HW01-Q1

April 17, 2022

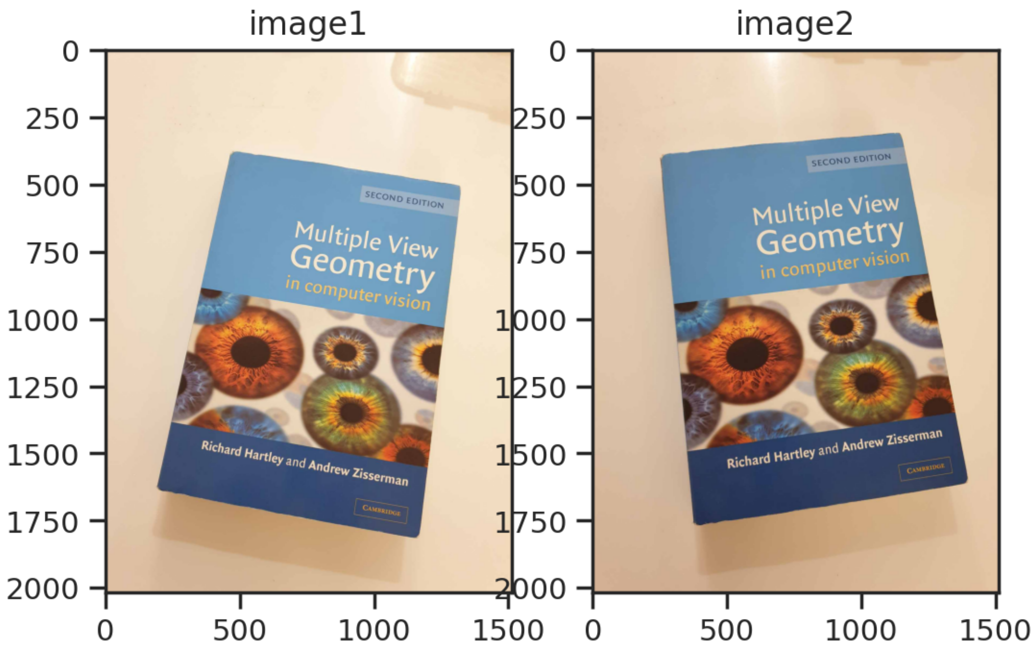
1 Harris Corner Detection and Matching

In this report we are going to implement harris corner Detection and after that extract feature matrix to match candidate corners. in the following we explaine each part in more deatils

1.1 show original images

at first we read and show the original images :

[32]:
```python
import cv2
import numpy as np
from matplotlib.patches import ConnectionPatch
import matplotlib.pyplot as plt
import seaborn as sns
sns.set(rc={"figure.dpi":100, 'savefig.dpi':300})
sns.set_context('notebook')
sns.set_style("ticks")
from IPython.display import set_matplotlib_formats
set_matplotlib_formats('retina')

img1=plt.imread('im01.jpg').astype('float64')
img2=plt.imread('im02.jpg').astype('float64')
plt.subplot(1,2,1)
plt.imshow((img1).astype('int32'))
plt.title("image1")

plt.subplot(1,2,2)
plt.imshow((img2).astype('int32'))
plt.title("image2")
plt.show()

#blured image before calculating gradins
blured_image = cv2.GaussianBlur(img1,(5,5),cv2.BORDER_DEFAULT)
blured_image2 = cv2.GaussianBlur(img2,(5,5),cv2.BORDER_DEFAULT)
```



1.2 Gradian calculator

in the following i write a function for calculating gradians of image . to calculate gradian of each channel i use `cv2.Sobel()` and set the depth of image to `CV_64F` to avoid overflow . after calculating gradian for each channel we set the gradian of image to the maximum absolute gradian of image in three channel using `np.maximum`

[33]:
```
def gradian(src):
    r,g,b=cv2.split(src)

    #gradian on x axis
    r_x=cv2.Sobel(r,cv2.CV_64F,1,0,3)
    g_x=cv2.Sobel(g,cv2.CV_64F,1,0,3)
    b_x=cv2.Sobel(b,cv2.CV_64F,1,0,3)
    grad_x=np.maximum(np.abs(r_x),np.abs(g_x),np.abs(b_x))

    #gradian on y axis
    r_y=cv2.Sobel(r,cv2.CV_64F,0,1,3)
    g_y=cv2.Sobel(g,cv2.CV_64F,0,1,3)
    b_y=cv2.Sobel(b,cv2.CV_64F,0,1,3)
    grad_y=np.maximum(np.abs(r_y),np.abs(g_y),np.abs(b_y))

    return grad_x,grad_y
```

1.3 Normilizer

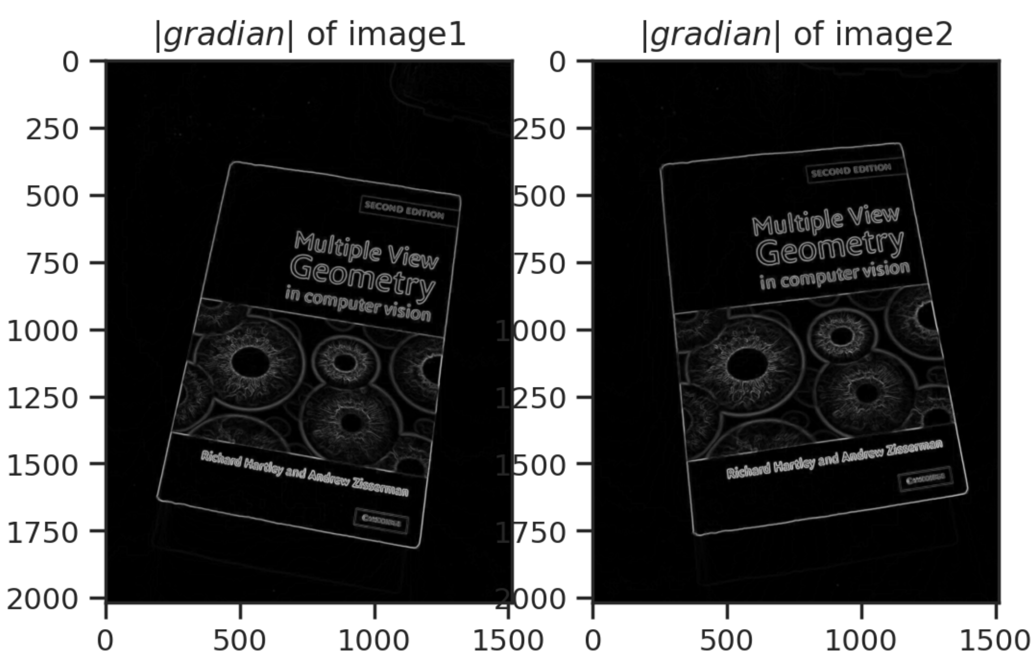
i define a function to normilize image between [0 1] when i want to show the images. the alogorithm is very straightforward and you can understand it by just looking at the code below :

[34]:
```
def normilizer(src):
    return (src-np.min(src))/(np.max(src)-np.min(src))
```

after calculating gradians of two image we show that magnitude of gradian by the formula : $|gradian| = \sqrt{I_x^2 + I_y^2}$.

you can see the magnitude of gradians for both image :

[35]:
```
ix,iy=gradian(blured_image)
mag=np.sqrt(ix**2+iy**2)

ix_2,iy_2=gradian(blured_image2)
mag_2=np.sqrt(ix_2**2+iy_2**2)

plt.subplot(1,2,1)
plt.imshow(normilizer(mag),cmap='gray')
plt.title(r'$|gradian|$ of image1')

plt.subplot(1,2,2)
plt.imshow(normilizer(mag_2),cmap='gray')
plt.title(r'$|gradian|$ of image2')

plt.show()
```



1.4 Derive Structure tensor from S_x^2, S_y^2, S_{xy}

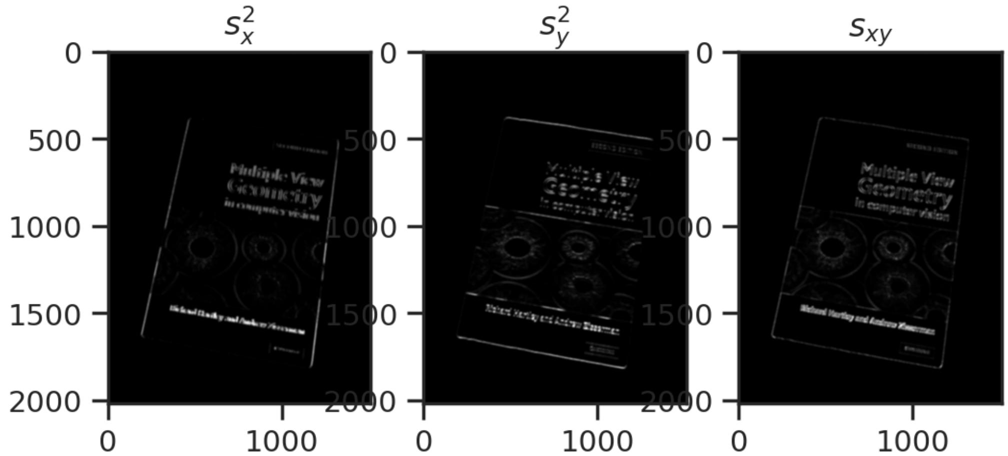
in this section we convolve gaussian filter with $I_x^2, I_y^2, I_X I_y$ using `cv2.GaussianBlur` to reach S_x^2, S_y^2, S_{xy} . note that i set standard deviation and size of filter as below :

$\boxed{\sigma = 4}$

$\boxed{size\ of\ kernel = 17}$

Structure tensor= $\begin{bmatrix} S_x^2 & S_{xy} \\ S_{xy} & S_y^2 \end{bmatrix}$

[36]:
```
sigma=4
k=17
sx2 = cv2.GaussianBlur(ix*ix,(k,k),sigma,sigma,cv2.BORDER_DEFAULT)
sy2 = cv2.GaussianBlur(iy*iy,(k,k),sigma,sigma,cv2.BORDER_DEFAULT)
sxy = cv2.GaussianBlur(ix*iy,(k,k),sigma,sigma,cv2.BORDER_DEFAULT)

sx2_2 = cv2.GaussianBlur(ix_2*ix_2,(k,k),sigma,sigma,cv2.BORDER_DEFAULT)
sy2_2 = cv2.GaussianBlur(iy_2*iy_2,(k,k),sigma,sigma,cv2.BORDER_DEFAULT)
sxy_2 = cv2.GaussianBlur(ix_2*iy_2,(k,k),sigma,sigma,cv2.BORDER_DEFAULT)
plt.subplot(1,3,1)
plt.imshow(normilizer(sx2),cmap="gray")
plt.title(r'$s_{x}^2$')

plt.subplot(1,3,2)
plt.imshow(normilizer(sy2),cmap="gray")
plt.title(r'$s_{y}^2$')

plt.subplot(1,3,3)
plt.imshow(normilizer(sxy),cmap="gray")
plt.title(r'$s_{xy}$')

plt.show()
```



1.5 Derive Harris function

after deriving structure tensor we calculate harris function using following formula :

$R = det(M) - K(trace)^2$ where

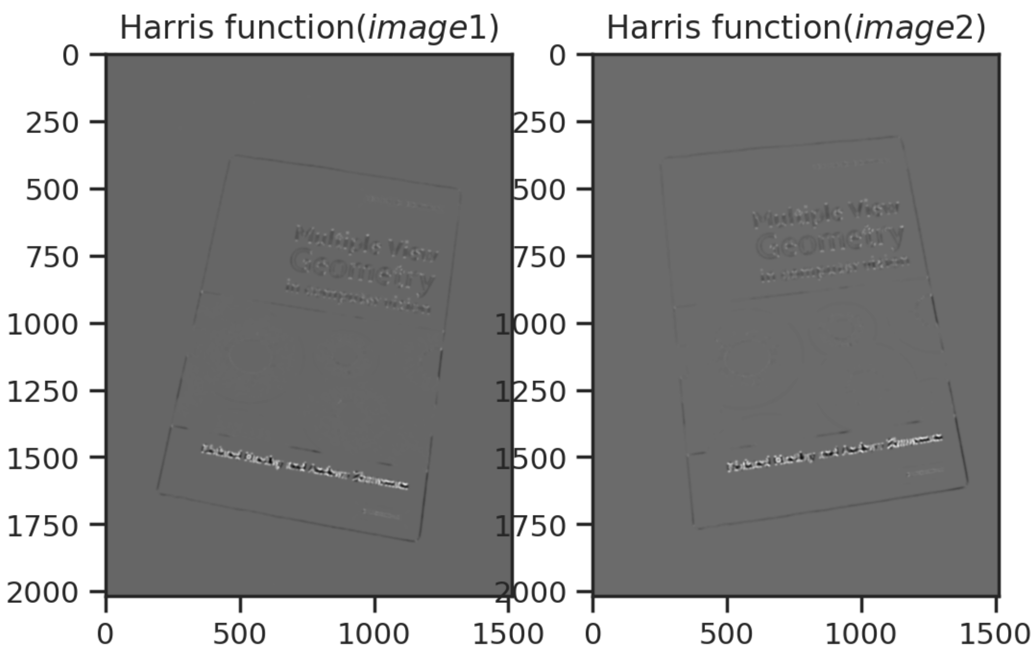
$det(M) = S_x^2 S_y^2 - Sxy^2$, $trace(M) = S_x^2 + S_y^2$

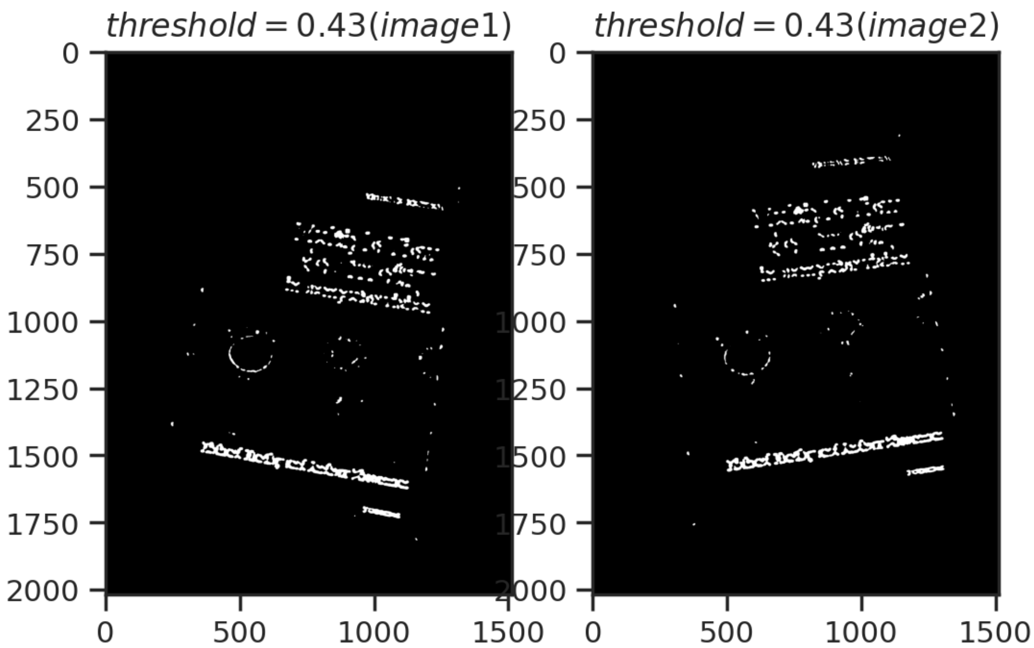
after that we use proper threshold to decrease the candidates that cen be corners becuase at first we have a lots of candidates . to implement threshold, i make a mask that has same size with images and if the pixel is below threshold it becomes 0 and if it's above threshold it becomes 1.

in this question i set $\boxed{K = 0.065}$, $\boxed{Threshhold1 = 0.43}$. in the following you can see the Harris function for two images before and after threshold :

[37]:
```
det=sx2*sy2-(sxy)**2
trace=sx2+sy2

det_2=sx2_2*sy2_2-(sxy_2)**2
trace_2=sx2_2+sy2_2

k=0.065
R=det-k*(trace)**2
R2=det_2-k*(trace_2)**2

plt.subplot(1,2,1)
plt.imshow(normilizer(R),cmap="gray")
plt.title(r'Harris function$(image1)$')

plt.subplot(1,2,2)
plt.imshow(normilizer(R2),cmap="gray")
plt.title(r'Harris function$(image2)$')
```

```
plt.show()
#-------------------------
#-------------------------

norm=normilizer(R)
norm_2=normilizer(R2)
matrix=norm > 0.41
matrix_2=norm_2 > 0.43

plt.subplot(1,2,1)
plt.imshow(matrix,cmap='gray')
plt.title(r'$threshold=0.43(image1)$')

plt.subplot(1,2,2)
plt.imshow(matrix_2,cmap='gray')
plt.title(r'$threshold=0.43(image2)$')

plt.show()
plt.imsave('temp.jpg',matrix)
plt.imsave('temp_2.jpg',matrix_2)
```





1.6 NMS(non-maximum suppression)

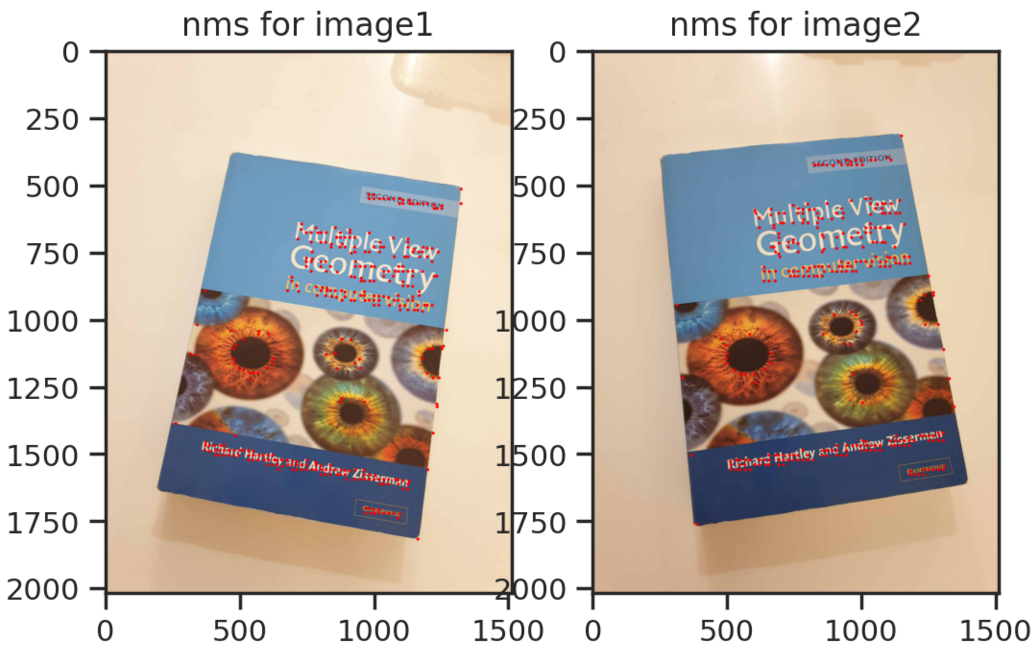
even after set the threshold , we can see in the harris function that pixels that are candidates to be corner are very close to each other . so beacuse of that i define a function `nms()`. the method is very simple : we set a window with proper size (in this question the size is 10px) and then we just save the pixel with highest score and ommit all other candidates in that window . we do this procedure until that window itterate all over the image .

[38]:
```
def nms(src):
    k=9
    result=np.copy(src)
    c1,c2=np.array(src).shape
    for i in range(k,c1-k,2*k):
        for j in range(k,c2-k,2*k):
            maximum=np.max(src[i-k:i+k,j-k:j+k])
            x,y=np.where(src[i-k:i+k,j-k:j+k]==maximum)
            result[i-k:i+k,j-k:j+k]=np.zeros([2*k,2*k])
            result[x[0]+ i,y[0]+j]=maximum

    return result
```

below are harris functions after passing from nms filter :

[39]:
```
nms1=nms(matrix)
nms2=nms(matrix_2)
```

```
res07=np.copy(img1)
res08=np.copy(img2)

x,y=np.where(nms1==1)
x1,y1=np.where(nms2==1)
color = (255, 0, 0)

for i in range(0,len(x)):
    cv2.circle(res07, tuple([y[i],x[i]]), 3, color, 3)

for i in range(0,len(x1)):
    cv2.circle(res08, tuple([y1[i],x1[i]]), 3, color, 3)


plt.subplot(1,2,1)
plt.imshow(res07.astype(np.uint8))
plt.title('nms for image1')

plt.subplot(1,2,2)
plt.imshow(res08.astype(np.uint8))
plt.title('nms for image2')

plt.show()


plt.imsave("res07_harris.jpg",res07.astype(np.uint8))
plt.imsave("res08_harris.jpg",res08.astype(np.uint8))
```



to show the effect of NMS in reducing the candidates i print candidates pixels before and after NMS

[40]:
```
x,y=np.where(matrix_2==1)
print(f"number of candidates before NMS: {len(x)}")

x,y=np.where(nms2==1)
print(f"number of candidates after NMS: {len(x)}")
```

```
number of candidates before NMS: 50074
number of candidates after NMS: 311
```

1.7 Extracting feature vector for each candidate

now, it's time to extract feature vector for corners . for each corner we choose a window with proper size(n) centered on that corner and consider the rgb value of that window as a information that is good for feasture . so we have a tensor of information with size (n,n,3) for each corner. at the end we convert this tensor to array using `np.ravel()` to reach feature vector. in this question $n = 80$

[41]:
```
n=80
feature_vector1=[]
feature_vector2=[]

x_image1,y_image1=np.where(nms1==1)
x_image2,y_image2=np.where(nms2==1)

for i in range(0,len(x_image1)):
      tensor_feature=img1[x_image1[i]-n:x_image1[i]+n,y_image1[i]-n:
 ↪y_image1[i]+n,:]
      vector=np.ravel(tensor_feature)
      feature_vector1.append(vector)

for j in range(0,len(x_image2)):
      tensor_feature2=img2[x_image2[j]-n:x_image2[j]+n,y_image2[j]-n:
 ↪y_image2[j]+n,:]
      vector=np.ravel(tensor_feature2)
      feature_vector2.append(vector)
```

1.8 Find corresponding points

to find corresponding points , for each corner in image 1 we have to find two nearest corners in image2 and vice versa.so i define `find_two_max()` function that find the two nerarest corners in images . in this function we itterate all over the image using `brute force` method. also we have to specify that which definition we use to compute distance between corners. in this question $\boxed{i\ use\ L_2\ distance}$. that is for two vectors u and v that their dimensions are $(1, n^2)$ we have : $\boxed{L_2(u,v) = \sqrt[2]{\sum_{i=1}^{n^2} |u_i - v_i|^2}}$

[42]:
```python
def find_two_max(feature1,feature2):

    ratio_im1=[]
    index_im1=[]
    ratio_im2=[]
    index_im2=[]

    #first part
    for i in range(0,len(feature1)):
         min_value_first=1e20
         min_value_second=1e20
         min_index=0
         for j in range(0,len(feature2)):
              dis=np.linalg.norm(feature1[i]-feature2[j])
              if(dis<min_value_first):
                  min_value_second=min_value_first
                  min_value_first=dis
                  min_index=j
                  dis-=1
              if(min_value_first<=dis<min_value_second):
                  min_value_second=dis
         ratio=float(min_value_first/min_value_second)
         ratio_im1.append(ratio)
         index_im1.append(min_index)

    #second part
    for i in range(0,len(feature2)):
         min_value_first=1e20
         min_value_second=1e20
         min_index=0
         for j in range(0,len(feature1)):
              dis=np.linalg.norm(feature2[i]-feature1[j])
              if(dis<min_value_first):
                  min_value_second=min_value_first
                  min_value_first=dis
                  min_index=j
                  dis-=1
              if(min_value_first<=dis<min_value_second):
                  min_value_second=dis
         ratio=float(min_value_first/min_value_second)
         ratio_im2.append(ratio)
```

```
        index_im2.append(min_index)

    return ratio_im1,index_im1,ratio_im2,index_im2
```

1.9 Rules for corresponding pairs

after finding two nearest corners in two image , we have to filter them based on three following rules :

- we have to set the threshold for ratio between first and second nearest corresponding for each corner . that is :

$r= \frac{L_2(first\ corresponding)}{L_2(second\ corresponding)} \leqslant Threshold$

$i\ set\ this\ Threshold\ to\ 0.95\ in\ this\ question$

- if B is the corresponding point for corner A , A should be corresponding point for B too
- if a corner is corresponding points for more than one corner in other image , we have to omit that corner from list of corresponding pairs

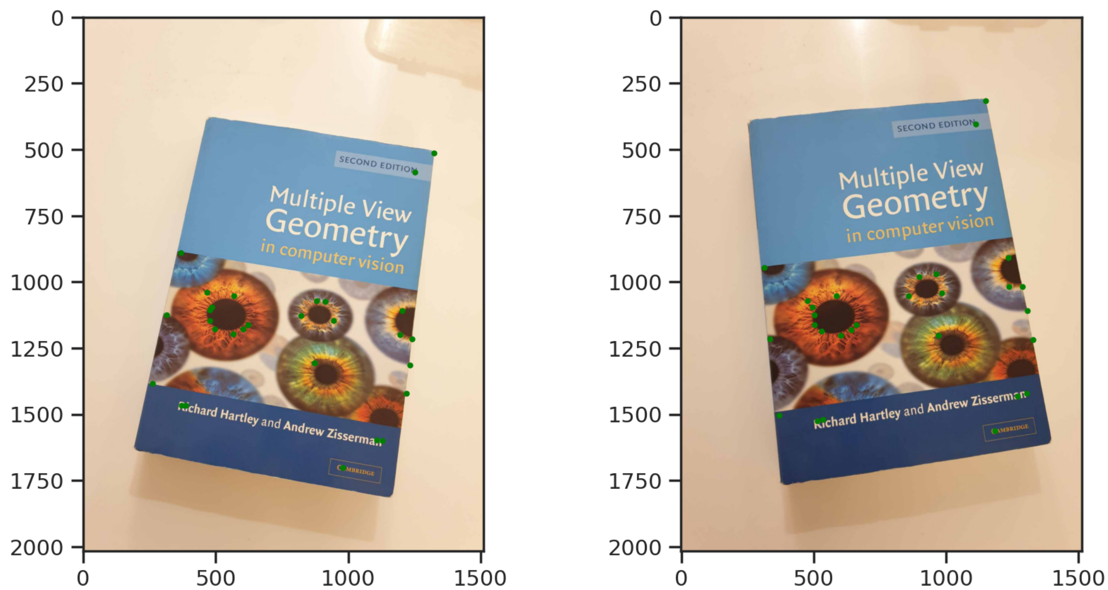
[43]:
```
rt1,ind1,rt2,ind2=find_two_max(feature_vector1,feature_vector2)

#pass from Threshold filter
index1=np.where(np.array(rt1)<(0.96))
index2=np.where(np.array(rt2)<(0.96))
same_index=[]
final_index=[]

#implement the rules above to find corresponding pairs
for i in range(0,int(np.array(index1).shape[1])):
      j= ind1[int(np.array(index1)[0,i])]
      if(ind2[j]==(np.array(index1)[0,i]) and j not in same_index):
          final_index.append((np.array(index1)[0,i]))
          same_index.append(j)
```

1.10 show final results

[44]:
```
fig = plt.figure(figsize=(10,5))
ax1 = fig.add_subplot(121)
ax2 = fig.add_subplot(122)

ax1.imshow(img1.astype('int32'))
ax2.imshow(img2.astype('int32'))
for i in range(len(final_index)):
  number=int(final_index[i])
  j=ind1[number]
  ax1.plot(y_image1[number],x_image1[number],'ro',markersize=2,color='green')
```

```
  ax2.plot(y_image2[j],x_image2[j],'ro',markersize=2,color='green')

plt.savefig("khkhk.jpg")
plt.suptitle(r"all of corresponding points")
plt.show()
```

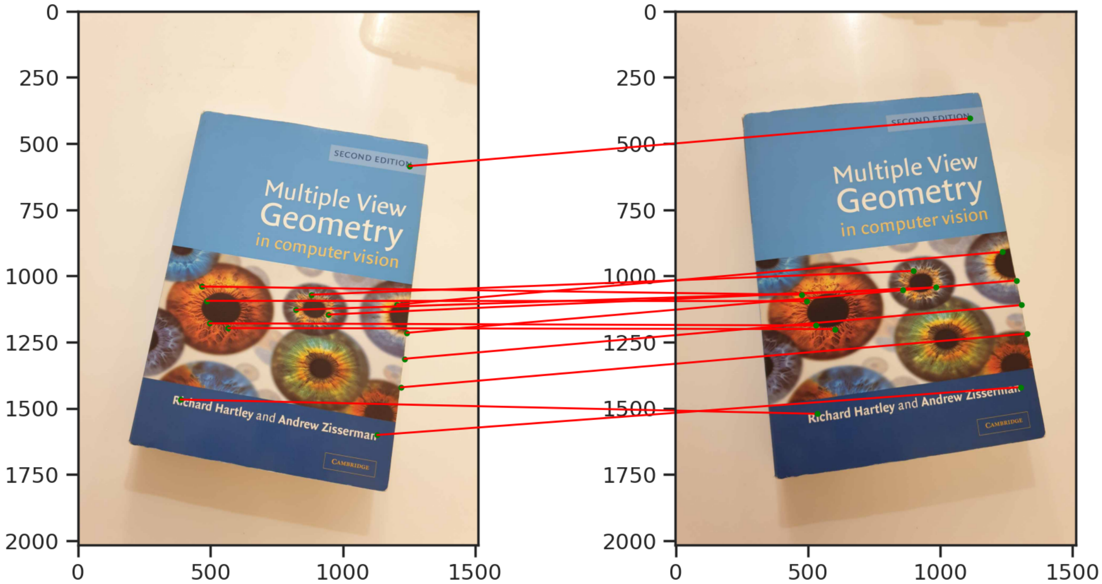
all of corresponding points



[45]:

```
fig = plt.figure(figsize=(10,5))
ax1 = fig.add_subplot(121)
ax2 = fig.add_subplot(122)
ax1.imshow(img1.astype('int32'))
ax2.imshow(img2.astype('int32'))
for i in range(1,len(final_index),2):
  number=int(final_index[i])
  j=ind1[number]
  xy = (y_image1[number],x_image1[number])
  xy1= (y_image2[j],x_image2[j])
  con = ConnectionPatch(xyA=xy, xyB=xy1, coordsA="data", coordsB="data",
                        axesA=ax1, axesB=ax2, color="red")
  ax2.add_artist(con)
  ax1.plot(y_image1[number],x_image1[number],'ro',markersize=2,color='green')
  ax2.plot(y_image2[j],x_image2[j],'ro',markersize=2,color='green')

plt.savefig("res11.jpg")
plt.suptitle(r"Half of the corresponding points connected with lines")
plt.show()
```

Half of the corresponding points connected with lines



1.11 Parameters

1) $standard\ deviation = \sigma = 4$

2) $size\ of\ guassian\ filter = 17$

3) $K = 0.065$

4) $N(size\ of\ window\ for\ feature\ vector) = 80\ pixel$

5) $Type\ of\ distance\ function = L_2\ norm$

6) $threshold1(used\ for\ res05\ and\ res06) = 0.43$

7) $threshold2(threshold\ for\ the\ ratio\ of\ d_1\ and\ d2) = 0.95$