Dear Editor

First of all, thank you for your attention and dear commenters. We hope that the changes made to the article and the answers prepared by the authors will attract the attention of the commenters. In any case, thank you very much for your time and the refereeing team.

Reviewer #2: The accurate calculation of capacitances is a significant problem that spans through multiple scientific areas: computational electromagnetics, computational physics, applied mathematics etc.

In the manuscript, starting from (4) it is now known what is the domain of double and quadruple integrals, namely (4), (5), (6), (9), (10) and (11). Reading thought the manuscript one can understand that the boundary surface is divided into multiple squares or tiles, (i.e., meshing of the boundary surface), which further implies that the constant surfaces charge is assumed on each mesh element. The authors should clearly indicate what is the domain of the integration of double and quadruple integrals. Further, the authors should clearly state what are the "basis functions" for the approximation of the distribution of the surface charges.

1 – In formulas (4), (5), (6), (9) and (10), domains of integrals are added. In (11), the primitive function of e ^ (-u2d2) is required and this is an indefinite integral. Therefore, this integral does not need to define a domain.

2 – In section 2 “Method of Moments for Capacitance Calculation” at third paragraph, the sentence “ Charge density of each segment is assumed to be constant for each segment.” is added to describe the charge density distribution.

The step between (11) and (12) seems to be the crucial for the presented work, yet very little is said about how that solution is obtained. The authors should provide further details, and comment how the accuracy of calculation of erf-function in (12) affects the overall accuracy of the calculation of capacitances. A comment on how many significant digits are used for erf-function is used is needed, too.

3 – Finding the primitive function in (11) can be done by common softwares. It can also be solved using Wolfram Alpha website. To do this, we first integrate over x1 and x2.

Integrate[Exp[-u^2 ((x1 - x2)^2 + y^2 + z^2)], x1, x2] : Here y means (y1 – y2).

And following answer is obtained.

(Sqrt[Pi] (-(1/(Exp[u^2 (x1 - x2)^2] Sqrt[Pi] u)) + x2 Erf[u (x1 - x2)] + x1 Erf[u (-x1 + x2)]))/(2 Exp[u^2 (y^2 + z^2)] u)

Now it is better to replace (x1 – x2) by x and y by (y1 -y2). Then we can integrate it over y1 and y2.

Integrate[(Sqrt[Pi] (-(1/(Exp[u^2 x^2] Sqrt[Pi] u)) + x2 Erf[u x] - x1 Erf[u x]))/(2 Exp[(u^2 (y1 - y2)^2 + z^2)] u), y1, y2]

And the exact answer is:

-(Sqrt[Pi] (1 + Exp[u^2 x^2] Sqrt[Pi] u x Erf[u x]) (-(1/(Exp[u^2 (y1 - y2)^2] Sqrt[Pi] u)) + y2 Erf[u (y1 - y2)] + y1 Erf[u (-y1 + y2)]))/(4 Exp[u^2 (x^2 + z^2)] u^3)

So the wolfram alpha now is referred in the paper:

“ It can be done by common mathematical softwares. The solution has been obtained by use of "Wolfram Alpha" “

4 – As mentioned in (17) and (18), the final answer is based on Arctan and Arcsinh or Log functions. All these functions can be calculated with standard numerical packages. We used NUMPY to compute numerical results. In fact, accuracy of calculating these functions is not the key point of problem and does not disturb our results.

Since the authors state that the accurate calculation of capacitances is of interest, complementary to Figs. 3 and 5 they should provide the number of significant digits of the results they obtained or some other numerical estimate of the accuracy of the capacitance that they calculate. Figs. 3 and 5 show monotonically increase of capacitance, but it is not clear if the results converge to the exact solution or to some other solution. The authors should compare their work the known results [34] and cross check the calculated capacitance if the number of tiles is one or two orders of magnitude higher than those presented in Figs. 3 and 5.

For the reasons stated above, the manuscript needs major revision.

5 – The answers obtained in this article are particularly accurate in coarse segmentation. Really this work is useful when the calculations are done quickly. For example, in non-destructive testing (NDT), we want to calculate the capacitance of a hypothetical geometric shape, and if the results do not match with the experimental results, the hypothetical shape is modified and another calculation is performed. In such cases the calculation speed is very important. After performing calculations in coarse segmentation we can increase the accuracy by increasing the number of segmentation.

Reviewer #3: Dear authors you have proposed the closed form solution for the quadruple integrals over rectangles for parallel and orthogonal case. The paper is well written but there is the following problem. This calculation has been addressed many times in literature. For example in the following paper

"Z. Song, F. Duval, D. Su, and A. Louis, "Stable partial inductance calculation for partial element equivalent circuit modeling," Applied Computational Electromagnetics Society (ACES) Journal, vol. 25, no. 9, 2010., vol. 25, pp. 738-749, 2010"

the closed form solution for parallel and orhotogonal rectangles can be already found without any limitation on the coplanarity (please see the Table I of this paper where formulas are given in the 4D-1 and 4D-2 tab). In addition, the singularities can be simply handled by using the atan2 function and by considering that lim x->0 x log (x) = 0.
I'm sorry but I don't see a clear advantage to use your formulas (with coplanar limitation). Can you clearly explain this point?

6 – In this article we have used boundary element method to calculate the capacitance of air gapped capacitors. For doing it we propose to use quadruple integration. And we used analytical solution of integrals. This method allows us to achieve accurate results even in coarse segmentation. (Figures 3 and 5).
Therefore, the main point of this paper is to use the boundary method in calculating the capacitance. And since the accuracy and speed of calculations in the boundary value method depends a lot on the computational speed of the base integrals, analytical integralization is proposed instead of numerical integralization. We think that knowing the path of integration is interesting and necessary for researchers. As mentioned, the correct and complete form of calculations has not been mentioned in previous articles. It seems that readers should have an overview of these calculations, so that they can solve other problems by making the necessary changes if necessary. In any case, the main innovation of this paper is to use the boundary element method to obtain accurate and fast electric capacitance.

Sincerely yours
Saeed Sarkarati