ISE 3293/5013 Laboratory 12: Hypothesis Testing

In this lab we will investigate hypothesis testing which fuses closely with two-sided confidence intervals. There are two hypotheses that you need to learn and understand:

1. The NULL hypothesis, H_0 which is the skeptic's choice – "there is nothing going on, there is no difference"
2. The Alternate hypothesis, H_1 which is the researchers hunch, "Something is going on, there is a difference"

The function of hypothesis testing is to establish if there is evidence against the NULL. This is carried out by assuming the NULL is true and then determining if the data collected is consistent with this assumption.

When performing Hypothesis testing there are two errors that are a part of the process. We call them type 1 and type 2 errors.

1. Type 1: Rejecting the NULL when it is true
2. Type 2: Accepting the NULL when it is false

We can represent these events with a table. The top row is the TRUTH

Action\ State of truth	NULL H_0 is ***TRUE***	NULL H_0 is ***FALSE***
Reject NULL	**Type 1 error $\boldsymbol{\alpha}$**	Correct $1-\beta$ (POWER)
Accept NULL	Correct $1-\alpha$	**Type 2 error $\boldsymbol{\beta}$**

The power of a test is defined as $P(reject\ H_0|H_0 is\ FALSE) = 1-\beta$

Tasks

All output made please copy and paste into **this word file**. Save and place in the dropbox when completed. Anything you are asked to make should be recorded under the question in this document. There will be two files you need to upload:

- a pdf of this document (pdf) or the word file (docx)
- a text file of all the code you used to create answers (txt)

Note: All plots you are asked to make should be recorded in this document.

You are expected to adjust the functions as needed to answer the questions within the tasks below.

- Task 1
  - Make a folder LAB12
  - Download the file "lab12.r"
  - Place this file with the others in LAB12.
  - Start Rstudio
  - Open "lab12.r" from within Rstudio.
  - Go to the "session" menu within Rstudio and "set working directory" to where the source files are located.
  - Issue the function `getwd()` and copy the output here.

```
"F:/Google Drive - Saied/Courses/02 OU/11 Fundamentals of Engin
eering Statistical Analysis/02 Labs/12 Lab 12"
```

- Create your own R file and record the R code you used to complete the lab.

- Task 2
  - This relates to one sample from a population where we want to test a specific NULL hypothesis. There are assumptions made for this test and all others in chapter 8.
  - Lab12.R contains code and data needed to carry out this task and the others.
  - Using the sample x1, and assume that you know neither the population mean nor the population variance, perform a one sample t test using the function `t.test()` with the following NULL hypotheses: (Say if the NULL is rejected in favour of the alternate or not and copy in the P-value and ci)
    - $H_0: \mu = 22, H_1: \mu \neq 22$

```
Rejected
> ci
[1] 23.30198 27.27320
attr(,"conf.level")
[1] 0.95
> pvalue=t.test(x1,mu=22)$p.value
> pvalue
[1] 0.002052426
```

    - $H_0: \mu = 23, H_1: \mu \neq 23$

```
Rejected
> ci
[1] 23.30198 27.27320
attr(,"conf.level")
[1] 0.95
> pvalue=t.test(x1,mu=23)$p.value
> pvalue
[1] 0.02542929
```

    - $H_0: \mu = 24, H_1: \mu \neq 24$

```
Not rejected
> ci
[1] 23.30198 27.27320
attr(,"conf.level")
[1] 0.95
> pvalue=t.test(x1,mu=24)$p.value
> pvalue
[1] 0.1951074
```

    - $H_0: \mu = 25, H_1: \mu \neq 25$

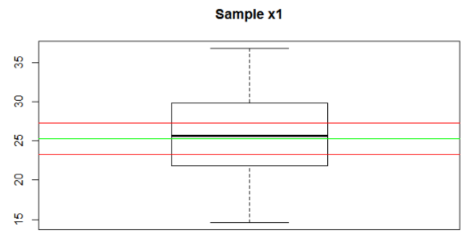
```
Not rejected
> ci
[1] 23.30198 27.27320
attr(,"conf.level")
[1] 0.95
>
> pvalue=t.test(x1,mu=25)$p.value
> pvalue
[1] 0.7691681
```

    - $H_0: \mu = 26, H_1: \mu \neq 26$

```
Not rejected
> ci=t.test(x1,mu=26)$conf.int
> ci
[1] 23.30198 27.27320
```

```
attr(,"conf.level")
[1] 0.95
>
> pvalue=t.test(x1,mu=26)$p.value
> pvalue
[1] 0.468963
```

- Make a boxplot of the data and plot the sample mean, and the 95% confidence interval (see the code in Lab 12.R)



- Follow the logic:
- We will investigate if the data is consistent with the NULL hypothesis by assuming that the data was generated from a population with mean $=\mu_0$. We will use a **pivotal statistic** to **transform the data** since we will be able to obtain distributional information from it.
- $T = \frac{\bar{X}-\mu}{\frac{s}{\sqrt{n}}}$ is the random variable with a t distribution and $\nu = n-1$ degrees of freedom.
- $t_{calc} = \frac{\overline{x1}-\mu_0}{\frac{s}{\sqrt{n}}}$
- $P\ value = prob. of\ a\ more\ extreme\ t|\ H_0$
- $P\ value = P(T \geq |t_{calc}|) + P(T \leq\ -|t_{calc}|)$
- If the P- value is small then the assumption that H_0 is true is unlikely or appears to contradict the sample data. If suitably small we will reject the NULL in favour of the alternate hypothesis H_1.
- Find t_{calc}, with $H_0: \mu = 24$

```
1.326252
```

- Use `mypvalue()` to create the plot of the p-value (alpha=0.05) using x1
- What is the rejection region?

```
|tcalc| > 2.04523
```

- What is the p-value that will determine if we reject H_0 or not?

```
0.1952
```

- Is t_{calc} in the rejection region?

```
No
```

- Construct bootstrap p-values using the same data set x1 for the following Hypotheses:
  - $H_0: \mu = 22$, $H_1: \mu \neq 22$

```
Rejected
$pvalue
[1] 0.002333333
$ci
    2.5%    97.5%
23.36578 27.13615
```

  - $H_0: \mu = 23$, $H_1: \mu \neq 23$

```
Rejected
$pvalue
[1] 0.027
$ci
```

```
    2.5%    97.5%
23.35295 27.15016
```

- $H_0: \mu = 24, H_1: \mu \neq 24$

```
Not rejected
$pvalue
[1] 0.193
$ci
    2.5%    97.5%
23.42505 27.10562
```

- $H_0: \mu = 25, H_1: \mu \neq 25$

```
Not rejected
$pvalue
[1] 0.7656667
$ci
    2.5%    97.5%
23.41679 27.24529
```

- $H_0: \mu = 26, H_1: \mu \neq 26$

```
Not rejected
$pvalue
[1] 0.4616667
$ci
    2.5%    97.5%
23.44446 27.21564
```

- Compare these results with the ones above. What do you conclude?

P-Value increases as H_0 gets close the mean.

- Task 3
  - Two sample t tests are done using the same function t.test(), you must be careful with the NULL hypothesis. Remember to express all NULL's as $H_0: \theta = \theta_0$, θ_0 is called the NULL value. You will use this in the `t.test(…,mu=`θ_0`)`.
  - Let x and y be defined as in the R code. i.e

```
set.seed(30);x=rnorm(15,mean=10,sd=7)
set.seed(40);y=rnorm(20,mean=12,sd=4)
```

  - Perform an equality of variance test using `var.test()`
  - What do you conclude?

```
As the p-value is less than 0.05 the null hypothesis is rejecte
d. i.e. var(x)/var(y) ≠ 3.33631.
```

  - What will you assign for `var.equal`= ### inside `t.test()`?

```
False
```

  - Perform a `t.test` for the following hypotheses
    - $H_0: \mu_y - \mu_x = 0, H_1: \mu_y - \mu_x \neq 0$

```
Not rejected
data:  x and y
t = -1.7945, df = 20.248, p-value = 0.08768
alternative hypothesis: true difference in means is not equal t
o 0
95 percent confidence interval:
 -7.9632156  0.5950525
sample estimates:
mean of x mean of y
 7.990343 11.674425
```

    - $H_0: \mu_y - \mu_x = 2, H_1: \mu_y - \mu_x \neq 2$

```
Rejected
data:  x and y
```

```
t = -2.7687, df = 20.248, p-value = 0.01176
alternative hypothesis: true difference in means is not equal t
o 2
95 percent confidence interval:
 -7.9632156  0.5950525
sample estimates:
mean of x mean of y
 7.990343 11.674425
```

- Summarize what you have learnt.

```
When the p-value is greater than the p critical, it is not poss
ible to reject the null hypothesis, but when the p-value < p-cr
itical, the null hypothesis would be rejected in favour of alte
rnative.
```

- Task 4
  - Let x and y be defined as in the R code, namely
    `set.seed(30);x=rnorm(15,mean=10,sd=4)`
    `set.seed(40);y=rnorm(20,mean=12,sd=4)`

  - Perform a test of the equality of variances.

```
data:  x and y
F = 1.0894, num df = 14, denom df = 19, p-value = 0.8454
alternative hypothesis: true ratio of variances is not equal to
1
95 percent confidence interval:
 0.4115743 3.1164915
sample estimates:
ratio of variances
          1.089407
```

  - What will you assign for `var.equal`= ### inside `t.test()`?

```
True
```

  - Perform a t.test for the following hypotheses
    - $H_0: \mu_y - \mu_x = 0, H_1: \mu_y - \mu_x \neq 0$

```
data:  y and x
t = 2.0623, df = 33, p-value = 0.04712
alternative hypothesis: true difference in means is not equal t
o 0
95 percent confidence interval:
 0.03803598 5.60756393
sample estimates:
mean of x mean of y
11.674425  8.851625
```

    - $H_0: \mu_y - \mu_x = 2, H_1: \mu_y - \mu_x \neq 2$

```
data:  y and x
t = 0.6011, df = 33, p-value = 0.5519
alternative hypothesis: true difference in means is not equal t
o 2
95 percent confidence interval:
 0.03803598 5.60756393
sample estimates:
mean of x mean of y
11.674425  8.851625
```

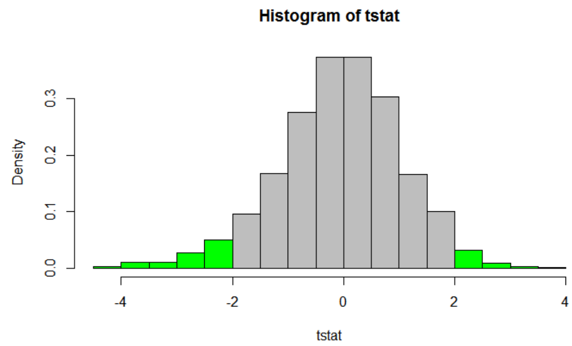
- Summarize what you have learnt.Task 5

```
It is better to perform a var test before t test to define if t
he variances are same or not. We may adjust the parameter for t
```

```
test. The p-value guide us to reject or not reject the null hypothesis.
```

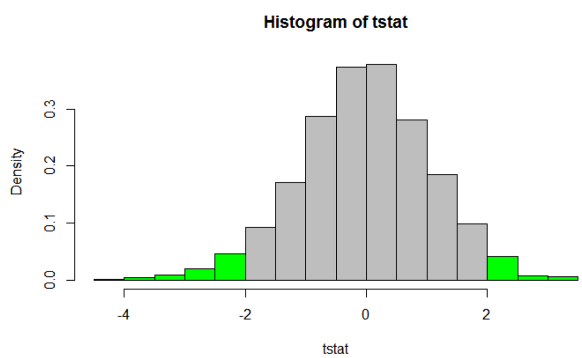
- Perform bootstrap testing using the function boot2pval() on Task 3 (No var.test() needed)
- Record p values and plots

```
0.104
```

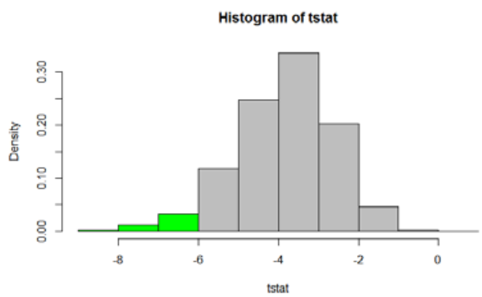
- 

- Task 6
  - Perform bootstrap testing using the function boot2pval() on Task 4 (No var.test() needed)
  - Record p values and plots

```
0.06033333
```




```
0.071
```



- Task 7
  - Explain the lines indicated with a #
  - > t.test(x1,mu=23) # A

```
Test for Null Hypothesis for mean = 23 and a sample of x1.
```

  - One Sample t-test # B

```
Here, we only have one sample x1, for hypothesis testing.
```

  - data: x1
  - t = 2.3563, df = 29, p-value = 0.02543 # C

We are declaring values for t = tcalc = 2.3563 , df = degree of freedom = 29, p-value = 0.02543.

- alternative hypothesis: true mean is not equal to 23 #D

  Alternate hypothesis(H1) is the opposite of Null hypothesis(H0) . The value of H1 is not the actual true mean.

- 95 percent confidence interval: #E

  Practically it means the actual value will be in the interval with 95% confidence.

- 23.30198 27.27320 #F

  The 95% confidence interval is 23.30198 27.27320

- sample estimates:
- mean of x
- 25.28759 #G

  The mean of sample data x is 25.28759 it is also called as the point estimator.

################## LAB FINISHES HERE ##############################

- Task 8: Extra for experts!
  - MAKE YOUR OWN BOOTSTRAP FUNCTION THAT WILL CALCULATE P-VALUES TESTING $\mu = \mu_0$ see MS page 384