6.40 a. From Exercise 6.13, we calculated:

$$E(x)=\frac{19}{12},\ E(y)=\frac{5}{12},\ \text{and } E(xy)=\frac{2}{3}$$

Therefore,

$$\operatorname{Cov}(x,\ y)=E(xy)-E(x)E(y)$$

$$=\frac{2}{3}-\left(\frac{19}{12}\right)\left(\frac{5}{12}\right)=\frac{2}{3}-\frac{95}{144}=\frac{96}{144}-\frac{95}{144}=\frac{1}{144}$$

b. $\operatorname{Cov}(x,\ y)=\dfrac{1}{144}=.0069444$

$$\sigma_1=\sqrt{\sigma_1^2},\ \sigma_1^2=E(x^2)-[E(x)]^2$$

$$E(x^2)=\int_1^2 x^2 f_1(x)dx=\int_1^2 x^2\left(x-\frac{1}{2}\right)dx=\int_1^2 x^3-\frac{x^2}{2}dx$$

$$=\frac{1}{4}x^4-\frac{x^3}{6}\Bigg]_1^2=\left(\frac{16}{4}-\frac{8}{6}\right)-\left(\frac{1}{4}-\frac{1}{6}\right)=\frac{32}{12}-\frac{1}{12}=\frac{31}{12}$$

$$\sigma_1^2=\frac{31}{12}-\left(\frac{19}{12}\right)^2=\frac{11}{144}$$

$$\Rightarrow \sigma_1=\sqrt{\frac{11}{144}}$$

$$\sigma_2=\sqrt{\sigma_2^2},\ \sigma_2^2=E(y^2)-[E(y)]^2$$

$$E(y^2) = \int_0^1 y^2 \left(\frac{3}{2} - y \right) dy$$

$$= \int_0^1 \frac{3y^2}{2} - y^3 \, dy = \frac{y^3}{2} - \frac{y^4}{4} \Big]_0^1 = \left(\frac{1}{2} - \frac{1}{4} \right) - 0 = \frac{1}{4}$$

$$\sigma_2^2 = \frac{1}{4} - \left(\frac{5}{12} \right)^2 = \frac{11}{144} \Rightarrow \sigma_2 = \sqrt{\frac{11}{144}}$$

$$\rho = \frac{\text{Cov}(x,\, y)}{\sigma_1 \sigma_2} = \frac{\frac{1}{144}}{\sqrt{\frac{11}{144}} \cdot \sqrt{\frac{11}{144}}} = \frac{\frac{1}{144}}{\frac{11}{144}} = \frac{1}{11}$$