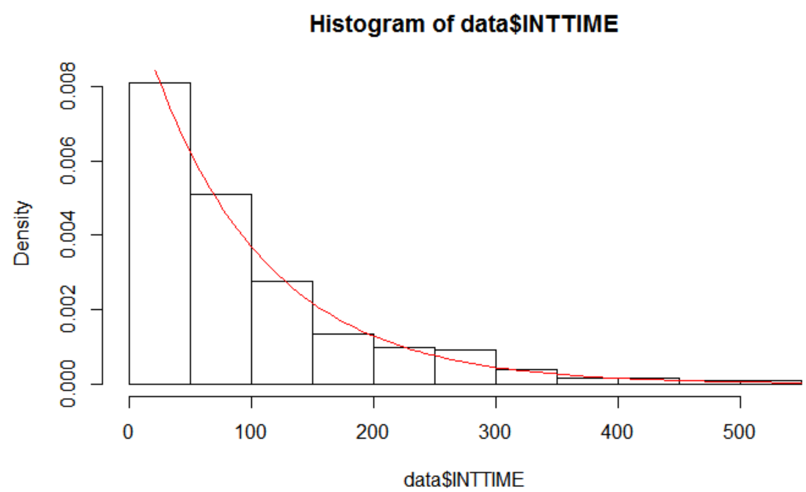
Q #1)

a) $P(y > 2) = 1 - P(y < 2) = 1 - F(2)$

$$F(y) = \int_{-\infty}^{+\infty} \frac{e^{-y/\beta}}{\beta} dy = -e^{-y/\beta}\Big]_0^y = 1 - e^{-y/\beta}$$

$$p(y > 2) = 1 - \left(1 - e^{-120/95}\right) = 0.2828$$

b) The mean value and standard deviation for data are 95.52 and 91.54 respectively. It is consistent with the exponential distribution with beta = 95. The following plot shows the data hypothetical exponential distribution is same frame:



Q #2)

a) $\mu = \alpha\beta = (3)(0.07) = 0.21$

$\sigma^2 = \alpha\beta^2 = (3)(0.07)^2 = 0.0147$

b) $\mu \pm 3\sigma = 0.21 \pm \sqrt{0.0147} = [0, 0.57]$

The value of 0.6 is beyond the 3 times of sd. The possibility of happening is not very much. We can also calculate the probability for values equal to or greater than this value which is 0.0078.

Q #3)

a) $\mu_A = \alpha_A\beta_A = (2)(2) = 4$

$\mu_B = \alpha_B\beta_B = (1)(4) = 4$

b) $\mu_A^2 = \alpha_A\beta_A^2 = (2)(2)^2 = 8$

$\mu_B^2 = \alpha_B\beta_B^2 = (1)(4)^2 = 16$

c) $P_A(y < 1) = 0.0902$

$P_B(y < 1) = 0.2212$

The gas B has higher probability.

Q #4)

a)
```
> a = 2; b = 4
> pweibull(2, shape = a, scale = b)
[1] 0.2211992
```

b)
```
> Mu = b^(1/a) * gamma((a+1)/a)
> Var = b^(2/a) *(gamma((a+2)/a) - gamma((a+1)/a)^2)
> Mu; Var
[1] 1.772454
[1] 0.8584073
```

c)
```
> SD = sqrt(Var)
> Mu-2*SD
[1] -0.08055165
> Mu+2*SD
[1] 3.625459
> pweibull(Mu+2*SD, shape = a, scale = b) - pweibull(Mu-2*SD, shape = a, scale = b)
[1] 0.5602273
```

d)
```
> 1 - pweibull(6, shape = a, scale = 2)
[1] 0.0001234098
```

Q #5)

a)
```
> a = 2; b = 9
> Mu = a / (a + b)
> Var = a * b / ((a + b)^2 * (a + b + 1))
> Mu; Var
[1] 0.1818182
[1] 0.01239669
```

b)
```
> 1 - pbeta(0.4, shape1 = a, shape2 = b)
[1] 0.0463574
```

c)
```
> pbeta(0.1, shape1 = a, shape2 = b)
[1] 0.2639011
```

Q #6)

a) Comparing the given distribution with Weibull distribution, alpha is 2 and beta is 16.

b)
```
> a = 2; b = 16
> Mu = b^(1/a) * gamma((a+1)/a)
> Var = b^(2/a) *(gamma((a+2)/a) - (gamma((a+1)/a))^2)
```

```
> Mu; Var
[1] 3.544908
[1] 3.433629
```

c)
```
> 1 - pweibull(6, shape = a, scale = b)
[1] 0.8688151
```

Q #7)

a) $p(x,y) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

b) $p_1(x) = \sum_{y=1}^{6} p(x,y) = 6 \times \frac{1}{36} = \frac{1}{6}$

$$p_2(y) = \sum_{x=1}^{6} p(x,y) = 6 \times \frac{1}{36} = \frac{1}{6}$$

c) $p_1(x|y) = \frac{p(x,y)}{p_2(y)} = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6}$

$$p_2(y|x) = \frac{p(x,y)}{p_1(x)} = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6}$$

d) $p(x,y) = p_1(x)p_2(y)$

Q #8)

		x			P(y)
		1	2	3	
y	1	1/7	2/7	1/7	4/7
	2	0	0	2/7	2/7
	3	0	0	1/7	1/7
P(x)		1/7	2/7	4/7	1

a) The table

b)

P(x)	1/7	2/7	4/7

c)

P(y)
4/7
2/7
1/7

d) P(y|1) = 1, p(y|2) = 1, p(y|3) = (1/4,2/4, 1/4)

Q #9)

a) $f(y) = \int_{-\infty}^{+\infty} f(x,y)dx = \int_{y}^{2y} \frac{e^{-\frac{y}{10}}}{10y} dx = \frac{e^{-\frac{y}{10}}}{10y} x \Big]_{x=y}^{x=2y} = \frac{e^{-\frac{y}{10}}}{10}$

The distribution is exponential distribution.

b) $E(y) = \int_{-\infty}^{+\infty} f(x,y)ydy = \int_{0}^{+\infty} \frac{e^{-\frac{y}{10}}}{10y} ydy = -e^{-\frac{y}{10}}\Big]_{y=0}^{y=\infty} = 1$

Q #10)

a) $F(y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x,y)dxdy = 1$

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} ce^{-x^2} dydx$$

$$= \int_{0}^{+\infty} \int_{0}^{x} ce^{-x^2} dydx$$

$$= \int_{0}^{+\infty} ce^{-x^2} y\Big]_{y=0}^{y=x} dx$$

$$= \int_{0}^{+\infty} cxe^{-x^2} dx$$

$$= -\frac{c}{2} e^{-x^2}\Big]_{x=0}^{x=\infty}$$

$$= \frac{c}{2} = 1$$

$$c = 2$$

b) $f_1(x) = \int_{-\infty}^{+\infty} f(x,y)dy = \int_{0}^{x} 2e^{-x^2} dy = 2ye^{-x^2}\Big]_{y=0}^{y=x} = 2xe^{-x^2}$

$$\int_{-\infty}^{+\infty} f_1(x)dx = \int_{0}^{+\infty} 2xe^{-x^2} dx = -e^{-x^2}\Big]_{x=0}^{x=\infty} = 1$$

c) $f_2(y|x) = \frac{f(x,y)}{f_1(x)} = \frac{2e^{-x^2}}{2xe^{-x^2}} = \frac{1}{x}$

Q #11)

$$E(XY) = \sum\sum xyp(x,y) = (-1)(-1)\left(\frac{1}{12}\right) + (-1)(1)\left(\frac{1}{12}\right) + (1)(-1)\left(\frac{1}{12}\right) + (1)(1)\left(\frac{1}{12}\right) = 0$$

$$E(X) = \sum xp_1(x) = (-1)\left(\frac{4}{12}\right) + (1)\left(\frac{4}{12}\right) = 0$$

$$E(Y) = \sum yp_2(y) = (-1)\left(\frac{4}{12}\right) + (1)\left(\frac{4}{12}\right) = 0$$

$$cov(X,Y) = E(XY) - E(X)E(Y) = 0$$

They are not independent. For example:

$$p(0,0) = 0 \neq p_1(0) \times p_2(0) = \frac{4}{12} \times \frac{4}{12}$$

Q #12)

a)
```
> a = 1; b = 3; n = 60
> Mu = (a + b) / 2
> Var = (b - a)^2 / 12 / n
> Mu
[1] 2
```

b)
```
> Var
[1] 0.005555556
```

c) Base on CLT, the distribution for sample is a normal distribution.

d)
```
> SD = sqrt(Var)
> pnorm(2.5, mean = Mu, sd = SD) - pnorm(1.5, mean = Mu, sd = SD)
[1] 1
```

e)
```
> 1 - pnorm(2.2, mean = Mu, sd = SD)
[1] 0.003645179
```

Q #13)

a)
```
> p = 0.4; n = 20; q = 1 - p
> Mu = n * p
> SD = sqrt(n*p*q)
> a = (1 + 0.5 - Mu) / SD
> pnorm(a)
[1] 0.001504434
>
> pnorm(1 + .5, mean = Mu, sd = SD)
[1] 0.001504434
```

b) > 1 - pnorm(11 - 0.5, mean = Mu, sd = SD)
[1] 0.1269165

c) > pnorm(1 + .5, mean = Mu, sd = SD)
[1] 0.001504434
> pbinom(1, size = 20, prob = .40)
[1] 0.0005240494
> 1 - pnorm(11 - 0.5, mean = Mu, sd = SD)
[1] 0.1269165
> 1 - pbinom(10, size = 20, prob = .40)
[1] 0.1275212

Q #14)

a) > Lead = c(1.32, 0, 13.1, .919, .657, 3.0, 1.32, 4.09, 4.45, 0)
> Copper = c(.508, .279, .320, .904, .221, .283, .475, .130, .220, .743)
>
> t.test(Lead, conf.level = 0.99)$conf.int
[1] -1.147845 6.919045
attr(,"conf.level")
[1] 0.99

b) > t.test(Copper, conf.level = 0.99)$conf.int
[1] 0.1518726 0.6647274
attr(,"conf.level")
[1] 0.99

c) The analysis shows that if we take a sample with probability of 99%, the lead content of water is les s than 6.92ppm and the Copper content is between 0.152 and 0.665 ppm.

d) 99% confident level means the if we take sample from the population, 99 sample out of 100 would have a value in interval range.

Q #15)

Sdfjfls
> data = read.csv("SOLARAD.csv")
> IW = data$IOWA
> ST = data$STJOS
> diff = ST - IW
> t.test(diff, conf.level = 0.95)$conf.int
[1] 156.8193 239.1807
attr(,"conf.level")
[1] 0.95

Q #16)

a) > data = read.csv("DIAZINON.csv")

```
> Day = data$DAY
> Night = data$NIGHT
> diff = Day - Night
> t.test(diff, conf.level = 0.90)$conf.int
[1] -58.89917 -18.91901
attr(,"conf.level")
[1] 0.9
```

b) The sample should be taken independently.

c) From part a we infer that the mean values for day and night samples are not same. We can say wit h 90% confident level that the residual at night is higher than day's at least 18.9 and at most 58.9.