Machine Learning based SAT Solvers for Cryptanalysis

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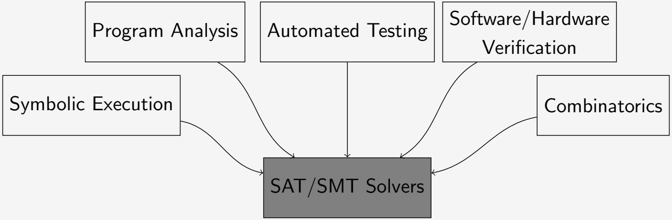
April 2nd, 2020

Motivation

- **SAT Solvers**: Powerful general purpose search tools



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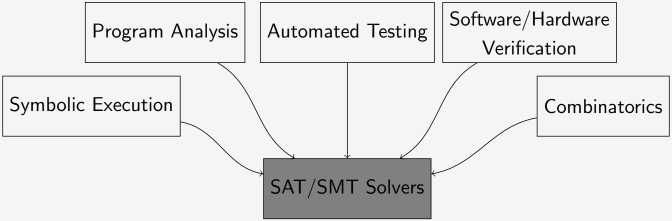
- **Cryptanalysis**: Searching a huge search space for a secret key/value



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- SAT/SMT solvers have increasingly been used in Cryptographic tasks
  - Finding cryptographic keys [Mas99, MM00]
  - Modular root finding [FMM03]
  - A collision attack [MZ06]
  - Preimage attacks [MS13], [Nos12]
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(Tailor internals for a specific cryptographic problem)

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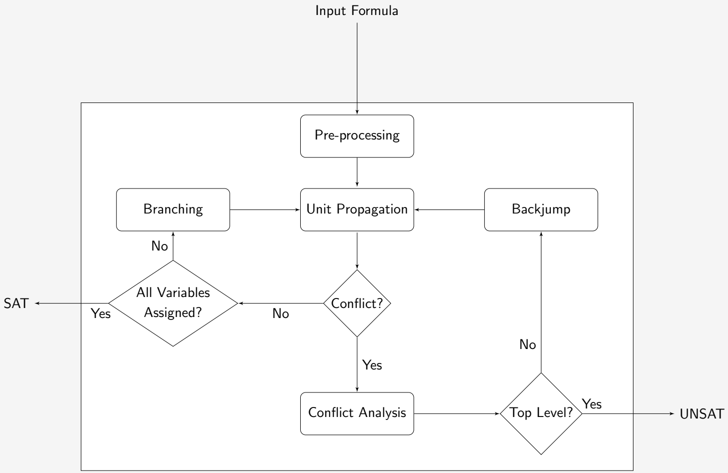

[LG+18]

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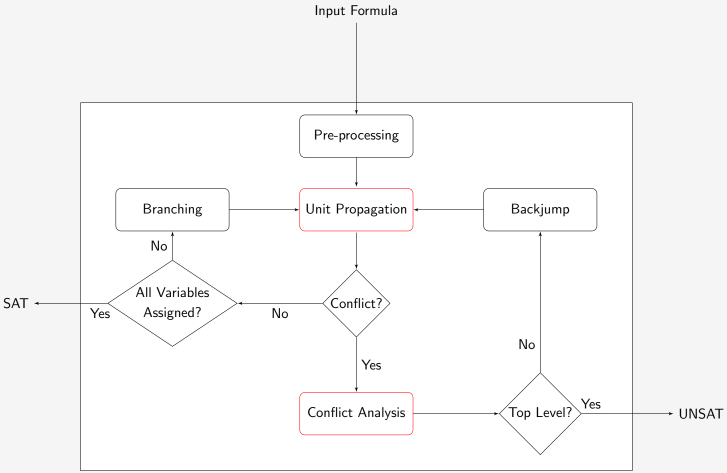

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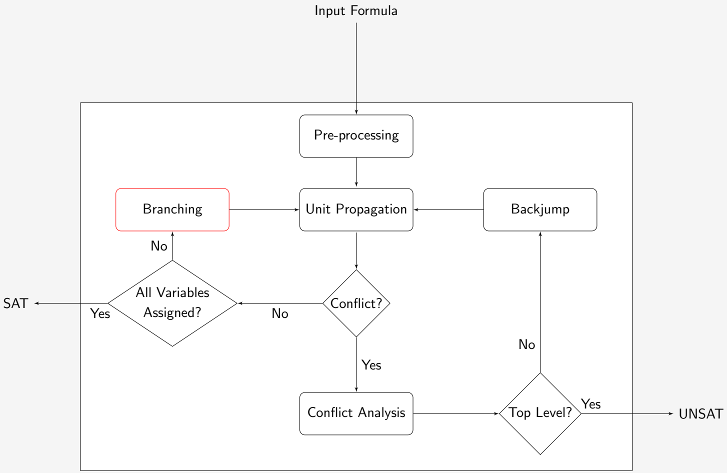

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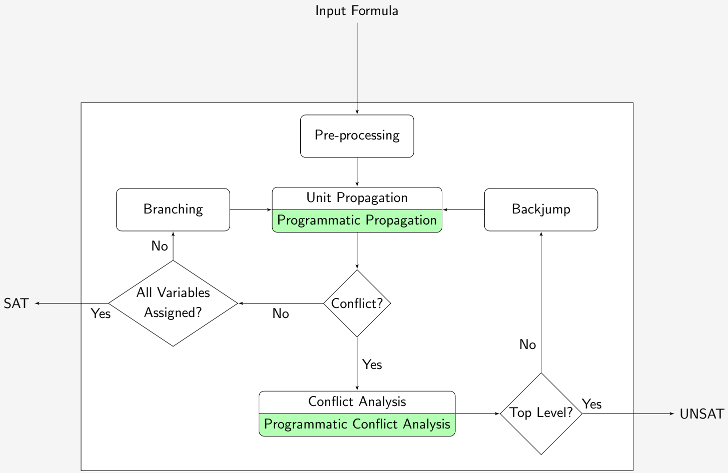

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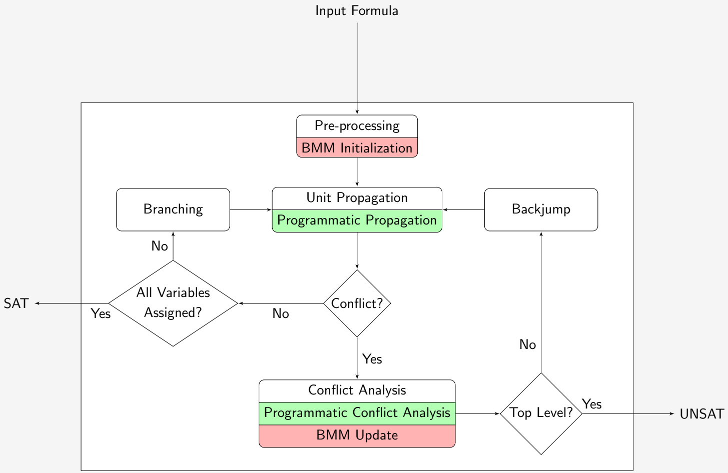

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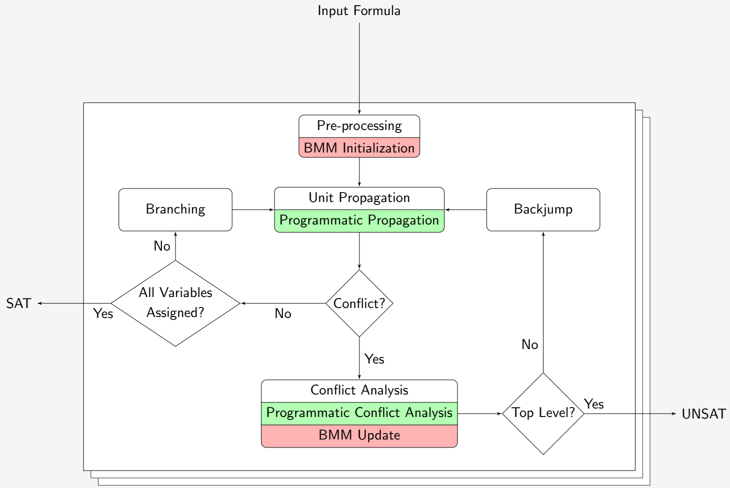

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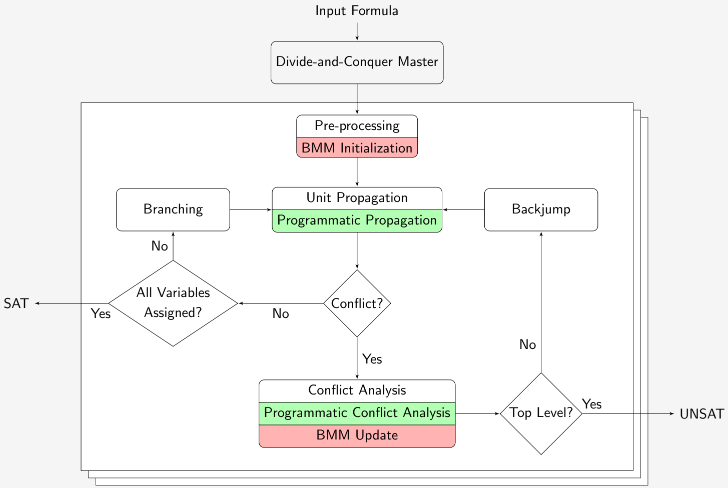

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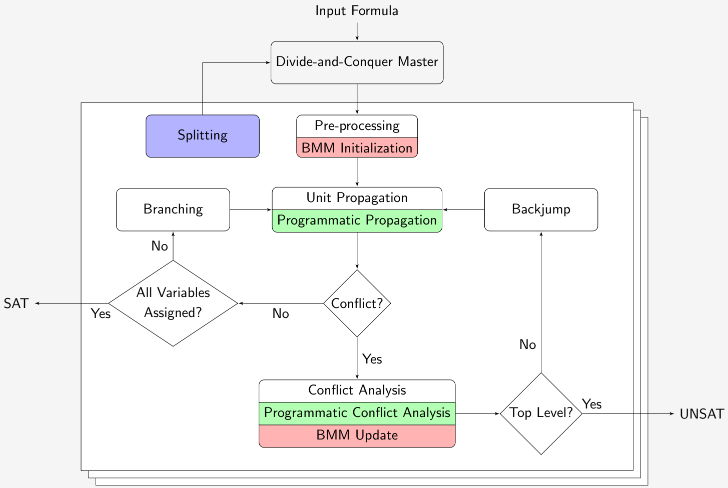

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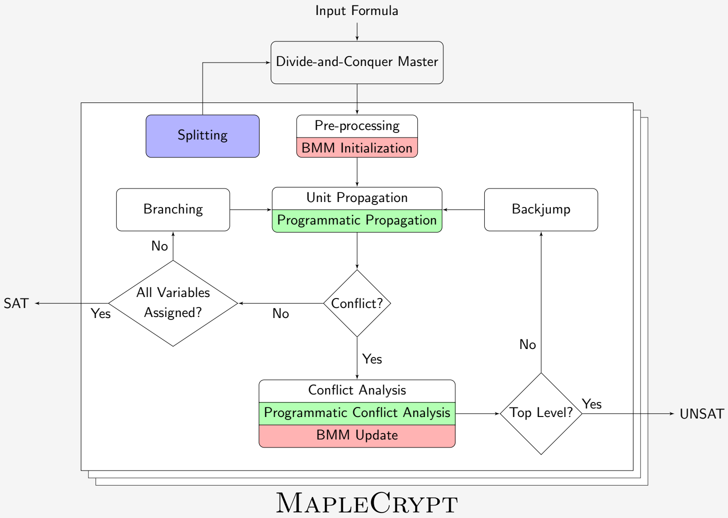
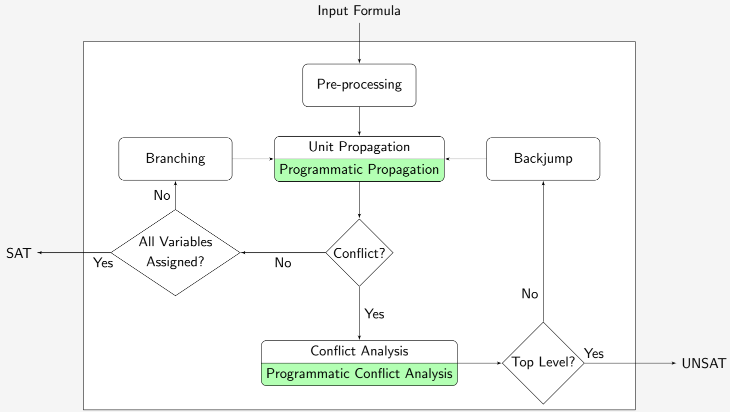


Outline of Contributions

1. Extending reasoning components for cryptographic problems
   - CDCL(Crypto) framework ([NG19])
   - Algebraic fault attack ([NHGG18])
   - Differential cryptanalysis ([NG19])
2. Improving search heuristics
   - Machine learning for search heuristics optimization problems
   - Sequencing: Splitting heuristics ([NLFG20, NNS$^+$17])
   - Initializing: Variable order and value selection (Branching heuristics) ([NDT$^+$20])

Part 1: CDCL(Crypto) Solvers

Overview



CDCL(CRYPTO): CDCL SAT solver with custom cryptographic reasoning

Lost in Translation

- When encoding a constraint into SAT, some higher level properties might be lost

- Example: consider a pseudo-Boolean constraint $C : x + y \leq 0, (x, y \in \{0, 1\})$
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  - No unit clause to propagate!

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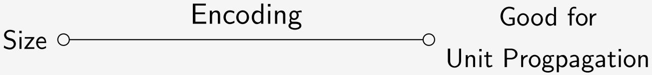

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- Using Programmatic SAT architecture [GOS+12]

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- It can be seen as as solver for hybrid "CNF+C" constraints.

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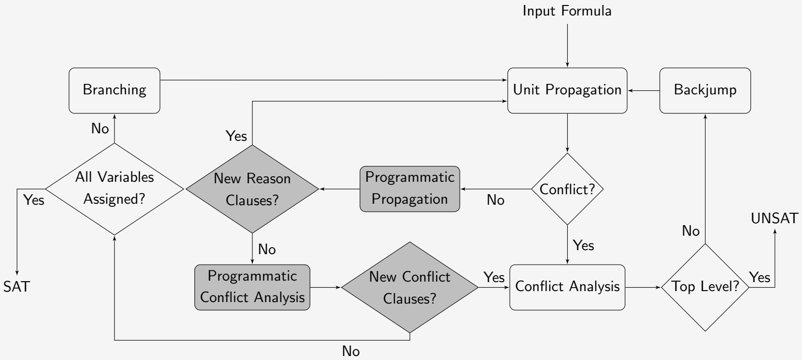

Case Studies

- Applied this framework to two cryptographic problems:
  - Algebraic Fault Attack on SHA-1 and SHA-256
  - Differential Cryptanalysis of round-reduced version of SHA-256

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- and repeat $SHA''(m) = H''$

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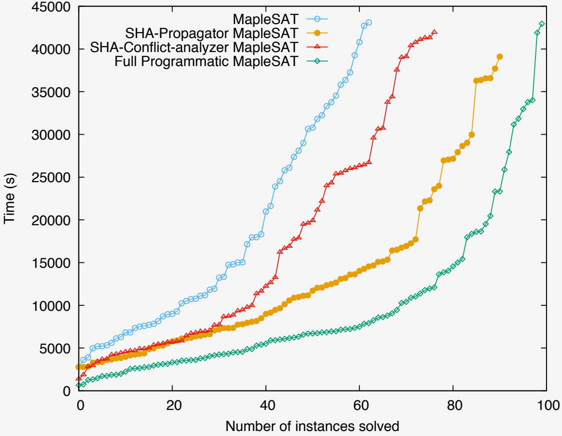
Algebraic Fault Analysis - Programmatic Approach

- Base SAT solver: MapleSAT
- Programmatic conflict analyzer
  - Embedding the verification loop
  - As soon as message word variables are set, they are ready to be verified
  - Early embedded check vs. Straightforward check after solving completely
- Programmatic propagator
  - Improving the propagation flow of multi-operand additions
  - Generating *reason clauses* in each column addition when output bits are missed

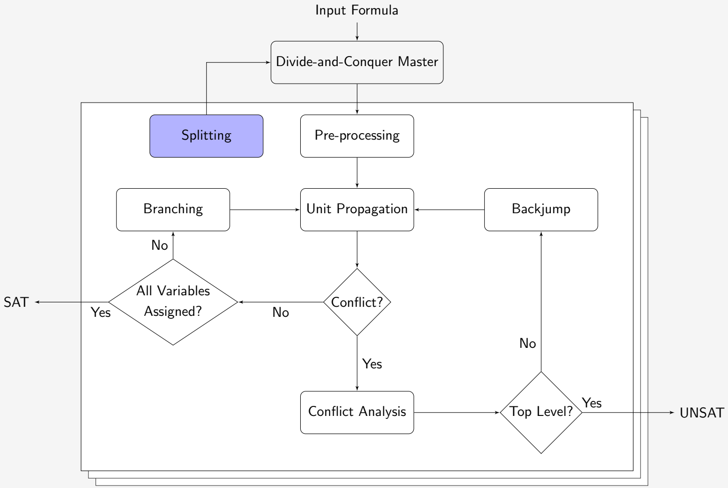
Algebraic Fault Analysis - Results

- Recovering SHA-256 message bits
- 14.3x speed-up on average
- 17 fewer faults were needed compared to the previous works



Part 2: Machine Learning based Splitting Heuristics in Parallel SAT Solvers

Overview



Input Formula
Divide-and-Conquer Master
Splitting
Pre-processing
Branching
Unit Propagation
Backjump
No
All Variables Assigned?
SAT
Yes
No
Conflict?
Yes
No
Conflict Analysis
Top Level?
Yes
UNSAT

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  - Repeat for ϕ_1 and ϕ_2
- ϕ is SAT: At least one solver returns SAT
- ϕ is UNSAT: All solvers return UNSAT

Search Space Splitting

- $\phi_1 = \phi \wedge \neg x_2 \wedge x_5 \wedge \neg x_1$
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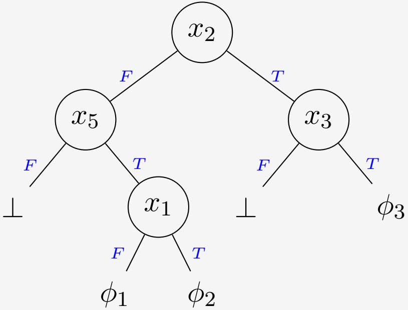

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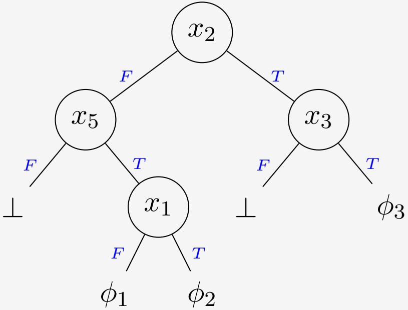

Question (Splitting Heuristic)

How to "divide" so the "conquer" becomes easier?

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- We define $pm(\phi, v)$: Total wall-clock runtime of solving ϕ when splitting once and solving $\phi[v]$ and $\phi[\neg v]$ in parallel.

Building the Splitting Heuristic

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- We don't know the runtime a priori
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- Observation: We are looking for a minimum element in a list of elements ordered by pm

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- Instead of predicting pm values for each item

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$$PW(\phi, v_i, v_j) = \begin{cases} 1, & pm(\phi, v_i) < pm(\phi, v_j) \\ 0, & otherwise \end{cases}$$

Learning PW

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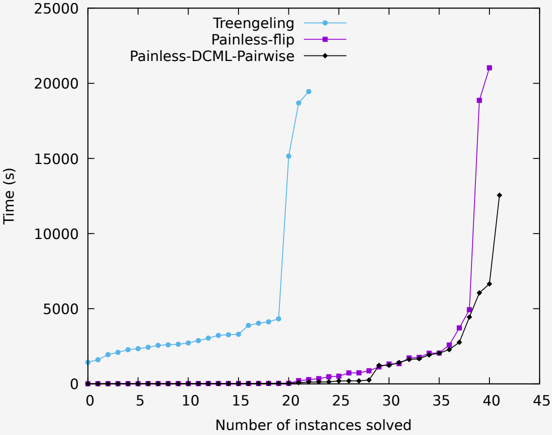
- Random Forest: accuracy 80.72%

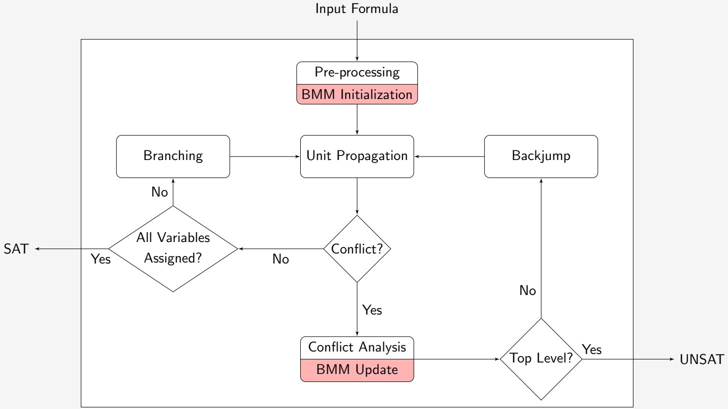
Experimental Results - Cryptographic benchmark

- Framework: Painless
- Baseline: Painless-DC w/ flip splitting heuristic
- SHA-1 preimage



Part 3: BMM-based Heuristic Initialization

Overview



Input Formula
Pre-processing
BMM Initialization
Branching
Unit Propagation
Backjump
No
All Variables Assigned?
Conflict?
SAT
Yes
No
Yes
No
Conflict Analysis
BMM Update
Top Level?
Yes
UNSAT

Heuristic Initialization

- Branching heuristics: variable selection and value selection (polarity)

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- Branching heuristics: variable selection and value selection (polarity)
- Usually *look-back*: make a decision based on the gathered search statistics
- At the start of search: no statistics available
- Goal: derive variable score and preferred value initial values, s.t. the runtime is improved.

Bayesian Moment Matching (BMM) for SAT

- For each variable: $P(x = T)$: probability of setting x to True

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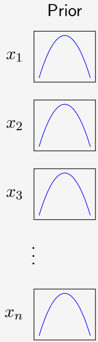

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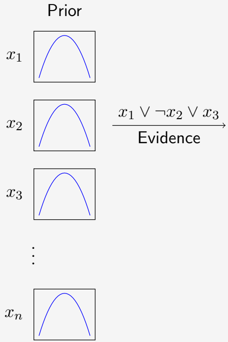

Bayesian Moment Matching (BMM) for SAT

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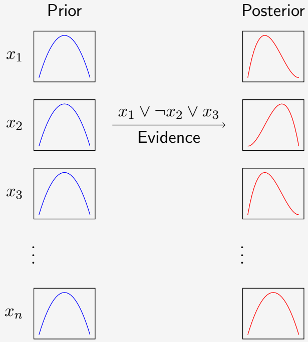

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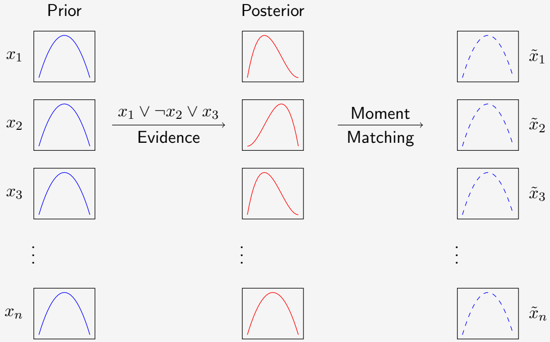
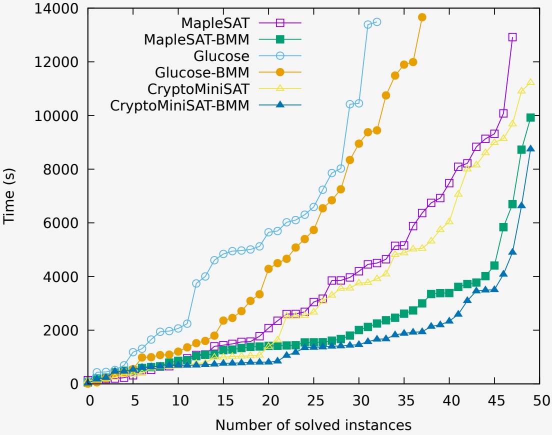


Heuristic Initialization

- Polarity
  - BMM probabilities collectively represent an assignment
  - $Polarity[x] = \begin{cases} False, & P(x = T) < 0.5 \\ True, & P(x = T) \geq 0.5 \end{cases}$
- Activity
  - Give higher priority to variables that BMM is more *confident* about its polarity
  - $Activity[x] = \begin{cases} 1 - P(x = T), & P(x = T) < 0.5 \\ P(x = T), & P(x = T) \geq 0.5 \end{cases}$

Experimental Results

- SHA-1 preimage benchmark
- Apple-to-apple comparison
- BMM on MapleSAT, Glucose and CryptoMiniSAT



Summary and Takeaways

Extending Reasoning Components

Improving Search Heuristics

- Key insights from literature
- Our designs
- Our results

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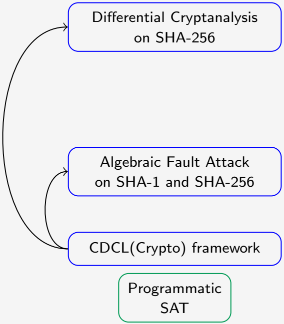

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Differential Cryptanalysis
on SHA-256

Recovering secret with 17 fewer faults
14.3x speedup

Algebraic Fault Attack
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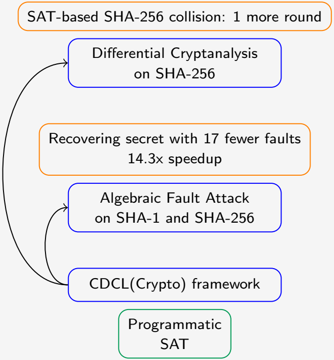

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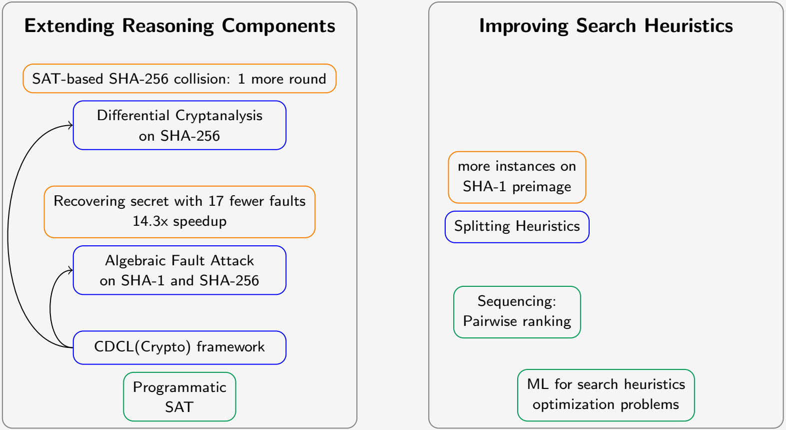

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more instances on
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Initialization of
Variable and Value
Selection

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2x speedup on SHA-1 preimage

Initialization of Variable and Value Selection

more instances on SHA-1 preimage

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Publications

[NLG+17] Nejati, Liang, Gebotys, Czarnecki, Ganesh
Adaptive restart and CEGAR-based solver for inverting cryptographic hash functions
VSTTE 2017

[NNS+17] Nejati, Newsham, Scott, Liang, Gebotys, Poupart, Ganesh
A propagation rate based splitting heuristic for divide-and-conquer solvers
SAT 2017

[NHGG18] Nejati, Horáček, Gebotys, Ganesh
Algebraic fault attack on SHA hash functions using programmatic SAT solvers
CP 2018

[NG19] Nejati, Ganesh
CDCL(Crypto) SAT solvers for cryptanalysis
CASCON 2019

[NDT+20] Nejati/Duan, Trimponias, Poupart, Ganesh
Online Bayesian Moment Matching based SAT Solver Heuristics
submitted to **ICML 2020**

[NLFG20] Nejati, Le Frioux, Ganesh
A Machine Learning based Splitting Heuristic for Divide-and-Conquer Solvers
submitted to **SAT 2020**

Thanks!
Questions?

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