Financial Frictions and Productivity Losses:

Importance of Default-Led Heterogeneity in Collateral and Loan Rates

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Abstract

I develop a model of entrepreneurship with default to quantitatively analyze the impact of financial frictions on total factor productivity (TFP). Default risk justifies the need for collateral as a means for securing loans, and heterogeneity in entrepreneurs' default likelihood generates heterogeneity in business loan collateral and interest rates across entrepreneurs. Consequently, the link between deposit rates and loan rates is broken. Entrepreneurs are charged higher loan rates if the value of their collateral is low, which favors wealthy over poor regardless of their talent, and discourages poor individuals from self-financing to start or expand their businesses. The resulting change in the dynamics of wealth accumulation and self-financing has important implications on misallocation and economic development. Backed by empirical evidence, I make an argument that self-financing is mainly a feature of developed financial markets, and fades away in both intensive and extensive margins in less-developed financial markets where collateral and loan rates are high due to frictions and inefficiencies. The weakening of self-financing channel can explain large losses from financial frictions while being consistent with high persistence of productivity which is a challenge with existing models of financial frictions.

Financial frictions in my model stem from three different sources: 1. limited enforceability related to recovery rate of collateral by financial intermediary, 2. informational frictions related to inefficiencies in financial intermediary's evaluation of entrepreneurs' default risks, and 3. frictions related to entrepreneurs expectations of future loan terms. I have used machine learning classification techniques to handle the quality of financial intermediary's evaluation of entrepreneurs' default risks. Also, individuals in my model

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are heterogeneous with respect to their creditworthiness. Entrepreneurs' expectations of future loan terms depend on assessments of their creditworthiness in the future which is governed by persistence of creditworthiness. My analysis shows sizeable losses from financial frictions, more than 40% in TFP losses for the U.S. if we were to replace its financial markets with a poorly functioning one. There are relatively large TFP losses due to amplification effects between the three sources of financial frictions. Without default and heterogeneity in collateral and loan rates my model would function similar to a neo-classical model, which justifies the small impact of financial frictions, only 7% loss in the U.S. TFP.

1 Introduction

The relationship between financial development and economic development is a well-established fact in the macroeconomics and finance literature.[1] The main related question is: How large are the TFP losses from financial frictions? There are both large and small losses documented using quantitative macro models in the literature, with the latter prevailing recently.[2] Self-financing, which is the process of wealth-accumulation to start or expand a business, has the pivotal role in these studies. The idea is that individuals can save and eventually overcome the financial constraints in order to start or grow their businesses. But for this to happen, their current productivity level should last a relatively long time. This high persistence of productivity, backed by empirical evidence, leads to strong self-financing motives[3] which dampens the impact of financial frictions.[4] However, there is ample empirical evidence against self-financing in countries with less-developed financial markets.[5] Quantitative models and theories in the literature have not succeeded to bring together the possibility of a highly persistent productivity with a diminishing self-financing motive, both of which are supported by empirical observations. As a result, quantitative studies that produce large TFP losses from financial frictions, mainly prioritize a weaker self-financing motive over a high persistence of productivity. On the other hand, the studies that produce small TFP losses, emphasize the high persistence of productivity over a weakening self-financing motive.

[1] See Levine (2005), Matsuyama, Gertler and Kiyotaki (2007), Townsend (2010) and Buera, Kaboski and Shin (2015) for comprehensive surveys on this literature.

[2] Buera, Kaboski and Shin (2011) and Buera and Shin (2013) documented large losses while Midrigan and Xu (2014) and Gopinath et al. (2017) documented relatively small losses.

[3] Throughout the paper, by self-financing motive I mean both ability and motive to accumulate wealth for self-financing.

[4] The role of persistence of productivity shocks is discussed in length by Moll (2014).

[5] The evidence is discussed extensively in section 2.

I develop a model of entrepreneurship with default that is consistent with empirical evidence on persistence of productivity and self-financing. That is, my model can account for high persistence of productivity while generating a weakening self-financing motive. This can explain large TFP losses from misallocation caused by financial frictions in my model. My results indicate that the U.S. economy, if it had a distorted financial market, would lose 43% of its TFP compared to undistorted case. Without default my model would be isomorphic to a Buera and Shin (2013) type model, which indeed, in the absence of their distortionary taxes, would function similar to a neo-classical model. Therefore, without default the impact of financial frictions is small, only 7% loss in the U.S. TFP.

Default risks justifies the need for collateral[6] as a means for securing loans. Heterogeneity in entrepreneurs' probability of default leads to heterogeneity in business loan collateral. This also leads to heterogeneity in loan interest rates across entrepreneurs and, as a result, the link between loan rates and deposit rates is broken. This disproportionately affects poor individuals with ideas worth implementing because they will end up paying higher interest on their loans because of their low collateral.[7] Therefore, a far-from-ideal financial market makes it nearly impossible for a talented but poor individual to pull herself up by her bootstraps, whether she wants to start or expand her business.

Consider an environment in which, unlike existing models in the literature, entrepreneurs face different loan interest rates depending on how much collateral they want to or are able to pledge. In this economy, individual loan rates will not be the same as the risk-free rate or the deposits rate. Therefore, a higher financial frictions would mean higher inefficiencies, say due to limited enforceability or informational frictions, which would result in higher collateral and loan rates[8] but not necessarily proportionally high deposit rates. This would have implications on wealth accumulation dynamics of agents across the spectrum. In order to clarify this, below, I discuss the wealth accumulation dynamics of potential entrants and existing entrepreneurs, both of which are important from the economic development perspective.
Extensive effect (Potential entrants): In a standard model[9] with high financial frictions, the talented but poor individuals would still be able to save their wages and earn interest in order

[6] Collateral in my model is defined as part of the working capital that will be transferred to financial intermediary in case an entrepreneur defaults. This capital can include loans as well as self investments.

[7] Also, there might be very little (or no loans) offered by financial intermediary to individuals with very low collateral. That is, there is a possibility of a borrowing cap for low levels of collateral.

[8] The increase in collateral and loan rates would vary across the agents, with smaller increase for some and larger increase for other agents.

[9] Midrigan and Xu (2014), or Buera and Shin (2013) with relatively high persistence of productivity.

to accumulate the wealth required to start their own businesses. However, in my environment with high financial frictions, these individuals would need to provide higher collateral to avoid paying extremely high interests on their loans when they start a business.[10] This would mean a lower expected earnings for prospective entrepreneurs and, considering a large gap between the deposit rates and loan rates, would discourage savings by potential entrants.[11] This would have two important implications on development: 1. it would further distort the entry towards wealthy away from the talented, and 2. it would reduce the aggregate capital in the economy which means a reduction in the productive capacity of the operating entrepreneurs especially those that heavily rely on external financing.
Intensive effects (Entrepreneurs): There is a similar situation evolving around incumbent entrepreneurs. High productivity entrepreneurs would want to accumulate wealth in order to expand their businesses. But with tighter financial constraints, entrepreneurs' borrowing capacity is more limited. The resulting lower return on their businesses makes wealth accumulation more challenging for poor entrepreneurs. In addition, all else equal, low collateral implies a higher loan rate. As a result, poor entrepreneurs with low levels of collateral are disproportionately affected by financial frictions.

The above-mentioned disproportionate effects on poor entrepreneurs and potential entrants are reminiscent of the phrase *tyranny of collateral* by Rajan and Zingales (2004). A combination of these extensive- and intensive-margin effects implies that financial frictions can have large and amplifying effects on aggregate output and TFP. Related, another important point is that financial frictions can create massive wealth inequality, especially amongst the wealthy agents[12] whose wealth matter the most for production, as well as amongst entrepreneurs.[13] Wealth inequality amongst entrepreneurs is generated by the intensive margin effect discussed above, and the inequality at the top of the wealth distribution is a result of the combination of intensive and extensive margin effects because the wealthiest are either entrepreneurs or potential entrants. The models introduced in the literature are not successful in producing the inequality amongst the entrepreneurs or the fraction of aggregate resources owned by the very

[10] Under-developed financial markets would mean higher collateral and loan rates but not necessarily proportionally high deposit rates (or wage rates). This is in contrast with the mechanism proposed in most models in the literature, e.g. Midrigan and Xu (2014) where deposit rates increase as a result of tightening financial constraint which, in fact, paves the way for potential entrants to save and overcome the financing barriers.

[11] The severity of this discouragement effect would also depend on preference-related factors.

[12] For the wealth inequality at the top we can think of different measures: for example, the wealth share of top 1% over the share of top 10%; the wealth share of top 1% over top 5%; or the wealth share of top 5% over top 10%, etc.

[13] For the wealth inequality amongst entrepreneurs we can think of the wealth share of top half entrepreneurs over the share of bottom half.

wealthiest.[14] Relying on empirical evidence, I will argue that inequality amongst entrepreneurs as well as the level of wealth concentration amongst the very wealthy matters a great deal for economic development.

Another contribution of this paper is that, my setup allows me to disentangle the effects of financial frictions due to three different sources: First, there is limited enforceability problem in the model. This is related to the ownership transfer cost or recovery rate of collateral by financial intermediary. A low collateral recovery rate means a higher cost for financial intermediary in case of default, which would drive up the loan rates and collateral. Second, there are informational frictions related to the ability of financial intermediary to accurately assess the default probabilities of the loan applicants. A less efficient evaluation of default risks by financial intermediary would cause large losses for them which would result in higher loan rates and collateral.[15] I have used a simple innovative approach borrowed from the machine learning field to handle the assessment ability of financial intermediary.[16] Third, there are informational frictions affecting entrepreneurs' assessment of future loan terms. Individuals in my model are heterogeneous with respect to their creditworthiness, which is a determinant of loan terms as entrepreneurs with high creditworthiness tend to avoid default harder than those with low creditworthiness.[17] If individuals' creditworthiness does not vary much through time, they can make reliable predictions of their future loan terms and, as a result, make self-financing plans for either starting or expanding their businesses.[18] That is, a high persistence of creditworthiness implies less uncertainty regarding future financing.[19] A low persistence of creditworthiness, on the other hand, means higher uncertainty for entrepreneurs and potential entrants regarding their financing in the future. This uncertainty discourages self-financing

[14] I have reproduced the results of several related works in the literature and analyzed their implications for the entrepreneurs' as well as top wealth inequality. Also, in the next section I will provide some empirical evidence on the inverse relationship between financial development and top wealth inequality which is consistent with the argument I have laid out.

[15] This is a case where financial intermediary does not know the true default risk and, for example, ends up assigning low risk and offering low rate to an actually high risk borrower, and vice versa.

[16] In the model, financial intermediary uses decision tree classifier to evaluate default probability for any given loan contract based on loan and borrower information available to them. The depth of the decision tree will control the quality of the assessment. I will explain the reasons for choosing decision trees over more sophisticated methods in the modeling section.

[17] The creditworthiness can be interpreted as different forms of customer-bank relationships which might make it easier to get better loan deals for some agents than others. It could also be seen as some limited type of credit score which informs financial intermediaries of the borrowers' default risk. See Chatterjee et al. (2020) for a quantitative theory of credit scores.

[18] This is assuming the fact that, productivity is also highly persistent.

[19] A more persistent creditworthiness can also be interpreted as a better credit registry in the economy and vice versa.

because individuals do not know if their savings would suffice to secure a loan with reasonably low rate in the near future.

To quantitatively discipline my model, I initially calibrate the parameters to reflect the U.S. aggregate and distributional data on wealth and entrepreneurship. Then, to distort the U.S. financial markets I change the financial friction parameters to match the first three moments of the collateral rate distribution[20] for countries in different per-capita income groups. That is, as if the U.S. with all its underlying characteristics had a less developed financial market. I will then use this exercise to analyze the effects of financial frictions on TFP, entrepreneurship and top wealth inequality.

In another exercise, I change the three financial market parameters one at a time to low levels of the previously mentioned exercise while keeping all other parameters at the U.S. level. Ownership transfer cost equivalent to that of the countries in the lowest income decile reduces the U.S. TFP by 9%. The quality of financial intermediary's access to credit information together with the persistence of creditworthiness account for 22% drop in the U.S. TFP, with the latter being more important; 5% vs. 12% respectively when they are considered independently. This implies a relatively significant role for enforceability, but even larger role for informational frictions in explaining TFP losses from financial frictions. Note that for all these exercises, the productivity shocks are highly persistent, consistent with empirical evidence.

This paper is organized as follows. After a review of the related literature in the following, I will extensively discuss the empirical evidence in section 2. In section 3, I introduce the quantitative model and its solution. The results of the model and how they relate to the observations from the data are explained in section 4. Concluding remarks and directions for future research is provided in section 5.

Related Literature

There is a large empirical, theoretical and quantitative literature trying to explore the links between financial development and economic development. Extensive surveys are conducted by Levine (2005), Matsuyama, Gertler and Kiyotaki (2007), Townsend (2010) and Buera, Kaboski and Shin (2015). Motivated by empirical observations, this paper contributes to our understanding of finance-development links using a quantitative model of financial frictions where

[20]This paper is the the first to use the distribution of the collateral rate as an indicator of financial frictions. I show in the data that the first three moments of the collateral rate distribution are informative for economic development.

aggregate outcomes are driven by individual decisions on occupation, financing, default and savings.

The effects of financial frictions are heavily dependent on the ability of individuals to accumulate wealth and overcome the financial constraints. This is, to a great extent, governed by the persistence of productivity in the models. There is a long tradition for persistence of productivity as part of the models in the context of firm dynamics, e.g. the seminal work of Hopenhayn (1992).[21] The emphasis on the role of persistence of the productivity in the context of financial frictions is also not recent, e.g. Cooley and Quadrini (2001). The fact that higher persistence of productivity shock dampens the effect of financial frictions on TFP was first elaborated by Caselli and Gennaioli (2013). My paper is the first to settle the empirical contrast between persistence of productivity and self-financing, by introducing default and heterogeneity in loan collateral and rates. As a result, the self-financing motive and ability weakens extremely with financial frictions while the productivity remains highly persistent. The dependence of self-financing ability to the persistence of productivity in the literature explains much of the variations in their results. See Restuccia and Rogerson (2017) for a discussion on this. Also see Moll (2014) for an extensive analytical assessment of persistence of productivity and self-financing.

Buera, Kaboski and Shin (2011) and Buera and Shin (2013) attribute large effects to financial frictions.[22] Different from my paper, the former develops a two-sector economy and analyzes the sectoral dynamics while the latter focuses on transition dynamics in a one-sector economy. Both works characterize financial frictions as a form of collateral constraint, but neither has default or heterogeneity in loan rates and collateral.[23] A main driver of the large effects of financial frictions in their models is the weakness of self-financing ability due to relatively low persistence of productivity shocks. In my model, consistent with empirical evidence, the persistence of productivity is high, but the self financing ability is strong for the developed financial markets and it is weak for the under-developed financial markets.

On the other hand, Midrigan and Xu (2014) argue in favor of small effects for financial frictions.[24] They develop a model with technological choice in an economy with formal and informal sectors. They provide evidence for high persistence of productivity, and reflect it in their quan-

[21] See Shaker Akhtekhane (2017) for an analysis of Hopenhayn's model in a continuous-time setting.

[22] Many other works in the literature have also documented large effects from financial frictions, e.g. Jeong and Townsend (2007), Amaral and Quintin (2010)

[23] Buera, Kaboski and Shin (2011) use an endogenous form of collateral constraint related to contract enforceability, but it has no implications on loan rate.

[24] See also Gopinath et al. (2017) who produce small losses from financial frictions.

titative analysis. The high persistence of productivity leads to a strong self-financing motive which makes the wealth accumulation easy especially when agents enter the productive sector, and this drives their results. As a result of default and heterogeneity in loans, the wealth accumulation dynamics is very different in my model, while following their lead in terms of productivity process.

More recently, the adoption of productivity processes that are different from the standard AR(1) has gotten attention in attempts to be consistent with high persistence of productivity while having a weaker self-financing motive. Ruiz-Garcıa (2020) has used a non-linear and non-Gaussian productivity,[25] and produced relatively large TFP losses from financial frictions. Different from his work where the productivity process drives the results, my results are driven by default and a rich heterogeneity in collateral and loan rates while using a standard AR(1)-type process.[26] Apart from the mentioned underlying differences in the mechanism, another reason that my model generates larger losses than Ruiz-Garcıa' is that in my model financial frictions arise from multiple sources, enforceability and informational frictions, each of which has different implications on certain margins as well as the amplifying effects, whereas his financial frictions stem from a size-dependent collateral constraint similar to the one in Gopinath et al. (2017).[27]

My paper also contributes to another strand of literature relating financial development and wealth inequality. In a closely related work, Cagetti and De Nardi (2006) show that more restrictive financial constraint would result in less wealth concentration. I focus on entrepreneurs' and top wealth inequality arguing that they are the most relevant for economic development. However, in my mechanism with heterogeneity in loan rates, financial frictions increase top wealth inequality, with the effects being more severe at the low levels of financial development.[28]

In a related paper, Chatterjee and Eyigungor (2020) rationalize the increase in firm concen-

[25] Jo and Senga (2019) also uses non-Gaussian productivity process to assess the policies that alleviate the financial burden of small and young businesses. De Nardi, Fella and Paz-Pardo (2020) use similar processes in the context of household earnings dynamics and welfare analysis.

[26] I use Ornstein-Uhlenbeck process which is the equivalent of AR(1) in continuous time.

[27] The use of different forms of collateral constraint has a long tradition in the literature. The earlier contributions on models of financial constraints that use collateralized assets as the basic cost of financing are Bernanke and Gertler (1989), Banerjee and Newman (1993), Kiyotaki and Moore (1997) and Berger and Udell (1990) to name a few.

[28] See Jalilian and Kirkpatrick (2005) for an empirical analysis on this. Also see Madsen, Islam and Doucouliagos (2018) who discuss the role of inequality on economic development conditional on financial development. In a sample of OECD data they show that inequality limits economic development when financial markets are under-developed, but it has little to no effect when financial markets are developed. This is consistent with my results and the evidence on declining self-financing ability in under-developed financial markets.

tration through low levels of risk-free rate in an environment with financial frictions. In my paper, financial frictions generate high firm concentration that I discuss in the context of wealth inequality amongst entrepreneurs. In their mechanism low interest rates benefit large firms, and in my environment larger firms (wealthy entrepreneurs) can enjoy lower interest because they can pledge high collateral, which generates higher firm concentration, i.e. higher wealth inequality amongst entrepreneurs.

Regarding the underlying sources of financial frictions, apart from enforceability,[29] my paper contributes to the literature that relates economic development to informational frictions. David, Hopenhayn and Venkateswaran (2016) develop a model where firms make production decisions under imperfect information,[30] and produce relatively sizeable TFP losses from informational frictions. My source of informational frictions mainly affects financial intermediaries and their ability to assess the default probabilities of loan applicants as well as the reliability and persistence of credit information. Although the target of informational frictions is different in my model, the magnitude of productivity losses from these frictions are comparable to their results.

My paper also contributes to the literature that provide different measures and indicators for financial development to explain economic growth and development. In an influential industry-level study Rajan and Zingales (1998) use the ratio of private credit to GDP and stock market capitalization, also as a ratio to GDP, to show that higher financial development facilitates economic growth.[31] Influenced by Rajan and Zingales' work, the external dependence as an indicator of financial development has become extremely popular in the quantitative literature where most models simply use this measure to govern the level of financial frictions in the economy.[32] The state of financial development in my economy is measured by the distribution of collateral rates. I argue that this distribution is very relevant in the context of economic development since it contains information from both financial intermediaries' and firms' side, e.g. information related to default risks and how they are evaluated. Across countries, the

[29]See for example, Amaral and Quintin (2010) for a study on the importance of limited enforcement for economic development.

[30]See Bloom et al. (2013) for another related work.

[31]Goldsmith (1969) was the first to use total assets over GDP as indicator of size of financial sector to show its positive correlation with economic growth. Many other works, have also used credit to GDP as measures of financial development in a similar context, e.g. Arcand, Berkes and Panizza (2015) and Dabla-Norris and Srivisal (2013)

[32]As argued by Čihák et al. (2012) and Aizenman, Jinjarak and Park (2015), there are other important features of financial systems that should be considered as indicators of financial development. They consider some forms of access and efficiency in addition to the depth of financial markets and institutions.

distribution of collateral can inform us about the underlying legal and institutional differences related to enforceability and informational frictions.[33]

Finally, in a broader sense, my paper is related to the literature that studies the effects of factor misallocation on aggregate outcomes. The leading works in this literature are Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). Reasonably large TFP losses from resource misallocation is documented by Hsieh and Klenow.[34] My paper also reports large losses from misallocation caused by financial frictions, particularly limited enforcement and informational frictions.

2 Empirical Considerations

In this section I will use firm-level data from World Bank's Enterprise Survey[35] in conjunction with other standard cross-country data sets[36] to provide evidence on: 1. self-financing and wealth inequality 2. importance of collateral and its implications for misallocation, 3. relevance of collateral rate distribution to financial and economic development, 4. moments of collateral distribution and the significance of the first three moments, and 5. the factors that shape the collateral rate distribution and how they affect its first three moments.

2.1 Self-financing and Wealth Inequality

Self-financing is perhaps the most crucial determinant of the severity of financial friction's impact on economic aggregates in the models with financial frictions, and its direct relationship with the persistence of productivity causes inconsistency in these models. As a result of this inconsistency as well as other model feature, these models predict that the wealth inequality at the top and amongst entrepreneurs decrease with tightening of financial constraints. Here, I first look at the existing evidence on self-financing and its relationship with financial development, and then I will discuss the wealth inequality at the top and amongst entrepreneurs.

[33] I extract various indicators from the collateral rate distribution and inspect their relevance to institutional indicators as well as economic development indicators. I find that the first three moments of the collateral rate distribution are the most relevant ones and they remain significant after controlling for several existing indicators of financial development.

[34] They calculated about 40 percent loss in manufacturing sector in India and China.

[35] See Appendix (A.1) for more information about this data set and my preparation steps.

[36] e.g. World Bank's World Development Indicators, Penn World Tables, International Monetary Funds' Financial Development Index, etc.

2.1.1 Self-financing

As mentioned, self-financing is somewhat feasible in developed financial markets, but it becomes much difficult if the financial markets are less-developed. This diminishing effect is worth considering in both intensive margin related to incumbent firms, and extensive margin related to potential entrants.

In the intensive margin, we are interested to learn about the ability of entrepreneurs to accumulate wealth and grow their businesses. Using the U.S. data Gentry and Hubbard (2004) and Quadrini (1999) document that there is an acceleration of wealth accumulation among entrepreneurs resulting in much higher wealth-to-income ratios compared to that of non-entrepreneurs. Similarly, using Thai data Pawasutipaisit and Townsend (2011) find higher saving rates for high productivity households and for those with higher returns on business assets, mainly for entrepreneurs. These findings indicate that after entry self-financing is somewhat feasible. However, it is more challenging in less developed countries. Multiple studies have shown large rates of returns for capital, far exceeding the market rates, from small grants to small businesses: De Mel, McKenzie and Woodruff (2008) in Sri Lanka; McKenzie and Woodruff (2008) in Mexico; Fafchamps et al. (2011) in Ghana, all find that small grants significantly increase the rate of return on capital for small entrepreneurs. Also, McKenzie (2015) finds similar results using much larger amounts of randomized grants. He also finds evidence for employment growth. The argument I just laid out casts severe doubt on the possibility of wealth accumulation for poor entrepreneurs in less-developed economies, because such high returns on capital (after getting small grants) should have attracted many of these entrepreneurs to save and accumulate wealth with the prospect of much higher earnings. The fact that these effects are not there absent the small grants clearly indicates the difficulty, and in some cases impossibility of self financing in the intensive margin in less-developed financial markets.

In the extensive margin, we can ask two related questions: 1. is wealth a determinant of entry? and 2. does a sudden, exogenous increase in wealth increase the probability of starting a business? The answers to these questions will clarify the extent to which self-financing is feasible. There are many studies addressing the first question. The most notable work using the U.S. data is Hurst and Lusardi (2004). They find that wealth is not a determinant of entrepreneurship except for the top 5% of the wealth distribution. They show that exogenous shocks to wealth are more relevant to entrepreneurship than the wealth itself. Nykvist (2008) runs a similar exercise to Hurst and Lusardi' using Swedish data and finds that liquidity constraints are somewhat more extensive than they are in the U.S. In Thailand, Paulson and Townsend

(2004) find that financial constraints play a crucial role in entrepreneurial activity. The differences are also stark between the wealthy region and poor region with the entrants in the latter being affected more severely by financial constraints. Therefore, the answer to the first question is that wealth, whether is accumulated through time or comes as an exogenous shock, is an important determinant of entry to entrepreneurship. Regarding the second question Hurst and Lusardi (2004) find that inheritance has a positive relationship with probability of starting a business. Using British data, Taylor (2001) shows that probability of entering entrepreneurship is an increasing function of the size of the windfall payments received. Similar results have been found using Swedish data, Lindh and Ohlsson (1996), and using German data, Schäfer, Talavera and Weir (2011). Also using Spanish lottery data, Bermejo et al. (2018) show that entrepreneurial activity increases in the regions with higher concentration of lottery winners. These clarify the answer to the second question that: exogenous increase in wealth significantly increases the probability of entry. The evidence provided for both questions shows that wealth by itself is not a strong determinant of entry in the U.S. while it is in other countries. Also, the probability of entering into entrepreneurship increases for the receivers of a windfall payment. Since it is unlikely that the windfall gains increase the probability of entry for wealthy (they already would have entered if they wanted to), the increase in entrepreneurial entry as a result of windfall gains can be attributed to the participation of the poor.[37] From another perspective, this might explain the results of Hurst and Lusardi (2004) who only find a relationship between wealth and entry at the top of the wealth distribution. This is because of the fact that the exogenous (inheritance) shock as an instrument does not have much effect amongst the wealthy as it does amongst the poor. Therefore, it might be the case that, controlling for exogenous increase in wealth can explain the increase in entrepreneurship amongst less wealthy, but it cannot explain the increase in entrepreneurship amongst wealthiest. Putting both extensive and intensive margin effects together, these findings indicate that self-financing is not as strong as one might think and it is particularly weaker in less wealthy countries and regions with many poor individuals.

2.1.2 Wealth Inequality at The Top

Related to the strength of self-financing ability, another important point is that the tightening of financial constraints can create massive wealth inequality, especially amongst the wealthy agents whose wealth matter the most for production, as well as amongst entrepreneurs. This

[37] That is, these are the poor who were financially constrained, and the windfall gains make them overcome the constraint and start their businesses. The wealthy would not be as desperate for the windfalls to start business as the poor would be.

is mostly neglected in the related literature, and most of the existing quantitative models produce inconsistent results. I argue that inequality amongst entrepreneurs as well as that amongst the top wealthy (say inequality at the top 10%, 20%, etc.) matters a lot for economic development. From a modeling perspective, the importance of inequality amongst entrepreneurs stems from the standard assumption of a decreasing returns-to-scale (DRS) technology. In a DRS environment, more entrepreneurs reaching higher production capacity would mean higher GDP and TFP than a case where some entrepreneurs reach very high capacity while others remain low. Regarding the importance of inequality amongst the top wealthy group, we should note that the agents in this group are either entrepreneurs or the wealthy who fund most of the production with almost no concern about their consumption. If the wealth becomes more concentrated amongst very few (say the top 1%), they will more likely become entrepreneurs[38] and take a large portion of funding away from worthy projects to their own, perhaps less productive ones.

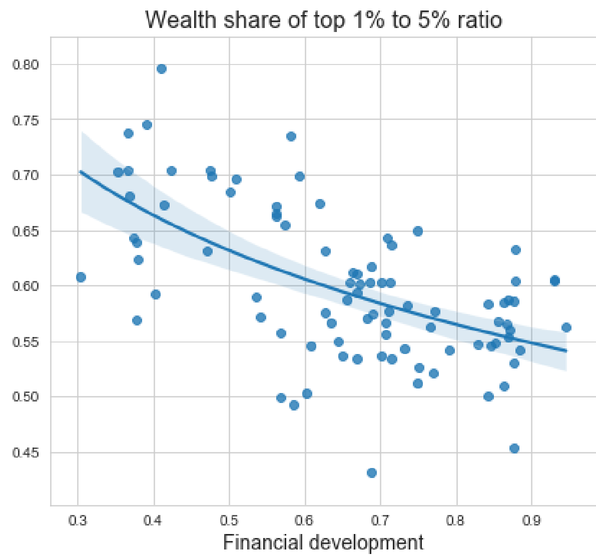
In order to explore the relationship between financial development and the inequality at the top, I use cross country wealth data of Credit Suisse Research Institute, in conjunction with the financial development index from the International Monetary Fund.[39] The wealth data used is for years 2015 to 2017. The correlation of financial development index with the wealth share of top 1% over the wealth share of top 5% is -0.56 which is large. This relationship is consistent with other wealth groups, say top 1% over top 10%, or top 5% over top 10%, etc. all of which exhibit strong negative correlation with financial development.

Because of the fact that, the wealth data from Credit Suisse Research Institute mostly contains developed countries, one might want to know the same relationship including less-developed economies. For this reason I use income data as a proxy for wealth data across countries for which there are richer data sets available. For income data I use World Income Inequality Database (WIID) of UNU-WIDER, from year 2000 to 2017. The correlation of financial development with income share of top 5% over the income share of top 10% is also large, about -0.52. This is also consistent when we use other income groups instead, say top 5% over top 20%, etc. Figure (2.1) shows the relationship of financial development with both top wealth inequality and top income inequality.

As we can see in figure (2.1), the relationship with income data is nonlinear indicating a

[38] This is consistent with the findings of Hurst and Lusardi (2004).

[39] Note, that throughout the paper I demonstrate the results related to financial development using the financial development index of International Monetary Fund, but I have also used other measures such as private capital to GDP, markets depth index, etc. all of which produce similar results.



(a) Top wealth inequality

(b) Top income inequality

Figure 2.1: Top inequality vs. financial development

stronger (negative) relationship for lower levels of financial development, something we cannot see using the wealth data as we do not have the data on less-developed economies. In a similar exercise, I use GDP per capita data from World Bank's WDI to check the relationship between top wealth inequality and GDP per capita. See figure (2.2). These figures along with the correlations mentioned earlier are robust for different wealth and income groups at the top as well as different measures of financial development. They also show strong negative correlation with GDP per capita.[40]

Regarding the wealth inequality amongst entrepreneurs, a cross country data that focuses only on the wealth shares of entrepreneurs or business owners would be very helpful, but such data is not available. Instead, as a proxy I look at the Herfindahl-Hirschman Index (HHI) across countries from World Bank's WITS database. This is a firm concentration measure, and to a very limited extent would inform us about the inequality amongst the entrepreneurs, if we can agree that there is a reasonably high correlation between the size of the businesses and the wealth of the business owners. The correlation of financial development with HHI is not very high, -0.22, but it still is an indicator of negative relationship. The coefficients of the simple regression of HHI against financial development index, GDP per capita and TFP are

[40] I have done simple regression analyses with top wealth and income inequality against financial development, GDP per capia and TFP, and have got negative and significance coefficients which strengthens my argument.

Figure 2.2: Top wealth inequality vs. GDP per capita.

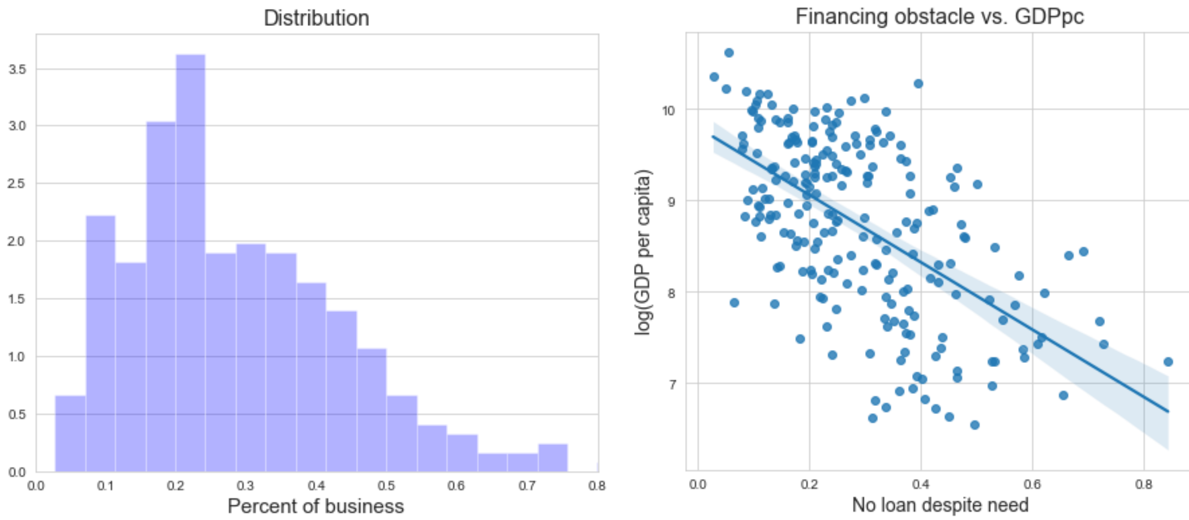
significant.[41] This implies, although weakly, that wealth concentration amongst the top wealthy entrepreneurs is much larger in economies with under-developed financial markets that it is in those with developed financial markets. The evidence from previous subsection on self-financing in the intensive margin would strengthen this idea.

2.2 Collateral and Misallocation

I use firm-level data from World Bank's Enterprise Survey to explore evidence on financial frictions and collateral constraints. This data set mainly consists of low-income and developing countries as well as few developed countries. Many businesses are surveyed about their financing as well as the limitations they face regarding their operations.[42] The most relevant piece of information in the survey to the purpose of this study is the loan and collateral values reported by firms as well as the main obstacles they face regarding financing.

In the entire sample of businesses, only 27% of the firms have applied for loans or lines of credit in the last fiscal year. About 60% of the remaining firms (the 73% that did not apply for loans) did not need any loans because they already had sufficient capital. The rest which is almost one third of all firms in the sample, did not apply for loans despite their needs. This is substantial and significant in terms of access and allocation of capital. It would cause severe misallocation in the intensive margin as a third of businesses in the sample are under-financed

[41] See Appendix (A.5) for regression results.
[42] See Appendix (A.1) for description of the data and my preparation steps.



(a) Distribution: no loan despite need

(b) Financing obstacle vs. GDP per capita

Figure 2.3: Variation and relevance of financing constraints

and are operating under their desired capacity. Note that these measures are related to the business in all countries in the sample pooled together, and obviously there is variation across countries regarding what percentage of the businesses that cannot get the needed funds. The misallocation of capital caused by such constraints can have serious implications on TFP and GDP per capita. Figure (2.3) illustrates the variation of this measure across countries as well as its relationship with GDP per capita.

Figure (2.3a) shows a large variation across countries for the percentage of businesses that were not able to find the capital they need. As we can see the average is about 30%, but the distribution is heavily right-skewed meaning that some countries are extremely far to the right. In those countries financing is out of reach for a great majority of businesses, which would create massive misallocation and would extremely impact the economic aggregates. For this reason, in figure (2.3b) I show the relationship between the percent of businesses that were not able to get the needed financing and the log GDP per capita for the countries in the sample. This relationship is very clear and strong, and somewhat convincing of the extreme misallocation effects of financial constraints and their impact on economic development.

In the survey the businesses are also asked about the underlying reasons that kept them from applying for the needed financing. The reasons listed are related to loan rates, collateral, application complexity, loan size and maturity and their expectations about loan approval. Again, in the entire sample, the breakdown of the reasons for not applying is reported in

Reason for not applying	% of non-applicants
unfavorable interest	35%
complex application procedures	20%
too high collateral requirement	16%
didn't think it would be approved	7%
insufficient size and maturity	5%
other	17%

Table 1: Reasons for not applying for loans despite need (entire sample)

table (1). This indicates that, interest rates and collateral are responsible for more than half of the firms deciding not to apply for loans. This is a conservative estimate since the other reasons for not applying, such as 'complex procedures', 'didn't think it would be approved' and 'others' are also very likely related to interest rates and collateral. For example, it can be argued that determining the right amount of collateral and, as a result, loan rates add significantly to the complexity of the loan application process. Similarly, not having enough assets to collateralize the loan might be the reason for those businesses that did not have any hope for their loan approval. Also note that the data only contains operating businesses, and as a result we can only observe the financing restrictions in the intensive margin. There might well be a large unobserved extensive margin related to those high productivity individuals that want to start a business but cannot do so because of the mentioned reasons. In order to have a grasp on the effects of these financial constraints, consider a country where, consistent with the observations in figure (2.3a), some 60% or 70% of the firms do not have access to the funds they need mainly because of the loan rates and collateral. On top of that, there are many individuals who would want to start businesses but cannot do so because of the same reasons. This would cause extreme misallocation of capital (and, as a result, labor) which would have very strong effects on output, TFP, investment, employment, etc. This also re-confirms, and gives context to the discussion laid out in section (2.1) about the relationship between financial development and wealth inequality amongst entrepreneurs and at the top. The reason is simply related to the growth prospects of existing entrepreneurs as well as the potential entrants' hopes for future entry. In an environment that a large portion of firms do not have access to the needed funds, only the wealthy can grow and expand (or start) their businesses, which would further widen the gap between wealthy and the poor, and more inequality at the top and more concentration would follow. The bottom line of this discussion is: although financial constraints might seem a minor issue in developed economies, they are extreme and can do serious damage to the development prospects of under-developed economies.

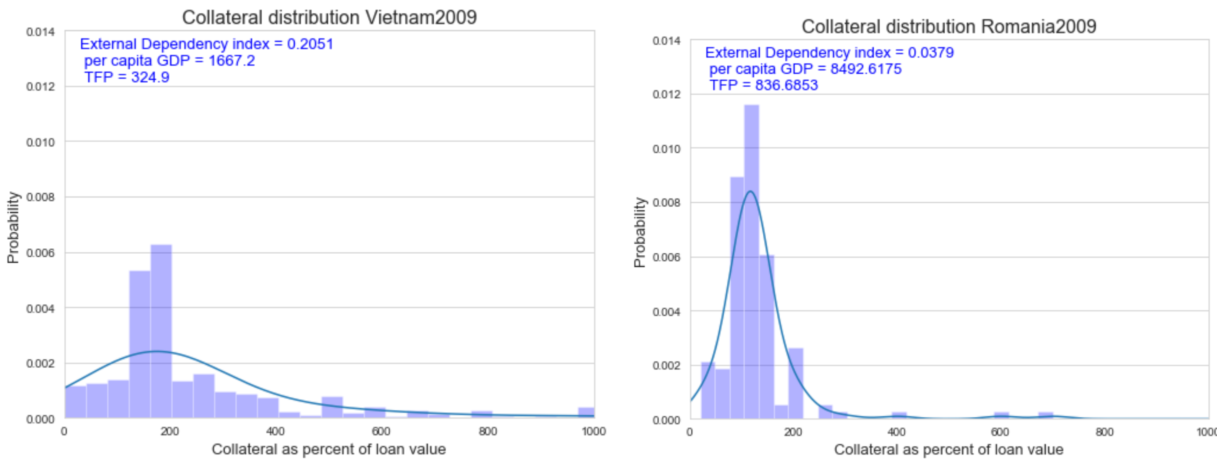
The percentages reported in table (1) are for the entire sample. For different countries in the sample, different reasons might be dominant and the orders might change from one country to another. That is, for any of the reasons provided in table (1) there is heterogeneity across countries. Related to this, a question of interest is: which one of the mentioned reasons (in table (1)) that deter business from financing is more relevant from economic development perspective? It is simply not possible to do a thorough analysis and assess causal links from the mentioned reasons to economic development without an extensive data set on multiple factors affecting each.[43] However, we can gauge the significance of these reasons regarding economic development relative to each other while controlling for some relevant factors for which data is available. To do so, I will utilize Random Forest Classification technique, and will use it to extract feature importance indexes for the mentioned reasons in table (1) based on how well they can explain TFP and GDP per capita. Random Forest Classification is a highly non-linear technique, and as the name suggests it is for classifying discrete categorical variables using any given explanatory variable (also called feature in that context). Since I want to use different variables and assess their strength in explaining continuous dependent variables (TFP and GDP per capita), I split the dependent variables into fine quantiles (say 10, 20, etc.) and then use the classifier to measure each explanatory variable's strength in classifying each quantile. The results of this exercise indicates the following ranking: 1. too high collateral, 2. not applied because had enough capital, 3. interest rate, 4. complex application procedure, 5. didn't think it would be approved, and 6. insufficient size and maturity.[44] In a similar exercise I have run a regression with TFP/GDP per capita as my dependent variables and these deterrent reasons as the explanatory variables. I have also controlled for financial development index as well as some other indicators of financial development by International Monetary Fund such as depth, access and efficiency indices of financial markets and institutions. I have found that both collateral and loan rates are significant as deterrents for business financing. The regression results are provided in Appendix (A.2).

2.3 Collateral Rates Heterogeneity

The discussion above provides us with the evidence that, loan rates and collateral are the main reasons deterring many businesses from obtaining the financing they need. Also, compared to other factors, collateral as an obstacle seems to be the strongest factor in explaining TFP and

[43] It would also be out the context of this paper.

[44] In addition to random forest, I have also used extra tree classifier which works similar to random forest and have got the same ordering.



(a) Collateral rate distribution: Vietnam 2009

(b) Collateral rate distribution: Romania 2009

Figure 2.4: Distribution of collateral rates

GDP per capita differences across countries. As a result of this, and the fact that the survey does not cover loan interest rates paid by the businesses, I will have a deeper look into the collateral and its variation within and across countries.

Within any given country, I observe a rich heterogeneity in collateral rates, defined as the value of collateral as a ratio of loan value. Figure (2.4) shows this heterogeneity through the distribution of collateral rates for Vietnam (2.4a) and Romania (2.4b). This rich heterogeneity is an indicator of default risks in the economy. If there was no default risk, the whole idea of collateral as a tool for securing loans would be obsolete. In an environment without default, agents could easily get the funds they need at the same risk free rate[45] which is the case for most models in the context of financial friction. However, the fact that agents in an economy vary with respect to their likelihood of default, creates the need for collateral. As a result, the financial intermediaries specialize in loans for customers with different underlying default risks. Since, financial intermediaries cannot lose money in the long run, there will be a balance between default risks, collateral rate, and the interest charged on loans. This will generate a rich heterogeneity in collateral rates (and loan rates) that we observe in the data.

Across countries, we observe fundamental differences between distributions of collateral rates. Figure (2.4) illustrates the differences between the distribution of collateral rates in Vietnam and that in Romania. As we can see in figure (2.4), there are many such differences that could

[45]There might be other costs such as depreciation, but the idea is that the rates would be identical for different agents.

be relevant for financial and economic development, e.g. mean, median, standard deviation, skewness, kurtosis, certain inter-quartile ranges, etc. Without any deeper investigation, we can see some stark differences between Romanian and Vietnamese collateral rate distributions, that might inform us about the state of financial development in these countries beyond what is measured by the existing indicators. Such information regarding financial development status of a country can help us explain cross-country differences in income, TFP, etc. Next, I turn to analyzing and extracting some relevant indicators from the collateral rate distribution.

2.4 Relevant Moments of Collateral Distribution

Here I explore whether certain features of the collateral distribution can explain economic development beyond what the existing indicators of financial development do. Despite the strong relationship between finance and development, there still remains a great deal of variation unexplained through the conventional indicators of financial development,[46] say external dependence.[47] I will show that differences in collateral distribution can help explain the variation in TFP and GDP per capita. Also, my choice of modeling is related to this, where the collateral distribution determines the level of financial development.[48] Cross-country data exhibits a relatively strong association between external dependency and economic development indicators such as GDP per capita and total factor productivity (TFP). This is depicted in figure (2.5) along with an example of the same two countries, Romania and Vietnam, displaying a completely different picture.[49]

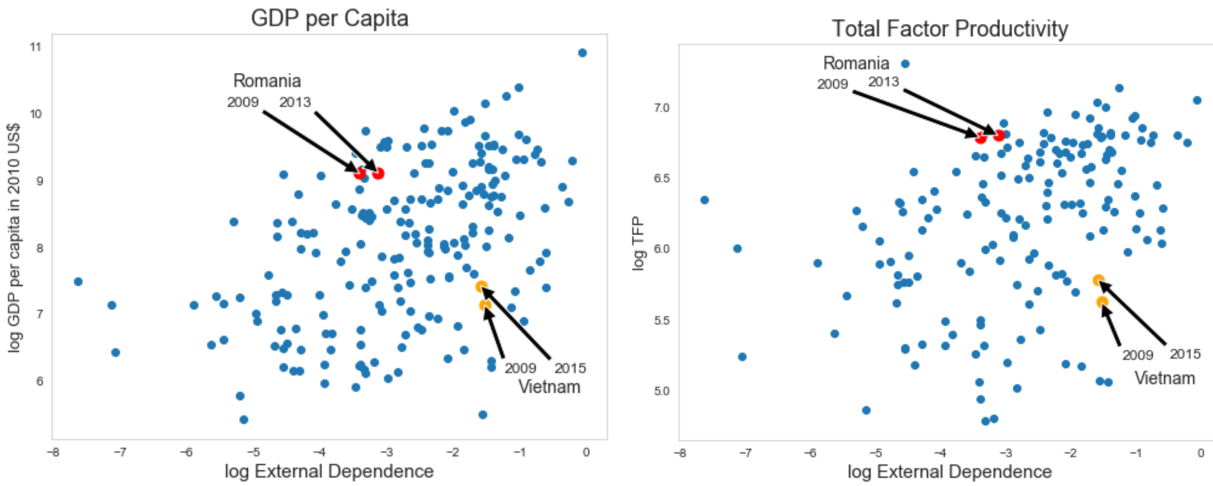
An important takeaway from figure (2.5) is that: Romania lags Vietnam in financial development but leads in economic development, and the differences are somewhat stark. Combining

[46]Some indicators do a better job than others, but there still remains unexplained variation in TFP and GDP per capita. Also, some of the existing indicators include information related to household financing, rather than business financing. As a result, even though some existing measures provide a relatively good fit for TFP or GDP per capita, my measures related to collateral distribution would still be very valuable as they are directly, and only related to firms financing.

[47]I use the financial markets' depth index from IMF as a measure of external dependence. I do so because the richness of IMF's cross-country data allows me to match most of the country-year observations in the sample of collateral distributions from World Bank's Enterprise Survey. Using financial institutions depth index also yields similar results. See Appendix (A.3) for an explanation on these indicators.

[48]This is different than the standard practice in the literature where mainly the external dependence indicator determines the level of financial development or financial frictions.

[49]Measure of GDP per capita is taken from World Bank's Development Indicators, TFP measure constructed using the same method of Klenow and Rodriguez-Clare (1997) using Penn World Tables 9.1, and external dependency is taken to be financial market's depth index from International Monetary Fund. Also note that in order to be consistent with my analysis throughout the paper, in figure (2.5) I have used the same countries across the same years for which data is available in the firm level data set of World Bank's Enterprise Survey.



(a) External dependence vs. GDP per capita

(b) External dependence vs. TFP

Figure 2.5: Distribution of collateral rates

this piece of information with that provided in figure (2.4), we can see the possibility that the information in collateral rate distribution can help explain the differences in income and productivity between Romania and Vietnam, as well as across other countries in the sample. After exploring such a possibility I observe that there actually is useful information related to economic development in the collateral rates distribution. To analyze this, I use different measures extracted from the collateral distribution, including simple mean, variance and higher moments of the distribution as well as some other more subtle features such as the moments within certain inter-quartiles of the distribution and other more complex measures such as entropy, divergence, etc. In order to visually inspect the significance of these features, I use Linear Discriminant Analysis (LDA), a tool from Machine Learning, to create an index from the information extracted from collateral distribution.[50] I call it the collateral distribution index. Note that I use this technique to reduces a large number of relevant variables into few (one in this case), and as a result, there will be some useful information lost in the process. However, this helps us visualize the relationship between the information extracted from collateral distribution and economic development indicators, and provides evidence on the relevance and significance of collateral distribution in explaining economic development. Figure (2.6) shows the relationship between the collateral distribution index and TFP as well as GDP per capita.

What we see in figure (2.6) is a clear association between the collateral rates distribution index and TFP and GDP per capita. Again, note that we lose some information related to the rich

[50]The method is explained in the Appendix (A.4).

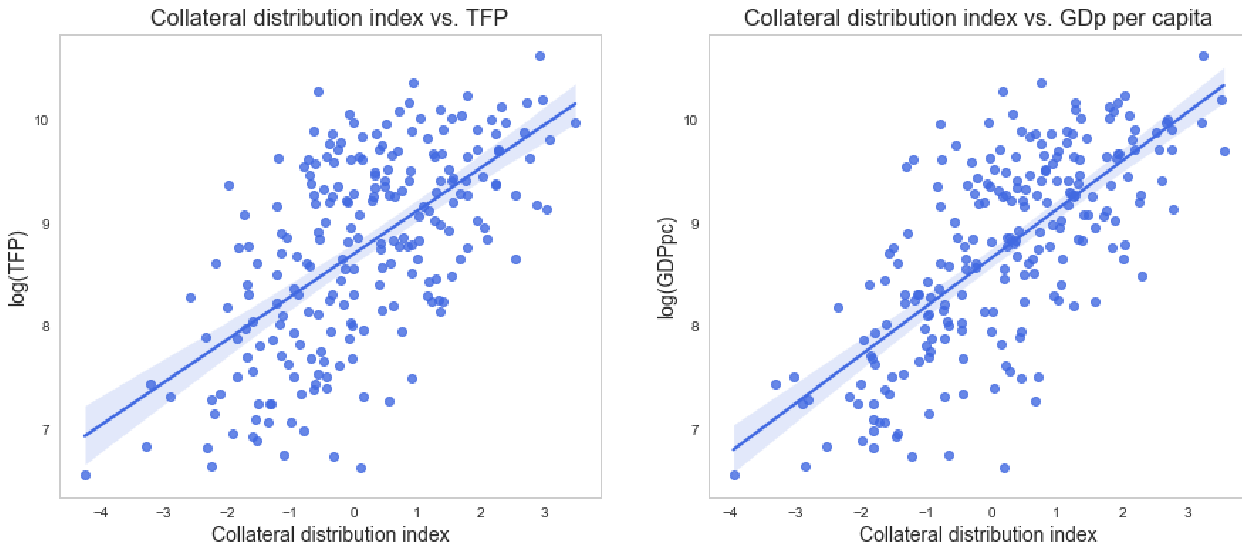


Figure 2.6: Collateral distribution index explaining development

distribution in the process of reducing it to a single variable. Despite this, the evidence on such a relationship is obvious. This strengthens the idea that the collateral rates distribution as a firm-related indicator of financial development, can well-explain economic development.

Since the collateral distribution index is more of an abstract measure and does not have a model counterpart, I will try to extract very few useful features of the distribution that can be related to my quantitative model. To do so, I will use Random Forest feature importance technique to identify the features of the collateral distribution that have the highest explanatory power regarding GDP per capita or TFP. I find that, the standard deviation of the bottom half of the distribution is the most relevant followed by skewness and mean of the distribution. There are many other features of the distribution that worth considering, but given the simplicity of these three moments as well as ease of matching with model counterparts, I will use these three features to summarize the information in the collateral rate distribution. See Appendix (A.4) for an explanation.

3 Model

In this section I discuss a model of entrepreneurship with financial frictions and default. The model follows the setup of Buera and Shin (2013), and for the within period loan structure the model is similar to Cooley and Quadrini (2001). The agents in my model make choices regarding their occupations, consumption-saving, financing and default.

3.1 Outline of the Model

Time is continuous. There are two main types of agents in my model where each of them make certain decisions. I will have individuals as well as a financial intermediary.

Individuals: There is a measure 1 of infinitely lived individuals who can choose to work for a wage or be entrepreneurs. Individuals are trying to maximize their lifetime utility out of consumption of a homogeneous good produced in the economy. Preferences are characterized by a CES utility form given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

The individuals are heterogeneous with respect to their wealth, a, productivity ξ, and creditworthiness, κ. Wage workers receive wage and make consumption-savings decisions only. Entrepreneurs will use capital and labor inputs to produce using the following technology:

$$y = \xi f(k, l) = \xi k^{\alpha} l^{\theta},$$

where $\alpha + \theta < 1$. The entrepreneurial productivity, ξ, has two components: a persistent part that is known at the beginning of period before making production decisions, z; and an unknown part, ϵ, that they find out about in the middle of the period after the production decisions are made.

$$\xi = z + \epsilon$$

The shocks take form of diffusion processes.

$$\begin{aligned} dz_t &= \mu_z(z_t)dt + \sigma_z^2(z_t)dW_t \\ d\epsilon_t &= \sigma_\epsilon^2(\epsilon_t)dW_t \end{aligned}$$

where W is a Brownian motion, μ_z and σ_z are drift and diffusion of the known part of the productivity process, and σ_ϵ is the diffusion of the unknown part of the productivity process. For z I will use an Ornstein-Uhlenbeck process which is equivalent of AR(1) in continuous time. I denote the persistence of z process by ρ_z.

Financing and financial intermediary: Entrepreneurs can borrow to finance their capital. There is a competitive financial intermediary that collects deposits from savers and offers loans at different rates to entrepreneurs. Financial intermediary offers loans based on four criteria: loan amount, b, own investment, d, collateral amount, x and borrowers' creditworthiness, κ.

The information set of financial intermediary is also limited to these four criteria. Working capital will be the sum of loan amount and own investment: $k = b + d$. Also, only working capital can be collateralized, because the financial intermediary does not have information on assets of entrepreneurs: $x \leq k$.

Then based on the available offers, entrepreneurs decide whether to get a loan or not and if they get they choose the contract. The entrepreneurs make the loan decisions knowing their wealth, a, creditworthiness, κ, and the known part of their productivity, z, while they don't know the uncertain part of the productivity, ϵ. This creates the risk of default. At the end of the period, after realization of ϵ, entrepreneurs can decide whether to repay their loan plus interest or to default. Creditworthiness also follows a similar diffusion process with drift and diffusion components as following:

$$d\kappa_t = \mu_\kappa(\kappa_t)dt + \sigma_\kappa^2(\kappa_t)dW_t.$$

Similarly, for creditworthiness I use an Ornstein-Uhlenbeck process with persistence denoted by ρ_κ.

Default: In the case of default, entrepreneur gives up all of the pledged collateral, x, but can keep any assets that were not part of the collateral. There is no additional financial consequences other than losing the collateral. But, there is a stigma for entrepreneurs in case of default. The stigma is a utility cost as a function of entrepreneur's creditworthiness. That means entrepreneurs with higher creditworthiness will try harder to avoid default.

Also, financial intermediary incurs an ownership transfer cost γ. That is, in case of default, only $(1 - \gamma)$ portion of the collateral is recoverable by the financial intermediary. This loss is related to limited enforceability of contracts. A higher transfer cost implies higher loan rates an collateral because financial intermediary is competitive and would not lose money in equilibrium.

Timing: The timing of the model is as the following: At the beginning of the period, knowing their wealth, creditworthiness and the known part of productivity, individuals make occupation decisions. Wage workers' problem easy as they earn wages and then make consumption and savings decision. Entrepreneurs, on the other hand, calculate how much capital they need given risk free rate and wages, and considering their productivity. Then they decide whether they want loan or not and if they choose to get loans, they decide on the loan contract. Then the unknown part of the productivity is realized and entrepreneurs produce given their

<table>
<tr><td>t</td><td colspan="8"></td><td>$t + dt$</td></tr>
<tr><td rowspan="2">a_t
z_t
κ_t</td><td rowspan="2">**Occupation choice**</td><td>E</td><td>**Choice of** $b_t, d_t, x_t,$ k_t, l_t</td><td>**Realization of** ϵ_t</td><td>**Production**</td><td>**Repay or default decision**</td><td rowspan="2">**Saving and consumption decision**
c_t, a_{t+dt}</td><td rowspan="2">**Realization of** κ_{t+dt} **and** z_{t+dt}</td><td rowspan="2">a_{t+dt}
z_{t+dt}
κ_{t+dt}</td></tr>
<tr><td>W</td><td colspan="4">**Receive wage,** w_t</td></tr>
</table>

Figure 3.1: Timing of the model

productivity. After production they decide whether to repay the loan plus interest or to default. See figure (3.1) for timing of the model.

3.2 Financial Frictions

Motivated by cross-country observations regarding the heterogeneity in collateral rates, I identify the state of financial frictions in the economy using the distribution of collateral distribution. This is not a conventional way to identify the level of financial development (or financial frictions) in the literature, as most work relate the financial frictions to a single parameter measured by external dependency. The main reasons for my choice of collateral distribution is that collateral and loan rates are inter-connected, and in the sample of countries in the World Bank's Enterprise Survey they account for more than half of the firms that avoid financing despite needing it. Another reason is that, as discussed in section (2), the collateral distribution contains information that provide a good fit for development indicators such as TFP and GDP per capita. Also, as discussed in section (2), I will choose the first three moments of the collateral distribution to proxy for the whole distribution. In the model, I need at least three parameters for financial frictions to match the first three moments of the collateral distribution. Ownership transfer cost, (γ), which is related to limited enforceability is one of the parameters. The other one is the persistence of creditworthiness, ρ_κ. This affects agents savings decisions because of the future uncertainty regarding their loans, and can shape the collateral distribution through savings and self-financing channels. The third parameter is related to informational frictions. This parameter is related to ability of financial intermediary to assess the default probabilities of loan-applicant. In order to control the accuracy of financial intermediary's default evaluations, I use Decision Tree Classification method. This works in the following way: Knowing the past outcome of default in their information space (b, d, x, κ), financial intermediary uses Decision Tree Classifier to classify the space into default and no-default zones and assigns default probability for any given point in the information space. I have chosen decision trees classifier because they are simple and it is easy to adjust the accuracy of the classification using the depth parameter, ζ. Therefore, a low value of ζ means a poor and inefficient assessment of the probability space and as depth increases, the classification becomes more accurate.[51] See Appendix (B.1) for a simple example on how decision tree classifier works in the environment of my model.

3.3 Loan Pricing

As discussed earlier, financial intermediary's information set is (b, d, x, κ). Also financial intermediary is competitive. Zero profit condition on each loan gives the following:

$$\mathbb{E}_{\epsilon_t}[R_b(b_t, d_t, x_t, \kappa_t; \epsilon_t; w_t, r_t; r_{b,t})] = (1 + r_t)b_t \tag{3.1}$$

where

$$R_b(.) = \begin{cases} (1 + r_b)b_t, & \text{no default} \\ (1 - \gamma)x_t, & \text{otherwise} \end{cases}$$

Loan rate, r_b, is the unique solution (if exists) for the above problem. This simply means that the expected earnings from loans should be equal to deposit payments and interests paid on deposits. The expected loan earnings will be a combination of earnings from defaulters as well as non-defaulters. The non-defaulting entrepreneurs simply pay back the loan plus the loan interest rate. For the defaulting entrepreneurs, the financial intermediary recovers $(1 - \gamma)$ fraction of their collateral.

Now, the financial intermediary solves an inference problem to assign default probabilities to the loan space. This is done using Decision Trees Classifier method where the Depth parameter of the decision tree, ζ, governs the efficiency of their assessment.[52]

Proposition 1. *Let $P^D(b, d, x, \kappa; \epsilon; \zeta)$ be the default probability of contract (b, d, x) for an entrepreneur with creditworthiness of κ that receives the unknown shock ϵ. We have the following loan pricing:*

$$r_b(b, d, x, \kappa) = \frac{rb + (b - (1 - \gamma)x)\mathbb{E}_\epsilon[P^D(b, d, x, \kappa; \epsilon; \zeta)]}{b(1 - \mathbb{E}_\epsilon[P^D(b, d, x, \kappa; \epsilon; \zeta)])} \tag{3.2}$$

[51] Not that too high values for depth might cause over-fitting issues which I avoid by choosing a smaller range for depth and not choosing too high values.

[52] We can also think of the depth parameter, ζ, as the quality or accessibility of credit information.

The proof is straightforward and follows from equation (3.1). From the perspective of the model solution, the loan pricing algorithm is as follows:

Given wages, w, and risk-free rates, r, begin with an initial guess for loan rates $r_b^0(b, d, x, \kappa)$,

1 Solve for the loan decisions and value function given prices and loan rates.

2 Solve for the distribution of agents across state space.

3 Identify the defaulters in the state space (a, z, κ).

4 Knowing the loan choices of the agents, identify defaulters in the financial intermediary's information space (b, d, x, κ).

5 Using the defaulters vs. non-defaulters and their corresponding density in the loan space, use Decision Tree Classifier to assign default probability to each possible loan in the space (b, d, x, κ).

6 Using the default probabilities from step 5, update the loan rates using equation (3.2). Go back to step 1.

Repeat until loan rates converge.

3.4 Occupation Decisions

Workers receive wages, w, and earn interest, r on their deposits. That is, every period they earn $w + ra$. I now consider the entrepreneurs options and profits which compared to the wages and interests earned by workers will help determine the occupational choice.

Given their knowledge about their wealth, known part of productivity and creditworthiness, and considering the loan menu as well as the uncertainty they will face, entrepreneurs will choose the optimal amount of production factors, as well as loan contract consisting of: amount of loan, own investment and collateral.

In order to make things simple in the model, I assume that the only inter-temporal feedback regarding the loan and production choice is related to default indicator, i.e. related to the values of unknown shocks ϵ that will lead to default. The entrepreneurs will solve the following problem to determine the loan and production factors.

$$
\begin{aligned}
\pi^E(a,z,\kappa) &= \max_{b,d,x,l} \mathbb{E}_\epsilon \{ f(z+\epsilon,k,l) - wl - \delta k + r(a-d) \\
&\qquad -(1-I_{\mathcal{D}}(a,z,\kappa,\epsilon)) r_b b - I_{\mathcal{D}}(a,z,\kappa,\epsilon)(x-b)\}, \qquad (3.3)
\end{aligned}
$$

subject to

$$
\begin{aligned}
0 \leq \quad & d \quad \leq a \\
0 \leq \quad & x \quad \leq k \\
r_b \quad &= \quad r_b(b,d,x,\kappa) \text{ , solution to (3.2)} \\
k \quad &= \quad b+d \\
I_{\mathcal{D}}(a,z,\kappa,\epsilon) \quad & \quad \text{is default indicator taking 1 and 0}
\end{aligned}
$$

Knowing the expected profits given by (3.3), and knowing the earnings for a wage worker, individuals will choose their occupation as the following:

$$
\Pi(a,z,\kappa) = \max\{\pi^E(a,z,\kappa), w+ra\}, \qquad \text{where } \pi^E \text{ is given by (3.3).} \qquad (3.4)
$$

After the realization of the unknown part of productivity, ϵ, we will have the following earnings for entrepreneurs and wage workers:

$$
\begin{aligned}
\tilde{\Pi}^E(a,z,\kappa;\epsilon) &= (z+\epsilon)f(k,l) - wl - \delta k + r(a-d) - r_b b, \qquad (3.5) \\
\tilde{\Pi}^W(a,z,\kappa;\epsilon) &= w + ra
\end{aligned}
$$

where all the loan and production decisions are given by (3.3). Savings for continuing individuals is given by:

$$
\dot{a} = \tilde{\Pi}^j(a,z,\kappa;\epsilon) - c, \qquad \text{for} \quad j \in \{E, W\} \qquad (3.6)
$$

3.5 Value Function

Because of the sudden change in the state and entrepreneurs incurring a utility cost (stigma) in case of default, I will model it as a stopping time problem. Individuals will solve the following problem, which is the expected life-time value:

$$
\begin{aligned}
&V^j(a,z,\kappa,\epsilon) = \max_{c_t,\tau} \left\{ E_0 \int_0^\tau e^{-\rho t} u(c_t) dt + e^{\rho\tau} V^{*j}(a,z,\kappa,\epsilon) \right\} \\
&\text{subject to} \\
&\dot{a}_t = \tilde{\Pi}^j_t(a_t, z_t, \kappa_t; \epsilon) - c_t \\
&dz_t = \mu_z(z_t)dt + \sigma_z^2(z_t)dW_t \\
&d\epsilon_t = \sigma_\epsilon^2 dW_t \\
&d\kappa_t = \mu_\kappa(\kappa_t)dt + \sigma_\kappa^2(\kappa_t)dW_t
\end{aligned}
\tag{3.7}
$$

where $\tilde{\Pi}^j(a,z,\kappa;\epsilon)$ is given by (3.5); and V^{*j} is the default value only available for entrepreneurs with $b > 0$.

In case of default, the collateral is transferred to the lender, and the defaulting borrower can keep whatever savings she has extra to the collateral value. The defaulting entrepreneurs will incur a utility cost which is a function of their creditworthiness. The switching value at default is given by the following:

$$V^*(a,z,\kappa,\epsilon) = V(a^D, z, \kappa, \epsilon) - h(\kappa) \tag{3.8}$$

where

$$a^D = a + b - x,. \tag{3.9}$$

The stigma cost is a function of entrepreneurs creditworthiness. To get the scale right, I will relate the stigma to the average value in the state space. I also assume $h()$ is an increasing function of creditworthiness and it is weakly convex. That is, the individuals will differ more at the highest levels of creditworthiness. I will use the following quadratic form for stigma function:

$$h(\kappa) = (h_0 + h_1\kappa + h_2\kappa^2)\bar{V}$$

where h_0, h_1 and h_2 are the parameters to be determined in calibration, and $\bar{V}$ is the average value across the state space.

I solve for value functions using the Hamilton-Jacobi-Bellman variavtional inequality (HJBVI). Since the individuals can make default decisions which leads to a sudden change in the state and the value function, I formulate the value function as a stopping time problem. To solve these problems we make some modifications on the main problem and obtain Hamilton-Jacobi-

Bellman variational inequality (HJBVI). Since ϵ shocks are independent Brownian incidents, we can solve for the value functions independently for different values of ϵ. Individual's HJB has the following form. For occupations $j \in \{W, E\}$, where W stands for wage workers and E stands for entrepreneurs.

$$\begin{aligned}
\rho V^j(a,z,\kappa;\epsilon,t) &= \max_c u(c) + \frac{\partial V^j}{\partial a}\left(\tilde{\Pi}^j(a,z,\kappa;\epsilon) - c\right) \\
&+ \frac{\partial V^j}{\partial z}\mu_z - \frac{1}{2}\frac{\partial^2 V^j}{\partial z^2}\sigma_z^2 + \frac{\partial V^j}{\partial \kappa}\mu_\kappa - \frac{1}{2}\frac{\partial^2 V^j}{\partial \kappa^2}\sigma_\kappa^2.
\end{aligned}$$

The HJBVI will be derived from this using the default value, given by $V^*(a,z,\kappa;\epsilon)$.

3.6 Distribution

Now we want to solve for the stationary distribution of the economy. The density function will be obtained using Kolmogorov Forward Equation (KFE), which is a partial differential equation similar to HJB, and similar to value function, it will be solved numerically using the finite difference method.

After solving the value function for different individuals using HJBVIs,[53] the relevant value functions will provide the areas in the state space where the individuals default. The KFE will be as the following for $j \in \{W, E\}$:

$$\begin{aligned}
\frac{\partial g^j(a,z,\kappa;\epsilon,t)}{\partial t} &= \frac{1}{2}\frac{\partial^2}{\partial z^2}\left(\sigma_z^2 g^j(a,z,\kappa;\epsilon,t)\right) - \frac{\partial}{\partial z}\left(\mu_z g^j(a,z,\kappa;\epsilon,t)\right) \qquad (3.10)\\
&+ \frac{1}{2}\frac{\partial^2}{\partial \kappa^2}\left(\sigma_\kappa^2 g^j(a,z,\kappa;\epsilon,t)\right) - \frac{\partial}{\partial \kappa}\left(\mu_\kappa g^j(a,z,\kappa;\epsilon,t)\right) \\
&- \frac{\partial}{\partial a}\left[\dot{a} g^j(a,z,\kappa;\epsilon,t)\right] \\
&- g^D(a,z,\kappa;\epsilon,t) + g^D(a^D,z,\kappa;\epsilon,t) \qquad (3.11)
\end{aligned}$$

where $\dot{a}$ is the evolution of a given by equation (3.6). Also, g^D is the distribution of entrepreneurs that default, and a^D is given by (3.9).

[53]The HJBVIs are solved as Linear Complementarity Problem (LCP) which is a technique based on finite difference method.

3.7 Equilibrium and Market Clearing

The stationary equilibrium of the model is obtained by joint solution of HJBVIs and KFEs given the occupational choice and optimal decision rules for consumption, saving, production and financing. I solve the HJBVIs as a Linear Complementarity Problem (LCP) which is based on the finite difference method. KFEs are also solved using the finite difference method. See Achdou et al. (2017) for a detailed explanation on the application of finite difference method on a heterogeneous agent problem.

Market Clearing Conditions:

After solving for the distribution and the value function, we can use them along with the decision rules to calculate the aggregates and update loan rates. In the outer loop when loan rates converge, we update wages and risk free rates using the market clearing conditions. For simplicity in notation let's denote a general state vector as $S = (a, z, \kappa)$. Also let's define $S' = (a, z, \kappa, \epsilon)$.

i. Loans Market: The zero profit condition for financial intermediary means that payments for deposits plus interest should be equal to loans plus interest received from borrowers who do not default and the recovered collateral from defaulting entrepreneurs. This gives the following loan market clearing condition:

$$\begin{aligned} &\int_{S'} (1+r)\, b(S) g(dS') = \\ &\int_{S'^{ND}} [1 + r_b(b(S), d(S), x(S), \kappa)]\, b(S) g(dS') + \int_{S'^{D}} (1-\gamma) x(S) g(dS') \end{aligned} \tag{3.12}$$

where S'^{ND} is the part of the state space that default does not occur, and S'^{D} is the area that default occurs. Note that this market clears as a result of financial intermediary's loan pricing given by (3.1).

ii. Capital Market: Here, the amount of capital used in production by entrepreneurs equals the amount deposited by all of the individuals. That is:

$$\int_{S'^{E}} (b(S) + d(S)) g(dS') = \int_{S'} a g(dS') \tag{3.13}$$

where S'^{E} is the space of entrepreneurs.

iii. Labor Market: The demand for labor by entrepreneurs equals the supply of labor by wage workers:

$$\int_{S'^E} l(S)g(dS') = \int_{S'^W} g(dS') \tag{3.14}$$

where similarly S'^W is the space of wage workers.

Algorithm:

A simplified algorithm for solving the equilibrium is described below.

Start with an initial guess for wages, w^0, and interest rate, r^0. Then, for $s = 0, 1, 2, ...$ we do as follows:

1. Start with a guess for loan interest rates, $r_b^0(b, d, x, \kappa)$.
2. Given the prices and loan rates, solve for the value function, and obtain the default regions at the state space (a, z, κ) for any value of ϵ.
3. Solve for the distributions using KFEs.
4. check for the loans market clearing, and update $r_b(b, d, x, \kappa)$ accordingly and then go to step 2. If the loan rates converge, go to next step.
5. Check for capital and labor market clearing conditions. Update the wages and risk-free rates accordingly, and go to step 1. Stop iteration if the markets clear.

This provides the stationary equilibrium of the economy.

4 Results

The most distinct ingredient of my model is the default risk, so it is worthwhile to check the default probabilities produced by the model. The probability of default is evaluated by financial intermediary in the space of (b, d, x, κ) and I map it to the state space (a, z, κ) using the outcome of loan decisions for entrepreneurs. To graphically illustrate the default probability across the state space (a, z, κ) I will show them across different dimensions of the state space for chosen two state variables at a time. Figure (4.1) shows the default probability in the asset-productivity space for individuals with low creditworthiness (left panel) as well as for those with high creditworthiness (right panel). As we can see in figure (4.1), for a given level of productivity, default risk decreases with assets. Also individuals with low creditworthiness are

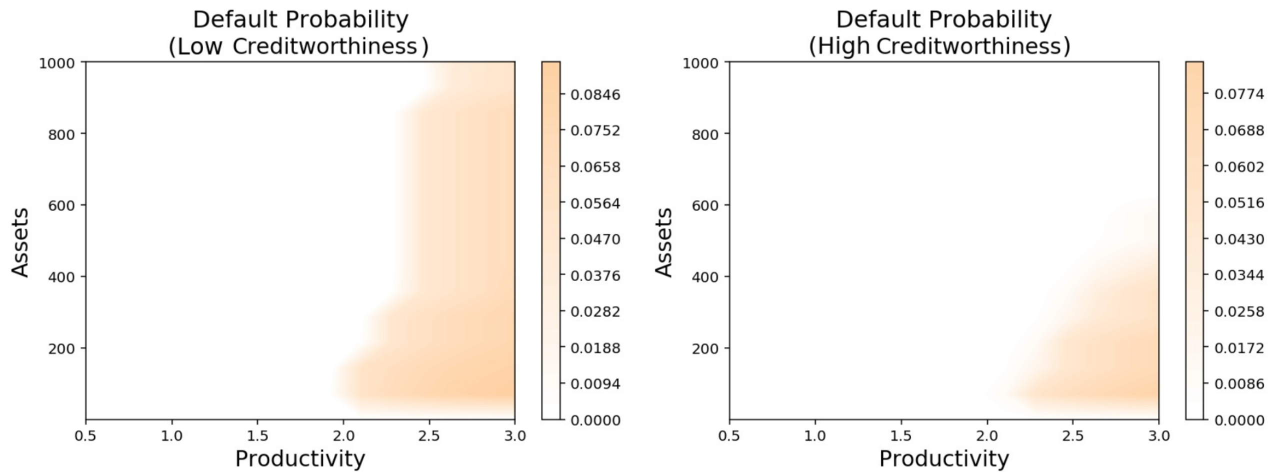


Figure 4.1: Default probabilities in the asset-productivity space

much more prone to default. Note that some of the white space in the figure belongs to wage workers who do not have a default option and default probability is not evaluated for them.

Figure (4.2) shows the default probability in the asset-creditworthiness space for both low productivity (left panel) and high productivity (right panel) individuals. It can be seen in the right panel of figure (4.2) that default probability decreases with both assets and creditworthiness, and there is almost no default risk at the top-right corner which belongs to agents with very high assets and creditworthiness. The reason that there is no default probability shown in the left panel is that, these are the lowest productivity individuals and none of them are entrepreneurs and as a result no default risk is evaluated for them.

Finally, figure (4.3) shows the default probabilities in the productivity-creditworthiness space for both low assets (left panel) as well as high assets (right panel). Similarly, it can be seen that default probability decreases with assets, productivity and creditworthiness conditional on entrepreneurship.

4.1 Calibration

As discussed in section (2), the first three moments of collateral distribution are empirically the most related (amongst higher moments) to TFP and GDP per capita. Also, the first three moments of collateral distribution are highly correlated with the three financial friction indicators in my model (collateral recovery rate, credit registry and access to credit information). The level of financial frictions in the economy is governed by three parameters: Ownership transfer

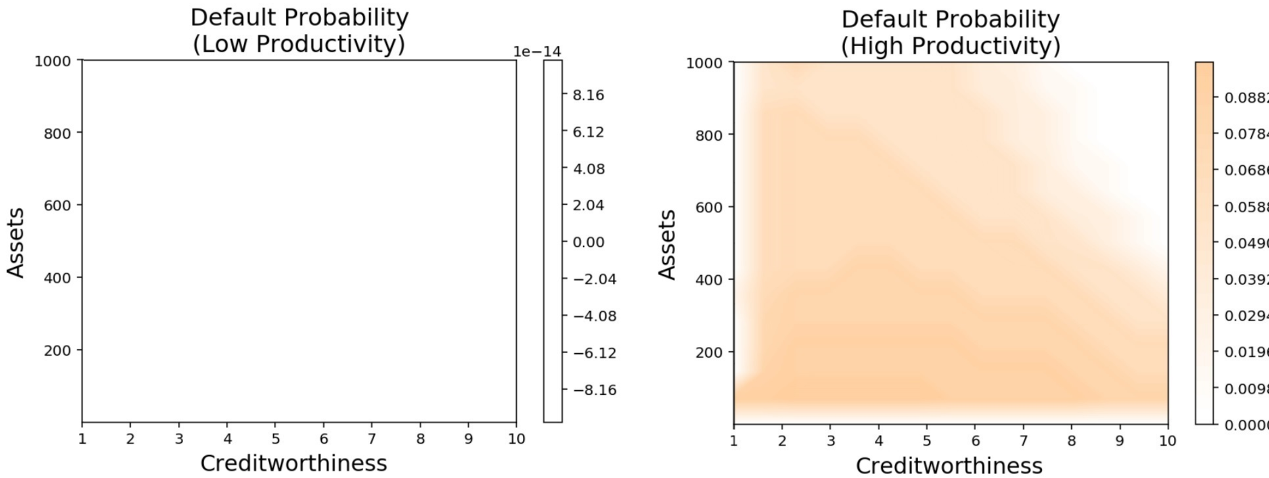


Figure 4.2: Default probabilities in the asset-creditworthiness space

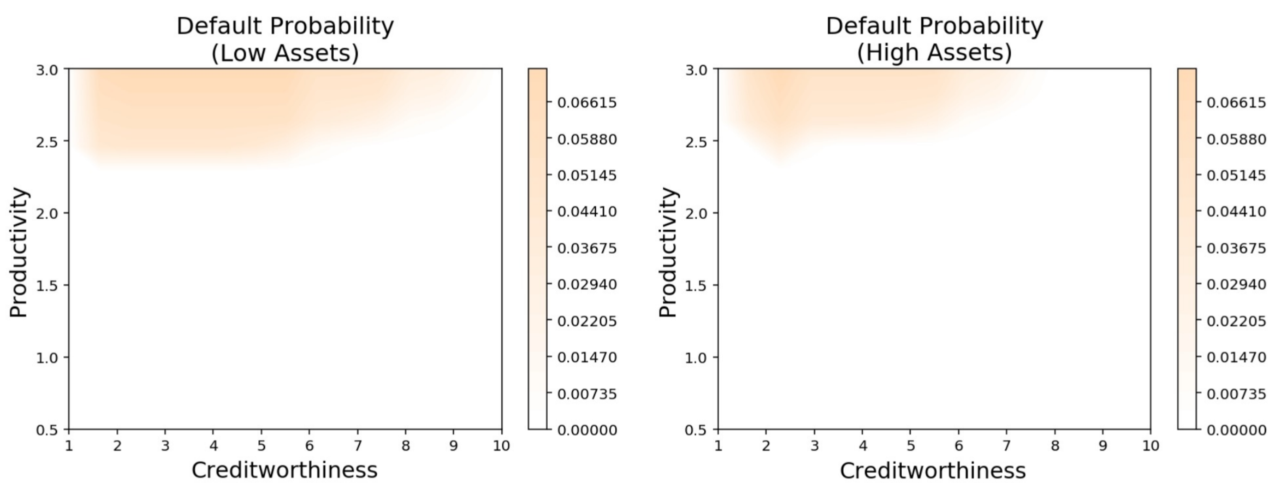


Figure 4.3: Default probabilities in the productivity-creditworthiness space

Parameter	Description	value
σ	(CRRA)	1.5
δ	Capital depreciation	0.05
γ	Collateral recovery rate	0.2
α	Capital share	0.3
θ	Labor share	0.5
ρ_κ	Creditworthiness persistence	0.98
h_0	stigma function parameter	0
h_1	Stigma function parameter	0
ζ	Decision Tree depth	20

Table 2: Freely Calibrated parameters

cost, γ, persistence of creditworthiness, ρ_κ and depth parameter of the decision tree classifier, ζ. For the calibration of the U.S. moments I will set these parameters freely, and then I will change them to distort the financial frictions for the U.S. and to match collateral distribution moments of other countries in the lower decile of per-capita income distribution. I have chosen, $\gamma = 0.2$ to be consistent with high collateral recovery rate of around 80% for the U.S. I have also set a very high value for the persistence of creditworthiness, $\rho_\kappa = 0.98$ to reflect the high quality of credit registry. For the depth of the decision tree that governs the accessibility of credit information, I have set $\zeta = 20$. It happens that values beyond 20 do not have significant effect on my results and I do not choose too high values for this parameter to avoid issues that may arise because of over-fitting of decision trees classifier. There are six other parameters that I set freely using the values from the literature. I set the coefficient of relative risk aversion for utility function, $\sigma = 1.5$, the depreciation rate of capital, $\delta = 0.05$, share of capital, $\alpha = 0.3$ and share of labor, $\theta = 0.5$. The labor and capital shares imply a decreasing returns to scale technology with scale of 0.8. Finally, to keep things simple I choose the parameters of stigma function, h_0 and h_1 both equal to zero. This means my stigma function will only have the quadratic part which will be calibrated jointly with other remaining parameters. I have listed the free parameters in table (2).

There remains eight parameters to be jointly calibrated to match distributional and aggregate moments of the U.S data. These parameters are: rate of time preference, ρ, persistence of productivity, ρ_z, volatility of productivity, σ_z, mean productivity, μ_z, volatility of creditworthiness, σ_κ, mean of creditworthiness, μ_κ, volatility of unknown shock, σ_ϵ and the quadratic coefficient of stigma function, h_2.

These parameters are jointly calibrated to match the following: Risk free rate, share of en-

Parameter	Description	Value
ρ	Time preference	.053
ρ_z	Productivity persistence	.97
σ_z	Productivity volatility	.39
μ_z	Productivity mean	2.28
σ_κ	Creditworthiness volatility	.57
μ_κ	Creditworthiness mean	3.06
σ_ϵ	volatility of unknown shock	.14
h_2	Stigma function parameter	.06

Table 3: Jointly Calibrated parameters

Targets	Model	Data
Risk-free rate	0.04	0.04
Entrepreneurs share pop. %	7.5	7.5
Entrepreneurs exit rate	0.1	0.1
Default rate %	2.3	2.85
Average collateral rate	1.34	1.4*
Wealth share top 1%	30	30
Wealth share top 5%	54	54
Wealth share top 10%	66	67

Table 4: Targeted moments

trepreneurs in the population, firms exit rate, default rate of entrepreneurs, average collateral rate,[54] wealth shares of top 1%, top 5% and top 10%. These moments and their values are listed in table (4). The jointly calibrated parameters are also reported in table (3).

4.2 Distorting U.S. Financial Markets

I will use the first three moments of the collateral distribution as my indicators of financial frictions to re-calibrate U.S. financial markets. I will adjust the three parameters related to financial frictions to change the collateral distribution of the U.S. so that it matches that of lower income countries. In the first exercise, I vary all three parameters at the same time to the level of countries in the middle 20% of GDP per capita amongst all countries in the sample. In another exercise I adjust the parameters to match the collateral distribution of the countries at the bottom 10% of GDP per capita distribution. Given these exercises I can look at multiple outcomes and analyze the changes. I am particularly interested at the top

[54] I did not have data on average collateral rate for the U.S. and instead I used the average collateral rate for the top 5% of the richest countries in the Enterprise Survey sample.

Parameters	U.S. Data	Benchmark Model (U.S.)	Distorted Middle 20%	Distorted Bottom 10%
γ		0.2	0.5	0.9
ζ		20	4	1
ρ_κ		0.98	0.88	0.71
Moments	**U.S. Data**	**Benchmark Model (U.S.)**	**Distorted Middle 20%**	**Distorted Bottom 10%**
Top 1%	30	30	36	45
Top 5%	54	54	59	65
Top 20%	81	79	83	87
Top 40%	94	90	91	94
% Entrepreneurs	7.5	7.5	6.8	5.3
TFP	1	1	0.87	0.57

Table 5: Distorted U.S. financial market

of the wealth distribution, TFP and fraction of entrepreneurs. Table (5) shows the values of distorted economy as well as the benchmark U.S. Starting from the TFP we observe that a mild distortion[55] reduces the U.S. TFP by 13% whereas a more severe distortion[56] reduces the U.S. TFP by a large amount, 43%. Entrepreneurship with mild distortion goes from 7.5% to 6.8%, and with severe distortion drops to 5.3%. Also another stark result is that the wealth share at the top of the distribution increases, and it increases disproportionately towards the top. That is, the top 1% gain more that the next 4%, and top 5% gain more than the next 5%. This is the issue of wealth inequality at the top that I have mentioned throughout the paper which other models are not successful at producing.

I another exercise, I change the parameters one at a time while keeping all other parameters at the benchmark U.S. level. Table (6) shows the results of this exercise. We observe that, persistence of the creditworthiness can cause a drop of 12% when distorted to the lowest level of previous exercise, while depth of decision tree can cause a 5% reduction, and ownership transfer cost can cause a 9% drop in TFP. If we add up these numbers we get only 26% drop at the isolated effects on TFP. This implies a relatively large amplification effect which happens when all three frictions are at work at the same time. This can have important policy implication because reducing a single friction not only can improve the TFP as a result of its direct effect, it can also improve a great deal through the amplification effects that happens in the presence of multiple frictions.

[55] A financial market similar to countries in the middle 20% of GDP per capita distribution.

[56] A financial market similar to countries at the bottom 10% of GDP per capita distribution.

Parameters	U.S. Data	Benchmark Model (U.S.)	Distorted Middle 20%	Distorted Bottom 10%
γ only		0.2	0.5	0.9
TFP	1	1	0.97	0.91
ζ only		20	4	1
TFP	1	1	0.99	0.95
ρ_κ only		0.98	0.88	0.71
TFP	1	1	0.95	0.88

Table 6: Isolated effects of distortions

4.3 A Model Without Default

As I have discussed in the paper, the model without default would be isomorphic to a standard model of entrepreneurship, where the persistence of productivity plays a crucial role in driving the results. In this exercise, I have created a version of my model without default, and have used a similar calibration strategy. I have used two different values for the persistence of the productivity shocks. A high persistence of productivity, similar to the benchmark model with default as well as a low one. The results of this exercise with high persistence of productivity are reported in table (7). As we can see, when we distort the financial markets using this model, we only get about 7% TFP losses. Also, the drop in the share of entrepreneurs is not as large. In the model without default the share of wealth held by top wealthy, say top 1%, 5%, etc. increases but this increase is not as large as the model with default. Another important point is that, in the model without default, the share of wealth at the top does not go to the wealthiest, i.e. top 1% share does not increase much compared to the share of next 4%, and similarly for the other segments at the top of the wealth distribution.

Parameters	U.S. Data	Benchmark Model (U.S.)	Distorted Middle 20%	Distorted Bottom 10%
Ext. Fin. GDP		2.5	0.5	0.1
Moments	U.S. Data	Benchmark Model (U.S.)	Distorted Middle 20%	Distorted Bottom 10%
Top 1%	30	27	27	28
Top 5%	54	59	59	60
Top 20%	81	94	95	95
Top 40%	94	99	99	99
% Entrepreneurs	7.5	8.2	8.1	7.9
TFP	1	1	0.96	0.93

Table 7: Model without default: high persistence, $\rho = 0.97$

The results of the exercise with relatively low persistence of productivity are reported in table (8). The distorted financial market in this case produces larger TFP losses which is consistent with the findings in the literature. In the case with lower persistence of productivity, the TFP losses are comparable to losses generated from the benchmark model with default.

Parameters	U.S. Data	Benchmark Model (U.S.)	Distorted Middle 20%	Distorted Bottom 10%
Ext. Fin. GDP		2.5	0.5	0.1
Moments	U.S. Data	Benchmark Model (U.S.)	Distorted Middle 20%	Distorted Bottom 10%
Top 1%	30	26	32	41
Top 5%	54	57	69	83
Top 20%	81	90	97	98
Top 40%	94	97	99	99
% Entrepreneurs	7.5	8.6	6.4	4.1
TFP	1	1	0.76	0.55

Table 8: Model without default: low persistence, $\rho = 0.70$

Similarly, we observe a very large drop in the fraction of entrepreneurs, from 8.6% to 4.1%. The wealth share at the top increases, but again the wealth does not become concentrated at the very top as a result of financial distortions. Therefore, the financial frictions in the model with no default cannot generate the right direction of wealth dynamics at the top.

5 Conclusion

In this paper I developed a model of entrepreneurship with default and heterogeneity in collateral and loan rates. My model generates relatively large losses from financial frictions, and it is consistent with high persistence of productivity as well as a declining self-financing motive. My model is also consistent with the dynamics of wealth inequality at the top of the wealth distribution. That is, consistent with empirical evidence, when financial frictions increase in my model, the wealth becomes more and more concentrated at the very top. The version of my model without default cannot be consistent with persistence of productivity shock and generate large losses from financial frictions. The model with no default is also not consistent with the increasing concentration of wealth at the very top of the distribution.

My model can also disentangle the effects of financial frictions due to enforceability and informational frictions. This makes my mode unique. Instead of a single parameter of external dependency which is used frequently in the literature, I use the collateral rate distribution to identify the level of financial frictions in the economy. Also, I can analyze the isolated effects of the sources of financial frictions as well as their amplifying effects. As indicated by my results, the amplifying impact on TFP is significantly large, which suggests that, improving financial markets might prove valuable even if that improvement occurs in one dimension or related to one source of financial friction.

For future directions, one might want to introduce entry costs to a financial frictions model with default. Adding entry cost would add extra value and might help us analyze the extensive margin effects in a model of financial frictions. Also, different physical adjustment costs such as capital adjustment cost would be very useful and can help us analyze another source of financial friction related to differences in valuation of collateral by financial intermediary and entrepreneurs for which there is ample empirical evidence. The idea is that, with a simple capital adjustment cost, the capital become more valuable for the entrepreneurs and that would be proportional the the amount of capital they use. That can reflect the fact that the repurchase value is greater than the book value (the one evaluated by financial intermediary), and since this valuation difference varies across countries, it can be considered as another related source of financial friction.

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Appendix

A Empirics

A.1 Data Description

I use firm level data of World Bank's Enterprise Survey. The sample covers years 2008 to 2020 with total of more than 160,000 observations in the entire sample. The firms surveyed from total of 148 countries, with some countries participated in one year and some in two or three years. There are a total of 285 country-year combinations in the sample. On average more than 550 firms are surveyed in each country-year sample. Some important variables that I use are the questions regarding the firms' loan applications. Questions about whether they applied for loans or lines of credit in the last fiscal year, and a list of reasons for not applying if they did not apply for loans. The reasons are listed in table (9).

Reason for not applying for loans or lines of credit
No need for a loan - establishment had sufficient capital
Interest rates were not favorable
Application procedures were complex
Collateral requirements were too high
Did not think it would be approved
Size of loan and maturity were insufficient
other

Table 9: Reasons for not applying for loans

The main variables that I will use are: the value of the most recent loan, and the value of collateral for the most recent loan. Using these two variables I create a variable named collateral rate defined as the value of collateral as percentage of the loan value. I will use this variable to create cross country observations related to collateral distribution within countries. Also, this data set will vary across years. Therefore I will use different indicators extracted from detailed collateral rate observations and will reduce the sample to 285 country-year observations.

Using firm-level observations of collateral rate, I create multiple measures related to collateral rates distributions for each country-year. These measures vary from simple mean, standard deviation and some higher moments to other inter-quantile moments as well as more complex measures of divergence and entropy of distributions. The measures I have extracted for each country-year are shown in table (10). For the complex measures such as distance, divergence

Collateral rate distributional features
mean
standard deviation
skewness
kurtosis
1st quartile
median
3rd quartile
standard deviation above median
standard deviation below median
inter-quartile range between 1st and median
inter-quartile range between median and 3rd
Jensen-Shannon Distance
Kolmogrov-Smirnov Distance
Mann-Whitney rank test
Cressie-Read power divergence statistic
Renyi entropy

Table 10: Features extracted from collateral rate observations for each country-year.

and entropy I have used the collateral rate distribution in the entire sample as a benchmark comparison point to the collateral rate distributions across all countries, and I have used the test statistic obtained from these tests as my extracted feature. Note that, I have used multiple other measures, but I do not report them as they were not significant in explaining TFP and GDP per capita.

After extracting the distributional features and other cross-country variables such as the reasons that deter firms from applying to loans, I have combined the obtained cross country data set with other standard cross-country data sets such as World Bank's WDI, Penn World Tables, International Monetary Fund's Financial Development Index, Credit Suisse Institute's Wealth Distribution and UNU-WIDER's World Income Inequality Data.

One important variable that I have used in the paper is the TFP measure for each country-year observation. I have calculated this measure from Penn World Tables data using Klenow and Rodriguez-Clare (1997) method. The TFP measure I create together with the GDP per capita measure from World Bank's WDI (similarly the one reported in Penn World Tables) are the main development indicators that I used in this paper.

Also, for financial development indicators I have used the financial development index data set which contains the measures for depth, accessibility and efficiency of financial markets and institutions.

A.2 Regression Results: Deterring Reasons

The regression results for TFP against the deterring reasons for financing is reported in table (11). In this regression TFP is the dependent variable, and the deterring reasons are explanatory variables.[57] Deterring reasons for any country-year is measured by the percent of firms that did not apply for loans because of that reason. This percentage is among the firms that did not apply for loans, and not the entire firms for that country-year. I have controlled for the financial markets and institutions indicator from IMF data. As we can see the mean collateral rate is less significant compared to other two measures and becomes insignificant when controlling for other indicators of financial development. Despite this fact, I keep the mean collateral rate as one of my indicators mainly because of its simplicity.

A.3 Other Indicators of Financial Development

Here I will briefly review different types of financial development indicators used in the literature, and discuss their differences with the features of collateral distribution I introduced in this paper.

A.3.1 External Dependence and Similar Measures

The size of financial sector relative to output has been a measure of financial development traditionally. See Goldsmith (1969) and McKinnon (1973) for instance. This is a measure of financial depth. In addition, King and Levine (1993) use the ratio of private credit to total domestic credit as well as the ratio of private sector credit to GDP as measures of financial market development, emphasizing the importance of credit distribution between private and state-owned firms. External dependence index of Rajan and Zingales (1998) is probably the most widely used measure of financial development in the recent literature. The external finance encompasses private sector credit and private bond market as well as stock market capitalization.[58]

[57] Similar results obtained when I use GDP per capita as dependent variable.

[58] Following Rajan and Zingales (1998) many researchers used ratio of external finance to GDP as an indicator of financial development: Buera, Kaboski and Shin (2011), Buera and Shin (2013), Moll (2014), Shaker Akhtekhane (2020) just to name few in a related subject to this paper's.

	Dependent variable: log(TFP)		
	(1)	(2)	(3)
No Loan (collateral)	-4.182***	-2.575***	-2.388***
	(0.787)	(0.889)	(0.748)
No Loan (rates)	-2.116***	-2.324***	-1.094**
	(0.479)	(0.462)	(0.426)
No Loan (complexity)		-1.811***	-0.286
		(0.639)	(0.568)
No Loan (no approval hope)		-0.177	1.579
		(1.653)	(1.43)
No Loan (size/maturity)		1.168	-0.654
		(2.563)	(2.212)
No Loan (other reasons)		-2.985***	-2.428***
		(0.727)	(0.637)
Fin. Inst. Index			1.467***
			(0.266)
Fin. Markets Index			0.576***
			(0.211)
constant	6.736***	6.996***	6.033***
	(0.076)	(0.089)	(0.161)
Observations	173.0	173.0	168.0
Adjusted R^2	0.282	0.371	0.561
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 11: Regression: deterring reasons vs TFP

A.3.2 Multi-factor Indicators

More comprehensive list of financial development indicators introduced by Beck, Demirgüç-Kunt and Levine (2000). Their indicators include size (depth), activity and efficiency of different types of financial intermediaries and markets. Also Čihák et al. (2012) introduced a data set on the characteristics of financial systems including a comprehensive list of indicators: size, access, efficiency and stability. Following Čihák et al., the International Monetary Fund (IMF) introduced Financial Development Index Database which is a comprehensive, and at the same time brief, list of financial development indicators. IMF's financial development index includes depth, access and efficiency for both financial institutions and markets. A break down of the different components of the index is as the following:

i. Depth
   - Financial Institutions: 1. Private-sector credit (% of GDP), 2. Pension fund assets (% of GDP), 3. Mutual fund assets (% of GDP), 4. Insurance premiums, life and non-life (% of GDP).
   - Financial Markets: 1. Stock market capitalization to GDP, 2. Stocks traded to GDP, 3. International debt securities government (% of GDP), 4. Total debt securities of nonfinancial corporations (% of GDP), 5. Total debt securities of financial corporations (% of GDP).

ii. Access to financing
   - Financial Institutions: 1. Branches (commercial banks) per 100,000 adults, 2. ATMs per 100,000 adults.
   - Financial Markets: 1. Percent of market capitalization outside of top 10 largest companies, 2. Total number of issuers of debt (domestic and external, nonfinancial corporations, and financial corporations).

iii. Efficiency
   - Financial Institutions: 1. Net interest margin, 2. Lending-deposits spread, 3. Non-interest income to total income, 4. Overhead costs to total assets, 5. Return on assets, 6. Return on equity.
   - Financial Markets: Stock market turnover ratio (stocks traded/capitalization)

See Svirydzenka (2016) for a discussion on the financial development index.

A.4 Why Use Collateral Rate Distribution?

There are some advantages for using the distribution of collateral rates as an indicator of financial development. First, this object (distribution of collateral rates) is obtained as an output of my model. Second, this distribution can be summarized by few features or moments that can directly be matched in the model. Third, one can argue that the collateral rates distribution contains information related to economic development that is not accounted for by other measures of financial development, mainly because it is only related to firms financing while other multi-factor measures include elements related to household financing. Also, in order to show the relevance of collateral rate distribution to TFP or GDP per capita, I condense the features extracted from the collateral distribution to create a one dimensional variable to easily illustrate it visually. I use the features shown in table (10) and apply Linear Discriminant analysis (LDA) to reduce the dimensionality of these features in the most related way to TFP and GDP per capita. To use LDA, I split TFP (same for GDP per capita) into ten quantiles. Then I apply LDA so that I get the most variation from the features of the collateral distribution in a way to achieve the best classification of the ten deciles of the TFP distribution. LDA is a widely used method for targeted dimensionality reduction in machine learning.

Given the relevance of collateral rate distribution, I choose three simple moments of as my main indicators. I choose mean, standard deviation at the bottom half, and skewness of the collateral distribution. One main reason is that these measures are straightforward and easily calculated as the outputs of my model. Another reason is that these measures are significant when regressing against TFP or GDP per capita. Table (12) shows this regression results controlling for several other indicators of financial markets and institutions from IMF's Financial Development data set. As we can see mean collateral rate is less significant and becomes insignificant when controlling for other indicators. However, I will use it as one of the indicators because it is simple and easily related to model outcome.

A.5 Firm Concentration vs Financial Development

In table (13), I report the regression results of Herfindahl-Hirschman Index (HHI) against financial development index. In another specification I control for GDP per capita and GDP per worker, and in the third specification I control for factors like human capital, average hours worked and investment as a ration of GDP. Note that these controls are taken from Penn World Tables. In all specifications we get a negative and significant coefficient for financial development. This strengthens the arguments laid out in the paper regarding the relationship

	Dependent variable: log(TFP)		
	(1)	(2)	(3)
Collateral mean	-0.001*	0.0	-0.0
	(0.0)	(0.0)	(0.0)
Collateral std. below median	-0.01***	-0.012***	-0.006**
	(0.003)	(0.002)	(0.003)
Collateral skewness	0.081***	0.031**	0.043**
	(0.021)	(0.014)	(0.02)
FinInstit.Efficiency.Index		0.382*	
		(0.218)	
FinInstitAccess.Index		1.349***	
		(0.135)	
FinInstitDepth.Index		0.587***	
		(0.192)	
FinMarketsAccess.Index			0.753***
			(0.235)
FinMarketsDepth.Index			0.41
			(0.309)
FinMarketsEfficiency.Index			0.146
			(0.177)
const	6.424***	5.62***	6.161***
	(0.113)	(0.142)	(0.116)
Observations	159.0	156.0	156.0
R2	0.193	0.626	0.318
Adjusted R2	0.178	0.611	0.291
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 12: Regression: features of collateral distribution vs TFP

	Dependent variable: HHI		
	(1)	(2)	(3)
Fin. Dev. Index	-1.581***	-1.297**	-1.7***
	(0.387)	(0.557)	(0.624)
GDP per capita		0.0	0.0
		(0.0)	(0.0)
GDP per woker		-0.0	-0.0
		(0.0)	(0.0)
Investment/GDP			-1.036
			(1.074)
Average hours			0.001**
			(0.0)
Human Capital Index			0.132
			(0.241)
const	-1.742***	-1.777***	-3.993***
	(0.113)	(0.115)	(1.008)
Observations	190.0	182.0	72.0
R2	0.082	0.088	0.215
Adjusted R2	0.077	0.073	0.142
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 13: Regression results HHI vs financial development

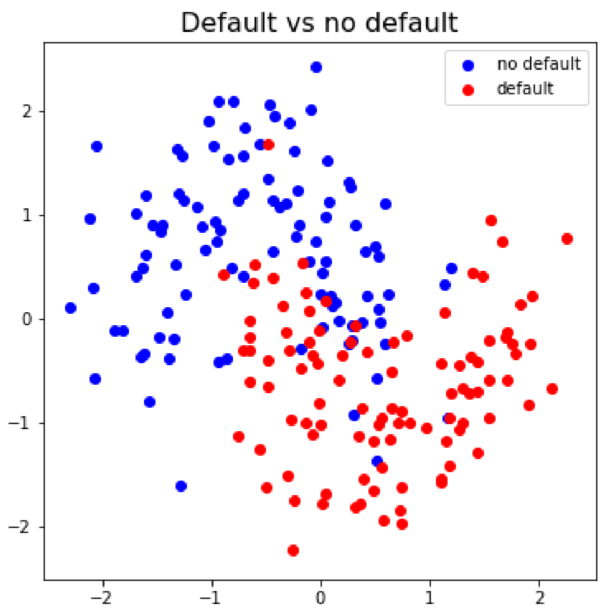
between financial development and wealth inequality amongst entrepreneurs.

B Model

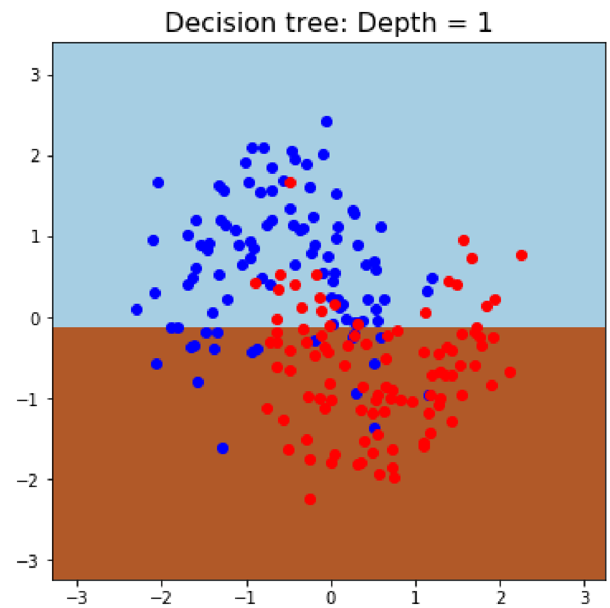
B.1 Assessment of Default Risk: Decision Trees

Here I will use a simple example to clarify the method used by financial intermediary to evaluate the default probabilities of loan applicants. The financial intermediary's information set is (b, d, x, κ). In order to illustrate the method visually I will use a hypothetical example in a two dimensional space (instead of the four dimensional space in the actual problem).

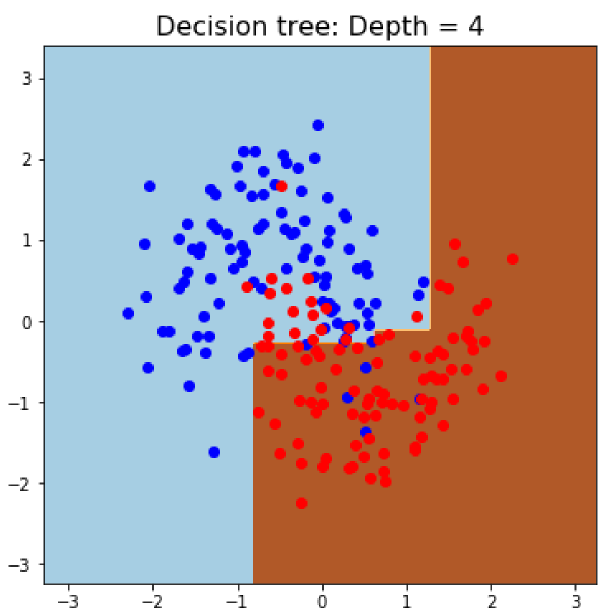
In the two dimensional space we have some observations for defaulters and some for non-defaulters. Also note the these observations will be different across observations of ϵ. For a given ϵ we will have observations scattered across the financial intermediary's information space. In figure (B.1a) consider the red colored dots for defaulters and the blue ones for non-defaulters. The decision trees classification for different values of depth is shown in figure (B.1), panels (b, c, d). As we can see the classification becomes more accurate when the depth increases, but if we increase the depth too much we may encounter issues with over-fitting. Since this is a low dimensional and simple example we can see some minor issues at the depth of 10. However, given the dimension and complexity of the original problem, higher levels of depth work fine. Given the classification, we can assign probability to the entire information space of financial intermediary, and take the expectation over different realizations of ϵ to obtain the default probability in the entire information space. Note that other classification methods also work in this environment, and they are more accurate than decision trees. The only reason I use decision tree is that I want a range of assessment abilities for financial intermediary, from very inefficient to more and more efficient assessments. Other complex methods generally become very accurate as depth increases slightly and do not provide much room for such variation in assessment. I have tried to use Random Forest instead of decision trees, but I do not get much variation in efficiency of assessment with depth parameter of Random Forest, as it becomes very close to the most efficient case at the depth of 2 or 3.



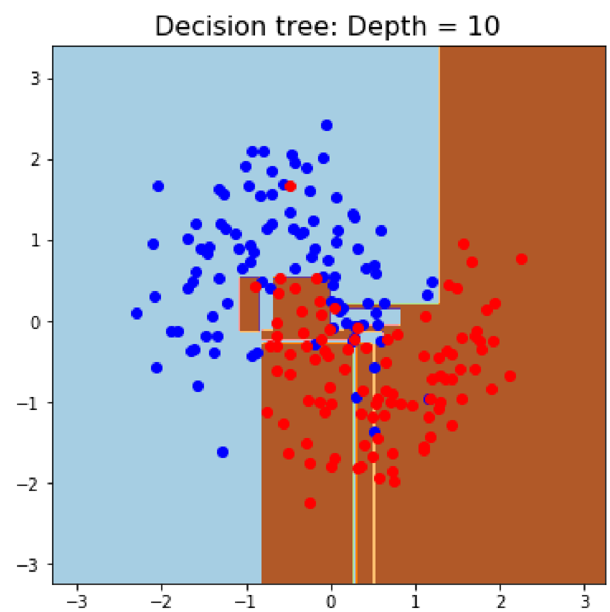
(a) Default Observations



(b) Decision Trees Classification, $\zeta = 1$



(c) Decision Trees Classification, $\zeta = 4$



(d) Decision Trees Classification, $\zeta = 10$

Figure B.1: Default assessment with different depth levels of decision trees