Comparison of Capacitance Computation by Different Methods

M.Dhamodaran, *Member, IEEE*, R.Dhanasekaran, *Senior Member, IEEE*

Abstract— **In this paper calculation of capacitance per unit length can be analyzed by different method like method of moment (MOM), Boundary Element Method (BEM), Finite element method (FEM), Finite difference method (FDM), Charge Simulation method(CSM), point matching method and Surface charge method. To illustrate the usefulness of this technique, we apply these methods to the computation of capacitance of cube. This paper reviews the results of computing the capacitance-per-unit length with the other methods and the capacitance of a metallic cube is reviewed by different method.**

Index Terms— **Capacitance, Conductors, Cube, Rectangular subdomain, Method of Moments, Finite element method.**

I. Introduction

Various methods used for analyzing capacitance computation include the method of moment (MOM), Boundary Element Method (BEM), Finite element method (FEM), Finite difference method (FDM), Charge Simulation Method (CSM), point matching method, Surface charge method.

Capacitance extraction has found extensive use for systems involving sets of long parallel transmission lines in multi-dielectric environment as well as integrated circuit package including three-dimensional conductors located on parallel planes. This paper starts by reviewing fundamental aspects of transient electromagnetics followed by the governing differential and integral equations to motivate the application of numerical methods as Method of Moments, Finite Element Method etc. Among these numerical tools, the surface-based integral-equation methodology - MoM is ideally suited to address the problem.

The method of moments is a technique used to solve electromagnetic boundary or volume integral equations in the frequency domain. Because the electromagnetic sources are the quantities of interest, the MOM is very useful in solving radiation and scattering problems. In this paper, we focus on the practical solution of boundary integral equations of radiation and scattering using this method.

M.Dhamodaran, Department of Electronics and Communication Engineering, Syed Ammal Engineering College, Ramanathapuram, Tamilnadu, India (e-mail: dhamodaranm@yahoo.com).

R.Dhanasekaran, Department of Electrical and Electronics Engineering, Syed Ammal Engineering College, Ramanathapuram, Tamilnadu, India (e-mail: rdhanashekar@yahoo.com).

The finite difference method uses the method of finite differences to solve Maxwell's Equations in the time domain. Application of the FDM method is usually very straightforward: the solution domain is typically discretized into small rectangular or curvilinear elements, with a "leap frog" in time used to compute the electric and magnetic fields from one another. FDM excels at analysis of inhomogeneous and nonlinear media, though its demands for system memory are high due to the discretization of the entire solution domain, and it suffers from dispersion issues as well and the need to artificially truncate the solution boundary. FDM finds applications in packaging and waveguide problems, as well as in the study of wave propagation in complex dielectrics.

The finite element method is a method used to solve frequency domain boundary valued electromagnetic problems by using a variation form. It can be used with two- and three-dimensional canonical elements of differing shape, allowing for a highly accurate discretization of the solution domain. The FEM is often used in the frequency domain for computing the frequency field distribution in complex, closed regions such as cavities and waveguides. As in the FDM method, the solution domain must be truncated, making the FEM unsuitable for radiation or scattering problems unless combined with a boundary integral equation approach.

The charge simulation method is very useful to solve partial differential equations in electrical engineering, and has been studied and developed by a lot of researchers. The method is easy to understand, and can be applied only by solving a system of simultaneous linear equations. Many examples show that the method makes possible to get rather precise solutions for the boundary value problems with respect to domains bounded by smooth curves.

II. Methods

Ghosh et al proposed that Method of Moments with pulse basis function and point matching is employed to calculate the charge distribution on the surface and hence the capacitance. This paper presents the results for capacitance of different conducting shapes, e.g., square, rectangular, circular, annular circular disk, T-shaped, L-shaped, triangular, annular triangular, etc. A simple and efficient numerical procedure is presented for treating electrostatic problem involving complex geometrical shaped planar conducting bodies. [1]

Karthikeyan et al describes free space capacitance of dielectric-coated conducting curvilinear surfaces is obtained using method of moments. The surfaces are discretized using

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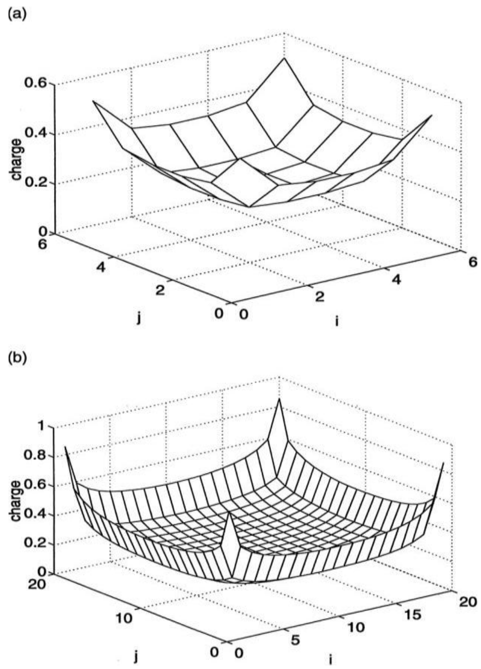

Fig.1(a) 5x5 sub area (b) 19x19 sub area

triangular subsections. The curvilinear surfaces are represented by quadratic parametric elements. The MOM based on the pulse basis function and point matching is used. In this article, capacitance for a dielectric-coated cylinder of finite axial length and a truncated cone is presented. The moment method with the parametric elements is used for the computation. The integration is performed numerically for both the regular and the singular part of the integral. The numerical results obtained are compared with the published results to validate the analysis. Results obtained show good agreement with the previous results.[2]

Mohamed ouda describes capacitance calculation of arbitrary-shaped conducting bodies is an important step in the prediction of electrostatic discharge and quasi static analysis of various system. In this study, the capacitances of arbitrary-shaped conducting surfaces are evaluated based on the Characteristic Basis function method (CBF) in conjunction with Integral Equation Method (IEM). Computer programs based on the CBF method and the classical IEM had been developed to determine the charge distribution and capacitance of general arbitrary shaped conducting structures.[3]

Milovanovic et al describes calculations of the capacitance per unit length of one or multilayer dielectric lines are presented. Special attention is given to the calculations of the capacitance per unit length of lines with rectangular cross sections, whose electrodes may be in different or the same layers of a two layer dielectric line. For the purpose of performing the above, several numerical methods are used and simple approximate expressions are proposed. This paper presents an instructive review of different techniques for calculations of the capacitance per unit length of lines with multilayer medium, especially lines with rectangular cross section. Several numerical methods and two program packages are used.[4]

Wei Bai et al describe evaluating the capacitance of a cube using MATLAB program. Choice of this software program is based on its wide availability in universities and industries and its ease of use. In order to use the method of subareas, author divide the total surface area a^2 of one side of a cube into a number of subareas. In Fig.1, the calculated charge distribution on one of the plates is calculated. Initially, the voltage on all of the some areas was specified to have a numerical value of 1. The calculation was repeated for two values of the parameter which indicates the number of subareas N x N in which the area of one side has been subdivided. The results of this calculation indicate that the distribution of charge on that plate is very non uniform. In addition, since the voltage on each area was chosen to have the numerical value of 1, these graphs also indicate the spatial distribution of the incremental capacitances.[5]

Cesar Monzon describes a simple formula for the capacitance of a finite length, perfectly-conducting right-circular cylindrical tube coated with a homogeneous dielectric sleeve. The formula may be used to determine capacitance for all practical aspect ratios (radius/length) and is validated by comparisons with available data calculated from numerical solutions of the coupled set of integral equations. A novel and elementary closed-form approximate expression for the capacitance of a finite duct with a dielectric sleeve has been presented and successfully validated against independent numerical data. The expression employs an approximate formula for the capacitance of a finite hollow duct which is extracted from the related problem of a solid cylinder. Based on comparison with available data, the expression is believed to be accurate for all parameters of practical interest.[6]

Chakrabarty et al describes Capacitance of dielectric coated metallic cylinder and truncated cone are evaluated using the method of moments based on pulse function and point matching. The analysis is based on boundary condition for the potential on the conductor surface and normal component of displacement density at the dielectric-free space interface. The total free charge on the conductor surface is found from the inversion of a matrix partitioned into submatrices. Numerical data on capacitance and charge distribution are presented. Agreement of the results justifies the validity of the analysis.[7]

Prarthan et al describes a method for the evaluation of the capacitance of the dielectric coated metallic plates forming a corner using the method of moments based on the pulse basis function and the point matching. Two integral equations are formed based on the boundary conditions for the potential on the conductor surface and continuity of the normal component of the displacement flux density at the dielectric-free space interface. A set of simultaneous equations are formed from the two integral equations using the method of moments. The total free charge on the conductor surface is found from the solution of the set of simultaneous equations. Numerical data on the capacitance and the charge distribution are presented. The validity of the analysis has been justified by comparing the data on the capacitance that is available in the literature for a metallic structure with the data on the capacitance computed with the present method for a similar structure considering a very low dielectric constant as well as a very thin dielectric coating. Further validation has been carried out by comparing the capacitance data of the specific case when the angle between two metallic plates is 180^o forming a dielectric coated metallic rectangular plate for which the capacitance data are available in the literature. The validity of the analysis has been further justified by the close agreement between the data on capacitance computed with the present method.[8]

Williams et al describes calculation of capacitance with two new techniques, one utilizing the resistance of the line and other that of a resistor embedded in the line. A technique for directly comparing the capacitance of two similar transmission lines is demonstrated. Various methods for determining the capacitance per unit length C of quasi-TEM lines were found to approximately agree with each other.[16]

III. COMPARISON OF METHODS

Many numerical methods have been applied to extract the electrical parameters of the interconnects and packages. These methods can be generally classified into two categories:
Differential equation methods and integral equation methods. The common features of these methods include discretization of the unknown function, approximations, and the solution based on matrix solution techniques. The approximations result in inaccuracies. The differential equation methods, such as Finite Element Method (FEM) and Finite Difference Method (FDM) divide an interconnect cell into meshes and lead to a large scale sparse matrix equation. Two types of basis functions used in FEM are node-based and edge-based. Node-based functions are commonly used for simplicity of analysis. The numerical calculation of electrostatic fields requires the solution of a large set of coupled linear equations, one for each active node in the solution volume which require the set up of boundary condition for the whole solution volume. The method uses a 3-D finite element model in which the conductor charges are approximated by a piece-wise linear function on a web of edges located on the surface of the conductors.

The integral equation methods, such as the Method of Moments and the Boundary Element Method, divides the surfaces of conductors and the interfaces of dielectric layers into meshes and lead to a comparatively smaller but full matrix. This procedure is robust and has many advantages over finite difference or finite element schemes, including good conditioning, reduction in dimensionality, and the ability of treating arbitrary regions. However, when the number of conductors and dielectric layers increase, the analysis procedure will also be too costly in terms of computing time and memory needs due to high cost of working with large dense matrix. Therefore, while the numerical solution of the integral equation delivers good performance in the a two-dimensional analysis, its application in three-dimensional capacitance extraction have been limited to small problems. Solutions have been obtained for problems involving infinite dielectric regions by using Green's function techniques similar to those used in the two-dimensional calculations. This yields a system of Green's function integral equations that is solved by a novel approximate matrix inversion technique. Though the compressed storage technique and some efficient sparse matrix equation solvers maybe applied, the solving process is still time-consuming and needs vast memory resources.

Solutions have been obtained for problems involving infinite dielectric regions by using Green's function techniques similar to those used in the two-dimensional calculations. It is well known that none of these methods is the most efficient in all cases.

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As we can see from above, all the algorithms suffer from at least one limitation. In order to overcome this difficulty, Hybrid method combined differential methods and integral methods. The result was a good trend for 3D capacitance extraction development to keep good accuracy with enlarged application range of 3D geometries For example Hongchuan Wei et al [9] presents a new algorithm of coupling computation of BEM and the finite difference method to avoid a significant error that occurs if some dielectrics do not contain a conductor. This algorithm still uses BEM for dielectrics which contain conductors, but employs FDM for those without. The numerical results indicate that this algorithm reduces calculation error greatly. It can be used for fast parameter extraction of multi-layer and multi-conductor interconnects in VLSI circuit. Compared to some reference solutions, both the memory requirement and the CPU time are significantly reduced while maintaining a relatively high accuracy.

IV. APPLICATION AND DEVELOPMENT OF MOM

The MoM is a powerful and versatile numerical method for the solution of partial integral equations. Applications of MoM to electromagnetic are discussed in many recent papers since 1970's. The advantage of the MoM over purely numerical methods is that there is still a large part that remains analytic like the Green's functions. Yet, it remains a numerical method based on a matrix inversion technique and therefore, convergence issues need to be examined. The convergence of the MoM is closely related to the choice of basis functions and, although to a lesser extends, to the choice of testing functions. There are essentially two families of basis functions: entire domain basis functions yielding a good convergence of the method without mesh process and sub-domain basis functions rely on a proper meshing of the geometry giving more exibility of the geometry. Finally, we can mention that point matching, which is easy to grasp and straightforward to implement, may not yield an optimal convergence. In most of the applications, the Galerkin technique is better, which consists in choosing the same testing functions as the basis functions. This applies to both sub-domain and entire domain functions.

MoM involves a good amount of preprocessing of Maxwell's equations because it makes use of the Green's function. It's proposed as a surface-based integral-equation methodology. It leads to a well-conditioned system with reduced size, as compared to volumetric methods, but the system of equations generated is inherently dense, thereby creating a time and memory bottleneck. Several fast iterative techniques have been developed to efficiently sore and solve a MoM system with linear time and memory complexity. All these methods, including QR-based approaches, fast-multipole methods (FMMs), and FFT-based techniques etc.

QR-based fast iterative solver (IES3) which adopts a binary tree multi-level decomposition of the geometry and consequent low-rank compression of the MoM sub-matrices which represent the interaction between well-separated geometrical regions.This scheme is particularly attractive for circuit problems. Dipanjan Gope et al[12]presents an improved matrix-compression technique for fast iterative solution of such dense systems, which applies QR de-composition on multilevel Oct-tree-based interaction sub-matrices. It combined the regular-tree structure of the fast-multipole method with the rank-revealing QR-based matrix-compression scheme to achieve superior time and memory efficiency. As is demonstrated by the numerical-simulation results presented herein, the new algorithm is found to be faster and more memory efficient than both existing QR-based methods and FastCap. The fast multipole method (FMM), while originally developed for particle simulation problems, can be combined with iterative techniques to solve the dense integral equation matrices that arise from the Laplace equation. Parameter extraction programs such as FastCap and FastHenry use the FMM for accelerating the dense matrix-vector products required by an iterative solver. FastCap and FastHenry employ the FMM that was originally developed for particle simulation problems.

Until now, several commercial as well as public domain tools such as TMA's Raphael based on Finite Difference Method, Ansoft's SPICELINK based on Finite Element Methodand MIT's FastCap based on multipole accerated Boundary Element Method are available to calculate the statistic capacitance of various interconnects. Then, considering the transmission line, since the transverse electric fields of a TEM mode (the fields in the xy-plane whilst the wave propagates along the z-axis) are curl-free (have the electric potential) and satisfy Laplace's equation in the domain with no charges, similar observation is valid for the transverse magnetic fields, with the charges residing on metal conductors, the Laplace's equation for the corresponding electric potential ' is transformed to the well-known Poisson's equation. This circumstance allows us to find the static capacitance per unit length between two or more conductors using established numerical or analytical electrostatic techniques. The analytical techniques are based on conformal mapping in 2D space.

V. OTHER METHODS

The measured equation of invariance (MEI) method can be considered as a variation of FDM. To solve the infinite-domain model of capacitance extraction, the MEI method terminates the meshes very close to the object conductors and still preserves the sparsity of the finite-difference (FD) equations. The geometry-independent measured equation of invariance (GIMEI) is proposed for the capacitance extraction of the general 2D and 3D interconnects by using the free-space Green function only. The MEI method has now been developed to the on-surface level, where a surface mesh is used to minimize the number of unknowns. The stochastic method is based on the random-walk theory and can effectively handle complex 3D structures.

VI. RESULTS

The results of various research works discussed in this study is summarized in Table I. Using boundary element method with extrapolation, the most accurate value (0.6606785pF) obtained.

TABLE I Values for the Capacitance of the Unit Cube

Method	Capacitance(pF)
BEM	0.6555
FDM	0.661
Surface Charge Method	0.6606747
BEM	0.6632
Passage Algorithm	0.660675
BEM	0.6606785
Monte Carlo Estimator	0.660683

VII. CONCLUSION

In this paper, several calculation techniques of capacitance are reviewed. Their advantages and disadvantages are analyzed. The accuracy of the method of rectangular subareas can be improved by increasing the number of sections which leads to significant increase of memory storage and computational requirements. The future challenge is how to efficiently and accurately solve the problem taking into consideration of capacitance calculation. In order to overcome this challenge, hybrid method like combination of differential methods and integral methods have to be developed.

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