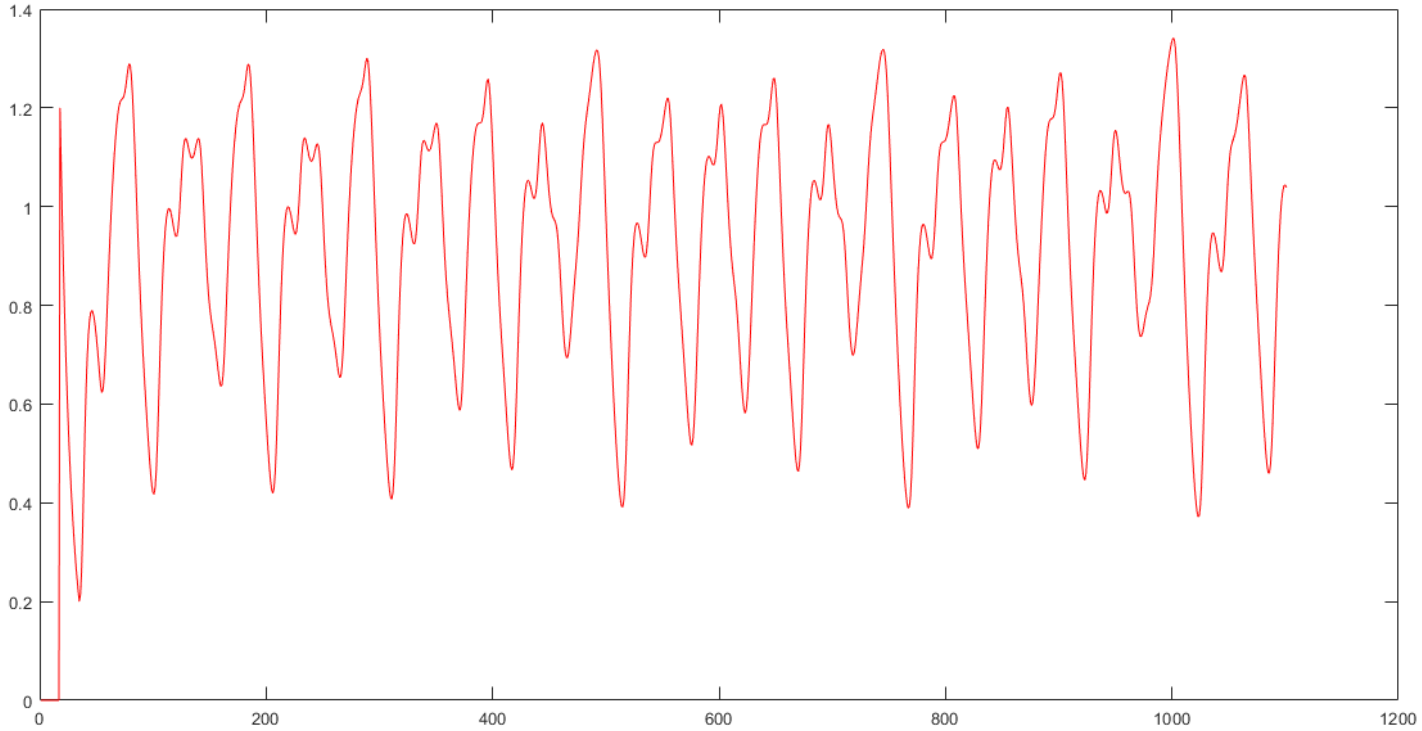
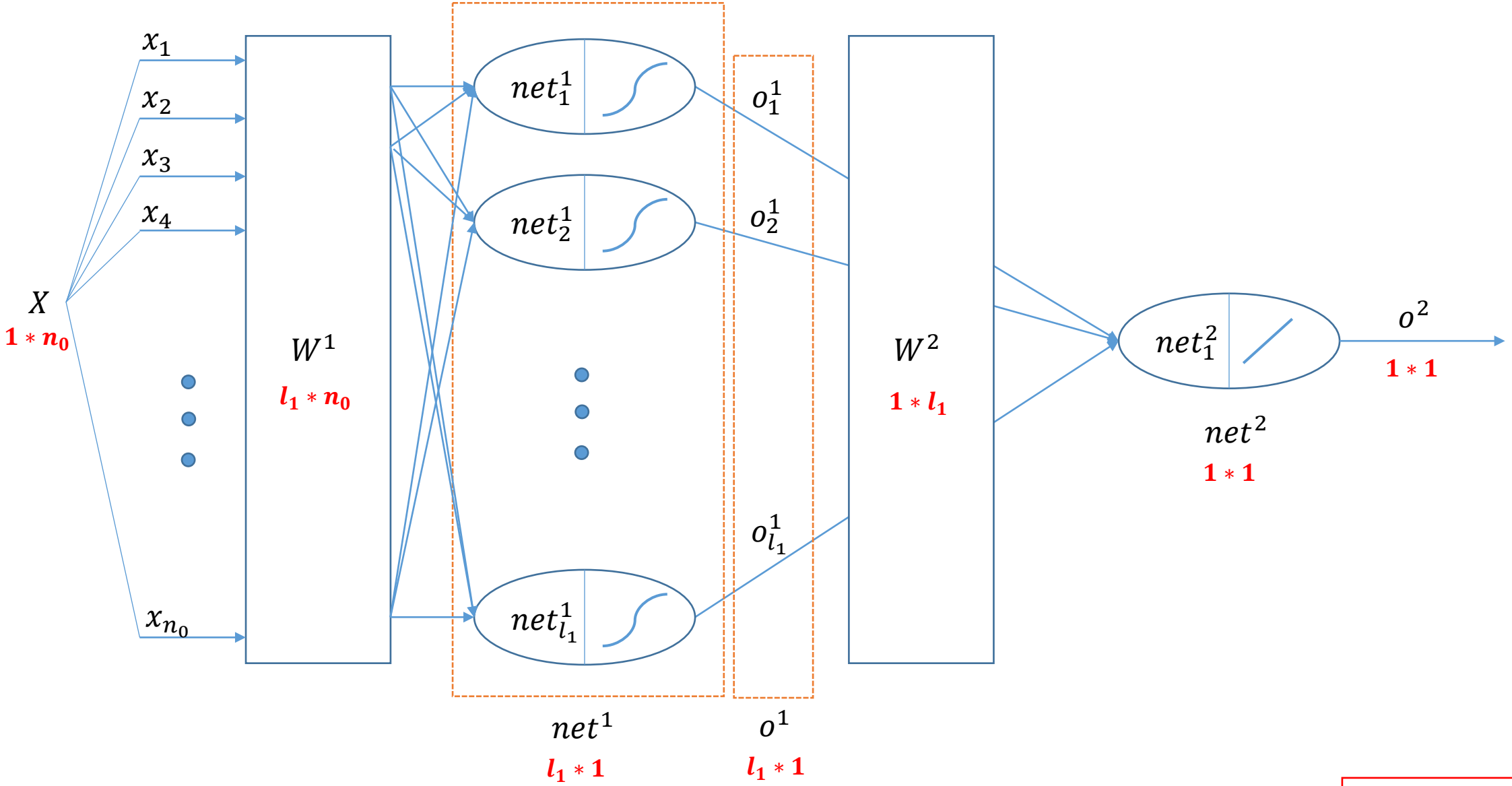
$$x(t+1) = x(t) + \beta \frac{x(t-\tau)}{1 + x^n(t-\tau)} - \gamma x(t)\,, \qquad \begin{cases} \beta = 0.2 \\ \gamma = -0.1 \\ n = 10 \\ \tau = 17 \\ t_{max} = 1100 \end{cases}, \qquad x(t) = 0 \; if \; t < 18$$




**Mackey-Glass**


$x_1$
$x_2$
$x_3$
$x_4$
$x_{n_0}$
$X$
$1 * n_0$
$W^1$
$l_1 * n_0$
$net_1^1$
$net_2^1$
$net_{l_1}^1$
$net^1$
$l_1 * 1$
$o_1^1$
$o_2^1$
$o_{l_1}^1$
$o^1$
$l_1 * 1$
$W^2$
$1 * l_1$
$net_1^2$
$net^2$
$1 * 1$
$o^2$
$1 * 1$

**2 Layer MLP**

## Feed Forward:

$net^1 = W^1 * X^T$ $\quad l_1 * 1 = l_1 * n_0 * n_0 * 1$

$o^1 = f^1(net^1) = sigmoid(net^1)$ $\quad l_1 * 1$

$net^2 = W^2 * o^1$ $\quad 1 * 1 = 1 * l_1 * l_1 * 1$

$o^2 = f^2(net^2) = net^2$ $\quad 1 * 1$

2 Layer MLP

## Back Propagation:

$$\underset{1*l_1}{W^2} = \underset{1*l_1}{W^2} - \eta\left(\frac{\partial E}{\partial e}\frac{\partial e}{\partial o^2}\frac{\partial o^2}{\partial net^2}\frac{\partial net^2}{\partial W^2}\right) = \underset{1*l_1}{W^2} - \eta(\underset{1*1}{e} * -1 * 1 * \underset{l_1*1}{o^1})$$

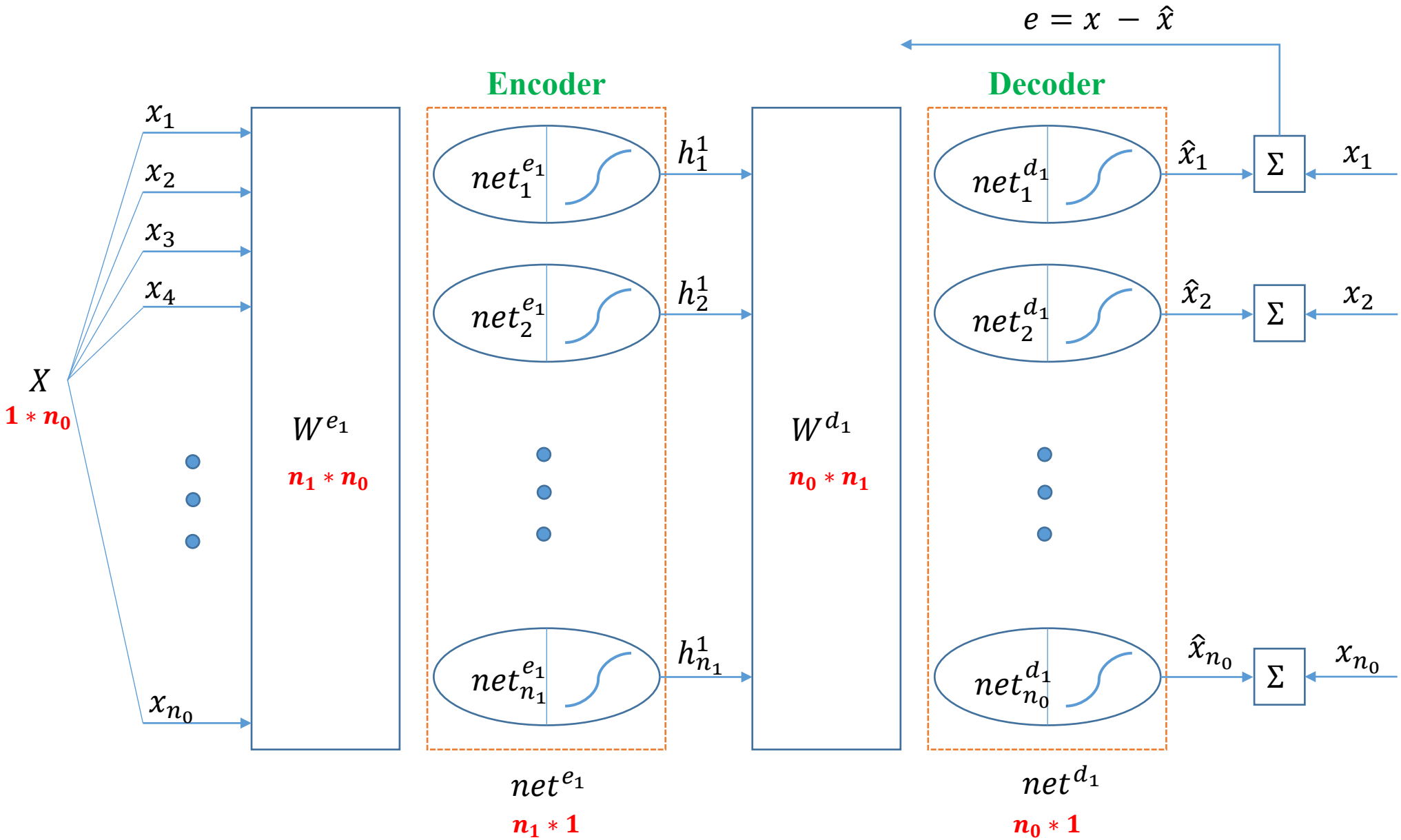
$$\underset{1*l_1}{W^2} = \underset{1*l_1}{W^2} + \eta * e * \underset{1*l_1}{o^{1^T}}$$

$$\underset{l_1*n_0}{W^1} = \underset{l_1*n_0}{W^1} - \eta\left(\frac{\partial E}{\partial e}\frac{\partial e}{\partial o^2}\frac{\partial o^2}{\partial net^2}\frac{\partial net^2}{\partial o^1}\frac{\partial o^1}{\partial net^1}\frac{\partial net^1}{\partial W^1}\right) =$$

$$\underset{l_1*n_0}{W^1} - \eta(\underset{1*1}{e} * -1 * 1 * \underset{1*l_1}{W^2} * \underset{l_1*l_1}{f^{1'}} * \underset{n_0*1}{X^T}) = \underset{l_1*n_0}{W^1} + \eta\left(\underset{1*1}{e} * (\underset{l_1*1}{W^2} * f^{1'})^T * \underset{1*n_0}{X^{T^T}}\right)$$

$$\underset{l_1*n_0}{W^1} = \underset{l_1*n_0}{W^1} + \eta * \underset{1*1}{e} * \underbrace{(W^2 * f^{1'})^T * X}_{l_1*n_0}$$

2 Layer MLP


$e = x - \hat{x}$
Encoder
Decoder
$x_1$
$x_2$
$x_3$
$x_4$
$x_{n_0}$
$X$
$1 * n_0$
$W^{e_1}$
$n_1 * n_0$
$net_1^{e_1}$
$net_2^{e_1}$
$net_{n_1}^{e_1}$
$h_1^1$
$h_2^1$
$h_{n_1}^1$
$W^{d_1}$
$n_0 * n_1$
$net_1^{d_1}$
$net_2^{d_1}$
$net_{n_0}^{d_1}$
$\hat{x}_1$
$\hat{x}_2$
$\hat{x}_{n_0}$
$\Sigma$
$x_1$
$x_2$
$x_{n_0}$
$net^{e_1}$
$n_1 * 1$
$net^{d_1}$
$n_0 * 1$

1 Layer AE

# Feed Forward:

$$net^{e_1} = W^{e_1} * X^T \qquad n_1 * 1 = n_1 * n_0 \quad * \quad n_0 * 1$$

$$h^1 = f^{e_1}(net^{e_1}) = sigmoid(net^{e_1}) \qquad n_1 * 1$$

$$net^{d_1} = W^{d_1} * h^1 \qquad n_0 * 1 = n_0 * n_1 \quad * \quad n_1 * 1$$

$$\hat{x} = f^{d_1}\left(net^{d_1}\right) = sigmoid(net^{d_1}) \qquad n_0 * 1$$

1 Layer AE

## Back Propagation:

$$W^{d_1}_{n_0 * n_1} = W^{d_1}_{n_0 * n_1} - \eta \left( \frac{\partial E}{\partial e} \frac{\partial e}{\partial \hat{x}} \frac{\partial \hat{x}}{\partial net^{d_1}} \frac{\partial net^{d_1}}{\partial W^{d_1}} \right) = W^{d_1}_{n_0 * n_1} - \eta (e_{1 * n_0} * -1 * f^{d_1}{}'_{n_0 * n_0} * h^1_{n_1 * 1})$$

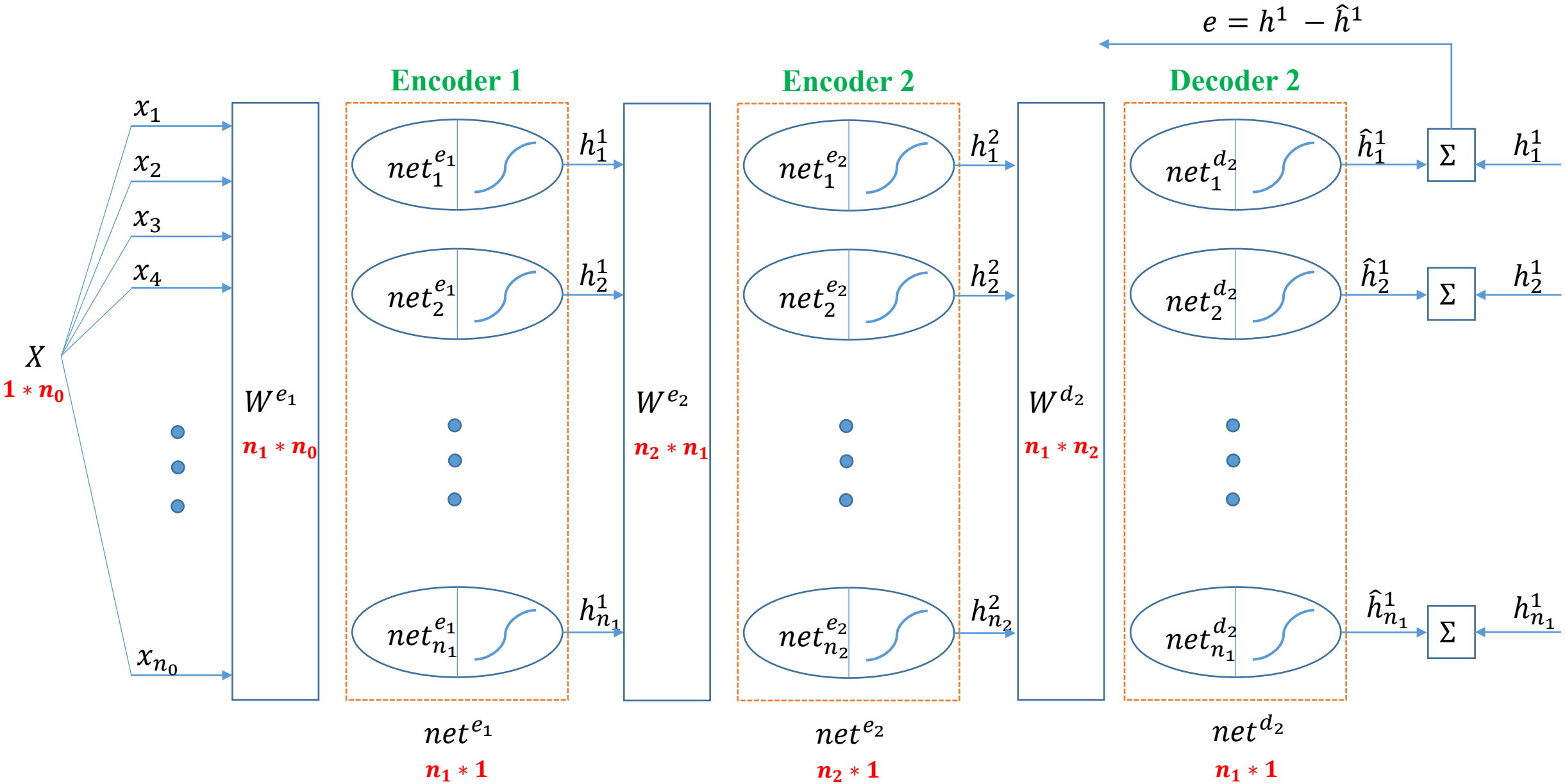
$$W^{d_1}_{n_0 * n_1} = W^{d_1}_{n_0 * n_1} + \eta * (h^1_{n_1 * 1} * e_{1 * n_0} * f^{d_1}{}'_{n_0 * n_0})^T$$

1. $W^{e_1} = W^{d_1}{}^T$
2. $W^{e_1}_{n_1 * n_0} = W^{e_1}_{n_1 * n_0} - \eta \left( \frac{\partial E}{\partial e} \frac{\partial e}{\partial \hat{x}} \frac{\partial \hat{x}}{\partial net^{d_1}} \frac{\partial net^{d_1}}{\partial h^1} \frac{\partial h^1}{\partial net^{e_1}} \frac{\partial net^{e_1}}{\partial W^{e_1}} \right) =$

$$W^{e_1}_{n_1 * n_0} - \eta \left( e_{1 * n_0} * -1 * f^{d_1}{}'_{n_0 * n_0} * W^{d_1}_{n_0 * n_1} * f^{e_1}{}'_{n_1 * n_1} * X^T_{n_0 * 1} \right)$$

$$W^{e_1}_{n_1 * n_0} = W^{e_1}_{n_1 * n_0} + \eta * (X^T_{n_0 * 1} * e_{1 * n_0} * f^{d_1}{}'_{n_0 * n_0} * W^{d_1}_{n_0 * n_1} * f^{e_1}{}'_{n_1 * n_1})^T$$

1 Layer AE


$e = h^1 - \hat{h}^1$
Encoder 1
Encoder 2
Decoder 2
$x_1$
$x_2$
$x_3$
$x_4$
$x_{n_0}$
$X$
$1 * n_0$
$W^{e_1}$
$n_1 * n_0$
$net_1^{e_1}$
$net_2^{e_1}$
$net_{n_1}^{e_1}$
$h_1^1$
$h_2^1$
$h_{n_1}^1$
$W^{e_2}$
$n_2 * n_1$
$net_1^{e_2}$
$net_2^{e_2}$
$net_{n_2}^{e_2}$
$h_1^2$
$h_2^2$
$h_{n_2}^2$
$W^{d_2}$
$n_1 * n_2$
$net_1^{d_2}$
$net_2^{d_2}$
$net_{n_1}^{d_2}$
$\hat{h}_1^1$
$\hat{h}_2^1$
$\hat{h}_{n_1}^1$
$\Sigma$
$h_1^1$
$h_2^1$
$h_{n_1}^1$
$net^{e_1}$
$n_1 * 1$
$net^{e_2}$
$n_2 * 1$
$net^{d_2}$
$n_1 * 1$

**2 Layer AE**

# Feed Forward:

$$net^{e_1} = W^{e_1} * X^T \qquad n_1 * 1 = n_1 * n_0 \ * \ n_0 * 1$$
$$h^1 = f^{e_1}(net^{e_1}) = sigmoid(net^{e_1}) \qquad n_1 * 1$$

$$net^{e_2} = W^{e_2} * h^1 \qquad n_2 * 1 = n_2 * n_1 \ * \ n_1 * 1$$
$$h^2 = f^{e_2}(net^{e_2}) = sigmoid(net^{e_2}) \qquad n_2 * 1$$

$$net^{d_2} = W^{d_2} * h^2 \qquad n_1 * 1 = n_1 * n_2 \ * \ n_2 * 1$$
$$\hat{h}^1 = f^{d_2}\left(net^{d_2}\right) = sigmoid(net^{d_2}) \qquad n_1 * 1$$

2 Layer AE

## Back Propagation:

$$\underset{n_1 * n_2}{W^{d_2}} = \underset{n_1 * n_2}{W^{d_2}} - \eta \left( \frac{\partial E}{\partial e} \frac{\partial e}{\partial \hat{h}^1} \frac{\partial \hat{h}^1}{\partial net^{d_2}} \frac{\partial net^{d_2}}{\partial W^{d_2}} \right) = \underset{n_1 * n_2}{W^{d_2}} - \eta (\underset{1 * n_1}{e} * -1 * \underset{n_1 * n_1}{f^{d_2}{}'} * \underset{n_2 * 1}{h^2})$$

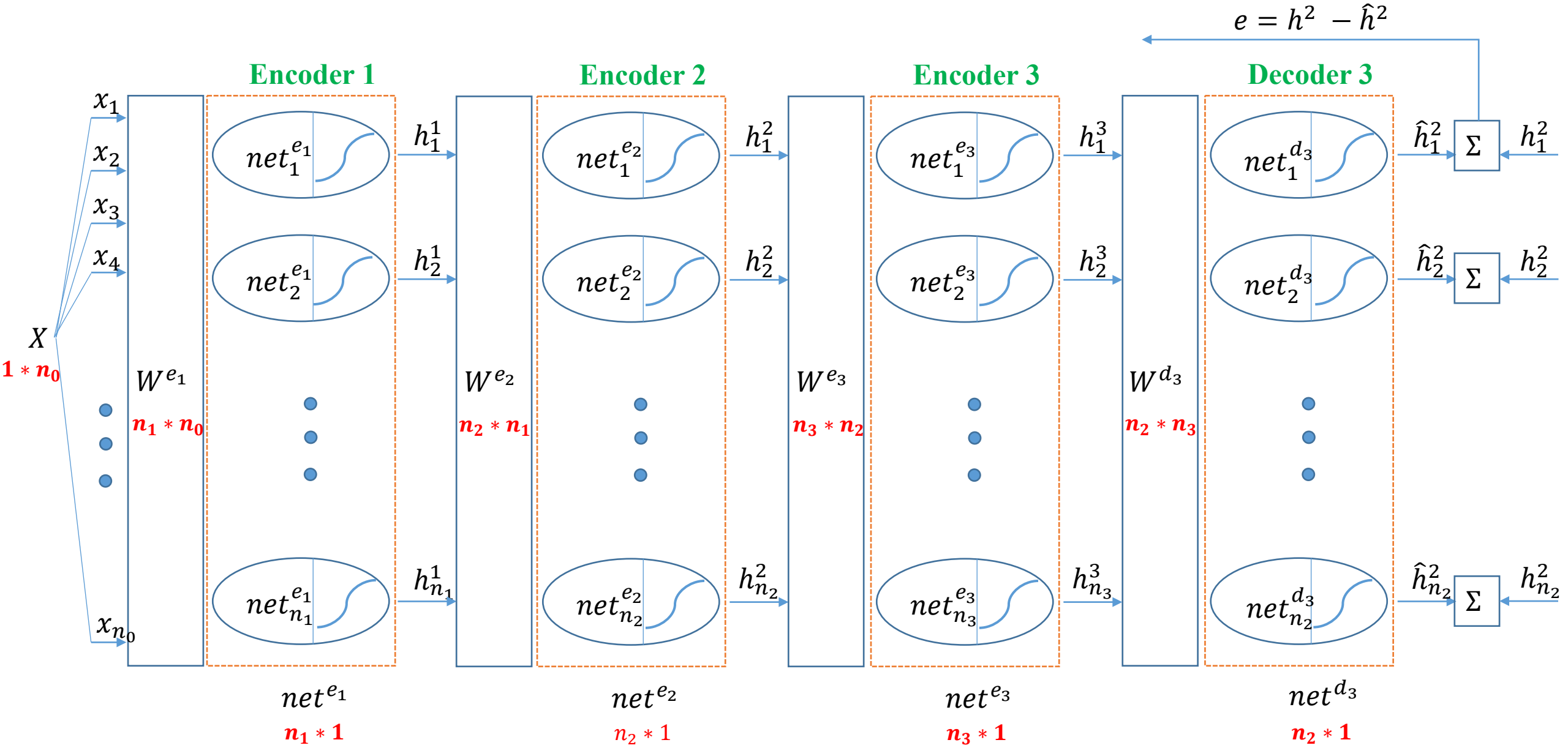
$$\underset{n_1 * n_2}{W^{d_2}} = \underset{n_1 * n_2}{W^{d_2}} + \eta * (\underset{n_2 * 1}{h^2} * \underset{1 * n_1}{e} * \underset{n_1 * n_1}{f^{d_2}{}'})^T$$

1. $W^{e_2} = W^{d_2{}^T}$
2. $\underset{n_2 * n_1}{W^{e_2}} = \underset{n_2 * n_1}{W^{e_2}} - \eta \left( \frac{\partial E}{\partial e} \frac{\partial e}{\partial \hat{h}^1} \frac{\partial \hat{h}^1}{\partial net^{d_2}} \frac{\partial net^{d_2}}{\partial h^2} \frac{\partial h^2}{\partial net^{e_2}} \frac{\partial net^{e_2}}{\partial W^{e_2}} \right) =$

$$\underset{n_2 * n_1}{W^{e_2}} - \eta \left( \underset{1 * n_1}{e} * -1 * \underset{n_1 * n_1}{f^{d_2}{}'} * \underset{n_1 * n_2}{W^{d_2}} * \underset{n_2 * n_2}{f^{e_2}{}'} * \underset{n_1 * 1}{h^1} \right)$$

$$\underset{n_2 * n_1}{W^{e_2}} = \underset{n_2 * n_1}{W^{e_2}} + \eta * (\underset{n_1 * 1}{h^1} * \underset{1 * n_1}{e} * \underset{n_1 * n_1}{f^{d_2}{}'} * \underset{n_1 * n_2}{W^{d_2}} * \underset{n_2 * n_2}{f^{e_2}{}'})^T$$

**2 Layer AE**


$e = h^2 - \hat{h}^2$
Encoder 1
Encoder 2
Encoder 3
Decoder 3
$x_1$
$x_2$
$x_3$
$x_4$
$x_{n_0}$
$X$
$1 * n_0$
$W^{e_1}$
$n_1 * n_0$
$W^{e_2}$
$n_2 * n_1$
$W^{e_3}$
$n_3 * n_2$
$W^{d_3}$
$n_2 * n_3$
$net_1^{e_1}$
$net_2^{e_1}$
$net_{n_1}^{e_1}$
$net_1^{e_2}$
$net_2^{e_2}$
$net_{n_2}^{e_2}$
$net_1^{e_3}$
$net_2^{e_3}$
$net_{n_3}^{e_3}$
$net_1^{d_3}$
$net_2^{d_3}$
$net_{n_2}^{d_3}$
$h_1^1$
$h_2^1$
$h_{n_1}^1$
$h_1^2$
$h_2^2$
$h_{n_2}^2$
$h_1^3$
$h_2^3$
$h_{n_3}^3$
$\hat{h}_1^2$
$\hat{h}_2^2$
$\hat{h}_{n_2}^2$
$\Sigma$
$net^{e_1}$
$n_1 * 1$
$net^{e_2}$
$n_2 * 1$
$net^{e_3}$
$n_3 * 1$
$net^{d_3}$
$n_2 * 1$

**3 Layer AE**

# Feed Forward:

$$net^{e_1} = W^{e_1} * X^T \qquad n_1 * 1 = n_1 * n_0 \ * \ n_0 * 1$$
$$h^1 = f^{e_1}(net^{e_1}) = sigmoid(net^{e_1}) \qquad n_1 * 1$$

$$net^{e_2} = W^{e_2} * h^1 \qquad n_2 * 1 = n_2 * n_1 \ * \ n_1 * 1$$
$$h^2 = f^{e_2}(net^{e_2}) = sigmoid(net^{e_2}) \qquad n_2 * 1$$

$$net^{e_3} = W^{e_3} * h^2 \qquad n_3 * 1 = n_3 * n_2 \ * \ n_2 * 1$$
$$h^3 = f^{e_3}(net^{e_3}) = sigmoid(net^{e_3}) \qquad n_3 * 1$$

$$net^{d_3} = W^{d_3} * h^3 \qquad n_2 * 1 = n_2 * n_3 \ * \ n_3 * 1$$
$$\hat{h}^2 = f^{d_3}\left(net^{d_3}\right) = sigmoid(net^{d_3}) \qquad n_2 * 1$$

3 Layer AE

## Back Propagation:

$$W^{d_3}_{n_2 * n_3} = W^{d_3}_{n_2 * n_3} - \eta \left( \frac{\partial E}{\partial e} \frac{\partial e}{\partial \hat{h}^2} \frac{\partial \hat{h}^2}{\partial net^{d_3}} \frac{\partial net^{d_3}}{\partial W^{d_3}} \right) = W^{d_3}_{n_2 * n_3} - \eta (e_{1 * n_2} * -1 * f^{d_3\prime}_{n_2 * n_2} * h^3_{n_3 * 1})$$

$$W^{d_3}_{n_2 * n_3} = W^{d_3}_{n_2 * n_3} + \eta * (h^3_{n_3 * 1} * e_{1 * n_2} * f^{d_3\prime}_{n_2 * n_2})^T$$

1 $$W^{e_3} = W^{d_3{}^T}$$

2 $$W^{e_3}_{n_3 * n_2} = W^{e_3}_{n_3 * n_2} - \eta \left( \frac{\partial E}{\partial e} \frac{\partial e}{\partial \hat{h}^2} \frac{\partial \hat{h}^2}{\partial net^{d_3}} \frac{\partial net^{d_3}}{\partial h^3} \frac{\partial h^3}{\partial net^{e_3}} \frac{\partial net^{e_3}}{\partial W^{e_3}} \right) =$$

$$W^{e_3}_{n_3 * n_2} - \eta \left( e_{1 * n_2} * -1 * f^{d_3\prime}_{n_2 * n_2} * W^{d_3}_{n_2 * n_3} * f^{e_3\prime}_{n_3 * n_3} * h^2_{n_2 * 1} \right)$$

$$W^{e_3}_{n_3 * n_2} = W^{e_3}_{n_3 * n_2} + \eta * (h^2_{n_2 * 1} * e_{1 * n_2} * f^{d_3\prime}_{n_2 * n_2} * W^{d_3}_{n_2 * n_3} * f^{e_3\prime}_{n_3 * n_3})^T$$

3 Layer AE

# Feed Forward:

$$net^{e_1} = W^{e_1} * X^T$$
$$h^1 = f^{e_1}(net^{e_1}) = sigmoid(net^{e_1})$$

Encoder 1

$$net^{e_2} = W^{e_2} * h^1$$
$$h^2 = f^{e_2}(net^{e_2}) = sigmoid(net^{e_2})$$

Encoder 2

$$net^{e_3} = W^{e_3} * h^2$$
$$h^3 = f^{e_3}(net^{e_3}) = sigmoid(net^{e_3})$$

Encoder 1

$$net^1 = W^1 * h^3$$
$$o^1 = f^1(net^1) = sigmoid(net^1)$$

Perceptron layer 1

$$net^2 = W^2 * o^1$$
$$o^2 = f^2(net^2) = net^2$$

Perceptron layer 2

3 Layer AE +2 Layer MLP + Global Train

## Back Propagation:

$$W^{e_1} = W^{e_1} - \eta\left(\frac{\partial E}{\partial e}\frac{\partial e}{\partial o^2}\frac{\partial o^2}{\partial net^2}\frac{\partial net^2}{\partial o^1}\frac{\partial o^1}{\partial net^1}\frac{\partial net^1}{\partial h^3}\frac{\partial h^3}{\partial net^{e_3}}\frac{\partial net^{e_3}}{\partial h^2}\frac{\partial h^2}{\partial net^{e_2}}\frac{\partial net^{e_2}}{\partial h^1}\frac{\partial h^1}{\partial net^{e_1}}\frac{\partial net^{e_1}}{\partial W^{e_1}}\right)$$

$$W^{e_2} = W^{e_2} - \eta\left(\frac{\partial E}{\partial e}\frac{\partial e}{\partial o^2}\frac{\partial o^2}{\partial net^2}\frac{\partial net^2}{\partial o^1}\frac{\partial o^1}{\partial net^1}\frac{\partial net^1}{\partial h^3}\frac{\partial h^3}{\partial net^{e_3}}\frac{\partial net^{e_3}}{\partial h^2}\frac{\partial h^2}{\partial net^{e_2}}\frac{\partial net^{e_2}}{\partial W^{e_2}}\right)$$

$$W^{e_3} = W^{e_3} - \eta\left(\frac{\partial E}{\partial e}\frac{\partial e}{\partial o^2}\frac{\partial o^2}{\partial net^2}\frac{\partial net^2}{\partial o^1}\frac{\partial o^1}{\partial net^1}\frac{\partial net^1}{\partial h^3}\frac{\partial h^3}{\partial net^{e_3}}\frac{\partial net^{e_3}}{\partial W^{e_3}}\right)$$

$$W^{1} = W^{1} - \eta\left(\frac{\partial E}{\partial e}\frac{\partial e}{\partial o^2}\frac{\partial o^2}{\partial net^2}\frac{\partial net^2}{\partial o^1}\frac{\partial o^1}{\partial net^1}\frac{\partial net^1}{\partial W^1}\right)$$

$$W^{2} = W^{2} - \eta\left(\frac{\partial E}{\partial e}\frac{\partial e}{\partial o^2}\frac{\partial o^2}{\partial net^2}\frac{\partial net^2}{\partial W^2}\right)$$

**3 Layer AE +2 Layer MLP + Global Train**

# Back Propagation:

$$W^{e_3} = W^{e_3} - \eta\left(\frac{\partial E}{\partial e}\frac{\partial e}{\partial o^2}\frac{\partial o^2}{\partial net^2}\frac{\partial net^2}{\partial o^1}\frac{\partial o^1}{\partial net^1}\frac{\partial net^1}{\partial h^3}\frac{\partial h^3}{\partial net^{e_3}}\frac{\partial net^{e_3}}{\partial W^{e_3}}\right)$$

$$\underset{n_3 * n_2}{W^{e_3}} = \underset{n_3 * n_2}{W^{e_3}} - \eta(\underset{1 * 1}{e} * -1 * 1 * \underset{1 * l_1}{W^2} * \underset{l_1 * l_1}{f^{1'}} * \underset{l_1 * n_3}{W^1} * \underset{n_3 * n_3}{f^{e_3}{}'} * \underset{1 * n_2}{h^2})$$

AE 3rd layer output = MLP 1st layer input

$$\underset{n_3 * n_2}{W^{e_3}} = \underset{n_3 * n_2}{W^{e_3}} - \eta(\underset{1 * 1}{e} * -1 * 1 * \underbrace{(W^2 * f^{1'} * W^1 * f^{e_3}{}')^T}_{n_3 * 1} * \underset{1 * n_2}{h^2})$$

**3 Layer AE +2 Layer MLP + Global Train**

# Back Propagation:

$$W^{e_2} = W^{e_2} - \eta\left(\frac{\partial E}{\partial e}\frac{\partial e}{\partial o^2}\frac{\partial o^2}{\partial net^2}\frac{\partial net^2}{\partial o^1}\frac{\partial o^1}{\partial net^1}\frac{\partial net^1}{\partial h^3}\frac{\partial h^3}{\partial net^{e_3}}\frac{\partial net^{e_3}}{\partial h^2}\frac{\partial h^2}{\partial net^{e_2}}\frac{\partial net^{e_2}}{\partial W^{e_2}}\right)$$

$$\underset{n_2 * n_1}{W^{e_2}} = \underset{n_2 * n_1}{W^{e_2}} - \eta(\underset{1 * 1}{e} * -1 * 1 * \underset{1 * l_1}{W^2} * \underset{l_1 * l_1}{f^{1'}} * \underset{l_1 * n_3}{W^1} * \underset{n_3 * n_3}{f^{e_3\prime}} * \underset{n_3 * n_2}{W^{e_3}} * \underset{n_2 * n_2}{f^{e_2\prime}} * \underset{1 * n_1}{h^1})$$

$$\underset{n_2 * n_1}{W^{e_2}} = \underset{n_2 * n_1}{W^{e_2}} - \eta(\underset{1 * 1}{e} * -1 * 1 * \underbrace{(W^2 * f^{1'} * W^1 * f^{e_3\prime} * W^{e_3} * f^{e_2\prime})^T}_{n_2 * 1} * \underset{1 * n_1}{h^1})$$

**3 Layer AE +2 Layer MLP + Global Train**

## Back Propagation:

$$W^{e_1} = W^{e_1} - \eta\left(\frac{\partial E}{\partial e}\frac{\partial e}{\partial o^2}\frac{\partial o^2}{\partial net^2}\frac{\partial net^2}{\partial o^1}\frac{\partial o^1}{\partial net^1}\frac{\partial net^1}{\partial h^3}\frac{\partial h^3}{\partial net^{e_3}}\frac{\partial net^{e_3}}{\partial h^2}\frac{\partial h^2}{\partial net^{e_2}}\frac{\partial net^{e_2}}{\partial h^1}\frac{\partial h^1}{\partial net^{e_1}}\frac{\partial net^{e_1}}{\partial W^{e_1}}\right)$$

$$\underset{n_1 * n_0}{W^{e_1}} = \underset{n_1 * n_0}{W^{e_1}} - \eta(\underset{1 * 1}{e} * -1 * 1 * \underset{1 * l_1}{W^2} * \underset{l_1 * l_1}{f^{1'}} * \underset{l_1 * n_3}{W^1} * \underset{n_3 * n_3}{f^{e_3\prime}} * \underset{n_3 * n_2}{W^{e_3}} * \underset{n_2 * n_2}{f^{e_2\prime}} * \underset{n_2 * n_1}{W^{e_2}} * \underset{n_1 * n_1}{f^{e_1\prime}} * \underset{1 * n_0}{X})$$

$$\underset{n_1 * n_0}{W^{e_1}} = \underset{n_1 * n_0}{W^{e_1}} - \eta(\underset{1 * 1}{e} * -1 * 1 * \underbrace{(W^2 * f^{1'} * W^1 * f^{e_3\prime} * W^{e_3} * f^{e_2\prime} * W^{e_2} * f^{e_1\prime})^T}_{n_1 * 1} * \underset{1 * n_0}{X})$$

**3 Layer AE +2 Layer MLP + Global Train**