Application of Moment Method to Study of Charge Distribution, Stability in Circular and Square Conductors

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Application of Moment Method for the Study of Charge Distribution, Stability in Circular and Square Conductors

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***Abstract*- This paper reports a study of charge distribution on circular and square conductors. A mathematical calculation of capacitance per unit length can be analyzed by moment method. This method is widely used in the electromagnetic and electric field. This work consists of solving the two-dimensional Poisson equation and calculating the surface charge density $\sigma(x, y)$ of a square plate brought to potential by the Moments method. The analysis is based on boundary conditions for the potential on the conductor surface and normal component of displacement density at the dielectric-free space interface. In this context, we developed a numerical algorithm in MATLAB for the study of load distribution, stability in circular and square conductors. The numerical data on capacitance and charge distribution are presented and compared with theoretical results.**

***Index Terms*- Numerical methods, MoMs method Electromagnetic, Square conductors, Charge density**

I. INTRODUCTION

Recently, various methods have been used to solve the problem caused by the complexity of the electromagnetic field. These different methods from numerical to analytical and depend of the complexity of the problem. The moment method is a numerical procedure for solving linear operator equation by transforming it into a system of simultaneous linear algebraic equation referred to as a matrix equation. Numerous problems in electromagnetic field problems can be cast in the form of integral equations. Following the development of technology in the computer the computational electromagnetic and the moment method have followed closely behind. In research area several methods are used to analyzing the capacitance variation in the different conductor and metal; method of moment, Boundary Element Method, Finite element method, Finite difference method, Charge Simulation Method, point matching method, Surface charge method [1]. Moreover, the use of the moment method in electromagnetism has become popular since the works of Richmond in 1965 and Harrington in 1967 [2]. This method has been successfully applied to a wide variety of electromagnetic problems of practical interest such as radiation due to thin elements, wires and networks [3]. The moment method has used in [4] to estimate the capacitance of different conducting bodies using rectangular subareas, calculation of surface electrostatic model for spacecraft charging application [5], approximate of capacitance of lines conductor [6], analysis of charge distribution and capacitance for the dielectric coated tilted plates isolated in free space [7].

This paper, deal about the evaluation and analysis of capacitances and charge distribution of different structures in the form of conductor wire, metallic square plat. Using the moment method to study the electromagnetic charge into the different conductor and calculated the capacitance. The square plate conductor is charged with length L and the potential fixed at $1V$. This conductor is subdivided into N square patches of side length $2a$ and area to $4a^2$ and assume the charge to be a constant value within each patch. We then choose independent observation points, each at the center of a patch. To order to use this method, it is necessary to break down the studied structure into several parts or cells [8]. As a result, the diffuser is

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replaced by a set of equivalent sources (current and charge density). An adequate formulation of the integral equations allows calculating the electric and/or magnetic fields (electric field methods of integral equation EFIE, magnetic field methods of integral equation MFIE). The obtained results compared with theoretical show the surface charge densities on the patches along the diagonal of the plate. As in the case of the thin wire, the electric charge accumulates near the corners and the edges of the plate, and our solution could benefit from additional discretization density in those areas.

II. THEORETICAL BACKGROUND

A. One-dimensional Cable by Poisson Equation

The Poisson equation in one variable is given in the equation (1), with φ is the potential voltage described in [0 1].

$$\frac{\partial^2\varphi}{\partial x^2} = -x \tag{1}$$

The important is to find the variable x following the boundary conditions φ(0)=φ(1)=0. For one filed we resolve l(φ)=g, where g(x)=-x, this problem is easily solved by direct integration, one finds:

$$\varphi(0) = \varphi(1) = 1 \tag{2}$$

For one variable we resolve $l(\varphi)=g$, where $g(x)=-x$, this problem is easily solved by direct integration, one finds:

$$\varphi = \frac{1}{2}x - \frac{1}{6}x^3 \tag{3}$$

Using the method of moments, we start with the choice of the general term of the functions φ_n as.

$$\varphi_n = x - x^{n+1} \tag{4}$$

With $n=1, 2, 3, \ldots\ldots, n$

Its solution can be generally writing in the following with respect to the boundary conditions.

$$\varphi = \sum_{n=1}^{N} \alpha_n \left(x - x^{n+1} \right) \tag{5}$$

We are choosing the weighting function that is the test function, and using the following scalar product:

$$\prec \varphi, g \succ = \int \varphi(x)\, g(x)\, dx \tag{6}$$

The integrals are evaluated as follows:

$$l_{mn} = \prec w_m, l\varphi_n \succ = \prec x - x^{n+1}, \frac{\partial^2}{\partial x^2}\left(x - x^{n+1} \right) \succ \tag{7}$$

$$\frac{\partial^2}{\partial x^2}\left(x - x^{n+1} \right) = n(n+1)x^{n-1} \tag{8}$$

$$l_{mn} = \int_0^1 \left(x - x^{n+1} \right) n(n+1) x^{n-1} dx \tag{9}$$

$$l_{mn} = \prec w_m, l\varphi_n \succ = \frac{mn}{m+n+1} \tag{10}$$

To calculate g_m we use the following integral.

$$g_m = \prec w_m, g \succ = \int_0^1 \left(x - x^{m+1} \right) dx \tag{11}$$

$$g_m = \prec w_m, g \succ = \frac{m}{3(m+3)} \tag{12}$$

We start with a single polynomial expression; this corresponds to the case Where $n=1$, the use of equations (6) and (7) gives a single equation for α_1

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$$\left\{\frac{1}{3}\right\}\{\alpha_1\}=\left\{\frac{1}{12}\right\} \qquad (13)$$

The first solution is $\alpha_1=1/4$ which leads to the general solution of the equation.

$$\varphi=\sum_{n=1}^{N}\alpha_n\varphi_n=\alpha\left(1-x^2\right)+\alpha\left(1-x^3\right) \qquad (14)$$

The obtained results previously by moment method, for different values of n presented in figure 1. These results are calculated via a numerical code realized using MATLAB software. From $n=2$ the results obtained by moment method coincide perfectly with those obtained theoretically.

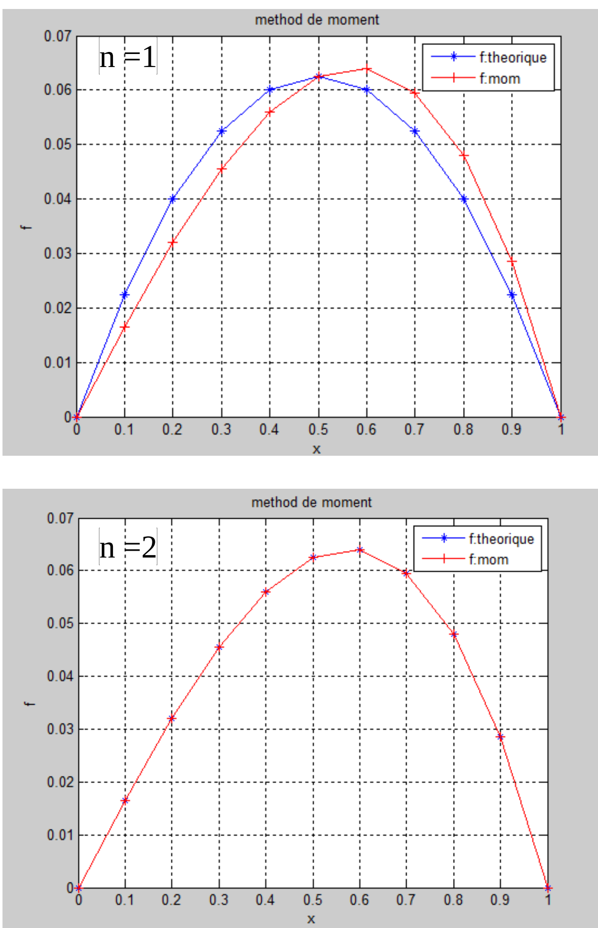

Fig.1. The obtained results by the MOM compared to the theoretical results obtained by the exact solution.

B. *Conductive plate loaded (square)*

The problem studied is to find the charge density $\sigma(x, y)$ unknown on a metal plate maintained at a constant potential [9]. So, let consider a square conductive plate $2a$ meters long and lying on the $z=0$ plane with a center at the origin Fig. 2.
The potential on the plate is given by the integral [10]:

$$\phi(x,y,z)=\int_{-a}^{a}\int_{-a}^{a}\frac{\sigma(x',y')}{4\pi R}dx'dy' \qquad (15)$$

Where $R=\sqrt{(x-y)^2+(x'-y')^2}$)

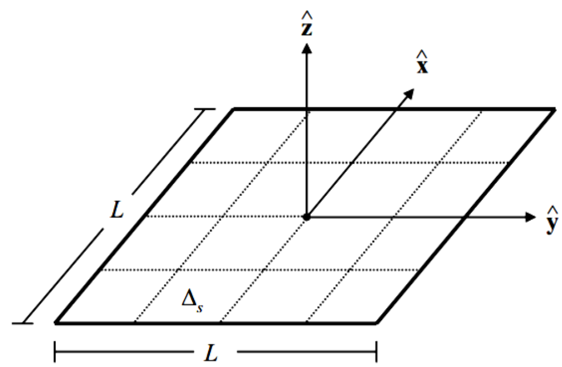

Fig.2. Square conductive plate divided into a number $NxN\Delta sn$.

The condition at the limit is (constant) on plate, the equation (10) becomes:

$$V=\int_{-a}^{a}\int_{-a}^{a}\frac{\sigma(x',y')}{4\pi\sqrt{(x-y)^2+(x'-y')^2}}dx'dy' \qquad (16)$$

Where, $|x|\prec a$ and $|y|\prec a$ the important thing here is to determine the charge density of the plate. The value of the capacitance is defined as [11].

$$C=\frac{q}{V}=\int_{-a}^{a}\int_{-a}^{a}\sigma(x,y)dxdy \qquad (17)$$

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The plate is subdivided into N*N square divisions of lateral length *2b* and area $\Delta S_n = 4b^2$, and we assume the constant charge in each division. We then choose *N*N* observation points (x_m, y_m) independent, each one is at the center of a division m=1,2, 3 …N.
The unknown charge density on the conductive surface developed as

$$\sigma(x,y) = \sum_{n=1}^{N} \alpha_n \varphi_n \quad (18)$$

With unknown coefficients α_n to be determined and φ_n are basic functions known as

$$\varphi = \begin{cases} 1 & \Delta S_n \\ 0 & elsewhere \end{cases} \quad (19)$$

By replacing equation (18) in equation (16), and satisfying the resulting equation at the middle

point (x_m, y_m) of each of the elementary surfaces Δs_m, we obtain the set of equations:

$$V = \begin{cases} \sum_{n=1}^{N} l_{mn} \alpha_n \\ m = 1, 2, 3, \ldots\ldots \mathrm{N} \end{cases} \quad (20)$$

Where

$$l_{mn} = \int_{\Delta x_n} dx' \int_{\Delta y_n} dy' \frac{1}{4\pi\varepsilon\sqrt{(x_m - x')^2 + (y_m - y')^2}} \qquad l_{mn} = \int \Delta x_n dx \quad (21)$$

We take note that l_{mn} is the potential at the center of ΔS_m for a uniform charge density and unit amplitude. A solution to the series (20) gives the α_n who in their turns and by using equation (18), give the load densities on each element ΔS_n. The corresponding capacity of the plate is calculated by (17) and given as:

$$c = \frac{1}{V} \sum_{n=1}^{N} \alpha_n \Delta s_n = \sum_{mn} {l_{mn}}^{-1} \Delta s_n \quad (22)$$

This result shows the capacity of an object is the sum of the capacities between each pair of subsections.
To translate the above results into the language of the linear spaces and the method of moments, we proceed as follows:

$$\varphi(x,y) = \sigma(x,y) \quad (23)$$

$$g(x,y) = V \, |x| < a \; and \; |y| < a \quad (24)$$

$$l(\varphi) = \int_{-a}^{-a} \int_{-a}^{-a} \frac{\varphi(x',y')}{4\pi\varepsilon\sqrt{(x-x')^2 + (y-y')^2}} dx'dy' \quad (25)$$

An appropriate scalar product is described in the following

$$< \varphi, g >= \int_{-a}^{-a} \int_{-a}^{-a} \varphi(x,y) g(x,y) dxdy \quad (26)$$

To apply the method of moments, one uses the functions (18), and one defines the functions test

$$w_m = \delta(x - x_m) - \delta(y - y_m) \quad (27)$$

Which is a two-dimensional Dirac function. The matrix of [g] is

$$[g_m] = \begin{bmatrix} V \\ V \\ \vdots \\ V \end{bmatrix} \quad (28)$$

The capacity (1.16) can be written as

$$C = \frac{<\sigma, \emptyset>}{V^2} \quad (29)$$

As $\phi = V$ on plate, Equation (28) is the classical stationary formula for the capacity of a conductive body.

C. *Evaluation of the matrix of elements*

When the observation point coincides with the center of the considered element, we then have

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m=n, the integral, in that case, presents a singularity and must be evaluated analytically. These elements of the matrix (*m=n*) are the most dominant and are called auto-terms [12]. The integral of self-term for the loaded plate is:

$$l_{mn} = \int_{-b}^{b}\int_{-b}^{b} \frac{1}{\sqrt{(x')^2+(y')^2}}\, dx'dy' \qquad (30)$$

Where, l_{mn} is the potential in the center of the element ΔS_n for a unit charge density. The first integration gives

$$l_{mn} = \int_{-b}^{b} log\left[\frac{\sqrt{b^2+(y')^2}+b}{\sqrt{b^2+(y')^2}-b}\right] dy' \qquad (31)$$

The second give
Which reduces to (for m=n)
For $m \neq n$, we use a simple approximation of the integral (1.29)

$$l_{mn} \approx \frac{\Delta s_n}{4\pi\varepsilon R_{mn}} = \frac{b^2}{\pi\varepsilon\sqrt{(x_m-x_n)^2+(y_m-y_n)^2}} \qquad (32)$$

Where x_n and y_n or (x_m and y_m) are chosen to be at the center of the source cells.

III. RESULTS

Recall that to study the density σ(x, y) of the square plate, by the MoM method, we divided the sides of it into N intervals, so its surface will be divided into NxN elemental surfaces Δsn. We represent here, as results, the charge densities of N elements Δsn, aligned in the direction of the x Fig.3, and lying in the middle of the plate. As well, we made a representation of the results in three dimensions, giving the charge density as a function of x and y Fig.4a. In these figures, the influence of the number N of elements Δsn, in the chosen model, on the results obtained is perfectly noted. Let us also note, taking advantage of the results obtained for the charge density, that we have calculated the capacitance C of this plate.

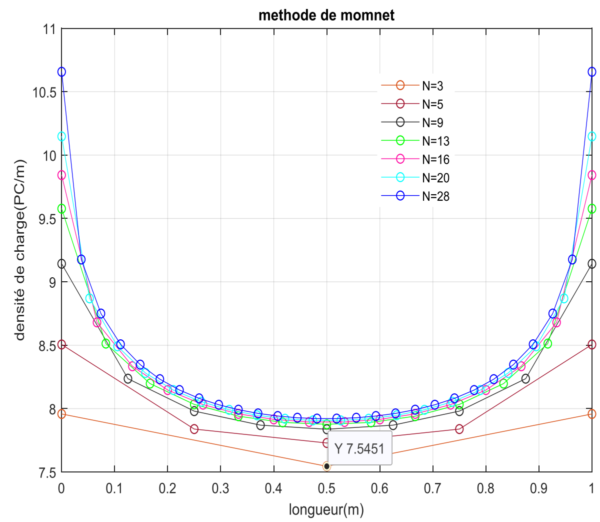
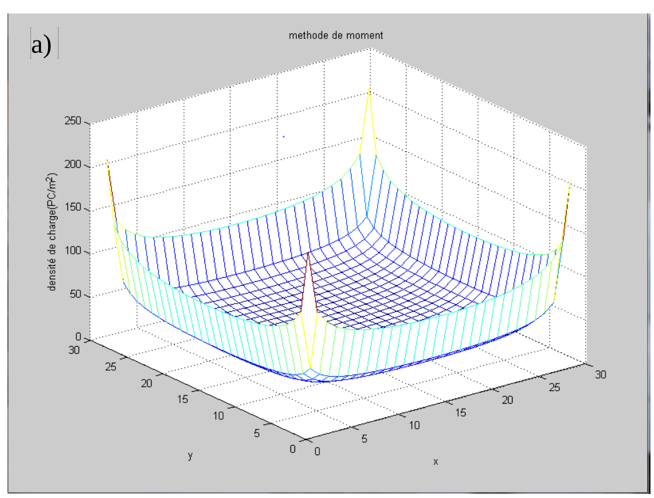


Fig.3. Density of charge along the cells Δsn, in the direction of the x, located in the middle of the conductive square plate. For N=14, N=20 and N=28.

Fig. 4b presents the approximation of surface charge density with given potential on the surface. The equations used are given in advance engineering electromagnetics by C.A. Balanis [13]. This figure illustrates the charge distribution in two dimensions with uniform charge distribution. Further, using L=1 side length of the square sheet, N=40; the number of square patches on the surface, V=1; potential in volts on the surface and th=0.05 as the thickness of the sheet in meters.



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b)

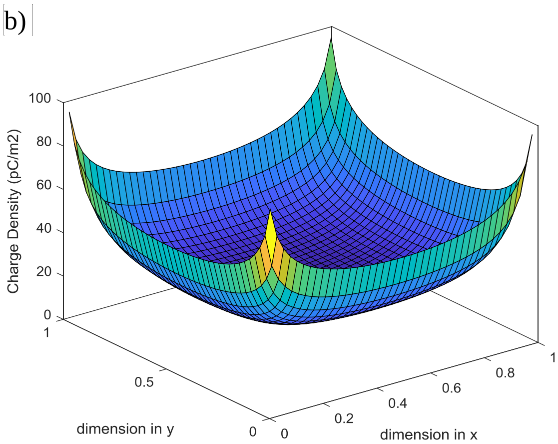

Fig.4. Representation of the charge density σ (x, y) of the conductive plate as a function of x and y, (a) our work, (b) problem given by C. A.Balanis [13].

In Fig.4b previously published by C.A. Balanis a comparison was made to verify the results obtained in Fig.4a.

IV. CONCLUSION

The Method of Moments is routinely used for the numerical solution of electromagnetic surface integral equations. This method is used to find the charge distribution in square plat conductor in its surface. So, this solution errors are inherent to any numerical computational method, and error estimators can be used to reduce these errors. The study conducted in this work is characterized by the use of numerical methods in two dimensions to find the charge density $\sigma(x, y)$ unknown on a metal plate maintained at a constant potential. We will study, in the first time, the Poisson equation and then we will expose in the second time to calculate the surface charge density of a square plate brought to a given potential. The obtained results show the charge are distributed in the medium and below in the board.

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