log_4 5} > n \rightarrow \boxed{n^c}$$

$$d) T(n) = 25T(n/5) + n^2$$

$$a=25 \quad b=5 \quad c=\log_5 25 = 2 \quad d=2$$

$$c=d \rightarrow \boxed{n^2 \log n}$$



$$e) T(n) = 4T(n/5) + \log n$$

$$a = 4 \quad b = 5 \quad c = \log_5 4$$

$$n^{\log_5 4} > \log n \rightarrow \boxed{\cancel{n^{\log_5 4}}} \boxed{n^c}$$

$$f) T(n) = 4T(n/5) + \log^5 n \sqrt{n}$$

$$a = 4 \quad b = 5 \quad c = \log_5 4$$

$$n^{\log_5 4} < \log^5 n \sqrt{n} \rightarrow \boxed{n^{\log_5 4} \cdot \log^5 n \sqrt{n}}$$

$$g) T(n) = 4T(\sqrt{n}) + \log^5 n$$

$$n = 2^m \rightarrow \textcircled{1}$$

$$T(2^m) = 4T(2^{m/2}) + m^5$$

$$T(2^m) = S(m) \rightarrow \textcircled{2}$$

$$S(m) = 4S(m/2) + m^5$$

$$a = 4 \quad b = 2 \quad c = \log_2 4 = 2$$

$$m^2 < m^5 \rightarrow m^5 \rightarrow \boxed{(\log n)^5}$$



$$H) T(n) = 4T(\sqrt{n}) + \log^2 n$$

$$n = 2^m \rightarrow ①$$

$$T(2^m) = 4T(2^{m/2}) + m^2$$

$$T(2^n) = S(m) \rightarrow ②$$

$$S(m) = 4S(m/2) + m^2$$

$$a = 4 \quad b = 2 \quad c = \log_2 4 = 2$$

$$m^2 \rightarrow m^2 \rightarrow m^2 \log m \rightarrow (\log n)^2 \log(\log n)$$

$$i) T(n) = T(\sqrt{n}) + 5$$

$$n = 2^m \rightarrow ①$$

$$T(2^m) = T(2^{m/2}) + 5$$

$$T(2^m) = S(m) \rightarrow ②$$

$$S(m) = S(m/2) + 5$$

$$a = 1 \quad b = 2 \quad c = \log_2 1 = 0$$

$$m^0 = 1 \rightarrow \log m \rightarrow \log(\log n)$$