Deeper Insight into iOS App

|  |  |
| --- | --- |
| **Overview Displaying Variables** | |
| **Main Screen**    Not displayed: increment timers | **Mathematics Screen**    Displays “Breakdown of Pay,” if ⓘ is pressed.  Caution: *a* and *α* are different quantities.  “[*R* or *E*]” means one or the other may be displayed. |

**Caution**

The following items are not accounted for:

* loan origination fees
* variable interest rates
* variable payments
* multiple loans
* true number of days in each month
* repayment penalties
* defaulting on loans
* loan consolidation

The following assumptions are made:

* students have already entered repayment
* any interest that would be capitalized has already been capitalized

Loan servicers may require a higher, minimum monthly payment.

**Formulae**

APR not compounded, *ε* = *ε*nominal = *APR*/12/100[[1]](#footnote-2)

APR compounded,

Absolute minimum:

*a*min = nint[*×*100 ]/100 + 0.01

10-year minimum[[2]](#footnote-3):

,

for APR > 0% and 0 < *α* ≤ 1

,

for APR > 0% and *α* = 0, **OR** APR = 0%

Net balance:

*Bj* = *Bj*-1 – , for *j* = 1, 2, 3, … and *a* > *a*min

Outstanding balance:

*Oj* = , for *j* = 1, 2, 3, …

Recurse both balances until *Bj* + **≤**  , at which point *j* = *n* – 1

Suggestion: compute nint(*×*100)/100 and nint(*×*100)/100 after each iteration,

otherwise and may lose precision.

Essentially, interest charged = , interest paid =, principal paid =, and outstanding interest = . *B*0 = *p*, and *O*0 = 0.

Final balance[[3]](#footnote-4):

*Bn* = *Bn*-1 – , for *j* = *n*,

where *a*f *= Bn*-1 + + *On*-1, and thus *Bn* = 0

Breakdown of Pay:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Prin. | + | Int. | = | *a*, for 1 ≤ *j* ≤ 4 |
| ⋮ |  | ⋮ |  | ⋮ |
| *Bn*-1 Prin. | + | + *On*-1 Int. | = | *a*f, for *j* = *n* |

*l*y =

*l*m = – 12*×l*y

Net and outstanding balance equations over *k* are identical to those over *j*, but *a* = *a*min.

Final balance equation over *k* is identical to that over *j*, but *n* = *n*\_min.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *C* | = |  |  | + *On*-1 |
| *C*min | = |  |  | + *On*\_min-1 |

*s* = *C*min – *C*

Δ*s* = *s*new – *s*old

*n* = 1 and *a* – [+ *O*0 ] > 0,

refund, *R* = *a* – [+ *O*0 ]

*n* = 1 and *a* – [+ *O*0 ] < 0,

pay extra, *E* = | *a* – [+ *O*0 ] |

Otherwise, refund and pay extra are not displayed.**Examples**

, and *N* = 20 ⇒ = 250

*APR* = 4.45% not compounded ⇒ *ε* = 4.45%/12/100 = 0.00370…

*APR* = 4.45% compounded ⇒ = 0.00371…

*APR* = 4.45% not compounded, *α* = 0.25 and *p* = 2,000

⇒ Absolute minimum:

*a*min = nint[*×*100 ]/100 + 0.01 = 185/100 + 0.01 = 1.86

⇒ 10-year minimum:

= 1762/100 = 17.62

*APR* = 4.45% not compounded, *α* = 0.25, *p* = 2,000 and *a* = 200

⇒ Net balance:

*B*1 = 2,000 –

= 2,000 –

= 1801.85

*B*2 = 1801.85 –

= 1801.85 –

= 1603.52

*B*3 = 1603.52 –

= 1603.52 –

= 1405.01

⋮

⇒ Outstanding balance:

*O*1 = 0

= 0

= 5.57

*O*2 = 5.57 n

= 5.57

= 10.58

*O*3 = 10.58 n

= 10.58

= 15.04

⋮

until 10.24 + = 10.24 + 4/100 = 10.28 **≤**  200,

at which point *j* = 10 = *n* – 1

(*B*10 = 10.24, and *O*10 = )

⇒ Final balance:

*B*11 = 10.24 –

= 10.24 –

= 41.07 –

*a*f *=* 10.24 + + 30.79

= 10.24 + + 30.79

= 41.07

⇒ *B*11 = 41.07 – 41.07 = 0

Breakdown of Pay:

(drawing from net and final balance equations)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| = 198.15 Prin. | + | = 1.85 Int. | = | | , for *j* = 1 | |
| = 198.33 Prin. | + | = 1.67 Int. | = | | , for *j* = 2 | |
| = 198.51 Prin. | + | = 1.49 Int. | = | | , for *j* = 3 | |
| = 198.70 Prin. | + | = 1.30 Int. | = | | , for *j* = 4 | |
| ⋮ |  | ⋮ | |  | | ⋮ |
| 10.24Prin. | + | + = 30.83 Int. | = | | 41.07, for *j* = 11 | |

*l*y = = = 0

*l*m = – 12*×*(0) = 11

Net and outstanding balance equations over *k* are identical to those over *j*, but *a* = *a*min:

*Bk* = *Bk*-1 – , for *k* = 1, 2, 3, … and *a* = 1.86

⇒ Net balance:

*B*1 = 1999.99, *B*2 = 1999.98, *B*3 = 1999.97, …

*Ok* = , for *k* = 1, 2, 3, …

⇒ Outstanding balance:

*O*1 = 5.57, *O*2 = 11.14, *O*3 = 16.71, …

until 1.59 + = 1.59 + 1/100 = 1.60 **≤**  1.86,

at which point *k* = 6171 = *n*\_min – 1

(*B*6171 = 1.59, and *O*6171 = )

Final balance equation over *k* is identical to that over *j*, but *n* = *n*\_min:

*Bn*\_min = *Bn*\_min-1 – , for *k* = *n*\_min,

where *a*f *= Bn*\_min-1 + + , and thus *Bn*\_min = 0

⇒ Final balance:

*B*6172 = 28438.68 –

*a*f *=* 28438.68

⇒ *B*6172 = 28438.68 – 28438.68 = 0

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *C* | = |  |  | + |
| *C* | = |  |  | + |
| *C* | = 2041.07 |  |  |  |
|  |  |  |  |  |
| *C*min | = |  |  | + |
| *C*min | = |  |  | + |
| *C*min | = 39916.74 |  |  |  |
|  |  |  |  |  |

*s* = *s*old = 39916.74 – 2041.07 = 37875.67

*APR* = 4.45% not compounded, *α* = 0.25, *p* = 2,000 and *a* = 250

⇒ *s* = *s*new = 37883.18

⇒ Δ*s* = 37883.18 – 37875.67 = ↑7.51

*APR* = 4.45% not compounded, *α* = 0.25, *p* = 2,000 and *a* = 150

⇒ *s* = *s*new = 37863.08

⇒ Δ*s* = 37863.08 – 37875.67 = ↓12.59

*APR* = 4.45% not compounded, *α* = 0.25, *p* = 2,000 and *a* = 2,010

⇒ *n* = 1

⇒ 2,010 – [ 2,000 + + 0 ]

= 2,010 – [ 2,000 + + 0 ]

= 2.58 > 0

⇒ refund, *R* = 2.58

*APR* = 4.45% not compounded, *α* = 0.25, *p* = 2,000 and *a* = 2,005

⇒ *n* = 1

⇒ 2,005 – [ 2,000 + + 0 ]

= 2,005 – [ 2,000 + + 0 ]

= -2.42 < 0

⇒ pay extra, *E* = | -2.42 | = 2.42

**Derivation of Formulae for 10-Year Minimum**

Derivation of , for APR > 0% and 0 < *α* ≤ 1

If APR > 0% and 0 < *α* ≤ 1, then *Bk* = *Bk*-1 – [ *a*min – *α*(*Bk*-1×*ε*) ].

Compute *Bk* for 1 ≤ *k* ≤ *n*\_min:

*B*1 = *B*0 – [ *a*min – *α*(*B*0×*ε*) ]

=> *B*1 = *p* – [ *a*min – *α*(*p*×*ε*) ]

*= p* + *α*(*p*×*ε*) – *a*min

= *p*(1 + *α*×*ε*) – *a*min

*B*2 = *B*1 – [ *a*min – *α*(*B*1×*ε*) ]

=> *B*2 = *B*1 + *α*(*B*1×*ε*) – *a*min

= [ *p*(1 + *α*×*ε*) – *a*min ] + *α*{ [ *p*(1 + *α*×*ε*) – *a*min ]×*ε* } – *a*min

= [ *p*(1 + *α*×*ε*) – *a*min ][1 + *α*×*ε*] – *a*min

= *p*(1 + *α*×*ε*)2 – (1 + *α*×*ε*)*a*min – *a*min

*B*3 = *B*2 – [ *a*min – *α*(*B*2×*ε*) ]

=> *B*3 = … = [ *p*(1 + *α*×*ε*)2 – (1 + *α*×*ε*)*a*min – *a*min ] + *α*{ [ *p*(1 + *α*×*ε*)2 – (1 + *α*×*ε*)*a*min – *a*min ]×*ε* } – *a*min

**=** [ *p*(1 + *α*×*ε*)2 – (1 + *α*×*ε*)*a*min – *a*min ][1 + *α*×*ε*] – *a*min

**=** *p*(1 + *α*×*ε*)3 – (1 + *α*×*ε*)2*a*min – (1 + *α*×*ε*)*a*min – *a*min

⁝

*Bn*\_min = *Bn*\_min-1 – [ *a*min – *α*(*Bn*\_min-1×*ε*) ]

=> *Bn*\_min = … = [ *p*(1 + *α*×*ε*)*n*\_min-1 – (1 + *α*×*ε*)*n*\_min-2*a*min – (1 + *α*×*ε*)*n*\_min-3*a*min – … – *a*min ][1 + *α*×*ε*] – *a*min

= *p*(1 + *α*×*ε*)*n*\_min – (1 + *α*×*ε*)*n*\_min-1*a*min – (1 + *α*×*ε*)*n*\_min-2*a*min – … – (1 + *α*×*ε*)*a*min – *a*min

Since *Bn*\_min = 0,

*p*(1 + *α*×*ε*)*n*\_min = (1 + *α*×*ε*)*n*\_min-1*a*min + (1 + *α*×*ε*)*n*\_min-2*a*min + … + (1 + *α*×*ε*)*a*min + *a*min

Multiply both sides of the equation by (1 + *α*×*ε*):

(1 + *α*×*ε*) × *p*(1 + *α*×*ε*)*n*\_min = (1 + *α*×*ε*)*n*\_min*a*min + (1 + *α*×*ε*)*n*\_min-1*a*min + … + (1 + *α*×*ε*)2*a*min + (1 + *α*×*ε*)*a*min

Subtract the equation for *p*(1 + *α*×*ε*)*n*\_min by the one for (1 + *α*×*ε*) × *p*(1 + *α*×*ε*)*n*\_min:

*p*(1 + *α*×*ε*)*n*\_min – (1 + *α*×*ε*) × *p*(1 + *α*×*ε*)*n*\_min = *a*min – (1 + *α*×*ε*)*n*\_min*a*min

Factor, simplify and rearrange both sides of the equation:

*p*(1 + *α*×*ε*)*n*\_min [ 1 – (1 + *α*×*ε*) ] = *a*min [ 1 – (1 + *α*×*ε*)*n*\_min ]

*p*(1 + *α*×*ε*)*n*\_min (-*α*×*ε*) = *a*min [ 1 – (1 + *α*×*ε*)*n*\_min ]

*α*(*p*×*ε*) × (1 + *α*×*ε*)*n*\_min = *a*min [ (1 + *α*×*ε*)*n*\_min– 1 ]

Thus:

, for ≠ 0

10 years being 120 months:

, for ≠ 0

Rounding any products, before obtaining the solution, will affect the precision and, hence, accuracy of the solution. So, round only the solution. Do so by rounding it up to the nearest two decimal places, or else one may not pay enough:

, for ≠ 0

If APR > 0% and 0 < *α* ≤ 1, then ≠ 0. Recall that *ε* = *APR*/12/100.

Derivation of , for APR > 0% and *α* = 0, **OR** APR = 0%

If APR > 0% and *α* = 0, then *Bk* = *Bk*-1 – [ *a*min – (0)(*Bk*-1×*ε*) ] = *Bk*-1 – *a*min.

If APR = 0%, then *ε* = (0)/12/100 = 0 and *Bk* = *Bk*-1 – [ *a*min – *α*(*Bk*-1×[0]) ] = *Bk*-1 – *a*min.

Regardless, compute *Bk* for 1 ≤ *k* ≤ *n*\_min:

*B*1 = *B*0 – *a*min=> *B*1 = *p* – *a*min

*B*2 = *B*1 – *a*min=>  *B*2 = *p* – *a*min – *a*min = *p* – 2*a*min

*B*3 = *B*2 – *a*min=> *B*3 = *p* – 2*a*min – *a*min = *p* – 3*a*min

⁝

*Bn*\_min = *Bn*\_min-1 – *a*min => *Bn*\_min = *p* – (*n*\_min – 1)*a*min – *a*min = *p* – *n*\_min *‧ a*min

Since *Bn*\_min = 0, *a*min = *p*/*n*\_min

10 years being 120 months:

Round the solution up to the nearest two decimal places, or else one may not pay enough:

**Derivation of Formula for**

If interest is compounded, unpaid interest will be compounded daily.

Let daily balance *Bd* = *Bd*-1 + *Bd*-1×, for 1 ≤ *d* ≤ , where is the average number of days each month. Compute *Bd* for 1 ≤ *d* ≤ :

*B*1 = *B*0 + *B*0×

=> *B*1 = *p* + *p*×

= *p*

*B*2 = *B*1 + *B*1×

=> *B*2 = *p* + *p*×

= *p*

= *p*

*B*3 = *B*2 + *B*2×

=> *B*3 = … = *p*

= *p*

⁝

= + ×

=> = … = *p*

Then, rewrite the equation as a function of monthly interest rate, *ε*:

= *p*(1 + *ε*)

Thus:

Solving for *ε*:

Do not round any terms in the equation or the solution itself.

1. Variable, *ε*, was utilized because the monthly interest rate is sought. APR and *APR* percentages are equivalent. [↑](#footnote-ref-2)
2. Minimum for repaying student loans within ten years, NOT for repaying student loans for at least ten years. [↑](#footnote-ref-3)
3. Unpaid interest accrues monthly and is paid the final month. [↑](#footnote-ref-4)