DSA 5113: Advanced Analytics and Metaheuristics

Homework #3
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Homework team:
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March 2017

1 Mr. X

1.1 *Part a*

$$y\epsilon\{0,1\} \tag{Eq 1}$$

$$x \geq 10y \tag{Eq 2}$$

$$x \leq 1000y \tag{Eq 3}$$

1.2 *Part b*

$$y\epsilon\{0,1\} \tag{Eq 4}$$

$$x \leq 15y + 100(1 - y) \tag{Eq 5}$$

$$x \geq 30(1 - y) \tag{Eq 6}$$

1.3 *Part c*

$$y_i\epsilon\{0,1\} \tag{Eq 7}$$

$$x = 12y_1 + 12.3y_2 + 87y_3 + 99.1y_4 \tag{Eq 8}$$

$$y_1 + y_2 + y_3 + y_4 = 1 \tag{Eq 9}$$

2 Inequalities

To find the first inequality; we can see that if

$$x_2 = x_4 = 0 \tag{Eq 10}$$

Then the master inequality would be:

$$3x_1 + 2x_3 + x_5 \leq -2 \tag{Eq 11}$$

Which is not possible, because the right-hand side cannot be less than zero; therefore the following equation is a valid inequality:

$$x_2 + x_4 \geq 1 \tag{Eq 12}$$

For the second one, if we have

$$x_1 = 1, \qquad x_2 = 0 \tag{Eq 13}$$

Then the master equality would be

$$0 \leq 2x_3 - 3x_4 + x_5 \leq -2 \tag{Eq 14}$$

Which is not possible. Hence the following inequality is valid:

$$x_1 \leq x_2 \tag{Eq 15}$$

3 Gizmos and Gadgets

The problem was formulated as has been asked in problem statement and an AMPL model was provided which is available as attachment. The results are:

Table 1. Purchased widget amounts from suppliers and related total costs

Demand	WII	WRS	WU	WOW	Total Cost
17,000	2,000	15,000	0	0	$55,750
18,000	3,000	15,000	0	0	$60,000
19,000	0	10,000	9,000	0	$63,600
28,000	3,000	0	0	25,000	$91,500
32,000	0	7,500	0	24,500	$101,375

4 Facility Locations

4.1 *Part a*

The problem was formulated as described in lecture note and then implemented as an AMPL model:

```
# AMPL model for Facility Location Problem
# Author:  Saied Hosseinipoor
# email:    saied@ou.edu
# Date:      March 28, 2017


# Initializing ------------------------------------------------------------------------

option cplex_options 'presolve=0  mipcuts=-1  splitcuts=-1 heuristicfreq=-1 mipsearch=1 timing=1 mipdisplay=5 mipinterval=1';

set I ordered;                                  # Districts
set J ordered;                      # Firehouse sites

# Parameters ------------------------------------------------------------------------

param p {I};                                    # Population in districs
param d {I,J};                                  # Distance from center of district i to site j
param B;                            # Budget limit
param c {J};                                    # Service cost
param f {J};                                    # Fixed cost

# Variables -------------------------------------------------------------------------

var x {I,J} binary;                 # 1 if destric i is assigned to site j; otherwise 0
var y{I} binary;                    # 1 if site j is selected
var z binary;                                   # 1 if restriction 1 activated; otherwise restriction 2 activated
var s {J} >= 0;                     # Population served
var D >=0;                          # Maximum distance of district i's to site j's
```

```
# Constrains --------------------------------------------------------------------------

subject to Unique_Assignment {i in I}:
        sum {j in J} x[i,j] = 1;

subject to Adjustment {j in J}:
        sum {i in I} x[i,j] <= 45 * y[j];

subject to Restriction_1:
        y[1] + y[2] >= 2 * z;
subject to Restriction_2:
        y[3] + y[4] >= 2 * (1 - z);

subject to Population_Served {j in J}:
        s[j] = sum {i in I} (p[i] * x[i,j]);

subject to Budget_Limit:
        sum {j in J} (c[j] * s [j] + f[j] * y[j]) <= B;

subject to Distances {i in I}:
        D >= sum {j in J} (d[i,j] * x[i,j]);


# Objective ---------------------------------------------------------------------------

minimize Worse_Case: D;
```

4.2 *Part b*

4.2.1 *Sub-Part i*

The AMPL model was solved by NEOS server online and following results were ontained:

α) The optimal solution selected the following site: 1, 3, 4, 5, 17, 19, and 22.
The objective value, minimized maximum distance, was 31.3.

β) $14,968,700 is used.

γ) Total solution time was 0.65 sec., and
6612 branch-and-bound nodes used in the algorithm.

δ) The root relaxation value was 6.13, and
The first incumbent value was 137.8 which found after 0.11 sec at node 727.

4.2.2 *Sub-Part ii*

The following graph was created from the results. The results were copied from the output page, then pasted into an excel file. The data was divided by text-to-column using space as delimiter. Finally, by filtering the data extra rows and columns were deleted. Remaining data were used to make the following graph:

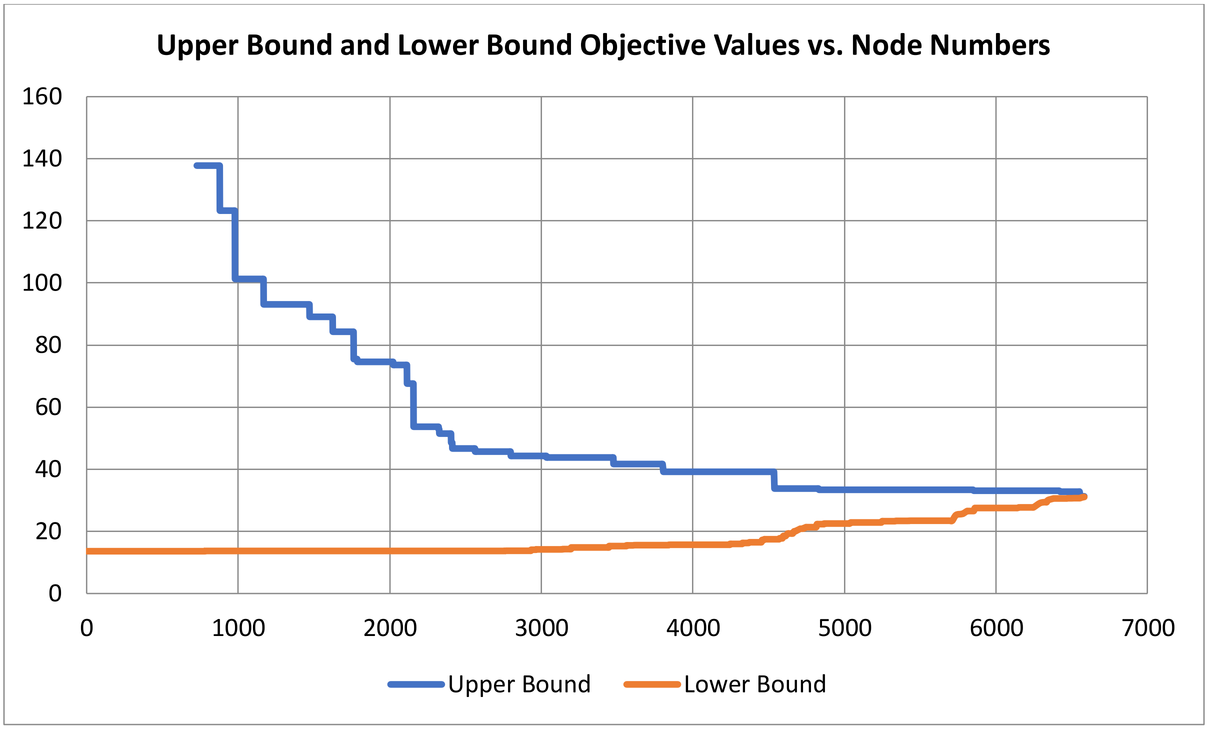

Figure 1. Upper Bound and Lower Bound Objective Values vs. Node Numbers

4.2.3 Sub-Part iii

The model with new option has been ran. The following table summarizes the results and shows the comparison:

Table 2. Comparison for results of part (i) and part (iii)

	Part i Solution	Part iii Solution
Optimal Solution (Selected sites)	1, 3, 4, 5, 17, 19, 22	1, 3, 4, 5, 17, 19, 22
Objective Value (Distance)	31.3	31.3
Used Budget	$14,968,700	$14,968,700
Solution Time	0.65	1.48
Number of B&B Nodes	6612	5610
Root Relaxation Value	6.13	6.13
First Incumbent Node	137.8	89.5
First Incumbent Value	727	483

4.3 *Part c*

The new criteria for objective was imposed to the model. AMPL model is:

```
# AMPL model for Facility Location Problem
# Author:  Saied Hosseinipoor
# email:    saied@ou.edu
# Date:      March 28, 2017

# Initializing ---------------------------------------------------------------------------

option cplex_options 'presolve=0  mipcuts=-1  splitcuts=-1 heuristicfreq=-1 mipsearch=1 timing=1 mipdisplay=5 mipinterval=1';

set I ordered;                                   # Districs
set J ordered;                      # Firehouse sites

# Parameters ---------------------------------------------------------------------------

param p {I};                                     # Population in districs
param d {I,J};                                   # Distance from center of district i to site j
param B;                            # Budget limit
param c {J};                                     # Service cost
param f {J};                                     # Fixed cost

# Variables ---------------------------------------------------------------------------

var x {I,J} binary;                 # 1 if destric i is assigned to site j; otherwise 0
var y{I} binary;                    # 1 if site j is selected
var z binary;                                    # 1 if restriction 1 activated; otherwise restriction 2 activated
var s {J} >= 0;                     # Population served
var D >=0;                          # Maximum distance of district i's to site j's


# Constrains ---------------------------------------------------------------------------

subject to Unique_Assignment {i in I}:
            sum {j in J} x[i,j] = 1;

subject to Adjustment {j in J}:
            sum {i in I} x[i,j] <= 45 * y[j];

subject to Restriction_1:
            y[1] + y[2] >= 2 * z;
subject to Restriction_2:
            y[3] + y[4] >= 2 * (1 - z);

subject to Population_Served {j in J}:
            s[j] = sum {i in I} (p[i] * x[i,j]);

subject to Budget_Limit:
            sum {j in J} (c[j] * s [j] + f[j] * y[j]) <= B;

subject to Distances:
            D = sum {i in I} (sum {j in J} (d[i,j] * x[i,j])) / 45;


# Objective ---------------------------------------------------------------------------

minimize Mean_Case: D;
```

The results were summarized in the following table:

Table 3. Comparison for results between p-center and p-median solutions

	p-center Solution	p-median Solution
Optimal Solution (Selected sites)	1, 3, 4, 5, 17, 19, 22	1, 3, 4, 5, 12, 17, 21, 22, 24
Objective Value (Distance)	31.3	10.99
Used Budget	$14,968,700	$14,982,300
Solution Time	0.65	0.15
Number of B&B Nodes	6612	1273
Root Relaxation Value	6.13	2.62
First Incumbent Node	137.8	11.06
First Incumbent Value	727	202