HW01-Q2

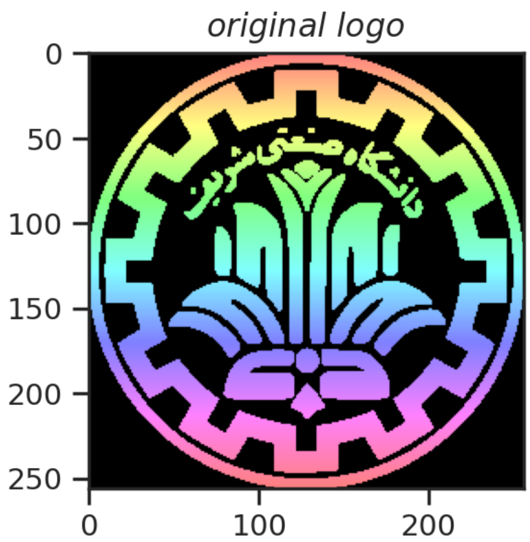
April 16, 2022

1 Perspective

In this report , we have an image from one poistion with specific angle, and our goal is to find the corresponding image from top down view . in the following we explain how to reach that goal in details .

1.1 Show original Logo

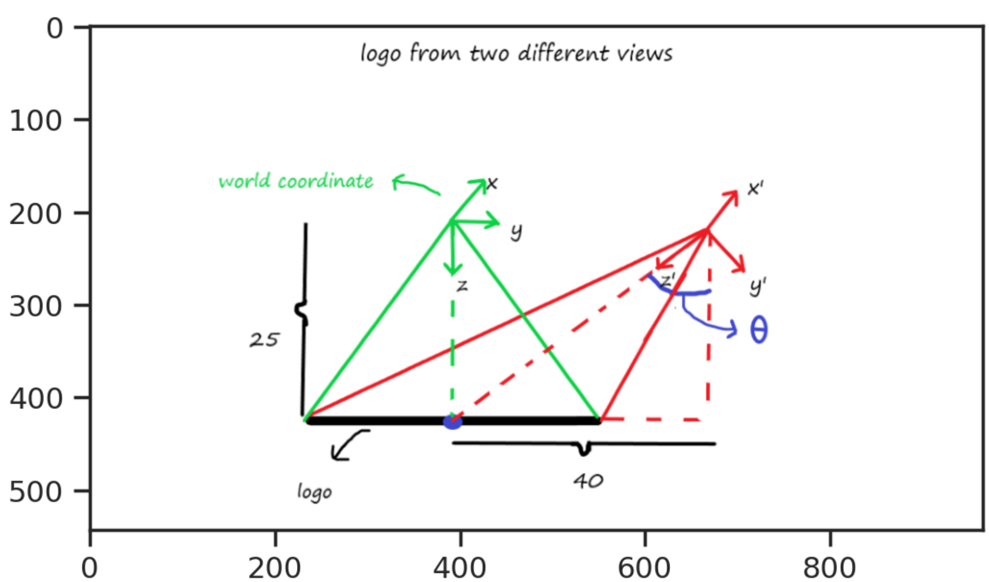
[ ]:
```
import cv2
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
sns.set(rc={"figure.dpi":100, 'savefig.dpi':300})
sns.set_context('notebook')
sns.set_style("ticks")
from IPython.display import set_matplotlib_formats
set_matplotlib_formats('retina')

logo=np.array(cv2.imread("logo.png"))
logo=cv2.cvtColor(logo, cv2.COLOR_BGR2RGB)
plt.figure(figsize=(3, 3))
plt.imshow(logo.astype(np.uint8))
plt.title(r"$original \ logo$")
plt.show()
```



1.2 Analyze the question more precisely

below you can see the soccer feild from the front view :

```
[ ]: img=plt.imread("/content/1.PNG")
     plt.figure(figsize=(6, 6))
     plt.imshow(img)
     plt.show()
```



as we can see , we have two changes in the second camera(red one) poistion :

- we have 40m movment on the possitive y axis
- we have a rotation that is equal to θ degree around x axis

1.3 How to reach a straightforward formula between two image from different view?

we now that we can use following formula to map 3d spacae coordinate to 2d image pixels :

$x = K[R|t]X$, where :

$x : coordinates\ of\ 2D\ image\ pixels$, $X : coordinates\ of\ 3D\ space$

$R : Rotation\ matrix\ with\ respect\ to\ world\ coordinate$

$T : Translation\ matrix\ with\ respect\ to\ world\ coordinate$

so for two cameras we can write following equations :

$x_{cam\ 1} = K_{cam\ 1}[R_{cam\ 1}|t_{cam\ 1}]X\ \ ,\ Eqation(1.1)$

$x_{cam\ 2} = K_{cam\ 2}[R_{cam\ 2}|t_{cam\ 2}]X\ \ ,\ Eqation(1.2)$

now suppose that all these matrix are invertible(they are not actually but we will handle this problem later), so we have :

$X = [R_{cam\ 1}|t_{cam\ 1}]^{-1}K^{-1}_{cam\ 1}x_{cam\ 1}\ \ ,\ Eqation(1.3)$

if we replace $Eqation(1.3)$ in $Eqation(1.2)$ we have :

$$\boxed{x_{cam\ 2} = K_{cam\ 2}[R_{cam\ 2}|t_{cam\ 2}][R_{cam\ 1}|t_{cam\ 1}]^{-1}K^{-1}_{cam\ 1}x_{cam\ 1}\ \ ,\ Eqation(1.4)}$$

$Eqation(1.4)$ is the straightforward formula between two camera. we will use this equation to reach homography matrix and then warp the origianl logo to get final image

1.4 How to compute R , T and K matrix ?

- $Translation\ matrix = T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$ where each component is movment with respect to positive x, y, z axis , so in this question according to above figure , we just have translation in $+y$ axis (40 meter) so out translation matrix in this question is like below :

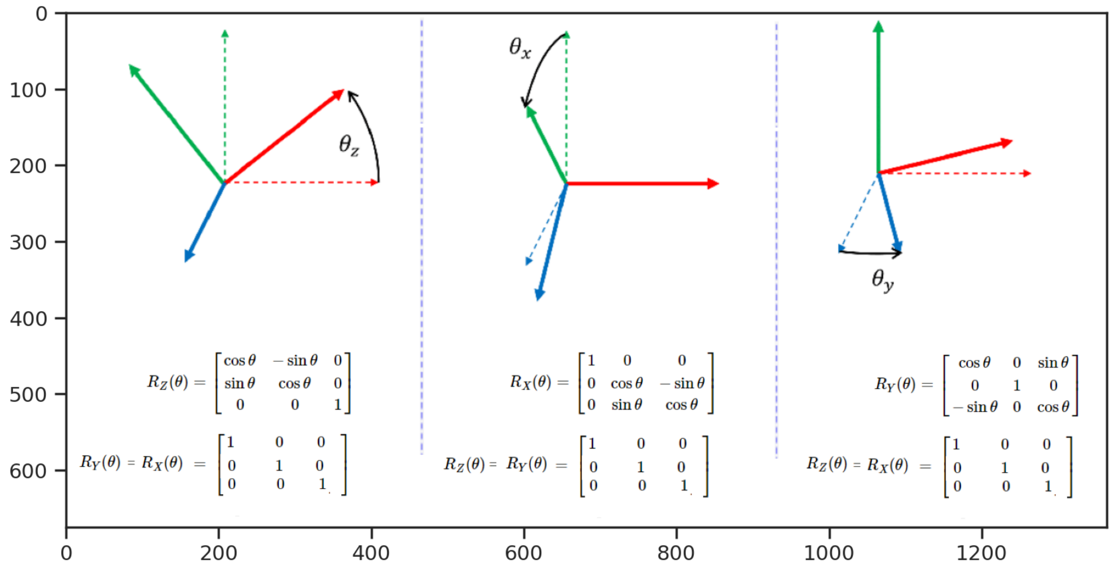
$$T = \begin{bmatrix} 0 \\ +40 \\ 0 \end{bmatrix}$$

- note that we assume that the camera from top down view is world coordinate

- $Rotation\ matrix = R = \begin{bmatrix} r_{11}, r_{12}, r_{13} \\ r_{21}, r_{22}, r_{23} \\ r_{31}, r_{32}, r_{33} \end{bmatrix}$

to compute this matrix we refer to the image below :

[ ]:
```
img1=plt.imread("/content/2.PNG")
plt.figure(figsize=(10, 10))
plt.imshow(img1)
plt.show()
```



we know that the Rotation matrix can be reached by following formula :

$Rotation\ matrix = R = R_x R_y R_z \ \ ,\ Equation(1.5)$

so with the help of $Equation(1.5)$ and above image about roation we can make our rotation matrix . note that as we can see in the first image our rotation angle in this question is $\theta_0 = tan^{-1}(\frac{width}{height}) = tan^{-1}(40/25)$

so our roation matrix for this problem that we have rotation about x axis is as below :

$$R = \begin{bmatrix} 1 \ \ ,\ \ 0 \qquad ,0 \\ 0, cos(\theta_0), -sin(\theta_0) \\ 0, sin(\theta_0), cos(\theta_0) \end{bmatrix}$$

$$calibration\ matrix = k = \begin{bmatrix} f_x & , & 0 & , & p_x \\ 0 & , & f_y & , & p_y \\ 0 & , & 0 & , & 1 \end{bmatrix}$$

1.5 Are T, R, K ... all sqaure matrices?

as we said before $Equation(1.4)$ is a formula to reach final image. but we can use this formula if all matrices are square in order to have inverse. we can see that $[R_{cam\ 1}|T_{cam\ 1}]$ and $[R_{cam\ 2}|T_{cam\ 2}]$ are $(3,4)$ matrices. to make these two matrices invertible (square) :

we append $[0,0,0,1]$ *wherever needed to make all matrices* $(4,4)$. *to say this more percisely we have* :

$$T = \begin{bmatrix} 1\ ,\ 0\ ,\ 0,\ 0 \\ 0\ ,\ 1\ ,\ 0,\ 40 \\ 0\ ,\ 0\ ,\ 1,\ 0 \\ 0\ ,\ 0\ ,\ 0,\ 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1\ \ ,\ \ 0\ \ \ ,0\ \ , \ \ \ \ 0 \\ 0, cos(\theta_0), -sin(\theta_0)\ \ ,\ 0 \\ 0, sin(\theta_0), cos(\theta_0)\ \ ,\ \ 0 \\ 0\ \ ,\ \ 0\ \ \ ,0\ \ , \ \ \ \ 1 \end{bmatrix}$$

```
[ ]: focal_length=500
     angle=-np.arctan(40/25)
     hx,hy=logo.shape[:2]
     px=hx/2
     py=hy/2
     K=np.array([[focal_length,0,px,0],[0,focal_length,py,0],[0,0,1,0]])
     Kinv = np.zeros([4,3])
     Kinv[:3,:3] = np.linalg.inv(K[:3,:3])*focal_length
     Kinv[-1,:] = [0, 0, 1]
     K=np.array([[focal_length,0,1000,0],[0,focal_length,1000,0],[0,0,1,0]])
```

```
[ ]: T = np.array([[1,0,0,0],
                    [0,1,0,40],
                    [0,0,1,0],
                    [0,0,0,1]])

     RX = np.array([[1,           0,            0, 0],
                    [0,np.cos(angle),-np.sin(angle), 0],
                    [0,np.sin(angle),np.cos(angle) , 0],
                    [0,           0,            0, 1]])

     RY = np.array([[ np.cos(0), 0, np.sin(0), 0],
                    [           0, 1,            0, 0],
                    [ -np.sin(0), 0, np.cos(0), 0],
                    [           0, 0,            0, 1]])

     RZ = np.array([[ np.cos(0), -np.sin(0), 0, 0],
                    [ np.sin(0), np.cos(0), 0, 0],
                    [           0,           0, 1, 0],
                    [           0,           0, 0, 1]])
```

```
        # Composed rotation matrix with (RX,RY,RZ)
R = np.linalg.multi_dot([ RZ , RY , RX ])
RT_inverse=np.linalg.inv(np.dot(R,T))
homograpgy_matrix=np.linalg.multi_dot([K, RT_inverse, Kinv])
homograpgy_matrix/=homograpgy_matrix[2,2]
print(f"homography matrix = {homograpgy_matrix}")
```

```
homography matrix = [[ 3.19579307e+00  5.42005420e+00  5.90938487e+02]
 [ 0.00000000e+00  7.11382114e+00 -6.99647441e+02]
 [ 0.00000000e+00  5.42005420e-03  1.00000000e+00]]
```

1.6 Homography matrix

the homography matrix that map image of camera 2(camera next to the soccer field) to camera 1 (top-down view camera in the center of soccer field) is as below :

$$H = \begin{bmatrix} 3.19579307e+00 \ , \ 5.42005420e+00 \ , \ 5.90938487e+02 \\ 0 \quad , \quad 7.11382114e+00 \quad , \quad -6.99647441e+02 \\ 0 \quad , \quad 5.42005420e-03 \quad , \quad 1 \end{bmatrix}$$

1.7 homography Normilizer

sometimes , our homography matrix map some of the pixel of warped image to negative value . so i define a function `norimilize_matrix` to remove this negative offset in homography to ensure that we are not losing any data .

[ ]:
```
def norimilize_matrix(src,hom):
    final=np.copy(hom)
    h,w=src.shape[:2]
    p=[[0,w,w,0],[0,0,h,h],[1,1,1,1]]
    p_prime=np.array(np.dot(hom, p))
    p_zegond=p_prime/p_prime[2,:]
    x_min=np.min(p_zegond[0,:])
    y_min=np.min(p_zegond[1,:])
    t=np.array([[1,0,0],[0,1,0],[0,0,1]])
    if(x_min<0):
       t[0,2]=x_min*-1.4
    if(y_min<0):
       t[1,2]=y_min*-1.4
    return np.dot(t,final)
```

1.8 Final result

[ ]:
```
homo=norimilize_matrix(logo,homograpgy_matrix)
homo/=homo[2,2]
dst=cv2.warpPerspective(logo, homo, (2000, 2000), cv2.INTER_NEAREST, cv2.
 ↪BORDER_CONSTANT, 0)
# Show the image
plt.imshow(dst.astype(np.uint8))
plt.title(r"logo from the top_down view")
plt.show()
plt.imsave("final.jpg",dst.astype(np.uint8))
```